~~CDS 110b, Winter 95~~

RMM 12 Feb 95
RMM 19 Feb 02

~~CDS 110b update~~
RMM 19 Nov 02
CDS 110a, L8.2

Lecture 9.2 - Control Design Example (24 Nov 04)

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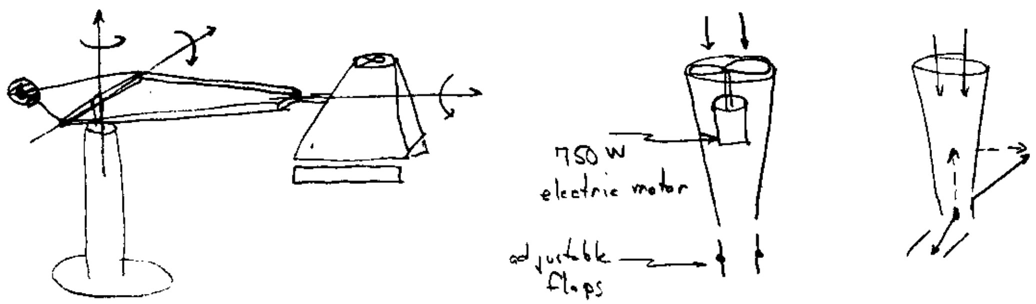
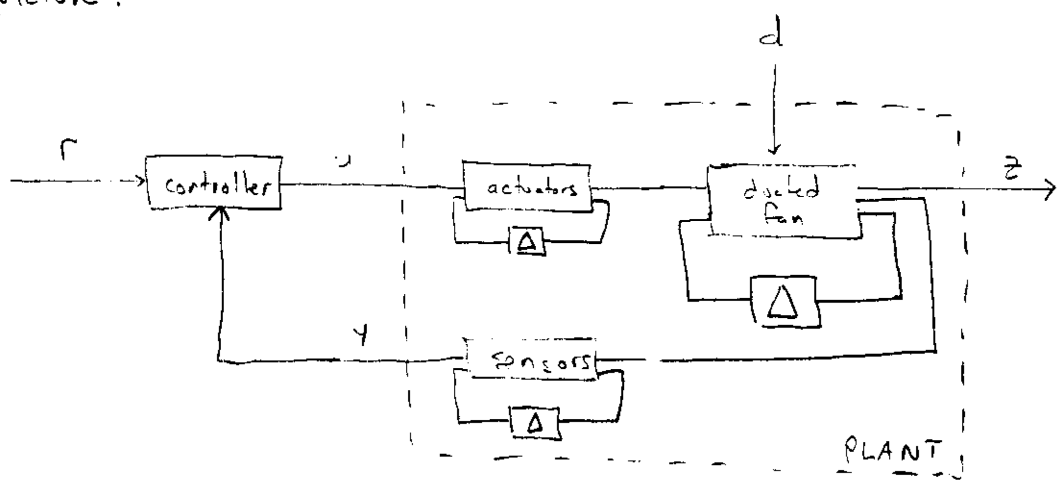
Example: ducted fan (vectored thrust aircraft)




Goal: track aggressive trajectories with high accuracy in presence of noise (wind), model uncertainty (variable motor dynamics, inertial parameters, etc)

Basic picture:




Δ = unknown params
d = disturbances
z = regulated outputs

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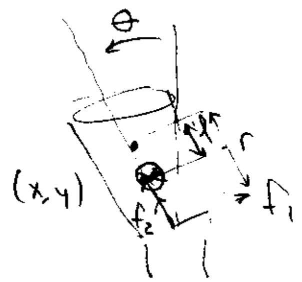
## Ducted fan dynamics




$$m\ddot{x} = \cos\theta\, f_1 - \sin\theta\, f_2$$
$$m\ddot{y} = \cos\theta\, f_2 + \sin\theta\, f_1 - mg$$
$$J\ddot{\theta} = r f_1 - mgl \sin\theta$$

