CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 101/110
Problem Set #2

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Fall 2002

Issued: 7 Oct 02
Due: 14 Oct 02

Note: In the upper left hand corner of the first page of your homework set, please put the class you are taking (CDS 101, CDS 110) and the number of hours that you spent on this homework set (including reading).

All students should complete the following problems:

1. For each of the following systems or subsystems, describe the "state", the "inputs" and "output", and the "dynamics". Your may give your answers in words, but please be as precise as possible.

   (a) Inverted pendulum

   (b) Lateral motion of an automobile

   (c) Microsoft Word

   (d) Flying insect

   (e) New York Stock Exchange

2. (MATLAB/SIMULINK) In this problem you will build a model of a vehicle in SIMULINK and control the vehicle using feedback control. The vehicle will consist of a body (chassis + wheels) and a drive train (engine + transmission). Assume that the vehicle dynamics are of the form

$$m\dot{v} = -bv + F_{\text{engine}} + F_{\text{hill}}$$

   where $m = 1000$ kg is the mass of the vehicle, $b = 50$ N sec/m is the viscous damping coefficient, and F_{engine} and F_{hill} represent the forces on the vehicle due to the engine and the terrain, respectively. We can implement this in SIMULINK as a two input, one output system, written in state space form as

$$\begin{aligned} \dot{x}_v &= \begin{bmatrix} -b/m \end{bmatrix} x_v + \begin{bmatrix} 1/m & 1/m \end{bmatrix} u_v \\ y_v &= x_v \end{aligned} \tag{1}$$

   where $x_v = v$ is the vehicle state, $u_v = \begin{bmatrix} F_{\text{engine}} & F_{\text{hill}} \end{bmatrix}^T$ is the vehicle input (two dimensional), and $y_v = v$ is the vehicle output (velocity). You should make this into a single SIMULINK block using the "State Space" block (under `Simulink`→`Continuous`→`State Space` in the Simulink Library Browser). You may also want to use the `Mux` block (under `Signals & Systems`).

   We will model the engine dynamics as a "first order lag". Let τ represent the engine torque and assume the engine has the following dynamics:

$$\begin{aligned} \dot{\tau} &= -a\tau + u_e \\ y_e &= K\tau \end{aligned} \tag{2}$$

   where $a = 0.2$ is the lag coefficient, $K = 5$ is the conversion factor between engine torque and force applied to the vehicle (representing the transmission) and u_e is the accelerator input (which we will assume has the proper units). You should also create a SIMULINK block for this subsystem.

   Finally, we include the effects of a hill. The hill simply exerts a force on the car that is based on the angle of the hill:

$$F_{\text{hill}} = -mg\sin(\theta) \tag{3}$$

   where $g = 9.8$ kg m/sec^2 and $\theta = \pi/18$ is the angle of the hill (10 degrees).

   (a) Plot the output of the vehicle model (1) for a step input of $F_{\text{engine}} = 500$ Newtons (assume $F_{\text{hill}} = 0$). What is the rise time (0 to 95% of the final value)?

(b) Plot the output of the engine model (2) for a step input of $u_e = 100$ Nm. What is the rise time?

(c) In the homework from last week, you built a simple cruise controller. Replace the vehicle/engine model in that system with your vehicle and engine models and plot the response for the default gains ($K_i = 50$, $K_p = 1000$). Make sure to set your simulation time to be sufficiently long.

(d) Now include the effect of a hill on your system. You should model the system so that the car is initially on a flat surface and then encounters the hill at $T = 100$ seconds. Plot the response of the system and compute the rise time.

Note: if you are having trouble figuring out how to create these blocks in SIMULINK, take a look at "hw1cruise.mdl" from last week's homework and see if you can modify it appropriately. It has all of the subsystems you will need (except for the `Mux` block). You may also find the following web-based tutorial helpful:

`http://www.engin.umich.edu/group/ctm/examples/cruise/cc.html`

(ignore the sections on transfer functions; we will get to these later in the class).

Only CDS 110a/ChE 105 students need to complete the following additional problems:

3. (MATLAB) Build a finite state controller for a traffic light system at an intersection. Your controller should take as inputs the sensor signals on the road that detect whether a car is present and should have as outputs the colors of the various signals. For simplicity, assume that all traffic is two-way (i.e. don't worry about left turn lanes, etc.). Implement your solution as a MATLAB function, which you can assume is called every 1 second of intersection operation. See the accompanying documentation on the course web page for more details:

   `http://www.cds.caltech.edu/~murray/cds101/L2.1_modeling.shtml#Homework`

4. Consider the following discrete time system

$$z[k+1] = Az[k] + Bu[k]$$
$$y[k+1] = Cz[k+1]$$

where

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \qquad A = \begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

In this problem, we will explore some of the properties of this discrete time system as a function of the parameters, the initial conditions, and the inputs.

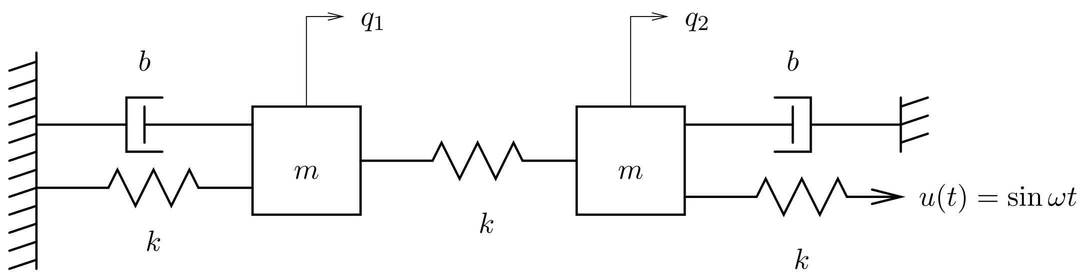
(a) Assume that the off diagonal element $a_{12} = 0$ and that there is no input, $u = 0$. Write a closed form expression for the output of the system from a nonzero initial condition $z[0] = (z_1[0], z_2[0])$ and give conditions on a_{11} and a_{22} under which the output gets smaller as k gets larger.

(b) Now assume that $a_{12} \neq 0$ and write a closed form expression for the response of the system from a nonzero initial conditions. Given a condition on the elements of A under which the output gets smaller as k gets larger.

(c) Write a MATLAB program to plot the output of the system in response to a unit step input, $u[k] = 1$, $k \geq 0$. Plot the response of your system with $z[0] = 0$ and A given by

$$A = \begin{bmatrix} 0.5 & 1 \\ 0 & 0.25 \end{bmatrix}$$

5. Consider the coupled mass spring system show in the figure below (the same one considered in class on Monday):



The input to this system is the sinusoidal motion of the end of rightmost spring and the output is the position of each mass, q_1 and q_2.

(a) Write the equations of motion for the system, using the positions and velocities of each mass as states.

(b) Rewrite the dynamics in terms of $z_1 = \frac{1}{2}(q_1 + q_2)$ and $z_2 = \frac{1}{2}(q_1 - q_2)$.

(c) Note that the resulting equations are diagonal. Solve these linear ODEs for $z_1(t)$ and $z_2(t)$ given initial conditions and input.

(d) Plot the amplitude and relative phase of the motion of the first mass as a function of the frequency of the sinusoidal input (you may assume a unit magnitude input). You can either solve this problem numerically (by building a simulation and measuring the results) or analytically (using the solution from part 5c and computing the motion of the first mass). If you use a numerical solution, you can use $m = 250$, $k = 50$, $b = 10$.