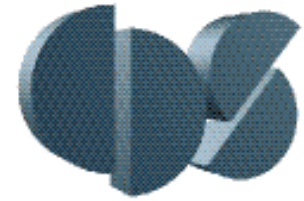




CDS 101: Lecture 8.1

Frequency Domain Design using PID



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19 November 2007

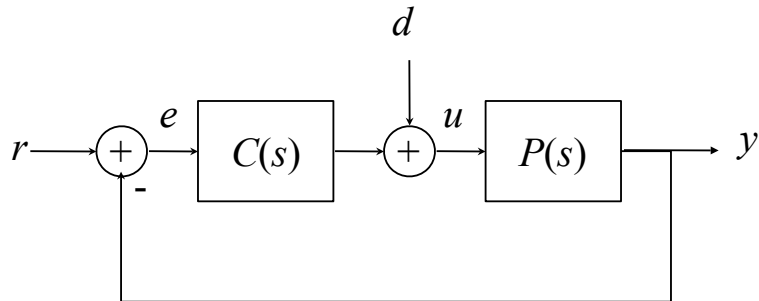
Goals:

- Describe the use of frequency domain performance specifications
- Show how to use “loop shaping” using PID to achieve a performance specification

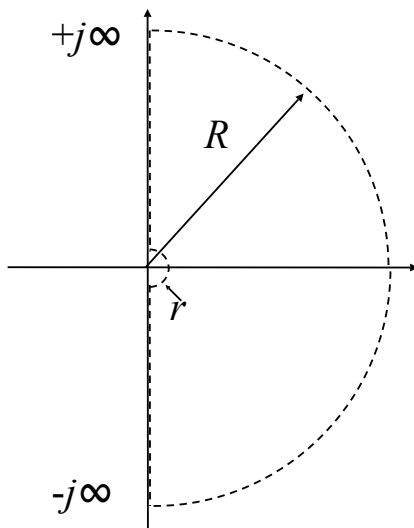
Reading:

- Åström and Murray, *Feedback Systems*, Ch 10

Review From Last Week



- Nyquist criteria for loop stability
- Gain, phase margin for robustness

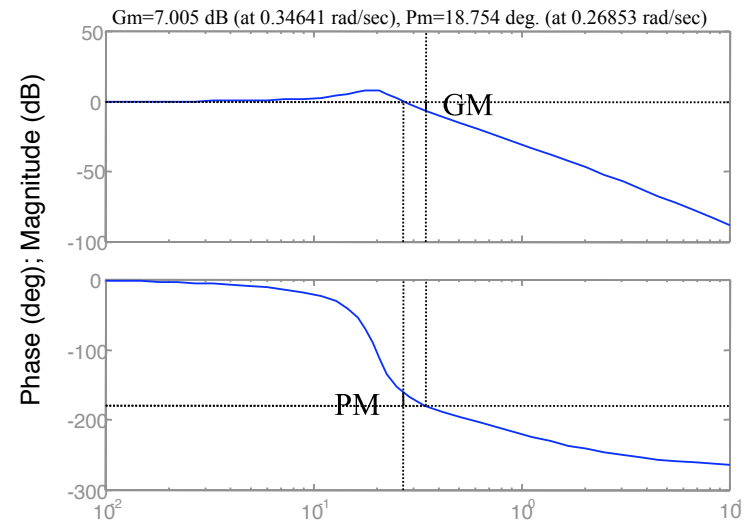


Thm (Nyquist).

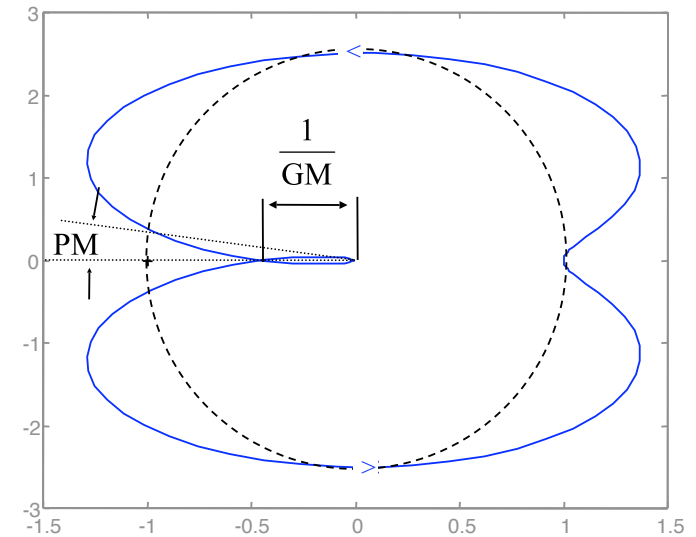
P # RHP poles of $L(s)$
 N # CCW encirclements
 Z # RHP zeros