Note: no damping

Linearize system around hover

$x = 0 \quad \dot{x} = 0 \quad f_1 = 0$

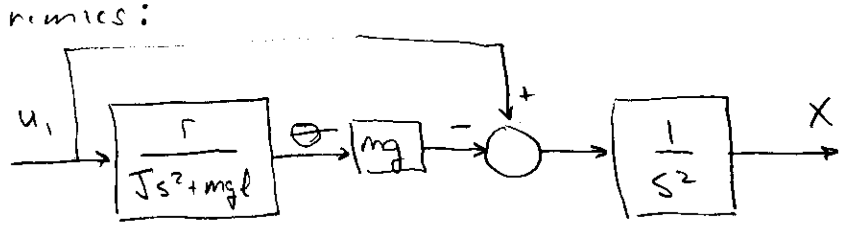
$y = 0 \quad \dot{y} = 0 \quad f_2 = mg$

$\theta = 0 \quad \dot{\theta} = 0$

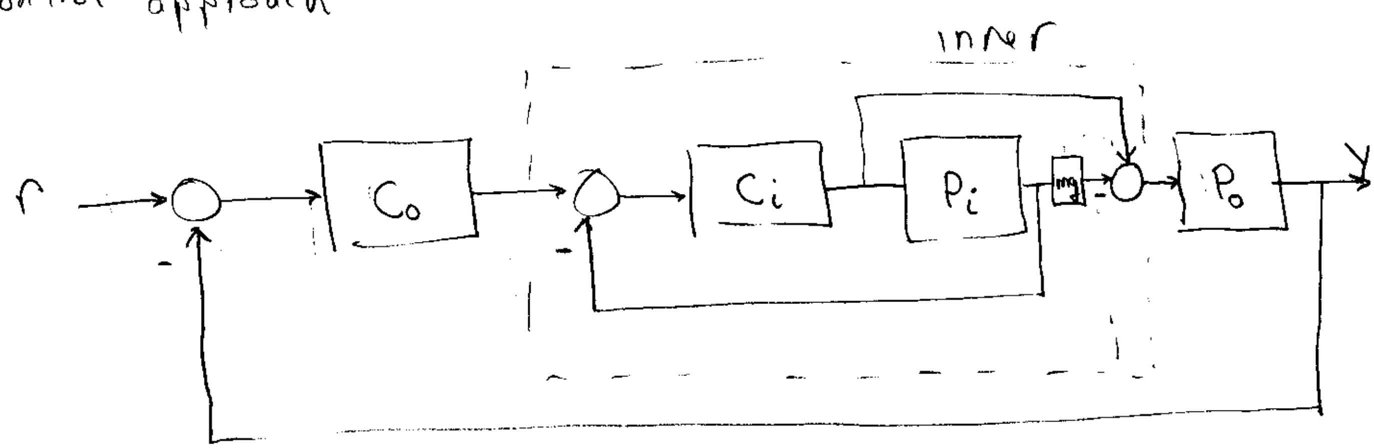
linearize $\Rightarrow$

$$m\ddot{x} = u_1 - mg\theta$$
$$m\ddot{y} = u_2$$
$$J\ddot{\theta} = r u_1 - mgl\theta$$

Lateral dynamics:



