CDS 101/110a: Lecture 10-1
Robust Performance

Richard M. Murray

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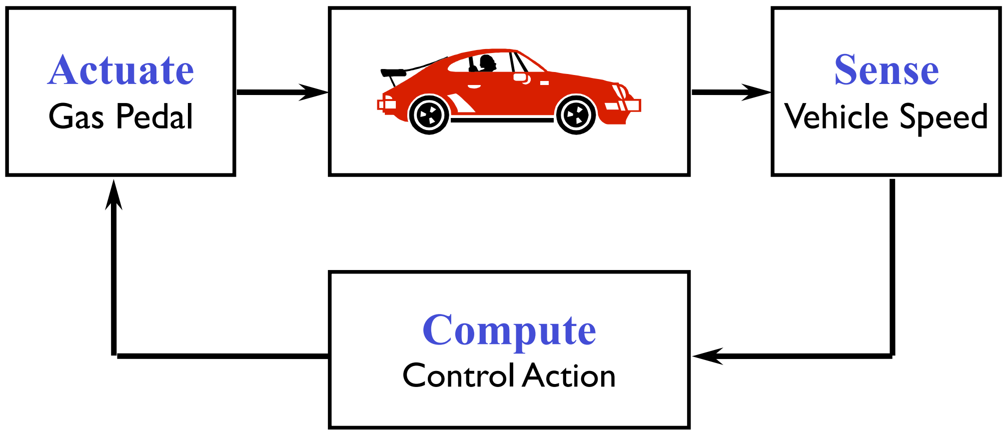
Goals:

- Describe how to represent uncertainty in process dynamics
- Describe how to analyze a system in the presence of uncertainty
- Review the main principles and tools for the course

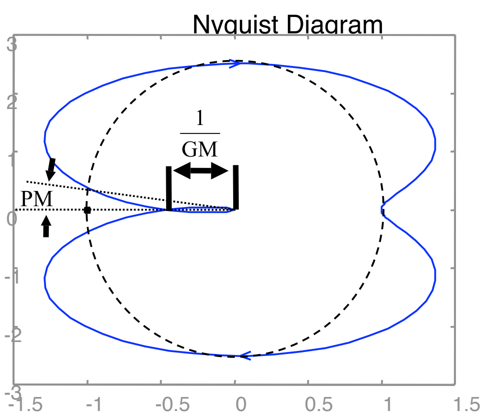
Reading:

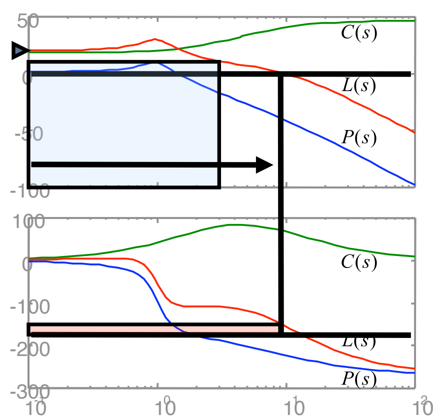
- Åström and Murray, Feedback Systems, Ch 12
- Advanced: Lewis, Chapter 9
- CDS 210: DFT, Chapters 4-6

Control = Sensing + Computation + Actuation



Goals: Stability, Performance, Robustness





Stability: bounded inputs produce bounded outputs

- Need to check all input/output pairs (Gang of Four/Six)
- Necessary and sufficient condition: check for nonzero solutions around feedback loop

Performance: desirable system response

- Step response: rise time, settling time, overshoot, etc
- Frequency response: tracking signals over given range