$$Z = N + P$$

Bode Diagram

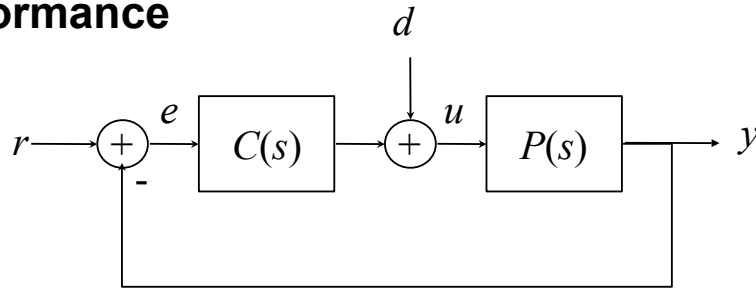


Nyquist Diagram

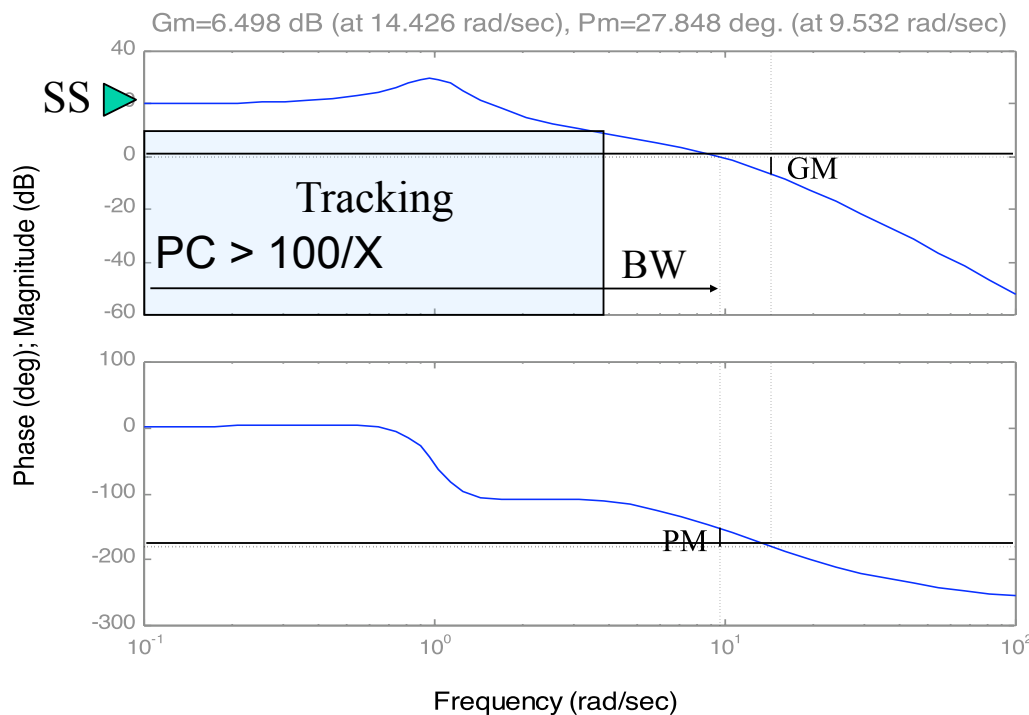


Frequency Domain Performance Specifications

Specify bounds on the loop transfer function to guarantee desired performance



Bode Diagrams



$$L(s) = P(s)C(s)$$

$$H_{er} = \frac{1}{1+L} \quad H_{yr} = \frac{L}{1+L}$$

- Steady state error:

$$H_{er}(0) = 1/(1+L(0)) \approx 1/L(0)$$

⇒ zero frequency (“DC”) gain

- Bandwidth: assuming $\sim 90^\circ$ phase margin

$$\frac{L}{1+L}(j\omega_c) \approx \left| \frac{1}{1+j} \right| = \frac{1}{\sqrt{2}}$$

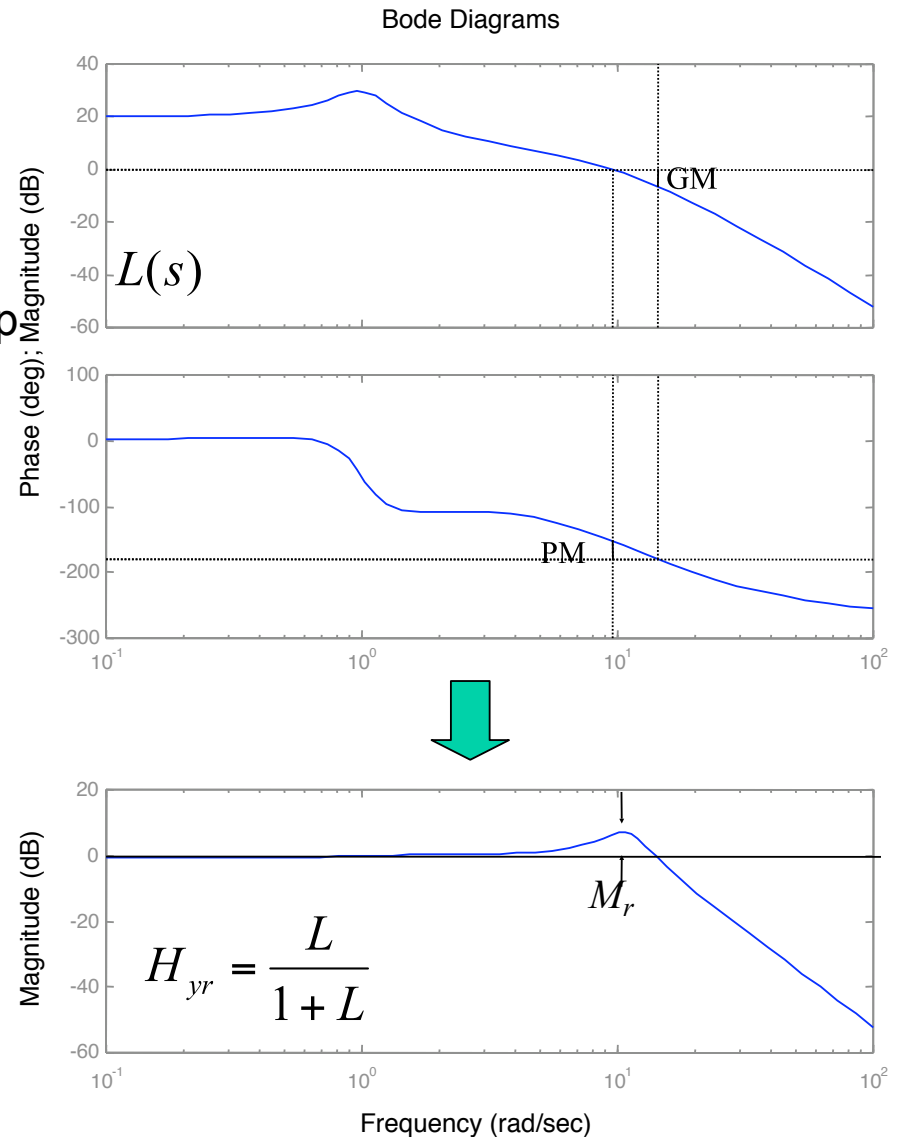
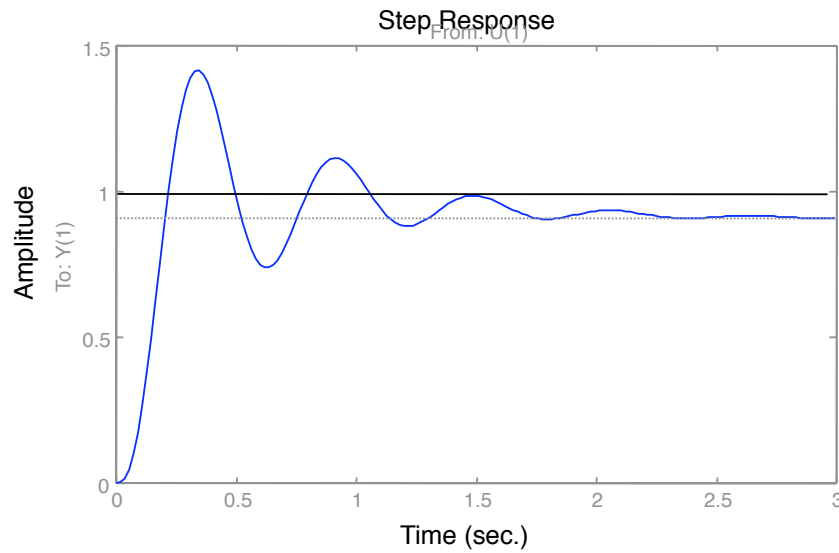
⇒ sets crossover freq

