CDS 101: Lecture 10.1
Uncertainty Analysis

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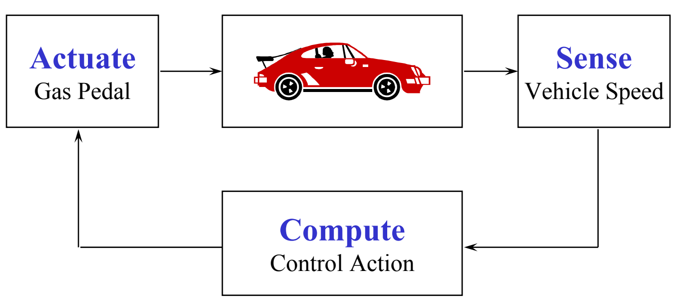
2 December 2002

Goals:
- Describe how feedback and control are used as tools for *uncertainty management*
- Summarize the main principles and tools for the course

Reading:
- None

Control = Sensing + Computation + Actuation = *Feedback*

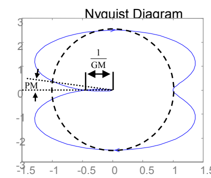


Goals: Stability, Performance, Robustness

Stability, Performance and Robustness

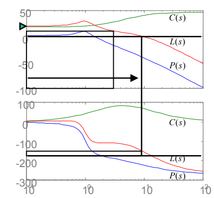
Stability: bounded inputs produce bounded outputs

- Necessary and sufficient condition: check for nonzero solutions around feedback loop
- Basic problem: positive feedback (internal or external)



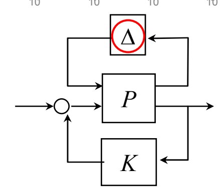
Performance: desirable system response

- Step response: rise time, settling time, overshoot, etc
- Frequency response: tracking signals over given range



Robustness: stability/performance in presence of uncertainties

- Need to check stability for *set* of systems



Modeling Uncertainty

Noise and disturbances

- Model the amount of noise by its signal strength in different frequency bands
- Can model signal strength by peak amplitude, average energy, and other norms
- Typical example: Dryden gust models (filtered white noise)

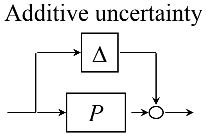
Parametric uncertainty

- Unknown parameters or parameters that vary from plant to plant
- Typically specified as tolerances on the basic parameters that describe system

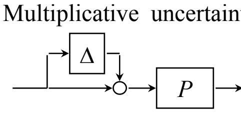
Unmodeled *dynamics*

- High frequency dynamics (modes, etc) can be excited by control loops
- Use bounded operators to account for effects of unmodelled modes:

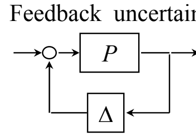
Additive uncertainty



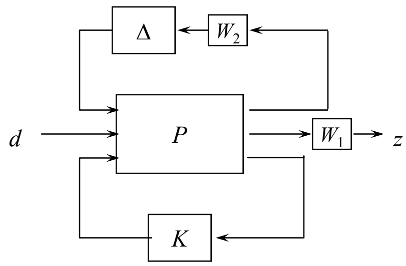
Multiplicative uncertainty



Feedback uncertainty



Robust Performance



d	disturbance signal
z	output signal
Δ	uncertainty block
W_2	performance weight
W_2	uncertainty weight

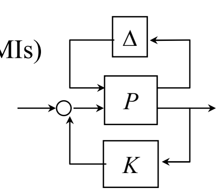
Goal: guaranteed performance in presence of uncertainty

$$\|z\|_2 \leq \gamma \|d\|_2 \quad \text{for all} \quad \|\Delta\| \leq 1$$

- Compare energy in disturbances to energy in outputs
- Use frequency weights to change performance/uncertainty descriptions
- “Can I get X level of performance even with Y level of uncertainty?”