Robustness: stability/ performance in presence of uncertainties

- Need to check stability for *set* of disturbances & system models

Modeling Uncertainty

Noise and disturbances

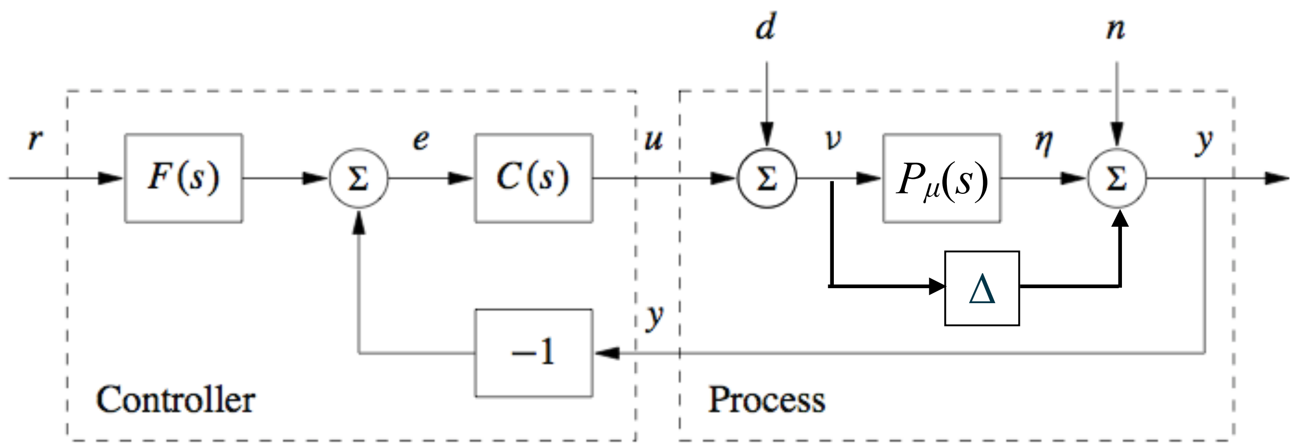
- Model the amount of noise by its signal strength in different frequency bands
- Can model signal strength by peak amplitude, average energy, and other norms
- Typical example: Dryden gust models (filtered white noise)

Parametric uncertainty

- Unknown parameters or parameters that vary from plant to plant
- Typically specified as tolerances on the basic parameters that describe system

Unmodeled dynamics

- High frequency dynamics (modes, etc) can be excited by control loops
- Use bounded operators to account for effects of unmodeled modes:



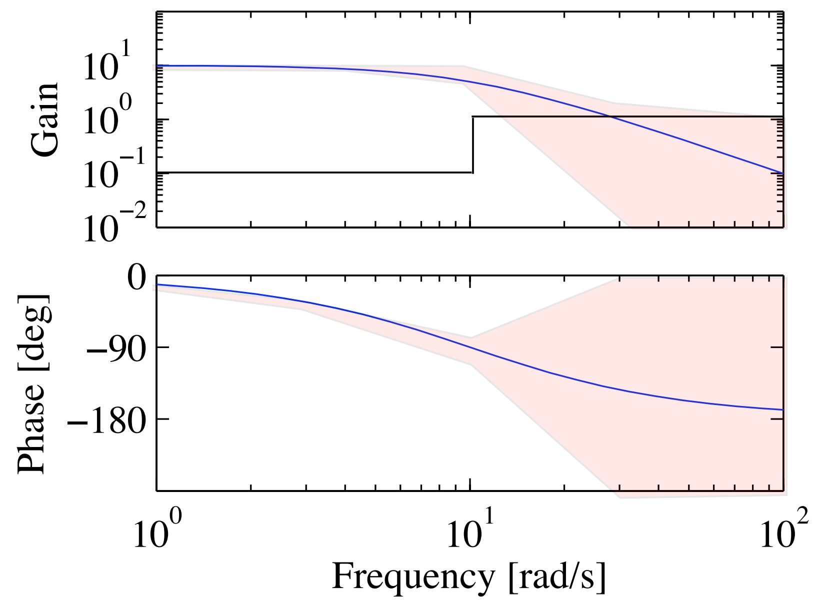
Unmodeled Dynamics

"Model" unknown dynamics through bounded transfer function

- Simplest case: additive uncertainty

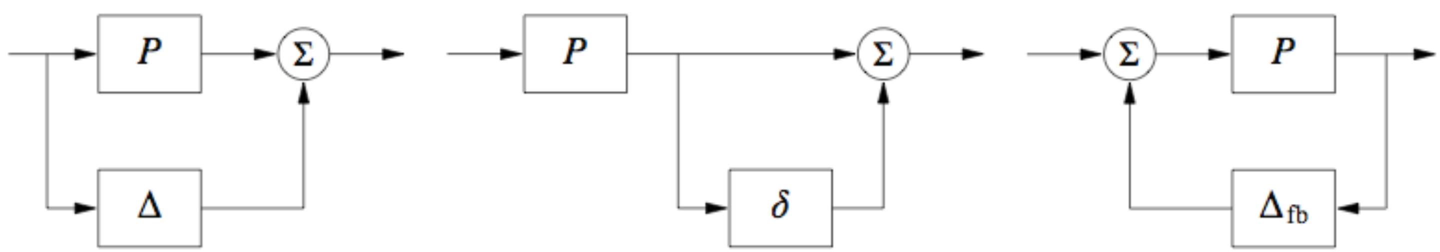
$$\tilde{P}(s) = P(s) + \Delta(s), \qquad |\Delta(s)| < W_2(s)$$

- $\Delta(s)$ is unknown, but bounded in magnitude
- Magnitude bound can depend on frequency; typically have a good model at low frequency



Different types of uncertainty can be used depending on where uncertainty enters

- Multiplicative: good for unknown gain or actuator dynamics
- Feedback: good for "leakage" effects (eg, in electrical circuits)



Additive: $P + \Delta$ Multiplicative: $P(1+\delta)$ Feedback: $P/(1+\Delta_{fb}\ \ P)$

Stability in the Presence of Uncertainty

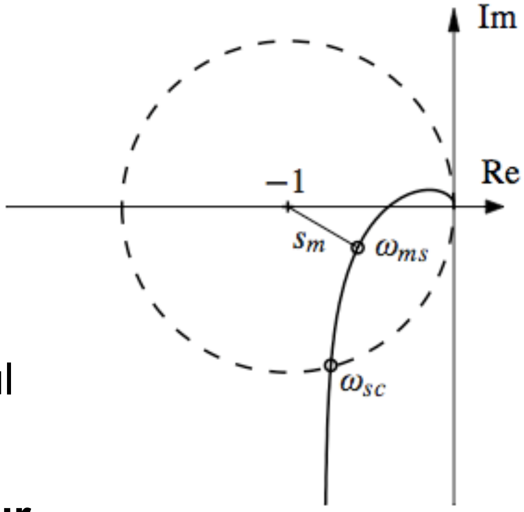
Characterize stability in terms of stability margin s_m

- Stability margin = distance on Nyquist plot to -1 point
- Stability margin = $1/M_s$ (M_s = maximum sensitivity)

$$M_s = \max |S(i\omega)| = \max \left| \frac{1}{1+L} \right|,$$

$$s_m = \min |(-1) - L| = \min |1 + L| = 1/M_s$$

- For robustness analysis, stability margin is more useful than classical gain and phase margins

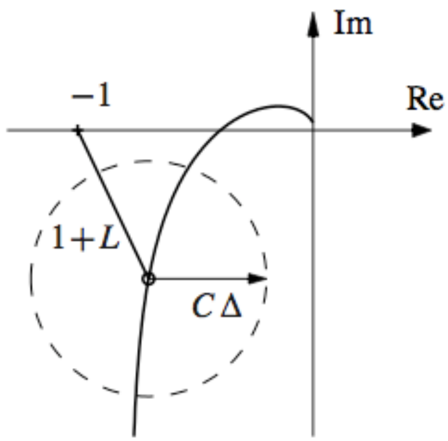


Robust stability: verify no new net encirclements occur

- Assume that nominal system is stable
- New loop transfer function: $\tilde{L} = (P + \Delta)C = L + C\Delta$
- No net encirclements as long as $|C\Delta| < |1 + L|$
- Can rewrite as bound on allowable perturbation

$$|\Delta| < \left| \frac{1+PC}{C} \right| = \left| \frac{P}{T} \right| \quad \text{or} \quad |\delta| = \left| \frac{\Delta}{P} \right| < \frac{1}{|T|}$$

- If condition is satisfied, then sm will never cross to zero => no new net encirclements