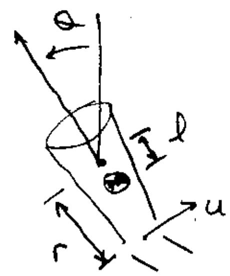

Control approach




Split design into two pieces: pitch + posn

inner outer

Example: ducted fan pitch axis




$$\frac{\Theta(s)}{u(s)} = \frac{r}{Js^2 + mgl}$$

$r$ = flap offset, 0.25 m

$J$ = mom. of inertia, 0.047 kg m²

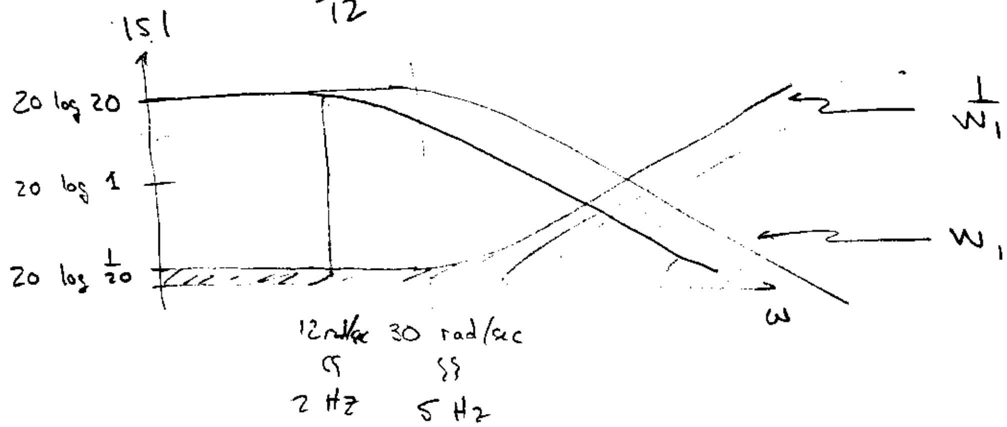
$g$ = 9.8 m/sec²

$l$ = center of mass offset, 0.05 m

$m$ = 1.5 kg

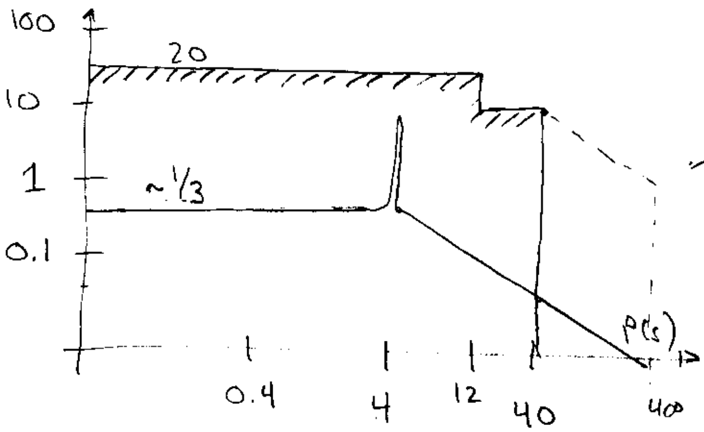
Nominal performance: $\leq 5\%$ error up to ~~5~~ 2 Hz
$\leq 10\%$ error up to 5 Hz

$$W_1 = \frac{20}{\left(\frac{s}{\cancel{30}\,12} + 1\right)^2} \qquad |W_1 S| \leq 1 \quad \forall j\omega$$

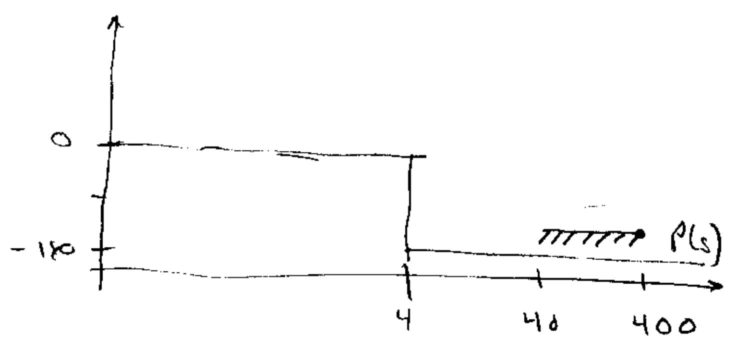


## Controller design

$$P_i(s) = \frac{r}{Js^2 + mgl} \approx \frac{0.25}{0.05s^2 + 0.75} \approx \frac{5}{s^2 + 16}$$



