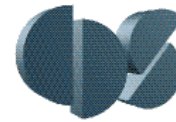




CDS 101: Lecture 7.1

Loop Analysis of Feedback Systems



Richard M. Murray
12 November 2007

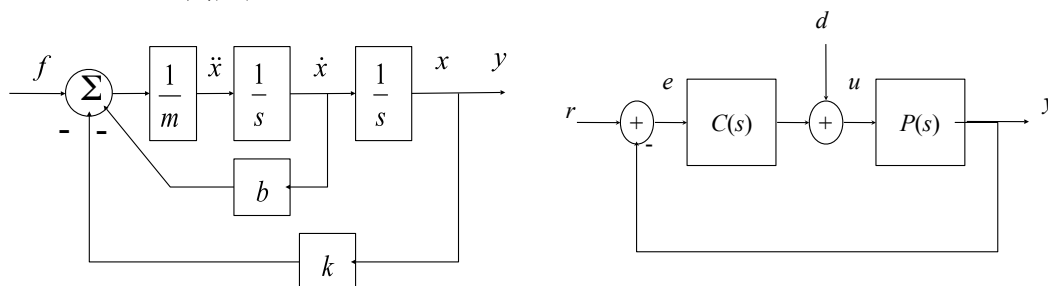
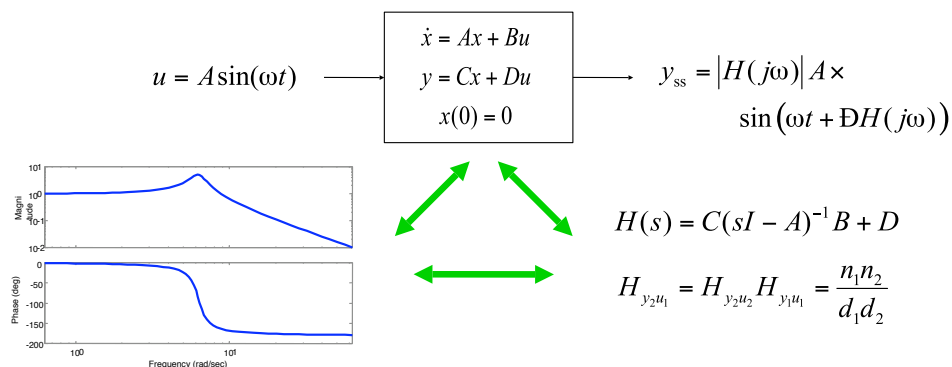
Goals:

- Show how to compute closed loop stability from open loop properties
- Describe the Nyquist stability criterion for stability of feedback systems
- Define gain and phase margin and determine it from Nyquist and Bode plots

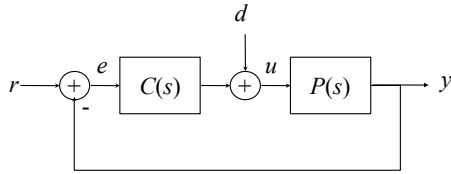
Reading:

- Åström and Murray, *Feedback Systems*, Ch 9

Review from Last Week



Closed Loop Stability



Q: how do open loop dynamics affect the closed loop stability?

- Given open loop transfer function $C(s)P(s)$ determine when system is stable

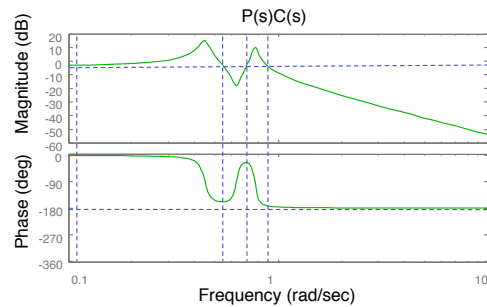
Brute force answer: compute poles closed loop transfer function

$$G_{yr} = \frac{PC}{1 + PC} = \frac{n_p n_c}{d_p d_c + n_p n_c}$$

- Poles of G_{yr} = zeros of $1 + PC$
- Easy to compute, but not so good for design

Alternative: look for conditions on PC that lead to instability

- Example: if $PC(s) = -1$ for some $s = j\omega$, then system is *not* asymptotically stable
- Condition on PC is much nicer because we can *design* $PC(s)$ by choice of $C(s)$
- However, checking $PC(s) = -1$ is not enough; need more sophisticated check