Example: Cruise Control

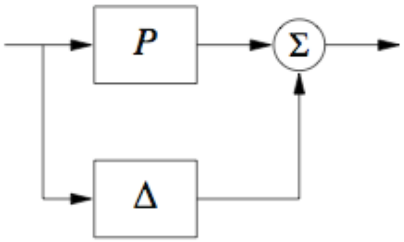
Question: how accurate does our model have to be

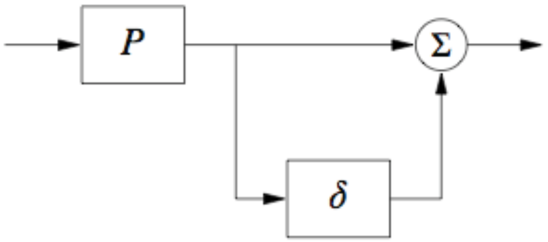
- Start with very simple model (25 m/s, 4th gear)

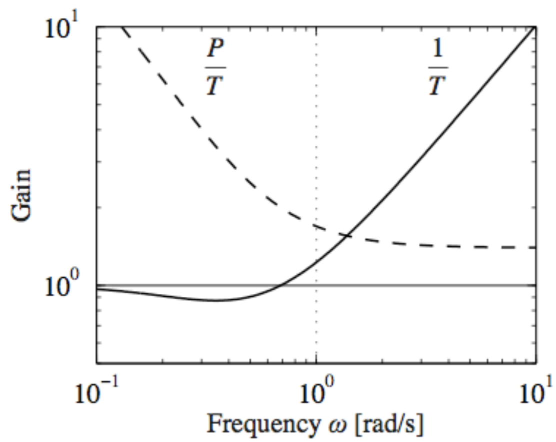
$$P(s) = \frac{1.38}{s + 0.0142} \qquad C(s) = 0.72 + \frac{0.18}{s}$$

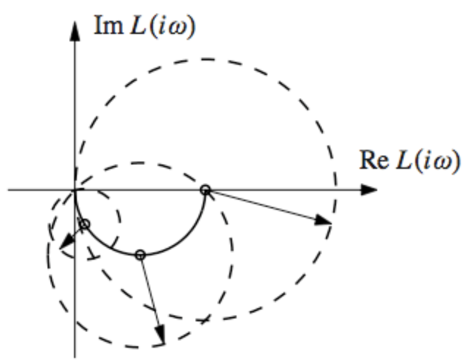
- Ignores details of engine dynamics, sensor delays, etc
- Model unknown dynamics as additive or multiplicative uncertainty (can convert bounds from one to the other)
- System will remain stable as long as

$$|\Delta| < \left|\frac{1 + PC}{C}\right| = \left|\frac{P}{T}\right| \qquad |\delta| = \left|\frac{\Delta}{P}\right| < \frac{1}{|T|}$$









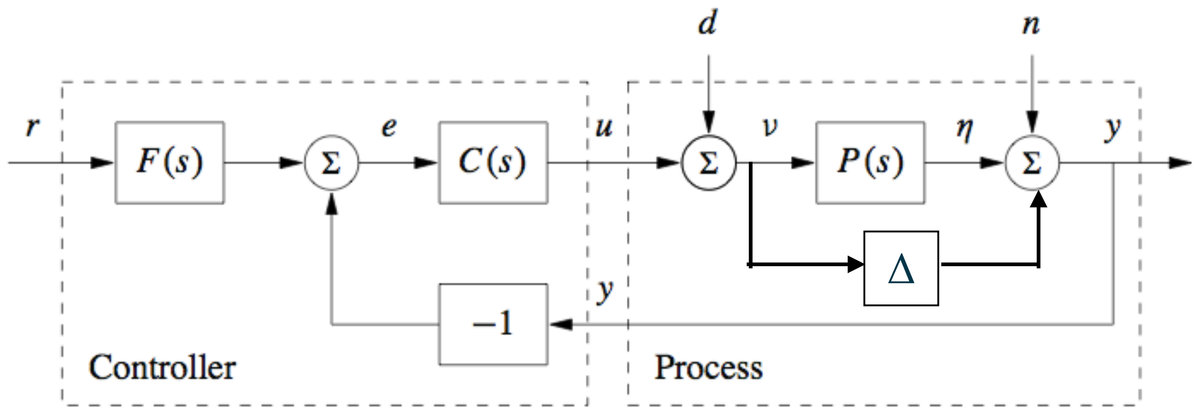
Remarks

- Conditions show why we can use simple models for designs
- Models must be accurate near critical point (eg, crossover)