- Tracking: $X\%$ error up to frequency $\omega_t \Rightarrow$ determines gain bound ($1+PC > 100/X$)

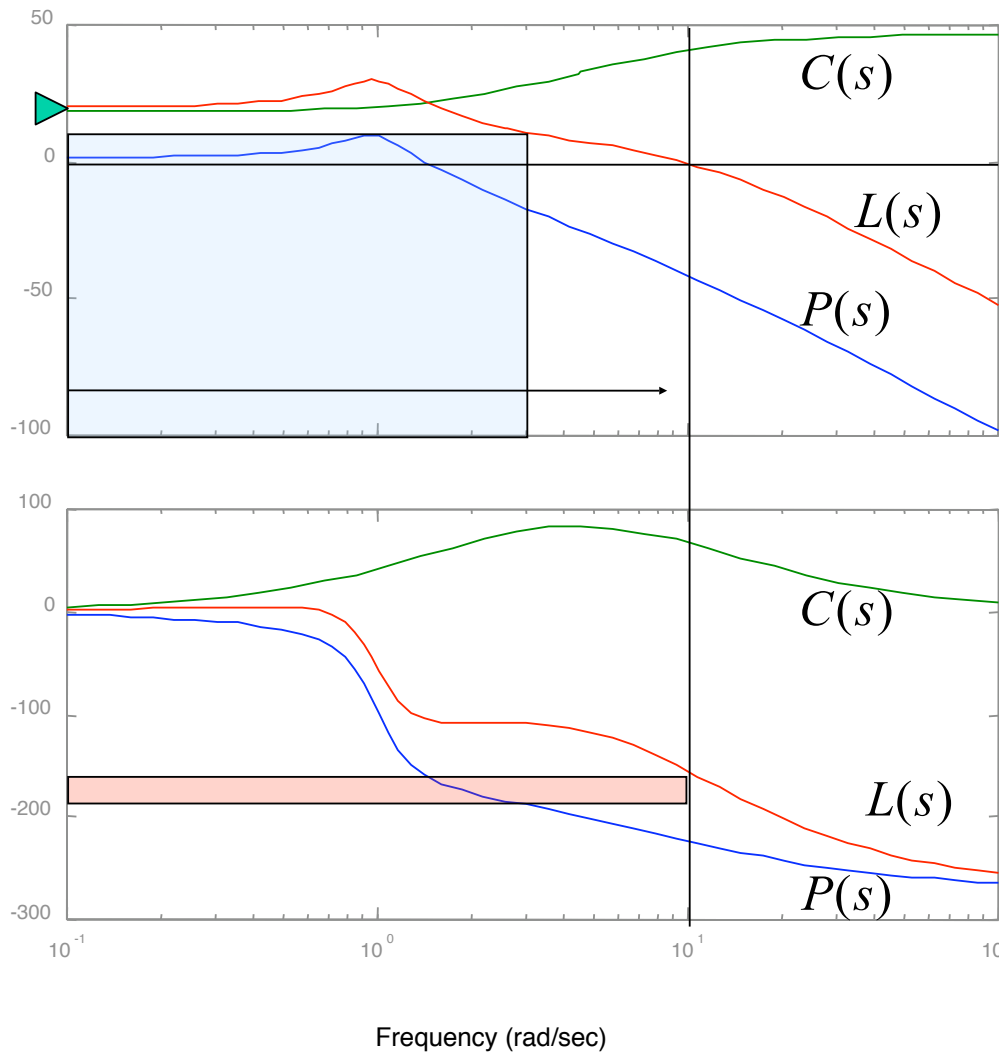
Relative Stability

Relative stability: how stable is system to disturbances at certain frequencies?

- System can be stable but still have bad response at certain frequencies
- Typically occurs if system has low phase margin $\Rightarrow$ get resonant peak in closed loop (M_r) + poor step response
- Solution: specify minimum phase margin. Typically 45° or more



Overview of Loop Shaping



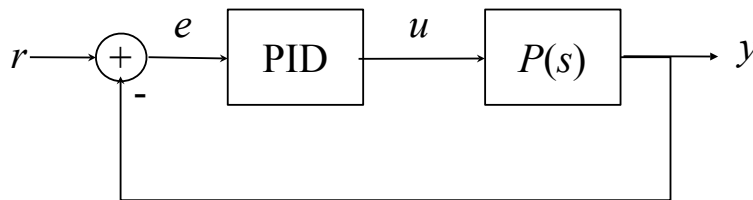
Performance specification

- ▶ Steady state error
- Tracking error
- Bandwidth
- Relative stability

Approach: “shape” loop transfer function using $C(s)$

- $P(s)$ + specifications given
- $L(s) = P(s) C(s)$
 - Use $C(s)$ to choose desired shape for $L(s)$
- Important: can't set gain and phase independently

Overview: PID control



$$u = K_p e + K_I \int e + K_D \dot{e}$$

Intuition

- Proportional term: provides inputs that correct for “current” errors
- Integral term: insures *steady state* error goes to zero
- Derivative term: provides “anticipation” of upcoming changes

A bit of history on “three term control”

- First appeared in 1922 paper by Minorsky: “Directional stability of automatically steered bodies” under the name “three term control”
- Also realized that “small deviations” (linearization) could be used to understand the (nonlinear) system dynamics under control

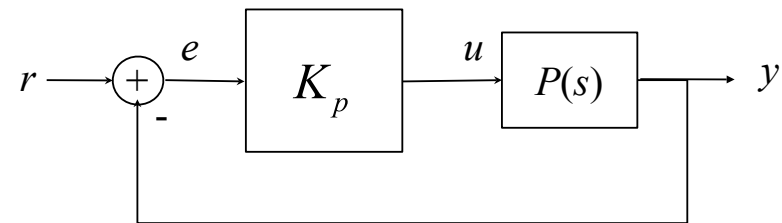
Utility of PID

- PID control is most common feedback structure in engineering systems
- For many systems, only need PI or PD (special case)
- Many tools for tuning PID loops and designing gains