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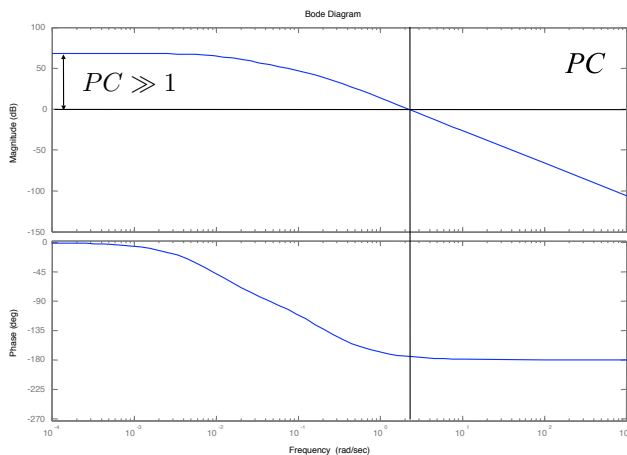
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Game Plan: Frequency Domain Design

Goal: figure out how to *design* $C(s)$ so that $1 + C(s)P(s)$ is stable *and* we get good performance

$$G_{yr} = \frac{PC}{1 + PC}$$

- Poles of G_{yr} = zeros of $1 + PC$
- Would also like to “shape” G_{yr} to specify performance at different frequencies



- Low frequency range:

$$PC \gg 1 \Rightarrow \frac{PC}{1 + PC} \approx 1$$

(good tracking)

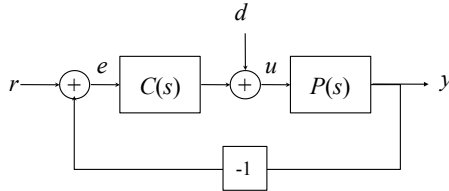
- Bandwidth: frequency at which closed loop gain = $\frac{1}{2}$
 $\Rightarrow$ open loop gain ≈ 1
- Idea: use $C(s)$ to *shape* PC (under certain constraints)
- Need tools to analyze stability and performance for closed loop given PC

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Nyquist Criterion



Determine stability from (open) loop transfer function, $L(s) = P(s)C(s)$.

- Use “principle of the argument” from complex variable theory (see reading)

Thm (Nyquist). Consider the Nyquist plot for loop transfer function $L(s)$. Let

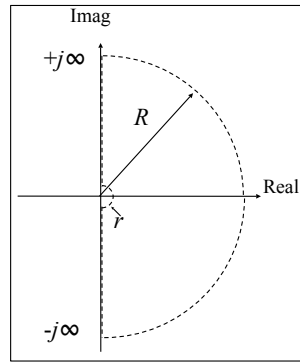
P # RHP poles of $L(s)$

N # CCW encirclements of -1

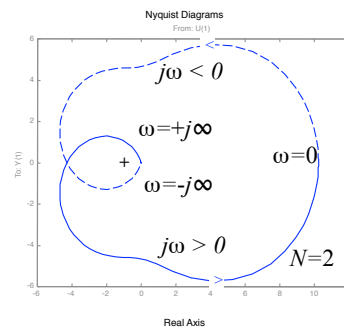
Z # RHP zeros of $1 + L(s)$

Then

$$Z = N + P$$



- Nyquist “D” contour
- Take limit as $r \rightarrow 0, R \rightarrow \infty$
- Trace from $+\infty$ to $-\infty$ along imaginary axis



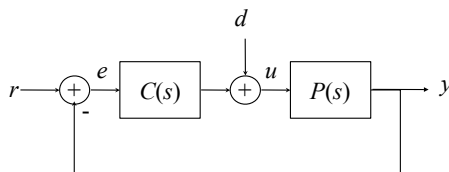
- Trace frequency response for $L(s)$ along the Nyquist “D” contour
- Count net # of CCW encirclements of the -1 point

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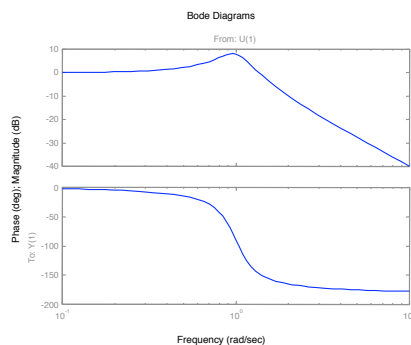
Simple Interpretation of Nyquist