Robust Performance Using Sensitivity Functions

Performance conditions

- What happens to performance in the presence of uncertainty?
- Start by looking at the *sensitivity of* transfer functions to process model



$$G_{yd} = \frac{P}{1+PC} = PS$$

$$\frac{dG_{yd}}{dP} = \frac{1}{(1+PC)^2} = \frac{SP}{P(1+PC)} = S\frac{G_{yd}}{P}$$

$$G_{yd} + \Delta_{yd} \approx \frac{P+\Delta}{1+(P+\Delta)C}$$
$$\approx \frac{P}{1+PC} + \frac{\Delta}{1+PC} = G_{yd} + S\Delta$$

Other measures

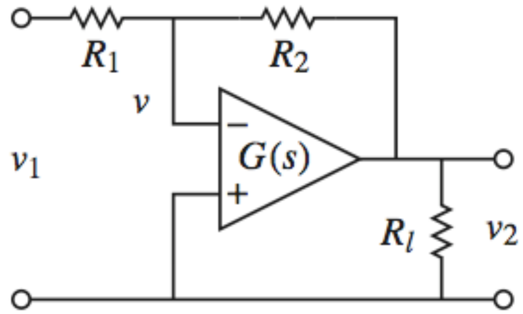
$$\frac{dG_{yr}}{G_{yr}} = S\frac{dP}{P}.$$

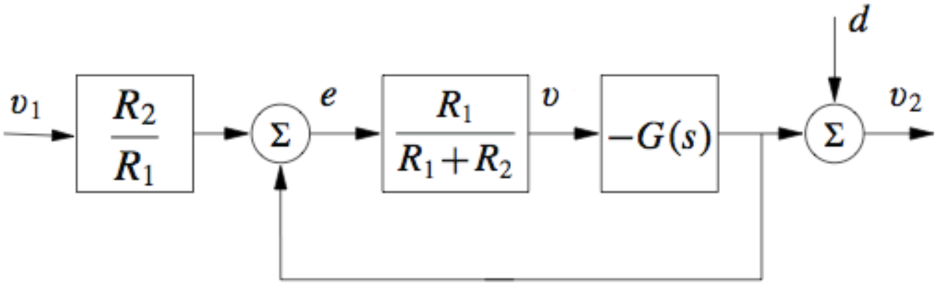
$$\frac{dG_{yd}}{G_{yd}} = S\frac{dP}{P}.$$

$$\frac{dG_{un}}{G_{un}} = T\frac{dP}{P}.$$

- Sensitivity functions actually come from this analysis

Example: Operational Amplifier





Note: disturbance enters in different location

Basic amplifier with unmodeled (high frequency) dynamics

- Start with low frequency model:

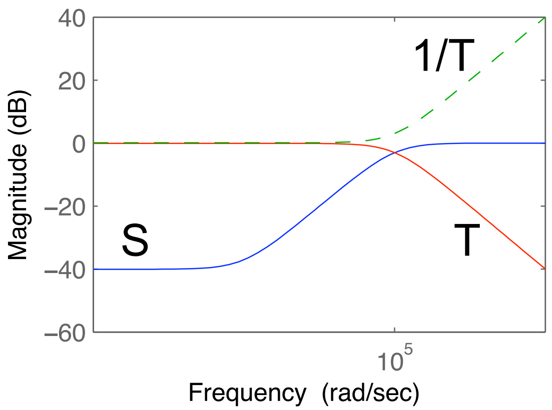
$$G_{v_2 v_1} = -\frac{R_2}{R_1}\frac{G(s)}{G(s) + R_2/R_1 + 1}. \qquad G(s) = \frac{b}{s+a}, \qquad \alpha = \frac{R_2}{R_1}$$

