CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 101/110
Homework Set #7

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All students should complete the following problems:

1. For the control systems below, determine that steady state error, the maximum frequency for which the closed loop system can track with less than 5% error, and the approximate bandwidth of the system.

   (a) Disk drive read head positioning system, using lead compensator:

   $$P(s) = \frac{1}{s^3 + 10s^2 + 3s + 10} \qquad C(s) = 1000\frac{s+1}{s+10}$$

   (b) Second order system with PD compensator:

   $$P(s) = \frac{100}{(100s+1)(s+1)} \qquad C(s) = s + 10$$

2. In this problem we will design a lead compensator for the pitch axis of the Caltech ducted fan. Use the following transfer function to represent the vehicle dynamics:

   $$P(s) = \frac{r}{Js^2 + bs + mgl} \qquad \begin{array}{lll} g = 9.8 \text{ m/sec}^2 & m = 1.5 \text{ kg} & b = 0.05 \text{ kg/sec} \\ l = 0.05 \text{ m} & J = 0.0475 \text{ kg m}^2 & r = 0.25 \text{ m} \end{array}$$

   Design a unity feedback controller with the following specifications

   - Steady state error of less than 1%
   - Tracking error of less than 5% from 0 to 1 Hz (remember to convert this to rad/sec).
   - Closed loop step response with maximum overshoot of 20%.
   - Closed loop frequency response with no more than 3 dB gain at all frequencies.
   - High frequency disturbance rejection from reference to output (H_{yr}) of at least 10X above 100 Hz.

   If you cannot meet all of the specifications, you should prioritize them in the order listed.

   (a) Plot the open loop Bode plot for the system and mark on the plot the various frequency domain constraints in the above specification.

   (b) Design a compensator for the system that satisfies the specification. You should include plots to show that all specifications are met.

   (c) *Optional*: Write down the state space representation of your controller. (I.e., write down a state space system whose transfer function is the control law you designed.)

   Hint: start with the controller given in class and see if you can modify it to satisfy the specification.

Only CDS 110a/ChE 105 students need to complete the following additional problems:

3. Consider the cart pendulum system with the pendulum hanging *down* (you can think of this as the problem of moving the cart without exciting the pendulum too much; similar to walking without

sloshing your coffee) The dynamics describing how the position of the cart depends on the applied force is given by the transfer function

$$P(s) = \frac{ls^2 + g}{Mls^4 + +bls^3 + (M+m)gs^2 + bgs}$$

$M = 0.5$ kg $m = 0.2$ kg
$l = 0.3$ m $b = 0.1$ N/m/sec
$g = 9.8$ kg m/sec^2

(you can verify this from the equations given on a previous homework set). In this problem you will design a control law that satisfies the following specifications:

- 0.1% steady state error
- Position (x) tracking within 10% up to 0.05 Hz
- Overshoot of less than 5% to step changes in x position
- Disturbance rejection of 10X for all disturbances above 10 Hz

If you cannot meet all of the specifications, you should prioritize them in the order listed (first specification has highest priority).

(a) Write the frequency domain portions of the specification as constraints on the loop transfer function in the appropriate frequeny ranges. Estimate the phase margin requirement imposed by the step response using a second order approximation. Show your results by sketching them on a Bode plot.

(b) Design a control law that satisfies the specification. Make sure to discuss how you determined the form of the control law and demonstrate that your control law satisfies all specifications. In addition to the standard Bode plots, you should plot the weighted sensitivity functions that demonstrate that you meet the specification given in (a).

4. Continuing the previous problem, we will now investigate the stability of your control law under perturbations.

(a) Determine whether your control law is robustly stable with respect to added sensor dynamics

$$G(s) = \frac{1}{s+1}.$$

These dynamics should be inserted in the feedback loop, as we did in Lecture 7.1. If your controller is not stable with for the perturbed system, redesign your control law to provide as much performance as you can for the nominal plant and still maintain stability for the perturbed plant.

(b) For the control law that provided robust stability in (a), check to see if the performance specifications are satisfied for the perturbed plant. If your controller is not robust, redesign your control law to provide robust stability and robust performance to the specified sensor dynamics. If necessary, relax the specifications according to the priorities.

(c) Consider now the case where we have a time delay of 1 second instead of $G(s)$. Determine whether your control law from part (b) provides robust stability and/or robust performance in the presence of the specified time delay. (You just need to check to see if it works; you don't need to redesign it.)