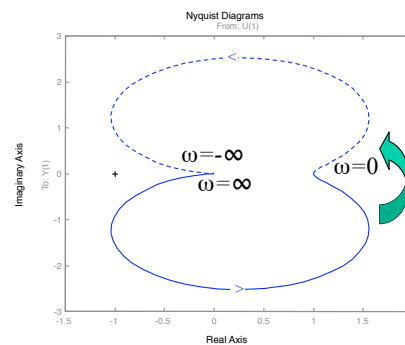
Basic idea: avoid positive feedback

- If $L(s)$ has 180° phase (or greater) and gain greater than 1, then signals are amplified around loop
- Use when phase is monotonic
- General case requires Nyquist

Can generate Nyquist plot from Bode plot + reflection around real axis



bode(sys)



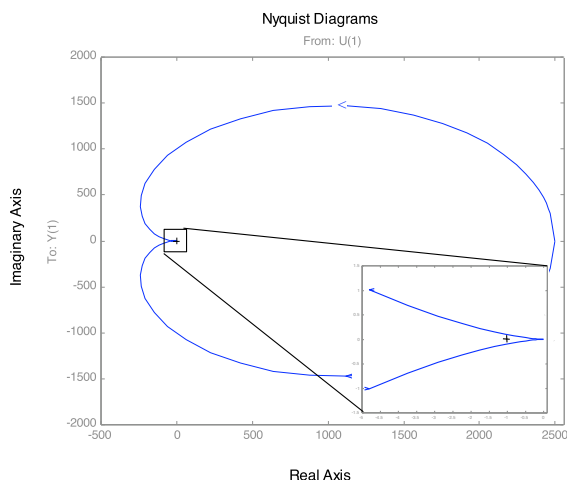
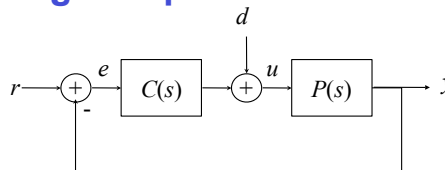
nyquist(sys)

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Example: Proportional + Integral* speed controller



$$P(s) = \frac{1/m}{s + b/m} \times \frac{r}{s + a}$$

$$C(s) = K_p + \frac{K_i}{s + 0.01}$$

Remarks

- $N = 0, P = 0 \Rightarrow Z = 0$ (stable)
- Need to zoom in to make sure there are no net encirclements
- Note that we don't have to compute closed loop response

* slightly modified; more on the design of this compensator in next week's lecture

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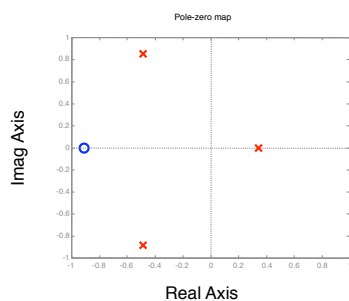
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More complicated systems

What happens when open loop plant has RHP poles?

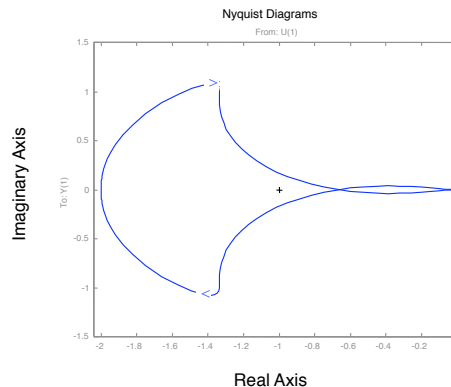
- $1 + PC$ has singularities inside D contour $\Rightarrow$ these must be taken into account



$$L(s) = \frac{s+1}{s-0.5} \times \frac{1}{s^2 + s + 1}$$

unstable pole

$$\frac{1}{1+L} = \frac{s+1}{(s+0.35)(s+0.07+1.2j)(s+0.07-1.2j)} \checkmark$$



$$N = -1, P = 1 \Rightarrow Z = N + P = 0 \text{ (stable)}$$

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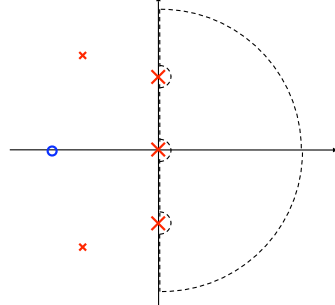
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Comments and cautions