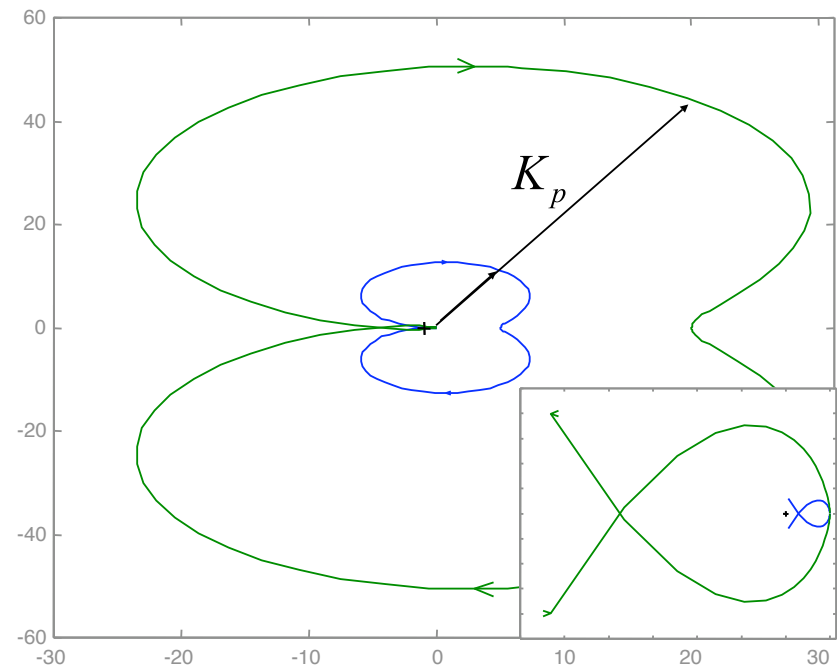
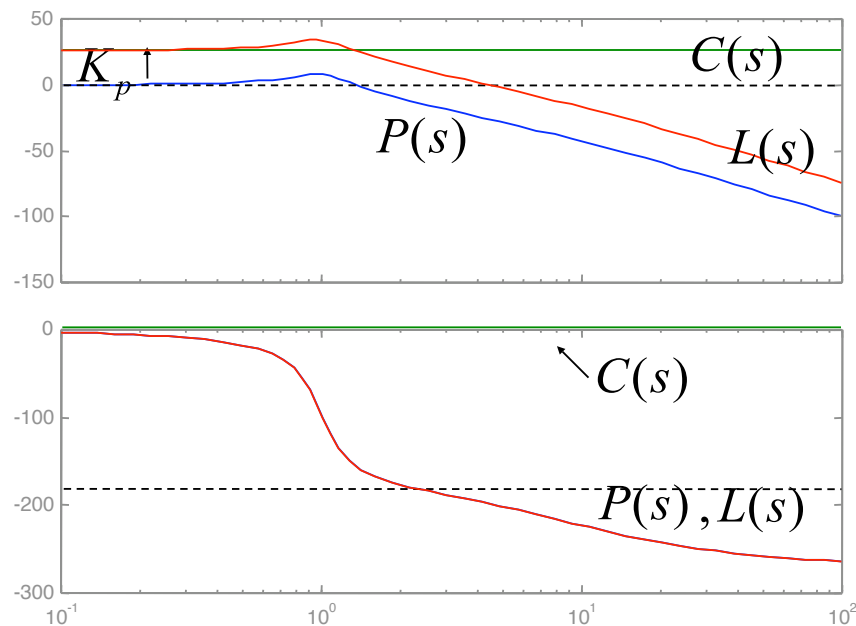
Proportional Feedback

Simplest controller choice: $u = K_p e$

- Effect: lifts gain with no change in phase
- Good for plants with low phase up to desired bandwidth
- Bode: shift gain up by factor of K_p
- Nyquist: scale Nyquist contour



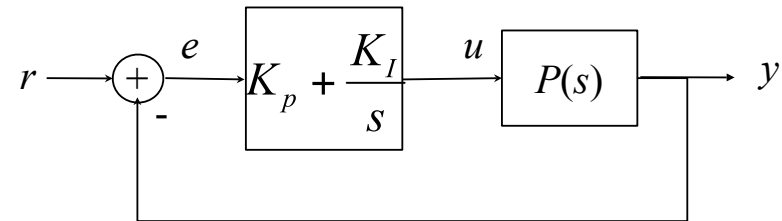
$$K_p > 0$$



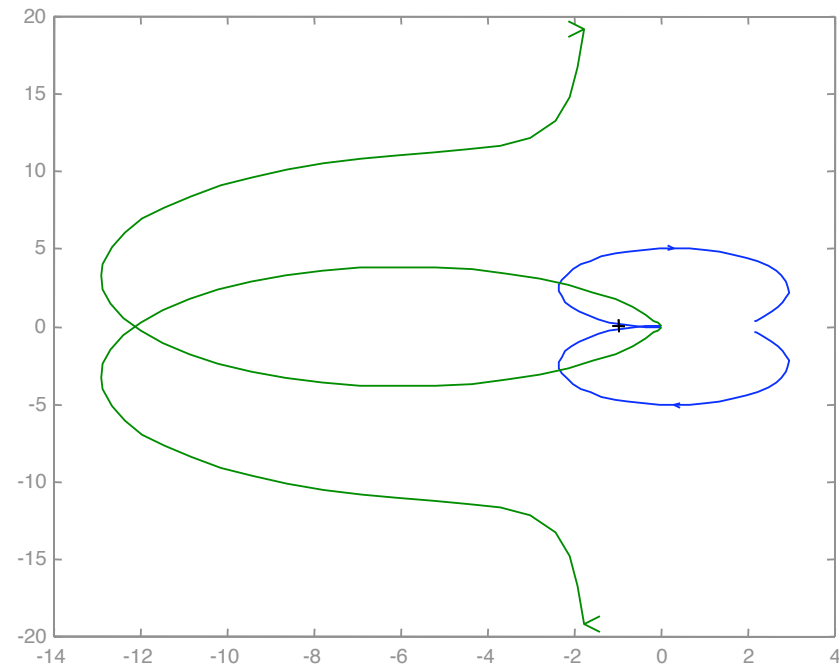
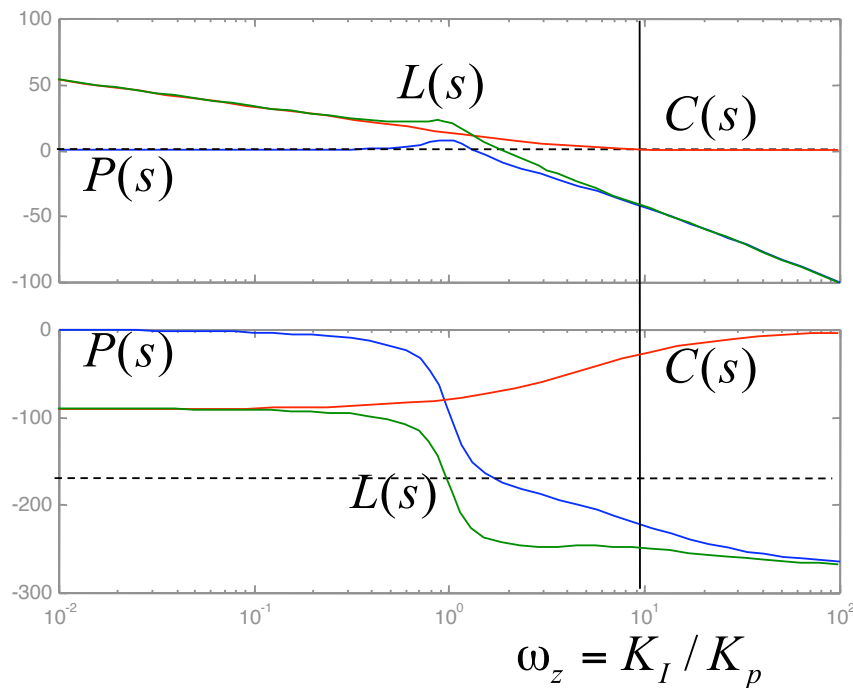
Proportional + Integral Compensation