Tools for Analyzing and Synthesizing Controllers

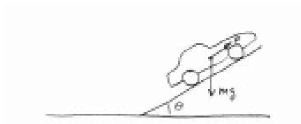
Robust Multi-Variable Control Theory
- Generalizes gain/phase margin to MIMO systems
- Uses operator theory to handle uncertainty, performance
- Uses state space theory to performance computations (LMIs)



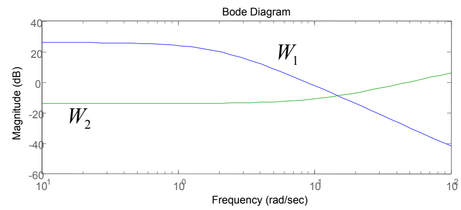
Analysis Tools
- H_∞ gains for multi-input, multi-output systems
- μ analysis software
  - Allow structured uncertainty descriptions (fairly general)
  - Computes *upper and lower bounds* on performance
  - Wide usage in aerospace industry

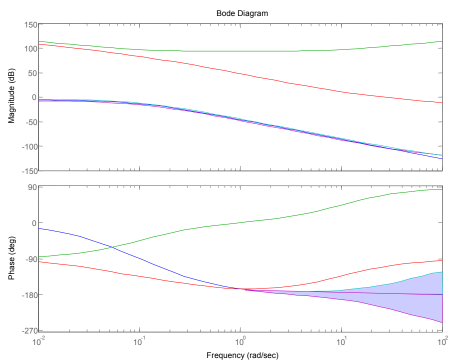
Synthesis Tools
- LQR/LQG + H_∞ “loop shaping”; modern tools for control engineers
- μ *synthesis* software; tends to generate high order controllers
- Model reduction software for reducing order of plants, controllers

Example: Robust Cruise Control



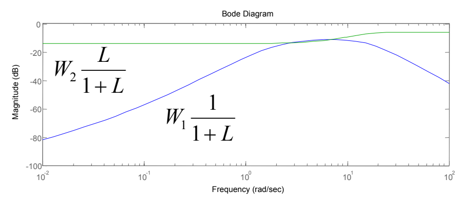
$$P(s) = \frac{1/m}{s + b/m} \cdot \frac{r}{s+a}\left(1 + W_2 \Delta\right)$$





Thm:

- Performance: $\left| W_1 \frac{1}{1+L} \right| < 1 \quad \forall \omega$
- Robust Stability: $\left| W_2 \frac{L}{1+L} \right| < 1 \quad \forall \omega$



Course Summary: Two Main Principles of Control

Robustness to Uncertainty through Feedback

- Feedback allows high performance in the presence of uncertainty
- Example: repeatable performance of amplifiers with 5X component variation
- Key idea: accurate *sensing* to compare actual to desired, correction through *computation* and *actuation*

Design of Dynamics through Feedback

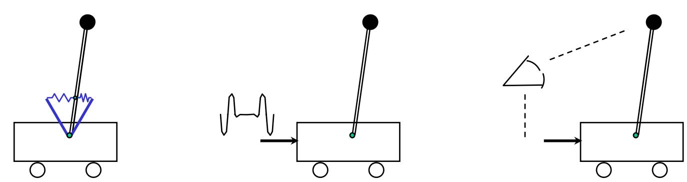
- Feedback allows the dynamics of a system to be modified
- Example: stability augmentation for highly agile, unstable aircraft
- Key idea: interconnection gives *closed loop* that modifies natural behavior

Example #1: Balancing an Inverted Pendulum

Passive Control
- Make structural modifications to change the plant dynamics
- Use this technique whenever it is a viable option: cheap, robust

Open Loop Control
- Exploit knowledge of system dynamics to compute appropriate inputs
- Requires *very* accurate model of plant dynamics in order to work well

Active (Feedback) Control
- Use sensors and actuators connected by a computer to modify dynamics
- Allows uncertainty and noise to be taken into account



