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CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 101/110
Homework Set #8

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All students should complete the following problems:

1. For the control systems below, design a PID control law that stabilizes the sytem. You may you any method (loop shaping, Ziegler-Nichols, sisotool, etc). For the closed loop system, determine that steady state error, the maximum frequency for which the closed loop system can track with less than 5% error, and the approximate bandwidth of the system.

   (a) Disk drive read head positioning system:

   $$P(s) = \frac{1}{s^3 + 10s^2 + 3s + 10}$$

   (b) Second order system:

   $$P(s) = \frac{100}{(100s+1)(s+1)}$$

2. In this problem we will design a PID compensator for the pitch axis of the Caltech ducted fan. Use the following transfer function to represent the vehicle dynamics:

   $$P(s) = \frac{r}{Js^2 + bs + mgl} \qquad \begin{array}{lll} g = 9.8 \text{ m/sec}^2 & m = 1.5 \text{ kg} & b = 0.05 \text{ kg/sec} \\ l = 0.05 \text{ m} & J = 0.0475 \text{ kg m}^2 & r = 0.25 \text{ m} \end{array}$$

   (a) Design a PID compensator that stabilizes the system. You can use any method that you choose. Plot the pole zero diagram, frequency response, and step response for the *closed loop* system.

   (b) Plot the root locus plot for the system. Mark the open and closed loop pole locations corresponding to the PID compensator at the default gain (from part (a)).

   (c) Use the root locus plot to choose a new gain such that the dominant poles have a settling time of half of the value of the settling time for the original compensator designed in part (a). Show the location of the the poles with the new gains on your root locus plot.

Only CDS 110a/ChE 105 students need to complete the following additional problems. These problems are from recent CDS and ME candidacy exams and can be done without using MATLAB (although its OK to use MATLAB if you like).

3. Consider the motion control problem for a motor with moment of inertia $J = 0.02$ kg m^2 and damping constant $D = 0.1$ kg m^2/sec. The transfer function from the motor torque to position is

   $$P(s) = \frac{1}{Js^2 + Ds}.$$

   (a) Draw a block diagram for the control system.

   (b) Plot the Bode plot for the plant transfer function.

   (c) Design a control law that gives zero steady state error and less than 5% tracking error from 0 to 0.5 Hz.

   (d) Plot or sketch the root locus diagram for your controller as a function of the loop gain.

   (e) Plot or sketch the Nyquist plot for your controller.

4. Consider a unity feedback system with a plant given by

$$P(s) = \frac{s+c}{(s+a)(s+b)} \qquad a > 0, \quad b, c < 0.$$

(a) Is it possible to stabilize the system with a real-valued, proportional gain?

(b) Design a stabilizing compensator for the system with $c < b < 0$. You do not need to compute an exact answer in terms of a and b, but you should describe what computations would need to be performed if given specific values of a and b.

(c) Assume that $a = -b$ and use a Nyquist plot to verify that your compensator stabilizes the system. (A qualitatively correct sketch is fine.)

(d) Suppose sensor noise of the form $n = A sin \omega t$ enters your system at the output of the plant. Describe the disturbance rejection properties of your controller as a function of A and ω. What is the worst case disturbance frequency from the point of view of attenuation of the disturbance.