Use to eliminate steady state error

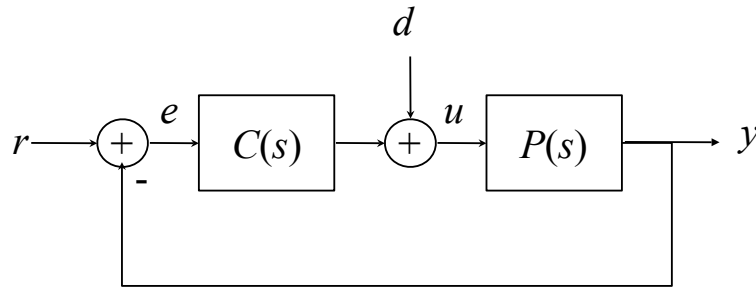
- Effect: lifts gain at low frequency
- Gives zero steady state error
- Bode: infinite SS gain + phase lag
- Nyquist: no easy interpretation
- Note: this example is *unstable*



$$K_p > 0 \quad K_I > 0$$



Proportional + Integral + Derivative (PID)



$$\begin{aligned} C(s) &= K_p + K_I \times \frac{1}{s} + K_D s \\ &= k \left(1 + \frac{1}{T_I s} + T_D s \right) \\ &= \frac{k T_D}{T_I} \frac{(s + 1/T_I)(s + 1/T_D)}{s} \end{aligned}$$

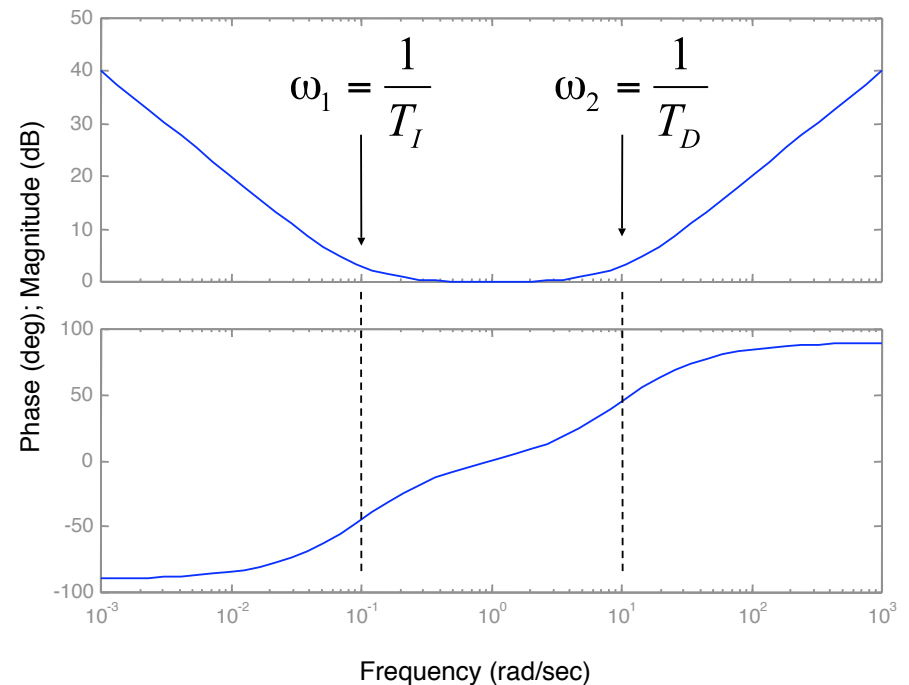
Transfer function for PID controller

$$u = K_p e + K_I \int e + K_D \dot{e}$$

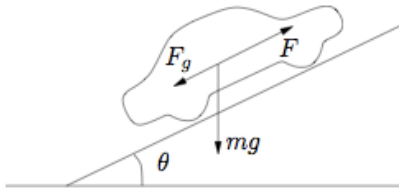
$$H_{ue}(s) = K_p + K_I \times \frac{1}{s} + K_D s$$

- Idea: gives high gain at low frequency plus phase lead at high frequency
- Place ω_1 and ω_2 below desired crossover freq

Bode Diagrams



Example: Cruise Control using PID - Specification



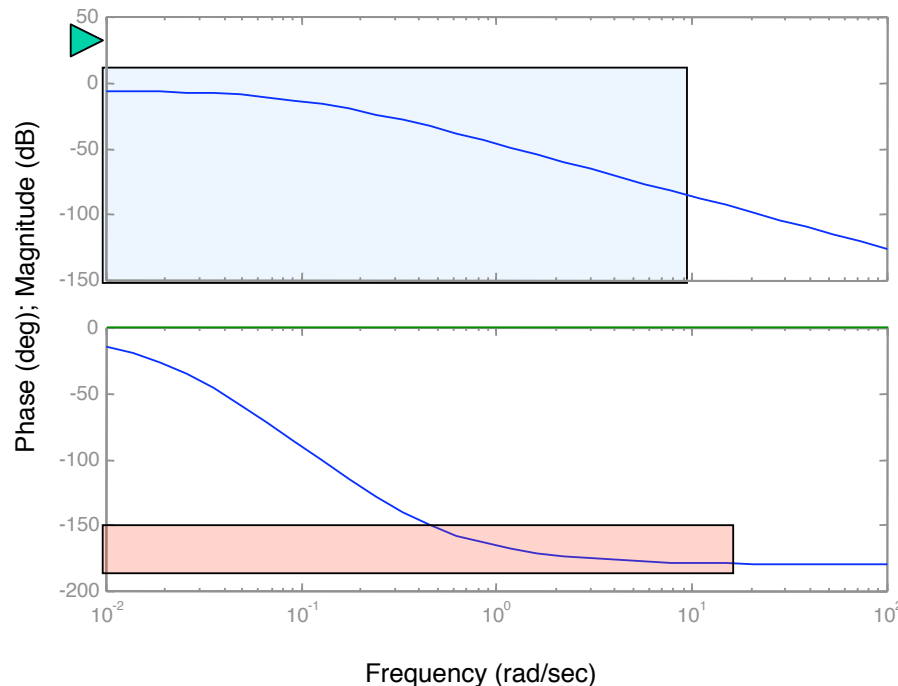
$$P(s) = \frac{1/m}{s + b/m} \times \frac{r}{s + a}$$