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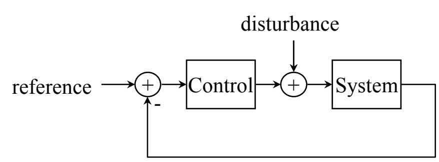
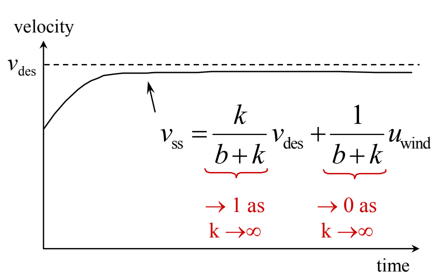
Example #2: Cruise Control



$$m\dot{v} = -bv + u_{\text{engine}} + u_{\text{hill}}$$
$$u_{\text{engine}} = k(v_{\text{des}} - v)$$



$$v_{ss} = \underbrace{\frac{k}{b+k}}_{\to 1 \text{ as } k \to \infty} v_{\text{des}} + \underbrace{\frac{1}{b+k}}_{\to 0 \text{ as } k \to \infty} u_{\text{wind}}$$

Stability/performance
- Steady state velocity approaches desired velocity as k → ∞
- Smooth response; no overshoot or oscillations

Disturbance rejection
- Effect of disturbances (hills) approaches zero as k → ∞

Robustness
- None of these results depend on the specific values of *b, m,* or *k* for *k* sufficiently large

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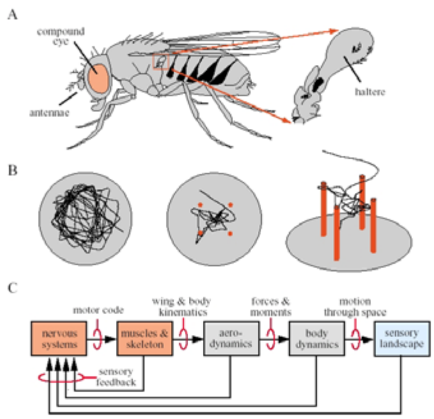
Example #3: Insect Flight (Dickinson)



Flight behavior in Drosophila

(a) Cartoon of the adult fruit fly showing the three major sensor strictures used in flight: eyes, antennae, and halteres (detect angular rotations)

(b) Example flight trajectories over a 1 meter circular arena, with and without internal targets.

(c) Schematic control model of the flight system

[from CDS Panel Report, Sec 3.4. Figure courtesy of Michael Dickinson, Caltech.]

Sensing:	antennae, eye, halters
Actuation:	wings, legs, body
Computation:	brain, nervous system
Effect:	robust flight in a variety of environments; fault tolerance

More information:

- M. D. Dickinson, Solving the mystery of insect flight, *Scientific American,* June 2001
- CDS 101 seminar : Friday, 11 Oct 02

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Example #4: Congestion Control and the Internet (Low)

Transmission control protocol (TCP)

- Source: send packet to destination
- Source: resend packet if no ACK rec'd
- Destination: ACK received packets
- Destination: reassemble packets
- Source: Adjust rate based on loss rate

Internet router operation:

- Receive packet from input link; place at end of queue, if not full
- Transmit packet from head of queue to next router on path
- Update route table based on link status

Sensing:	data, ACK packets
Actuation:	transmit rate, router paths
Computation:	src, dst, router processors
Effect:	high speed data transmission, tolerant of link failures

More information:

- www.howstuffworks.com
- CDS 101 seminar: Friday, 25 Oct 02

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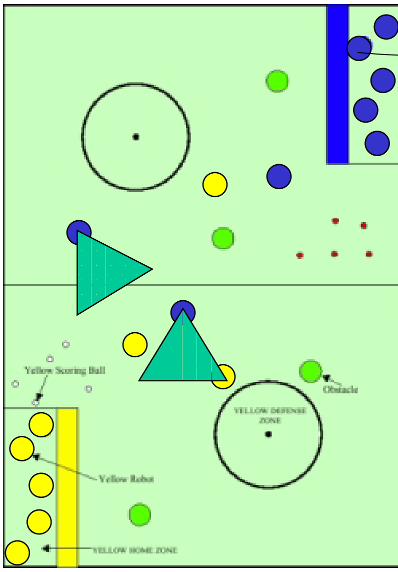
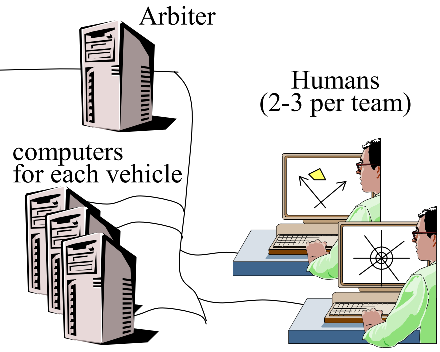
Example #5’: RoboFlag





Robot version of “Capture the Flag”

- Teams try to capture flag of opposing team without getting tagged
- Mixed initiative system: two humans controlling up to 6-10 robots
- SURF project (Cornell + Caltech)

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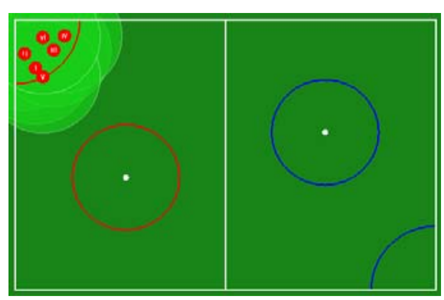
Hayes et al
ACC 2003 (p)

RoboFlag Demonstration



Caltech Multi-Vehicle Wireless Testbed

- Distributed computation on vehicles + command and control console
- Point to point networking (bluetooth) + local area networking (802.11)
- Overhead vision system provides global position data (LPS)

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CDS 101/110/111 Course Sequence

CDS 101 – Introduction to the *principles* and *tools* of control and feedback
- Summarize key concepts, with examples of fundamental principles at work
- Introduce MATLAB-based tools for modeling, simulation, and analysis

CDS 110a – Basic understanding of key concepts in control
- Detailed description of frequency domain and state space control concepts
- Provide enough knowledge to work with control engineers in a team setting

Fall

CDS 110b – Detailed design tools for control systems
- Estimation and robust control tools for systematic *synthesis* of control laws
- Control techniques for nonlinear systems

Winter

CDS 111 – Implementation of control systems for engineering applications
- Laboratory based *implementation* of computer control on flight vehicle

Spring

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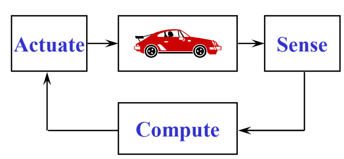
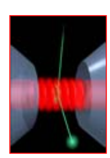
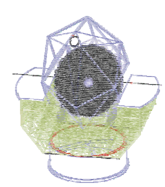
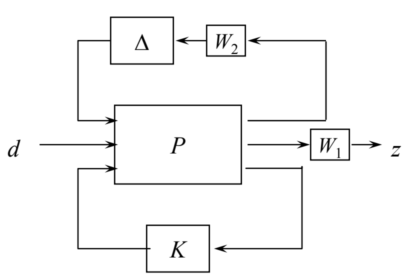
Summary: Uncertainty Management

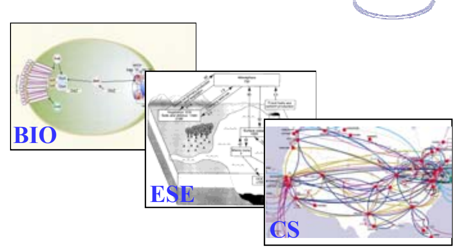
Feedback as a tool for uncertainty management
- Allows specification of performance in presence of noise, parametric error, unmodeled dynamics
- Sophisticated tools available for both analysis and synthesis
- Wide variety of applications











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