CDS 101: Lecture 8.1
Frequency Domain Design

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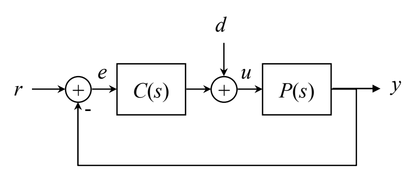
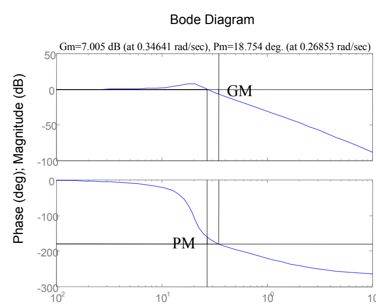
Goals:
- Describe the use of frequency domain performance specifications
- Show how to use “loop shaping” to achieve a performance specification
- Work through a detailed example of a control design problem

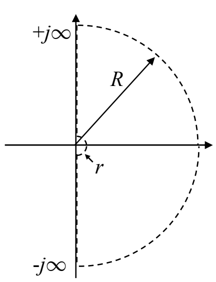
Reading:
- Åström and Murray, *Analysis and Design of Feedback Systems,* Ch 8
- *Advanced*: Lewis, Chapter 12

Review from Last Week





- Nyquist criteria for loop stability
- Gain, phase margin for robustness



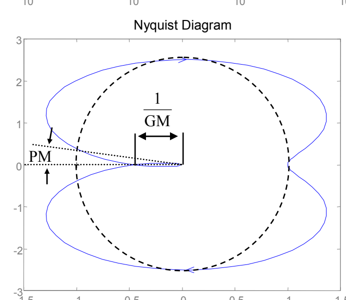
Thm (Nyquist).

P # RHP poles of $L(s)$

N # CW encirclements

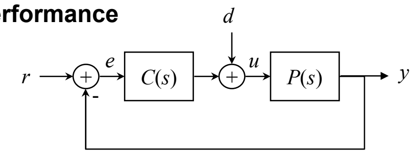
Z # RHP zeros

$$Z = N + P$$



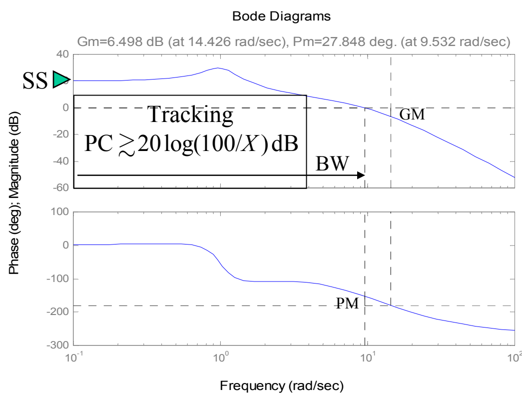
Frequency Domain Performance Specifications

Specify bounds on the loop transfer function to guarantee desired performance



$$L(s) = P(s)C(s)$$

$$H_{er} = \frac{1}{1+L} \qquad H_{yr} = \frac{L}{1+L}$$



- Steady state error:

  $H_{er}(0) = 1/(1+L(0)) \approx 1/L(0)$

  $\Rightarrow$ zero frequency ("DC") gain ▶
- Bandwidth: assuming ~90° phase margin

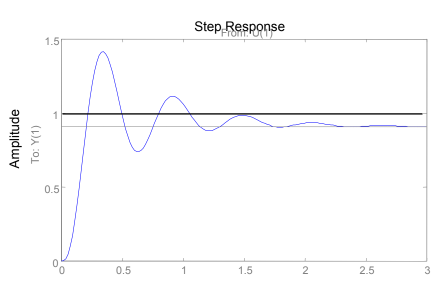
  $$\frac{L}{1+L}(j\omega_c) \approx \left|\frac{1}{1+j}\right| = \frac{1}{\sqrt{2}}$$