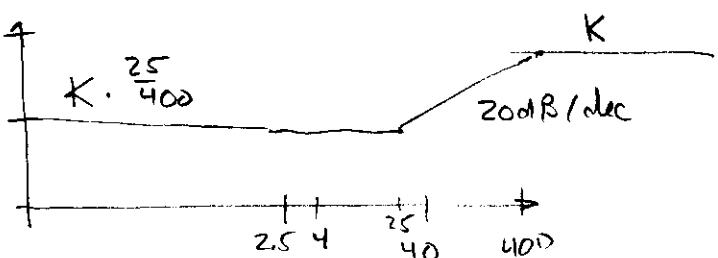

earliest crossover ~ 300 rad/sec (assuming 20 dB/decade rolloff)




PM > 45°

## Compensator design

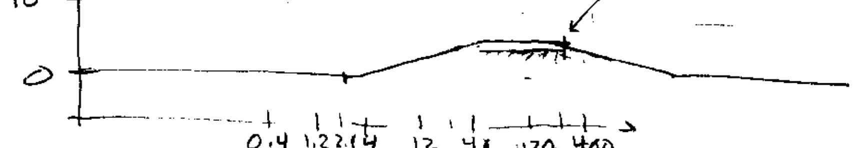
Need to add phase lead in 40-400 Hz range
Need to add gain up to 40 Hz




$K \cdot \frac{25}{400} = 60 \Rightarrow K \approx 1000$

$C_i(s) = K \frac{s+25}{s+400}$

45° @ ~40 rad/sec



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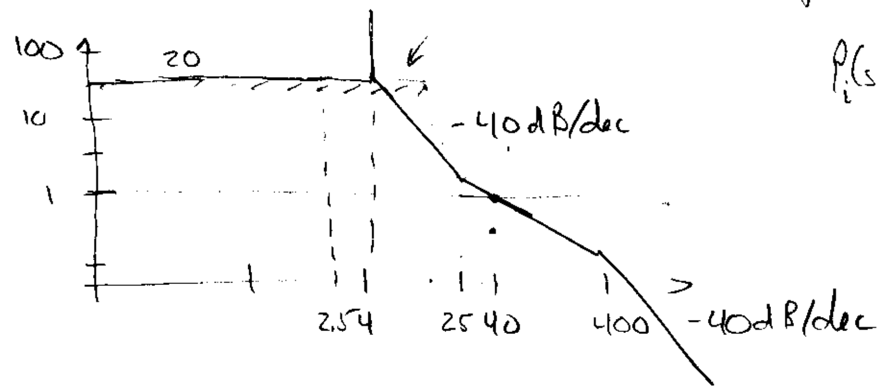
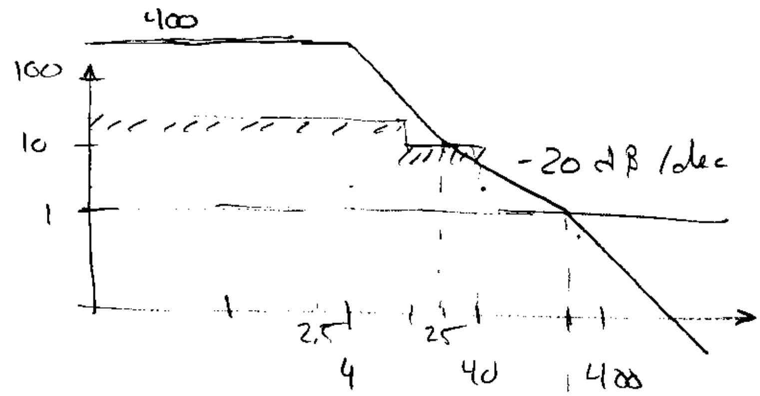
## Closed loop