- Typical parameters b » a » 1
- Robust stability: see how much uncertainty we can handle

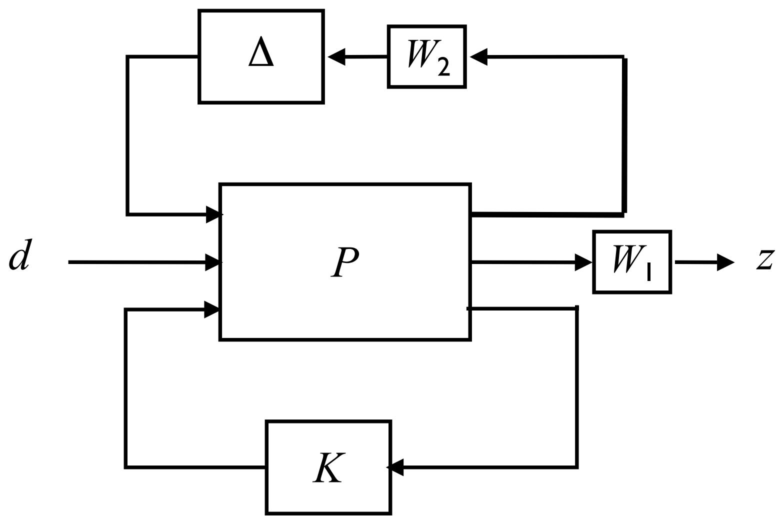
$$|\delta| = \left|\frac{\Delta}{P}\right| < \frac{1}{|T|} \qquad T = \frac{\alpha b}{s + a + \alpha b}$$

- Robust performance: effect on tracking, disturbance rej

$$\frac{dG_{yr}}{G_{yr}} \quad S\frac{dP}{P} \qquad \frac{dG_{yd}}{G_{yd}} = T\frac{dP}{P} \qquad S = \frac{s+a}{s+a+\alpha b}$$



Alternative Formulation for Robust Performance



d	disturbance signal
z	output signal
Δ	uncertainty block
W_2	performance weight
W_2	uncertainty weight

Goal: guaranteed performance in presence of uncertainty

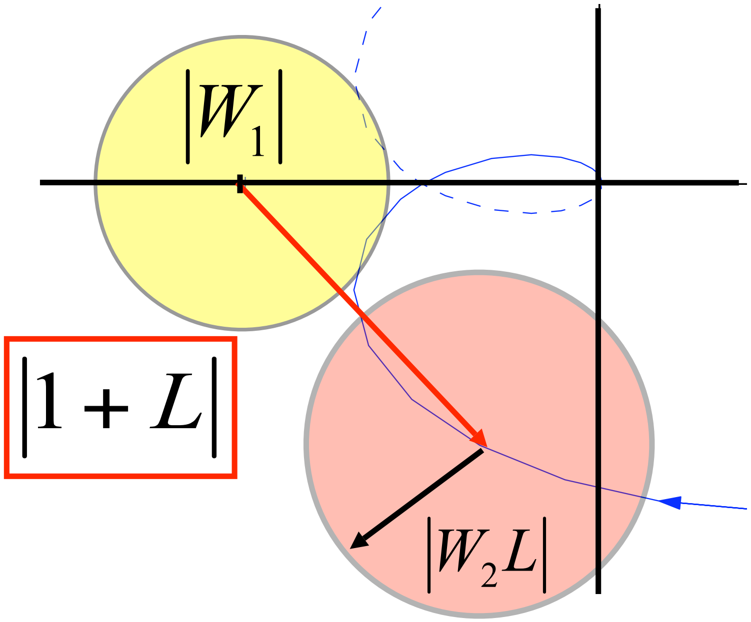
$$\|z\|_2 \leq \gamma \|d\|_2 \quad \text{for all} \quad \|\Delta\|_\infty \leq 1$$

$$\|u\|_2 = \sqrt{\int_0^\infty |u(t)|^2 \, dt}$$

$$\|H\|_\infty = \max_\omega |H(i\omega)|$$

- Compare energy in disturbances to energy in outputs
- Use frequency weights to change performance/uncertainty descriptions
- "Can I get X level of performance even with Y level of uncertainty?"