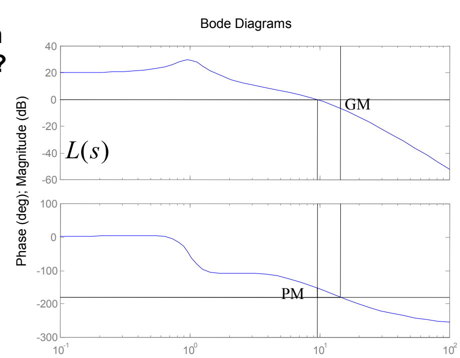
  $\Rightarrow$ sets crossover freq →
- Tracking: X% error up to frequency $\omega_t \Rightarrow$ determines gain bound ($1 + PC > 100/X$) □

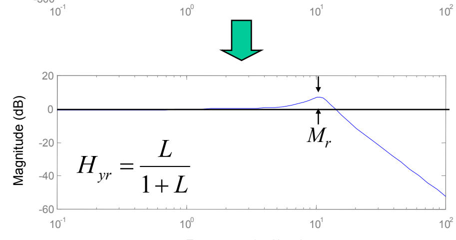
Relative Stability

Relative stability: how stable is system to disturbances at certain frequencies?

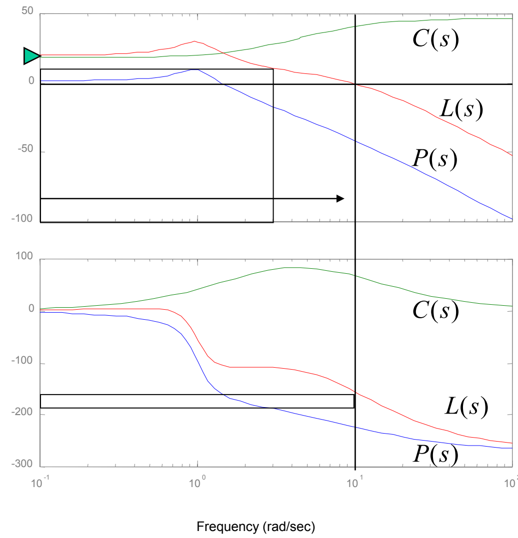
- System can be stable but still have bad response at certain frequencies
- Typically occurs if system has low phase margin $\Rightarrow$ get resonant peak in closed loop (M_r) + poor step response
- Solution: specify minimum phase margin. Typically 45° or more







Overview of Loop Shaping



Performance specification

- Steady state error
- Tracking error
- Bandwidth
- Relative stability

Approach: "shape" loop transfer function using $C(s)$

- $P(s)$ + specifications given
- $L(s) = P(s)\ C(s)$
  - Use $C(s)$ to choose desired shape for $L(s)$
- Important: can't set gain and phase independently

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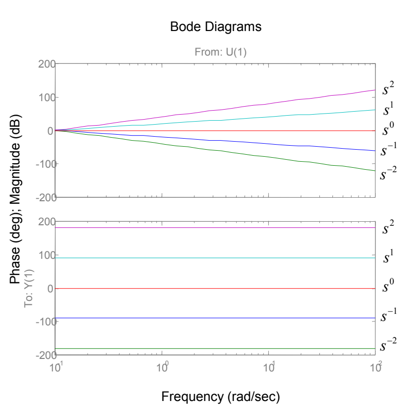
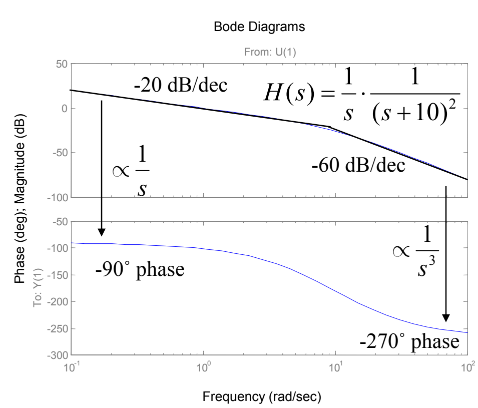
Gain/phase relationships