Performance Specification

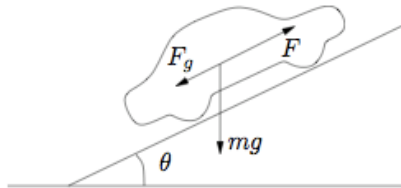
- $\leq 1\%$ steady state error
 - Zero frequency gain > 100
- $\leq 10\%$ tracking error up to 10 rad/sec
 - Gain > 10 from 0-10 rad/sec
- $\geq 45^\circ$ phase margin
 - Gives good relative stability
 - Provides robustness to uncertainty

Observations

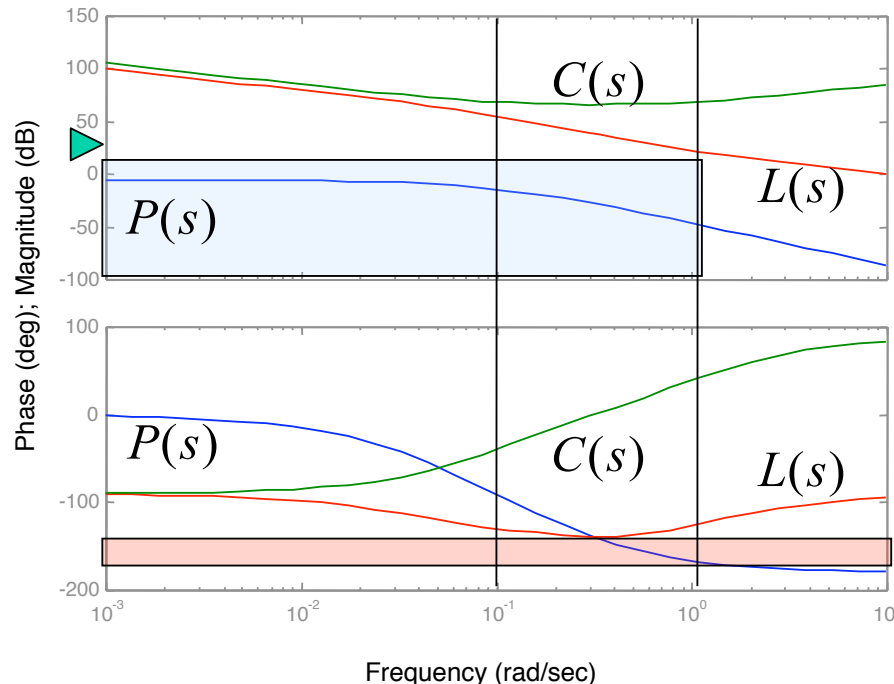
- Purely proportional gain won't work: to get gain above desired level will not leave adequate phase margin
- Need to increase the phase from ~ 0.5 to 2 rad/sec and increase gain as well



Example: Cruise Control using PID - Design



$$P(s) = \frac{1/m}{s + b/m} \times \frac{r}{s + a}$$



Approach

- Use integral gain to make steady state error small (zero, in fact)
- Use derivative action to increase phase lead in the cross over region
- Use proportional gain to give desired bandwidth

Controller

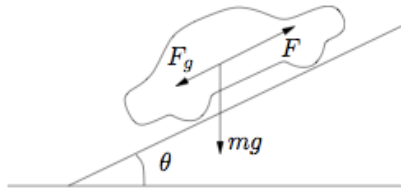
$$C(s) = 2000 \frac{s^2 + 1.1s + 0.1}{s}$$

$$= 2200 + \frac{200}{s} + 2000s$$

Closed loop system

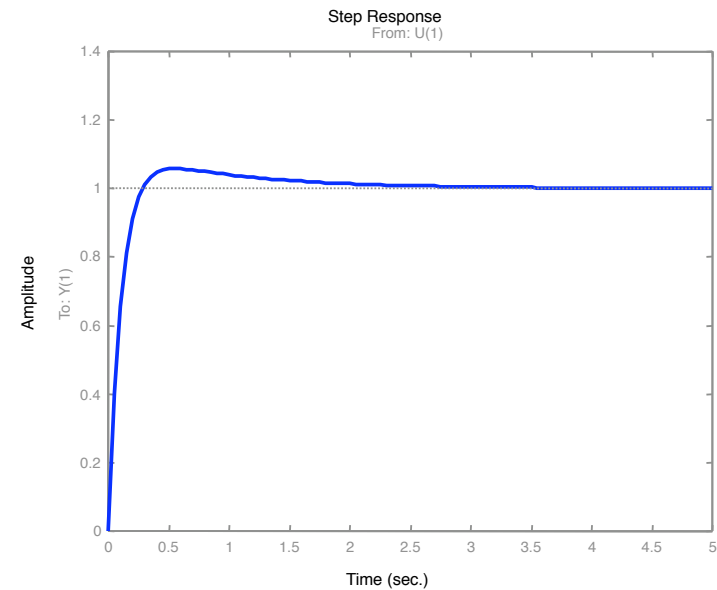
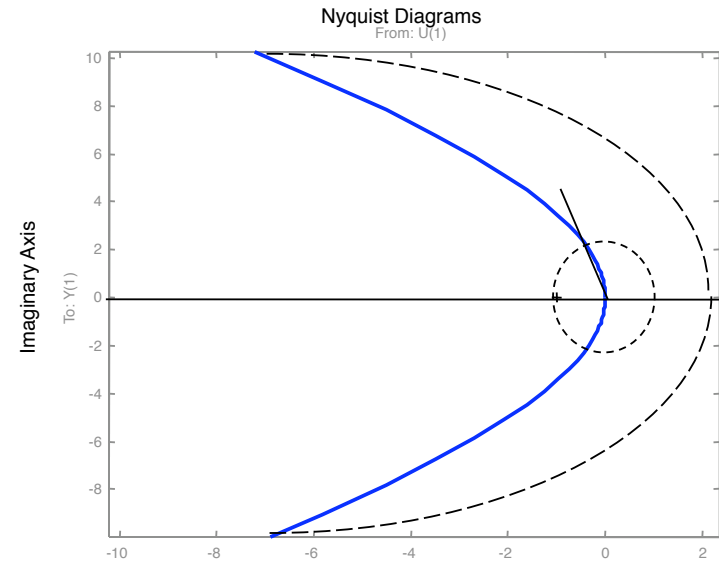
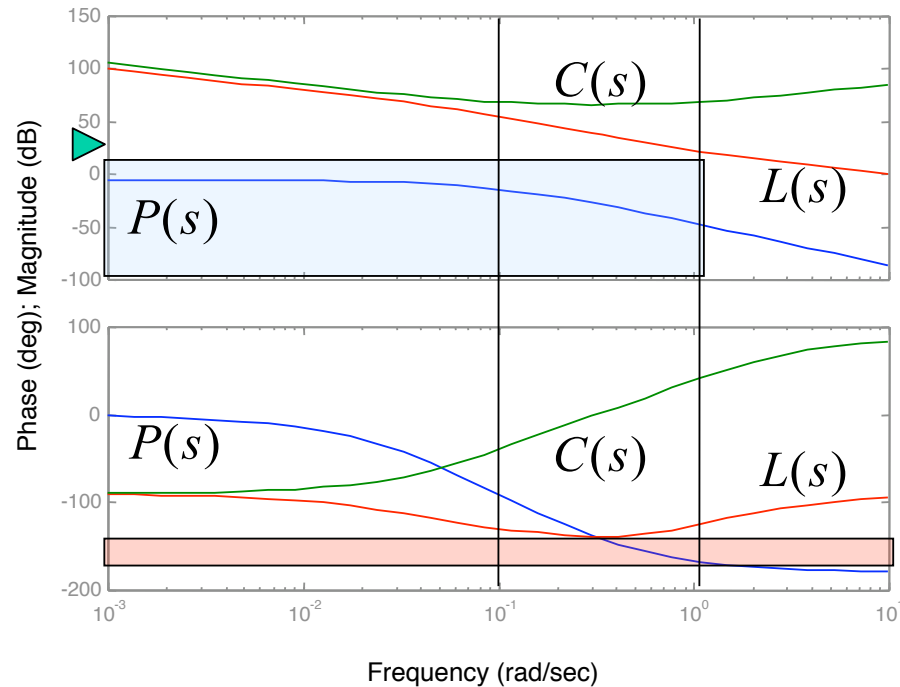
- Very high steady state gain
- Adequate tracking @ 1 rad/sec
- ~80° phase margin