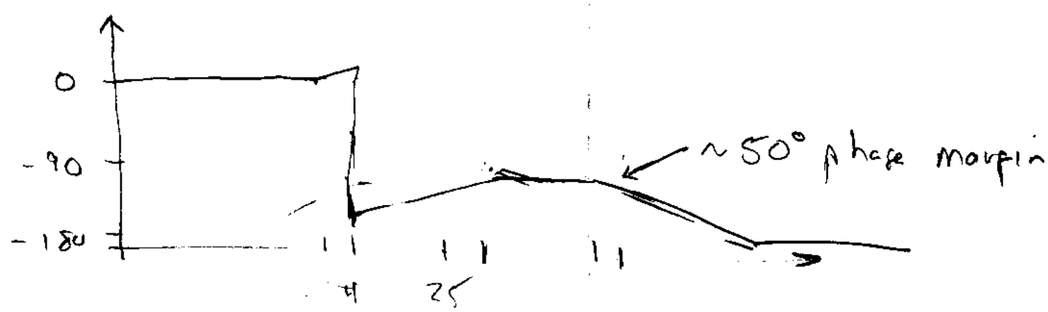
→ Problem: falls off too early
⇒ increase gain

$$P_i(s)\,C_i(s) = \frac{5}{s^2+16}\cdot\frac{1000(s+25)}{s+400}$$




Add factor of 20 gain



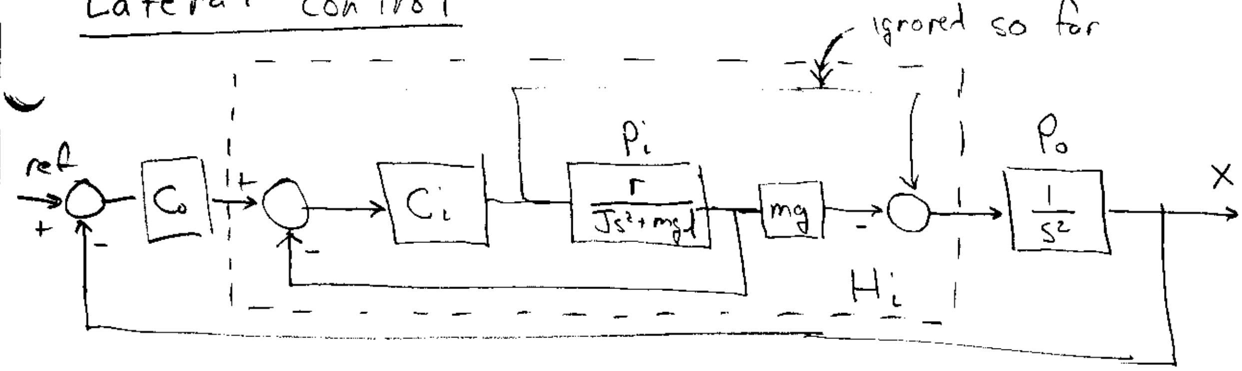




warning: very low phase here.
⇒ probably move back zero a bit

Check: $|W_1 S_i| < 1$ ✓

$|W_2 T_i| < 1$ ✓

## Lateral control




Pretend that pitch controller is perfect $\Rightarrow$ control $\theta$ directly

$$m\ddot{x} = u_1 - mg\theta$$

$$J\ddot{\theta} = ru_1 - mgl\theta \longrightarrow \theta_d \text{ given}$$

$$\text{SS } u_1 = \theta_d \frac{mgl}{r} = 0.2\, mg \cdot \theta_d$$