Gain and phase for transfer function w/ real coeffs are not independent

- Given a given shape for the gain, there is a unique "minimum phase" transfer function that achieves that gain at the specified frequencies
- Basic idea: slope of the gain determines the phase
- Implication: you have to tradeoff gain versus phase in control design





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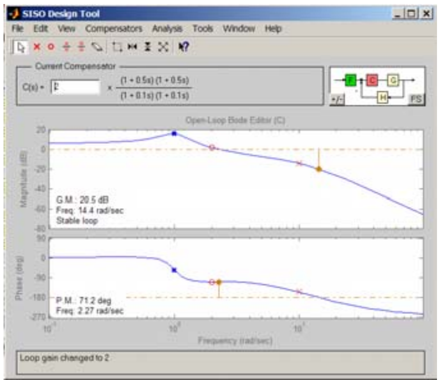
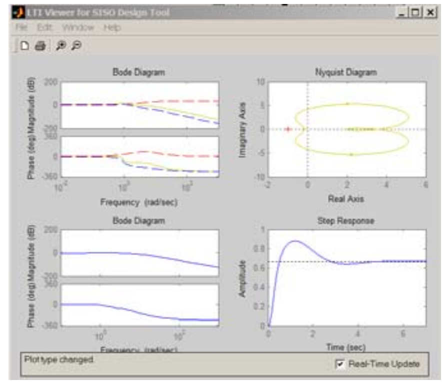
SISOtool (MATLAB 6.x)





Design Window

- Location of plant and controller poles and zeros shown
- Can move around controller poles using mouse

Analysis Window

- Dynamically plots various views, based on current design
- Choose different plot type, system, location, etc

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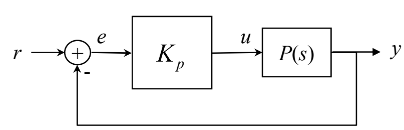
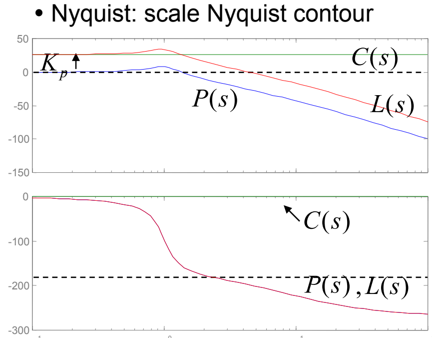
Proportional Feedback

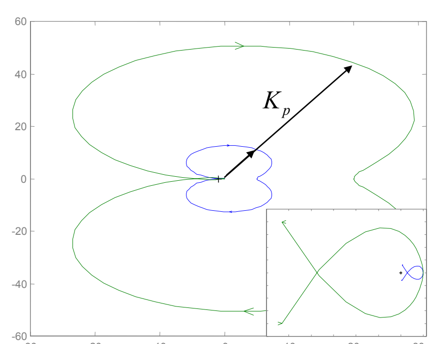
Simplest controller choice: $u = K_p e$

- Effect: lifts gain with no change in phase
- Good for plants with low phase up to desired bandwidth
- Bode: shift gain up by factor of K_p
- Nyquist: scale Nyquist contour



$K_p > 0$





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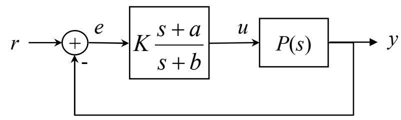
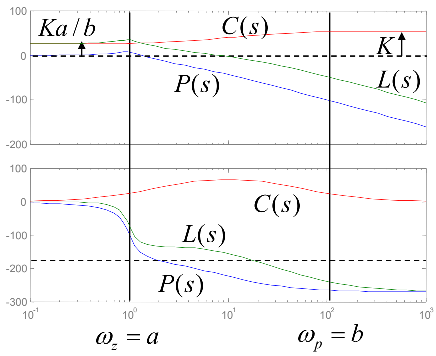
Lead compensation