Example: Cruise Control using PID - Verification

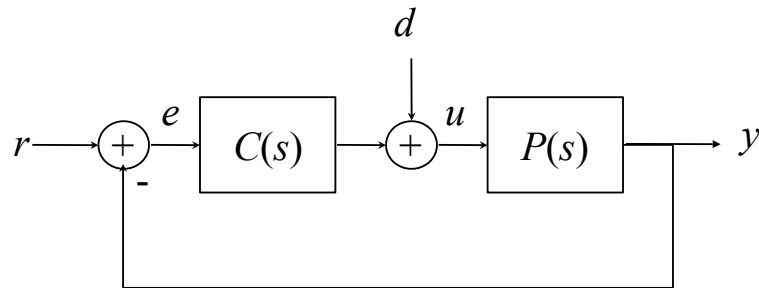


$$P(s) = \frac{1/m}{s + b/m} \times \frac{r}{s + a}$$

$$C(s) = 2000 \frac{s^2 + 1.1s + 0.1}{s}$$



Tools for Designing PID controllers



$$C(s) = K \left(1 + \frac{1}{T_I s} + T_D s \right)$$

Zeigler-Nichols tuning

- Design PID gains based on step response
- Works OK for many plants (but underdamped)
- Good way to get a first cut controller
- Frequency domain version also exists

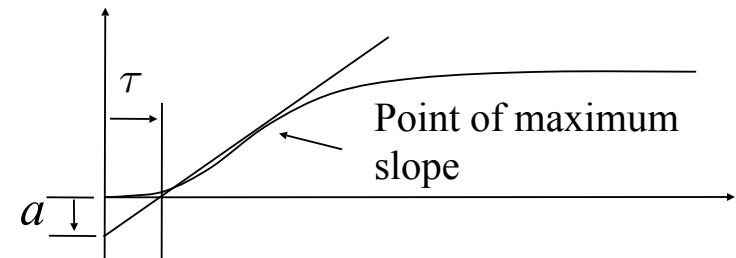
Caution: PID amplifies high frequency noise

- Sol'n: pole at high frequency

Caution: Integrator windup

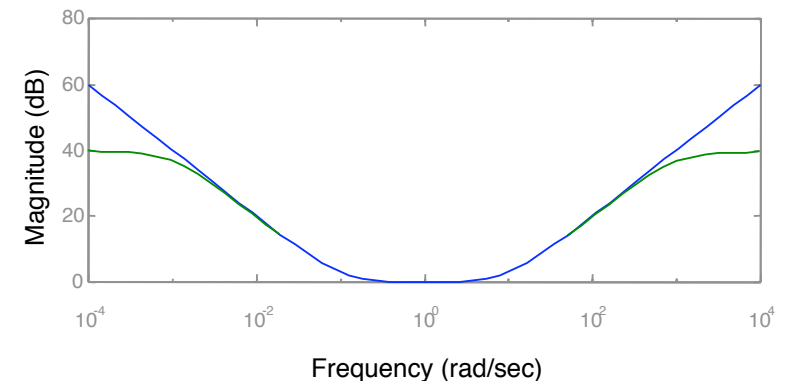
- Prolonged error causes large integrated error
- Effect: large undershoot (to reset integrator)
- Sol'n: move pole at zero to very small value
- Fancier sol'n: anti-windup compensation