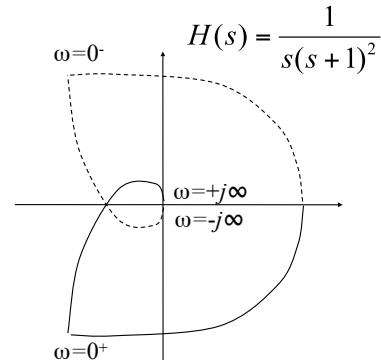
Why is the Nyquist plot *useful*?

- Old answer: easy way to compute stability (before computers and MATLAB)
- Real answer: gives *insight* into stability and robustness; very useful for reasoning about stability

Nyquist plots for systems with poles on the $j\omega$ axis



- chose contour to avoid poles on axis
- need to carefully compute Nyquist plot at these points
- evaluate $H(\delta+0j)$ to determine direction



Cautions with using MATLAB

- MATLAB doesn't generate portion of plot for poles on imaginary axis
- These must be drawn in by hand (make sure to get the orientation right!)
- Uses opposite orientation convention to AM08 (and ACM 95/100)

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Robust stability: gain and phase margins

Nyquist plot tells us if closed loop is stable, but not *how* stable

Gain margin

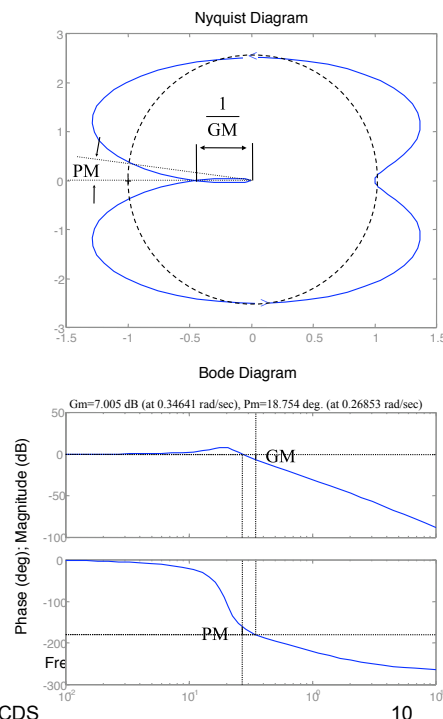
- How much we can modify the *loop gain* and still have the system be stable
- Determined by the location where the loop transfer function crosses 180° phase

Phase margin

- How much we can add "phase delay" and still have the system be stable
- Determined by the phase at which the loop transfer function has unity gain