Design for Robust Performance (ala DFT)



Basic idea: interpret conditions on Nyquist

- Performance: keep sensitivity function small

$$|W_1 S| < 1 \quad \Longrightarrow \quad |W_1| < |1+L|$$

- W_1 serves as performance weighting function
- Stability: avoid additional encirclements

$$|W_2 \delta| < \frac{1}{|T|} \quad \Longrightarrow \quad |W_2 L| < |1+L|,\ |\delta| < 1$$

- W_2 serves as uncertainty weighting function
- Individual conditions provide robust stability and (nominal) performance

Design loop shape to satisfy robust stability and performance conditions

- Nominal performance:

$$|W_1| < |1+L| \quad \text{for all} \quad \omega$$

- Robust stability:

$$|W_2 L| < |1+L| \quad \text{for all}$$

- Missing: robust performance...

Theorem: *robust* performance if circles don't intersect on Nyquest plot:

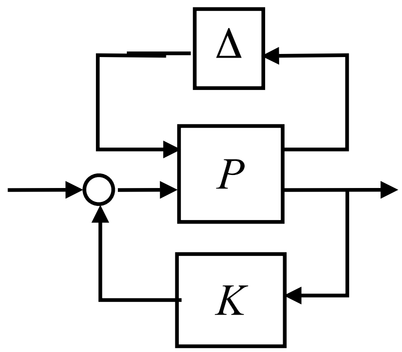
$$|W_1| + |W_2 L| < |1+L| \quad \text{for all} \quad \omega$$

- Holds for multiplicative uncertainty + weighted sensitivity (cf DFT)

Tools for Analyzing and Synthesizing Controllers

Robust Multi-Variable Control Theory

- Generalizes gain/phase margin to MIMO systems
- Uses operator theory to handle uncertainty, performance
- Uses state space theory to performance computations (LMIs)



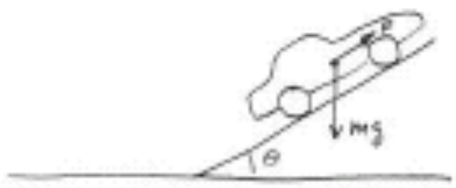
Analysis Tools

- H_∞ gains for multi-input, multi-output systems
- μ analysis software
  - Allow structured uncertainty descriptions (fairly general)
  - Computes upper and lower bounds on performance
  - Wide usage in aerospace industry
- SOSTOOLS: Nonlinear extensions