So, in steady state we can assume

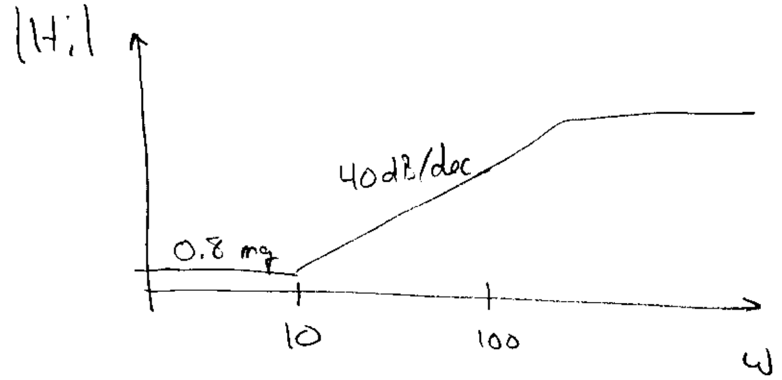
$$m\ddot{x} = (0.2\, mg - mg)\theta_d = \underbrace{-0.8\, mg\, \theta_d}_{v_1}$$

$v_1$ ← pretend we control this

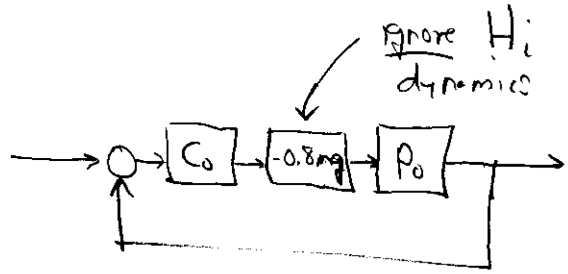
Q: How good an approximation is this?

A: Look at $H_i$

$$H_i = \frac{C_i}{1+C_iP_i} - mg\frac{C_iP_i}{1+C_iP_i} = \frac{C_i(1-mgP_i)}{1+C_iP_i}$$



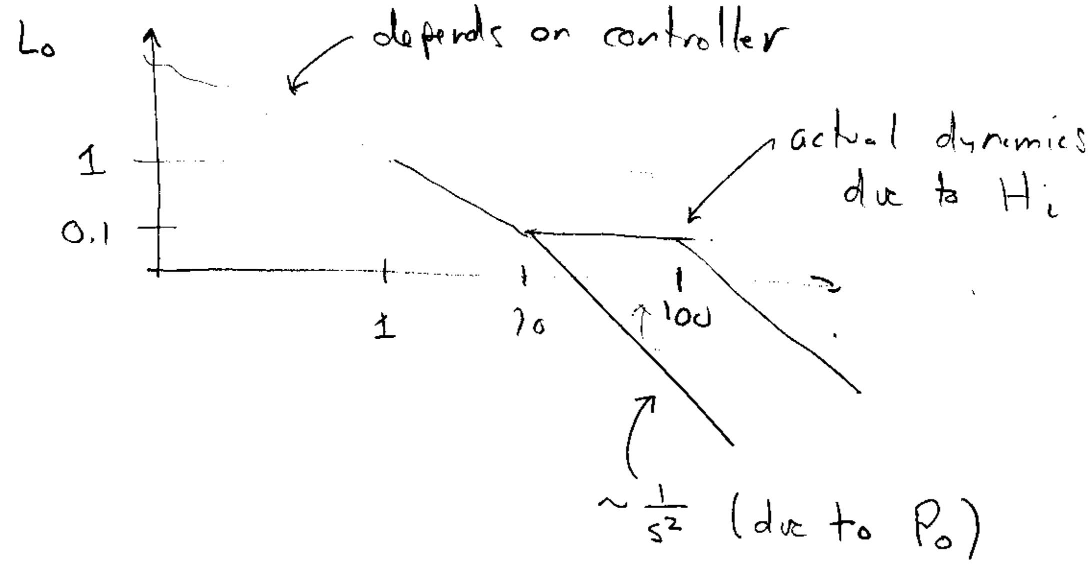

⇒ good approx up to 10 rad/sec ≈ 2 Hz




Outer loop design goals

- 0% steady state error
- BW = 1 rad/sec
- $|L_o| < \frac{1}{10}$ for $\omega > 10$ rad/sec