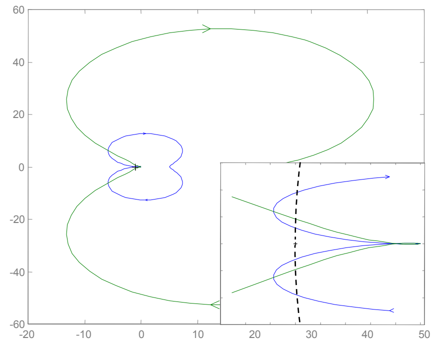
Use to increase phase in frequency band

- Effect: lifts phase by increasing gain at high frequency
- *Very* useful controller; increases PM
- Bode: add phase between zero and pole
- Nyquist: increase phase margin



$a < b \qquad K > 0$

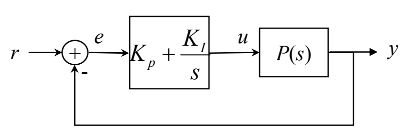




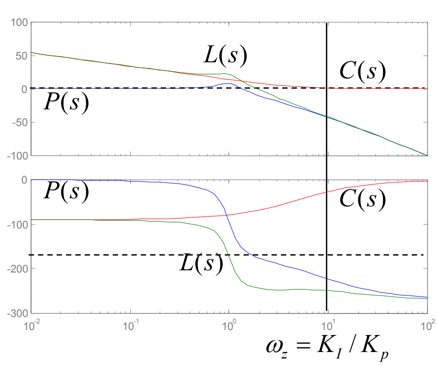
Proportional + Integral Compensation

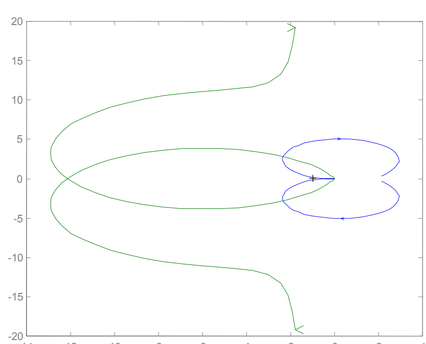
Use to eliminate steady state error

- Effect: lifts gain at low frequency
- Gives *zero* steady state error
- Bode: infinite SS gain + phase lag
- Nyquist: no easy interpretation
- Note: this example is *unstable*

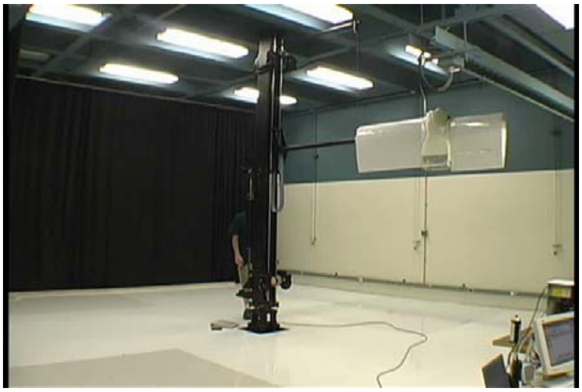


$K_p > 0 \quad K_I > 0$





Example: Pitch Control for Caltech Ducted Fan



System description

- Vector thrust engine attached to wing
- Inputs: fan thrust, thrust angle (vectored)
- Outputs: position and orientation
- States: x, y, θ + derivatives
- Dynamics: flight aerodynamics

Control approach

- Design "inner loop" control law to regulate pitch (θ) using thrust vectoring
- Second "outer loop" controller regulates the position and altitude by commanding the pitch and thrust
- Basically the same approach as aircraft control laws

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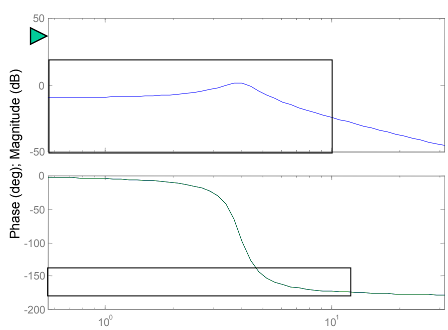
Performance Specification and Design Approach