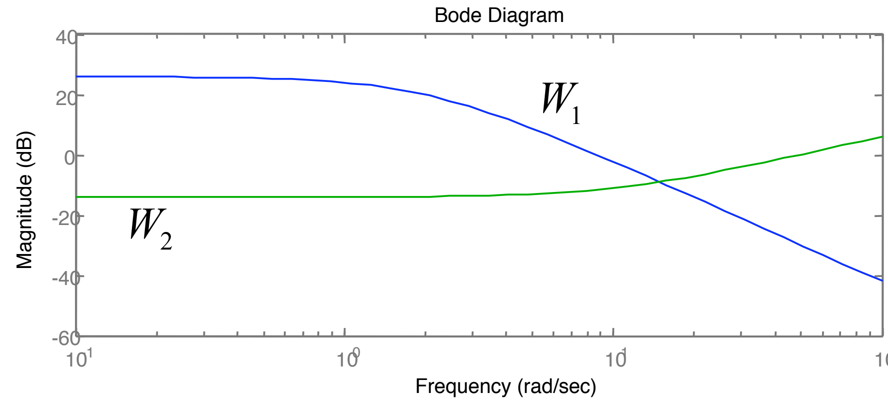
Synthesis Tools

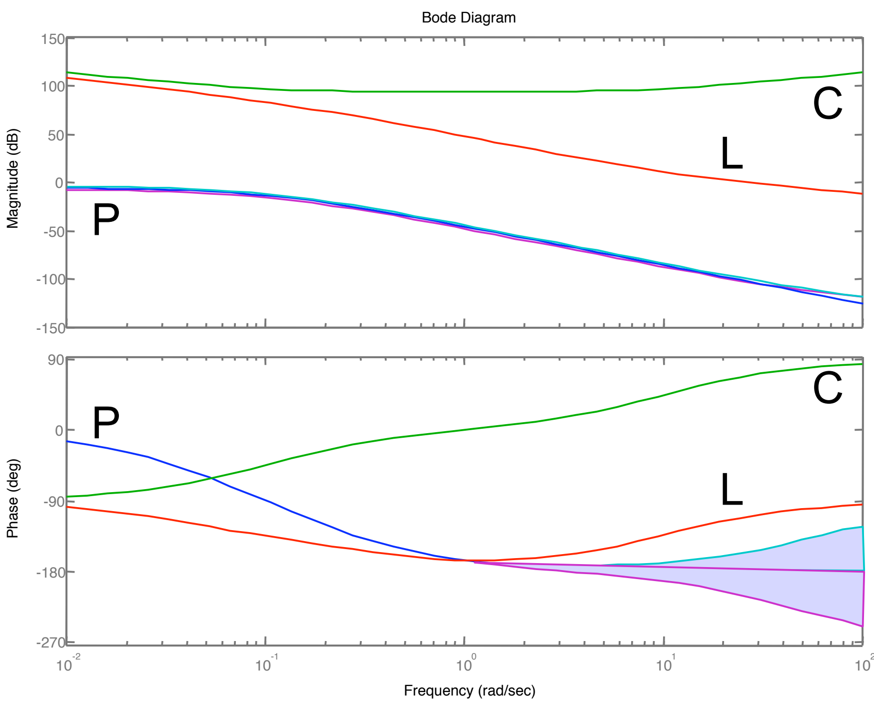
- LQR/LQG + H_∞ "loop shaping"; modern tools for control engineers
- μ synthesis software; tends to generate high order controllers
- Model reduction software for reducing order of plants, controllers

Example: Robust Cruise Control



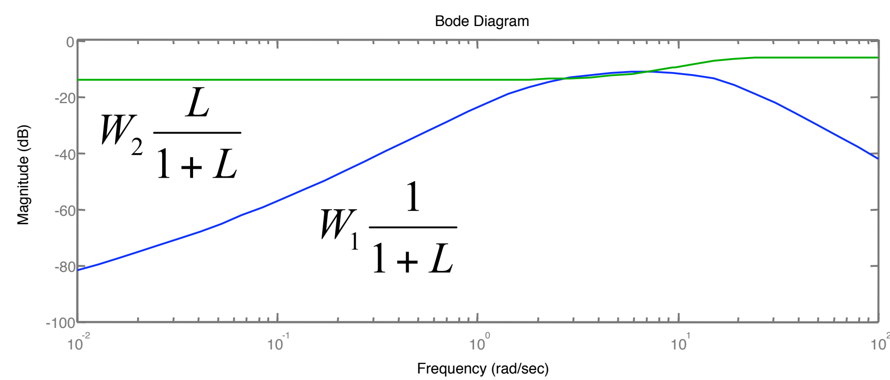
$$P(s) = \frac{1/m}{s + b/m} \cdot \frac{r}{s+a}\left(1 + W_2\Delta\right)$$





Theorem

- Performance: $\left| W_1 \frac{1}{1+L} \right| < 1 \quad \forall \omega$
- Robust Stability: $\left| W_2 \frac{L}{1+L} \right| < 1 \quad \forall \omega$



Course Summary: Two Main Principles of Control

Design of Dynamics through Feedback

- Feedback allows the dynamics of a system to be modified
- Key idea: interconnection gives closed loop that modifies natural behavior
- Tools: eigenvalue assignment, loop shaping

Robustness to Uncertainty through Feedback

- Feedback allows high performance in the presence of uncertainty
- Key idea: accurate sensing to compare actual to desired, correction through computation and actuation
- Tools: stability margins, sensitivity functions