  ⇒ roll off gain so that $H_i$ dynamics are not a factor

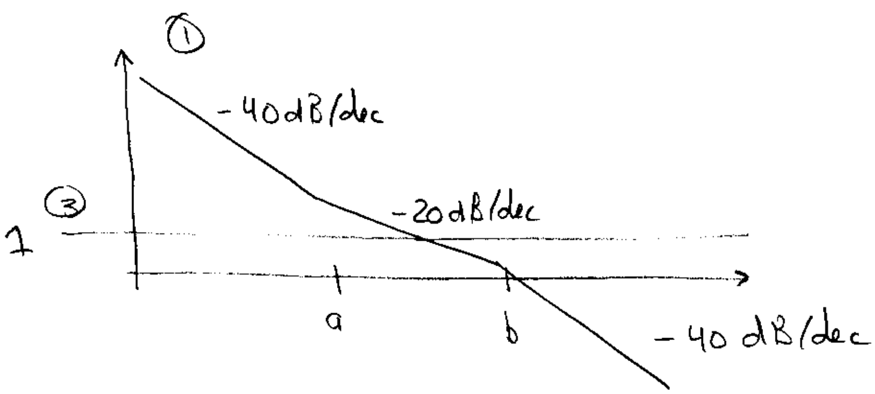


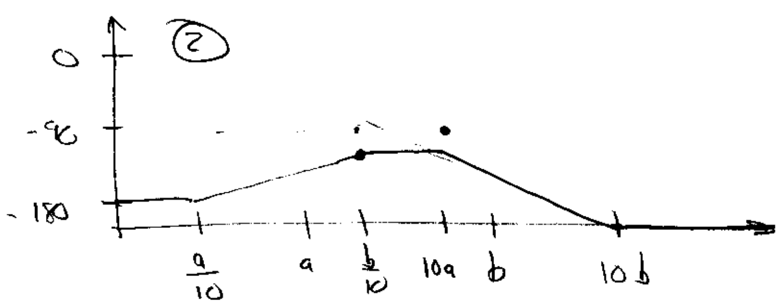
Outer loop design

$$H_2(0)\,P_o(s) = \frac{-0.8\,mg}{s^2}$$

$$C_o(s) = -\,K_o\,\frac{s+a_o}{s+b_o}$$

to get sign of gain correct




Choose crossover at $\sim \frac{b}{10} = 1$ rad/sec $\Rightarrow b = 10$

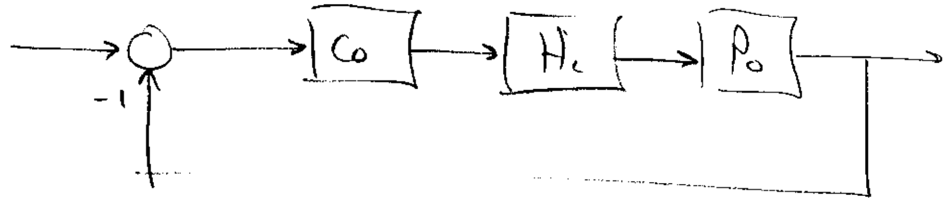
Choose zero at $\frac{b}{10} < 10a < b \;\Rightarrow\; \frac{1}{10} < a < 1$