Performance Specification

- ≤ 1% steady state error
  - Zero frequency gain > 100
- ≤ 10% tracking error up to 10 rad/sec
  - Gain > 10 from 0-10 rad/sec
- ≥ 45° phase margin
  - Gives good relative stability
  - Provides robustness to uncertainty

$$P(s) = \frac{r}{Js^2 + ds + mgl}$$

Design approach

- Open loop plant has poor phase margin
- Add phase lead in 5-50 rad/sec range
- Increase the gain to achieve steady state and tracking performance specs
- Avoid integrator to minimize phase

$$C(s) = K\frac{s+a}{s+b} \qquad a = 25, \quad b = 300, \quad K = 15 \cdot 300$$

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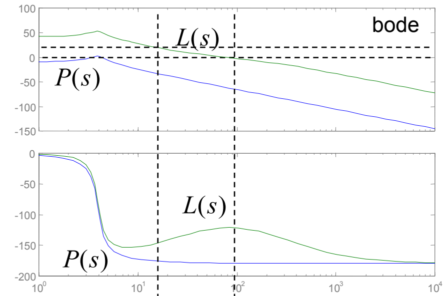
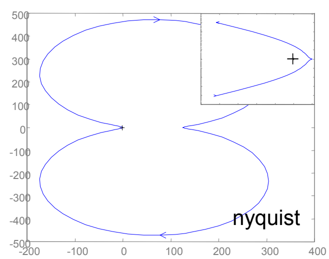
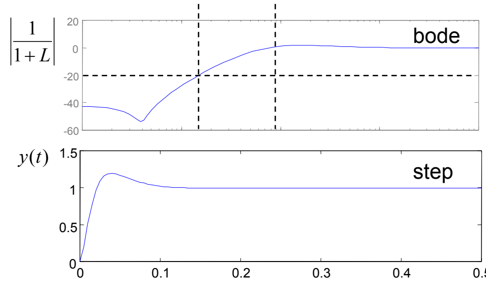
Control Design and Analysis

Select parameters to satisfy specs

- Place phase lead in desired crossover region (given by desired BW)
- Phase lead peaks at 10X of zero location
- Place pole sufficiently far out to insure that phase does not decrease too soon
- Set gain as needed for tracking + BW
- Verify controller using Nyquist plot, etc







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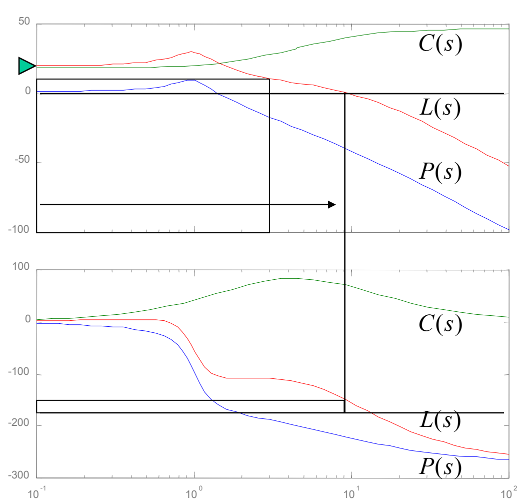
Summary: Frequency Domain Design

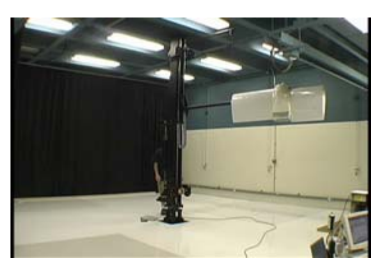
Loop Shaping for Stability & Performance

- Steady state error, bandwidth, tracking



Main ideas

- Performance specs give bounds on loop transfer function
- Use controller to shape response
- Gain/phase relationships constrain design approach
- Standard compensators: proportional, lead, PI



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