Step response

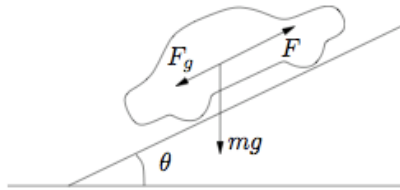


$$K = 1.2/a \quad T_i = 2 * \tau \quad T_d = \tau/2$$

Bode Diagrams



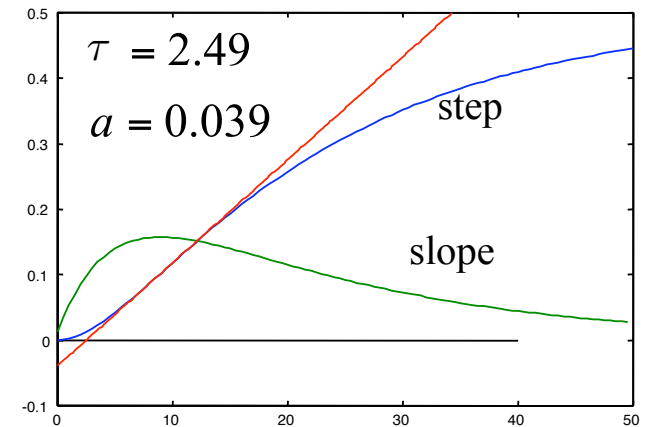
Example: PID cruise control



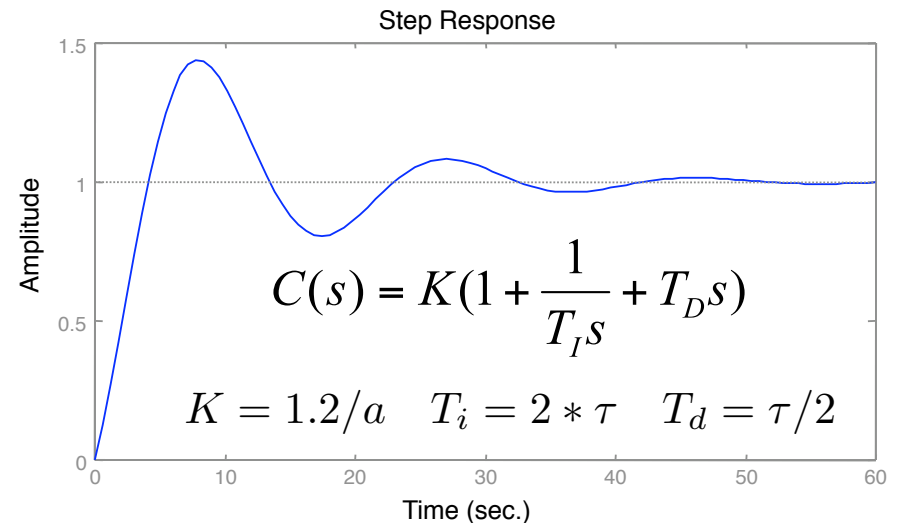
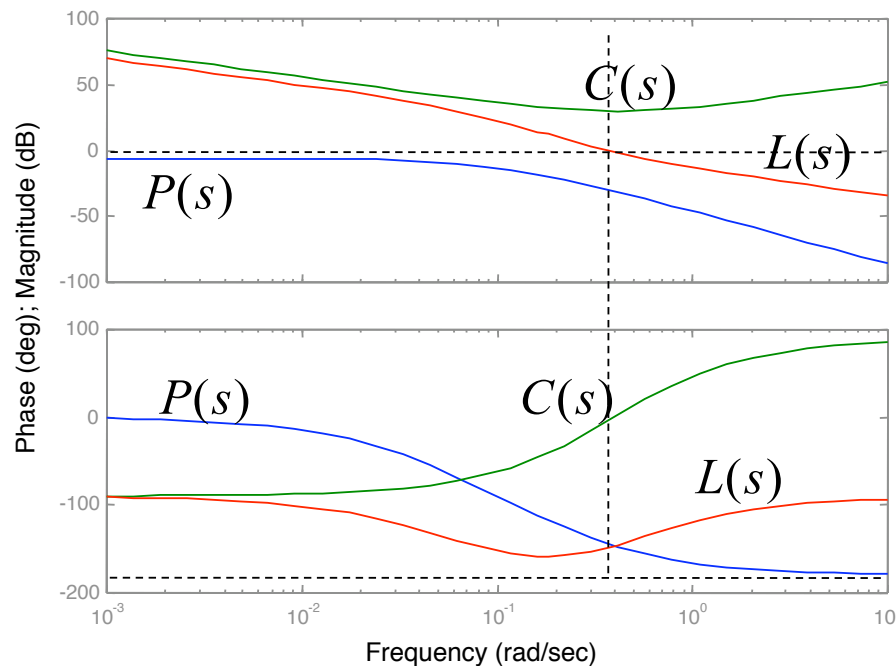
$$P(s) = \frac{1/m}{s + b/m} \times \frac{r}{s + a}$$

Ziegler-Nichols design for cruise controller

- Plot step response, extract τ and a , compute gains



Bode Diagrams



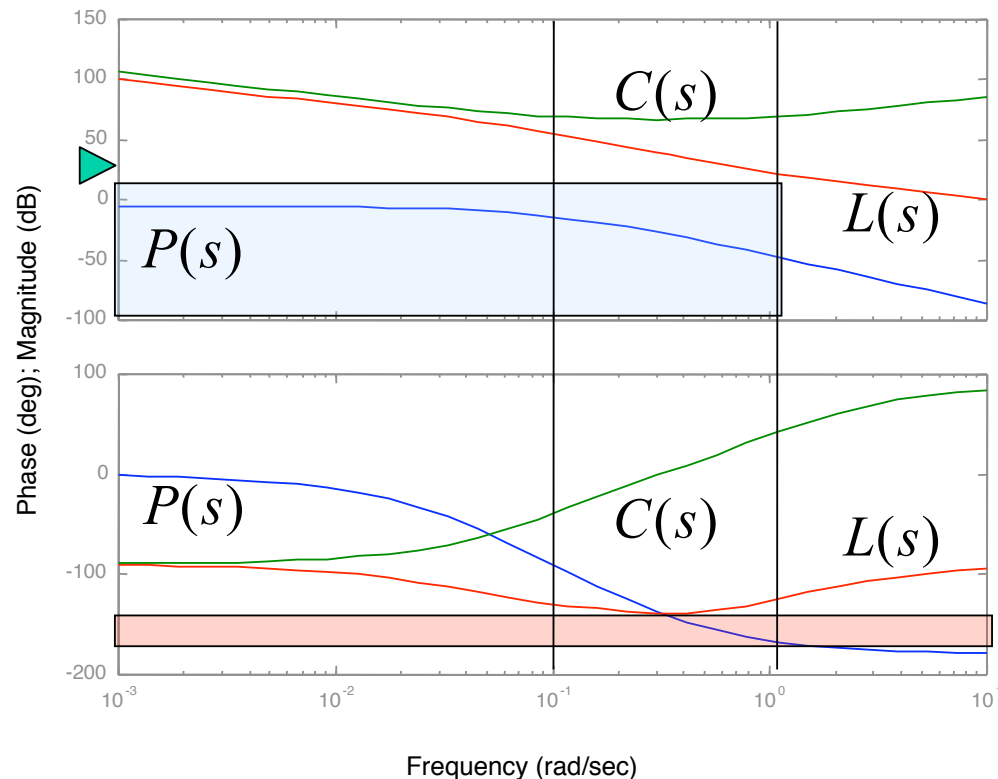
- Result: *sluggish* $\Rightarrow$ increase loop gain + more phase margin (shift zero)

Summary: Frequency Domain Design using PID

Loop Shaping for Stability & Performance

- Steady state error, bandwidth, tracking

$$H_{ue}(s) = K_p + K_I \times \frac{1}{s} + K_D s$$



Main ideas

- Performance specs give bounds on loop transfer function
- Use controller to shape response
- Gain/phase relationships constrain design approach
- Standard compensators: proportional, PI, PID