Try $a = 0.3$

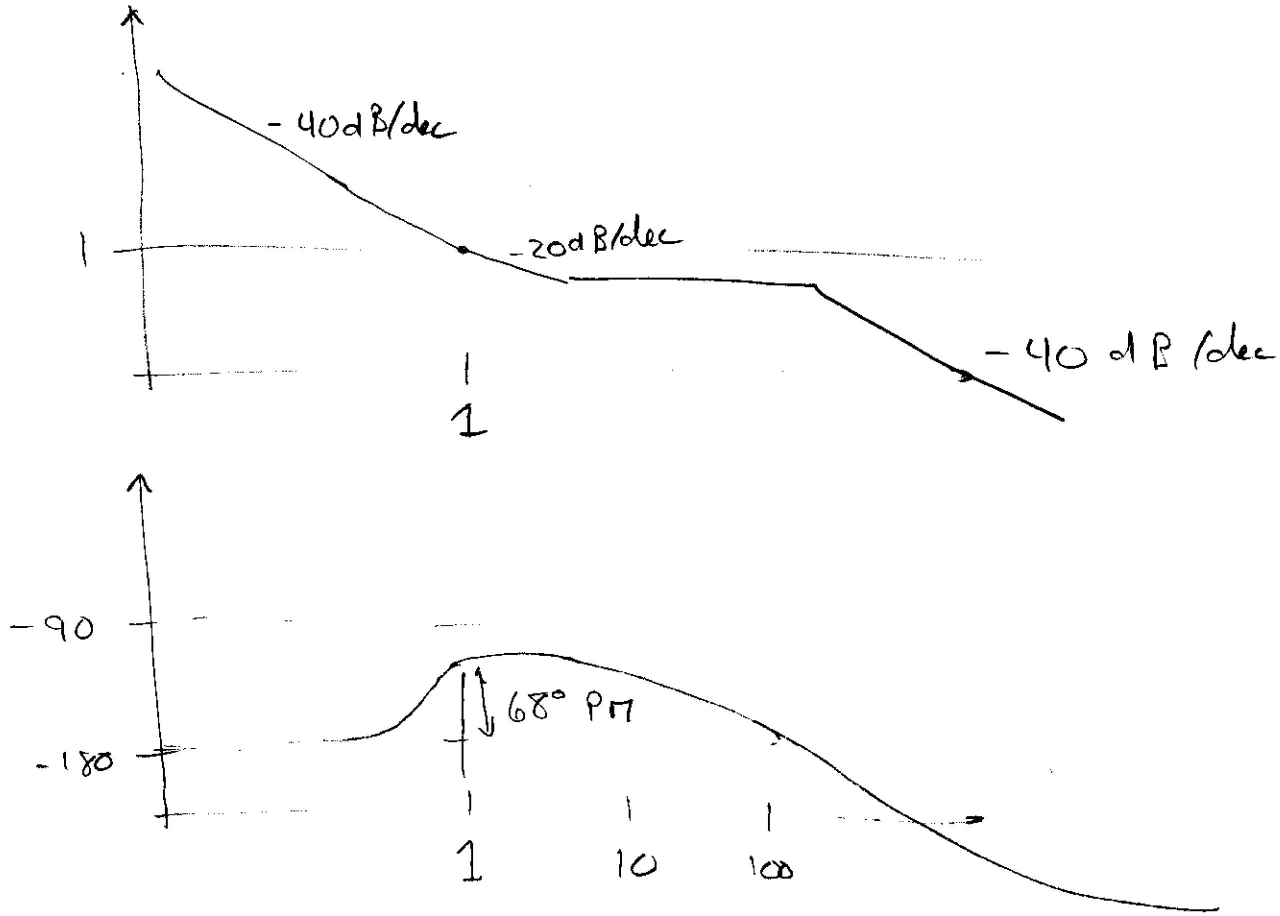
Set gain at $\omega_c = \frac{b}{10}$ to give $\left|H_2(0)\,P_o(j\omega_c)\,C(j\omega_c)\right| = 1$

$$+\,0.8\,mg\,\frac{1}{1}\cdot K\left|\frac{j+0.3}{j+10}\right| = 1 \;\Rightarrow\; K = 0.8$$

Final design check




$$\tilde{L}_o = C_o H_i P_o$$




Use Nyquist to verify stability

Poles: $-194 \pm 216j$, $-11.41$, $-10$, $-0.51 \pm 0.23j$

Zeros: $7.94$, $-10$, $-7.96$, $-0.3$

Note: some pz cancellations in MATLAB