Lecture 6-3: Frequency Response

RMM 1 Nov 06

Goals

1. Review the frequency response for a linear system using transfer functions
2. Describe how to sketch the frequency response by hand

Outline

I. Frequency response of a linear system

$u = A\sin\omega t \longrightarrow \boxed{\begin{matrix}\dot{x} = Ax + Bu \\ y = Cx + Du\end{matrix}} \longrightarrow y = B\sin(\omega t + \phi) + \text{transients}$ (ignore)

$$H(s) = C(sI - A)^{-1}B + Du \qquad \frac{B}{A} = |H(j\omega)| \qquad \phi = \tan^{-1}\left(\frac{\text{Im } H(j\omega)}{\text{Re } H(j\omega)}\right)$$

II How to sketch a Bode plot (Fall ~~2007 1997~~ 2002 notes)

A. Idea: separate mag & phase of each component

B. Bode: log mag $\Rightarrow$ addition, linear phase $\Rightarrow$ addition

C. Idea: sketch via plotting asymptotic approximations

III Examples

A. $H(s) = s^k \qquad k = \ldots, -2, -1, 0, 1, 2, \ldots$

B. $H(s) = \frac{1}{s+a}$

C. $H(s) = \frac{1}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$

D. $H(s) = \frac{s+b}{s+a} \qquad a < b$

E. $H(s) = \frac{k(s+b)}{(s+a)(s^2 + 2\zeta\omega_0 s + \omega_0^2)} \qquad a < \omega_0 < b$

Sketching Bode plots

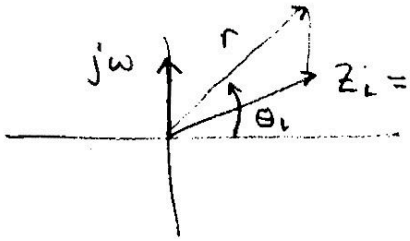
Given a transfer function $H(s) = \frac{n(s)}{d(s)}$, we can sketch Bode plot by looking at poles and zeros

$$H(s) = K \frac{(s+z_1)\cdots(s+z_m)}{(s+p_1)\cdots(s+p_n)} \qquad z_i, p_i \in \mathbb{C}$$

$$H(j\omega) = K \frac{(j\omega+z_1)\cdots(j\omega+z_m)}{(j\omega+p_1)\cdots(s+p_n)}$$

Each term can be written as $r_i e^{j\theta_i}$ where

r_i = magnitude, θ_i = phase



$$r = \sqrt{\alpha^2+\beta^2}$$
$$\sigma = \tan^{-1}\left(\frac{\beta}{\alpha}\right)$$

$$H(j\omega) = K \frac{r_1 e^{j\theta_1}\cdots r_m e^{j\theta_m}}{r_{m+1} e^{j\theta_{m+1}}\cdots r_{m+n} e^{j\theta_{m+n}}}$$

$$= K\, r_1 e^{j\theta_1}\cdots r_m e^{j\theta_m}\, r_{m+1} e^{-j\theta_{m+1}}\cdots r_{m+n} e^{-j\theta_{m+n}}$$

$$= \underbrace{K r_1 r_2 \cdots r_{m+n}}_{\text{mag}}\, e^{j\underbrace{(\theta_1+\cdots+\theta_m-\theta_{m+1}-\cdots-\theta_{m+n})}_{\text{phase}}}$$

Bode plot

- log amplitude ⇒ <u>add</u> $K, r_1, \ldots, r_{m+n}$
- linear phase ⇒ <u>add</u> phase of zeros, <u>subtract</u> phase of poles