Bode plot interpretation

- Look for gain = 1, 180° phase crossings
- MATLAB: `margin(sys)`

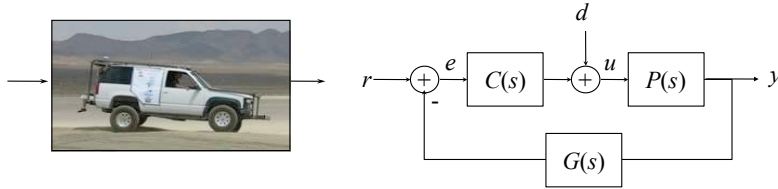


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Example: cruise control



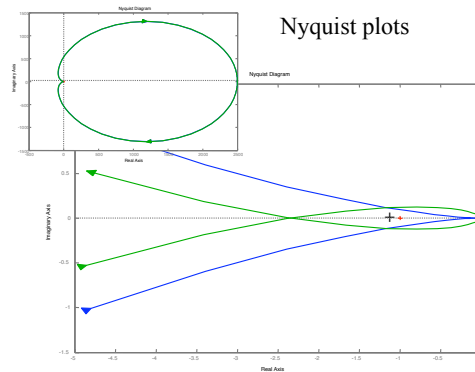
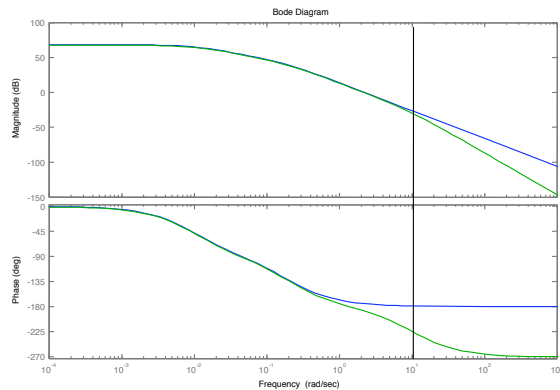
$$P(s) = \frac{1/m}{s + b/m} \times \frac{r}{s + a}$$

$$C(s) = K_p + \frac{K_i}{s + 0.01}$$

$$G(s) = \frac{10}{s + 10}$$

Effect of additional sensor dynamics

- New speedometer has pole at $s = 10$ (very fast); problems develop in the field
- What's the problem? A: insufficient phase margin in original design (not robust)

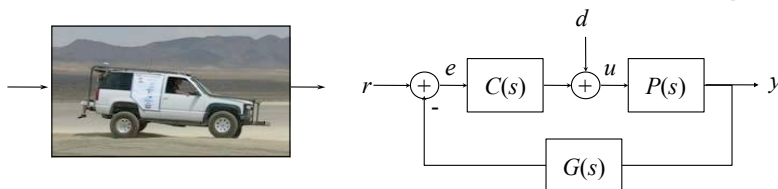


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Preview: control design



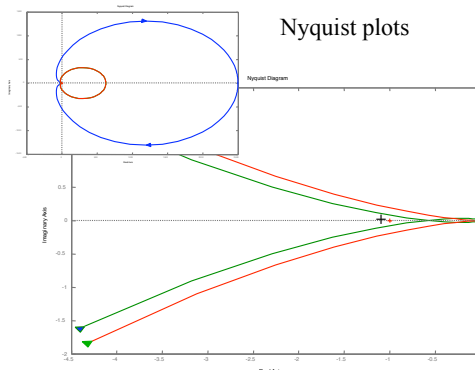
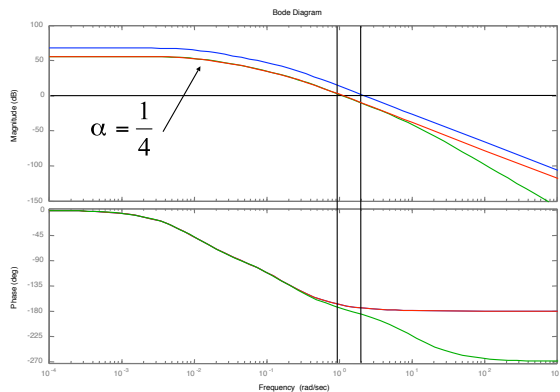
$$P(s) = \frac{1/m}{s + b/m} \times \frac{r}{s + a}$$

$$C(s) = \alpha \left(K_p + \frac{K_i}{s + 0.01} \right)$$

$$G(s) = \frac{10}{s + 10}$$

Approach: Increase phase margin

- Increase phase margin by reducing gain $\Rightarrow$ can accommodate new sensor dynamics
- Tradeoff: lower gain at low frequencies $\Rightarrow$ less bandwidth, larger steady state error

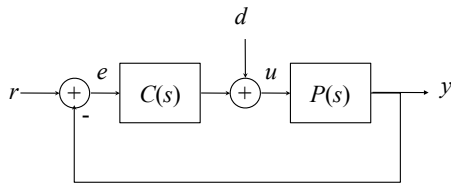


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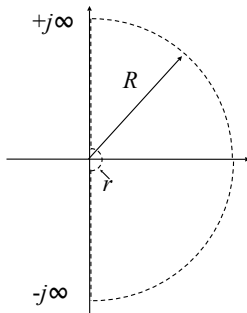
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Summary: Loop Analysis of Feedback Systems



- Nyquist criteria for loop stability
- Gain, phase margin for robustness



Thm (Nyquist).

P # RHP poles of $L(s)$

N # CCW encirclements

Z # RHP zeros

$$Z = N + P$$

