CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 101/110a
Final Exam

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4 December 2002

The exam consists of four questions, worth a total of 60 points. The point values for each section are shown on the right. The time limit is 3 hours, in one sitting. Budget your time to complete as much of the exam as possible. If time does not permit a complete answer, indicate how you would proceed as explicitly as possible.

The exam is open book. You may use any of the optional texts (Friedland, Franklin-Powell and Emami-Naeni, Leonard and Levin, or Kuo), course handouts, lecture and class notes, course problem sets and solutions, and *your own* handwritten notes. No other books are allowed. You may use a computer or calculator for carrying out numerical computations. MATLAB may be used but is not required. All of your answers must be hand written or hand drawn (do not turn in code or computer plots). If you use MATLAB, please make sure to provide descriptions of how you achieved your solution; do not just include the final answer.

You are not allowed to use the Internet during the exam (except for accessing local computing resources, such as MATLAB/SIMULINK), but you may download or print out copies of presentations, notes, FAQs, or other material posted on the course web site (CDS 101 or 110). You are not allowed to print out contents of other sites for use while taking the exam (although you can take handwritten notes on the sites and use your own notes in the exam).

The exam is due by 5 p.m. Friday, December 13, in the box outside 102 Steele.

Please write your solutions in a fresh exam book (blue book). We have to grade a large collections of exams in a short time and it makes things much simpler to manage if everyone uses a bluebook.

Please note that students in CDS 101 and CDS 110a/ChE 105 are required to answer different sets of questions (or portions of questions), depending on which course you are taking. Please be careful to answer only the questions for the course you are taking. No additional credit will be given for solutions provided to questions that are not for the course you are taking. Questions with no indication of the course should be answered by all students.

Problem 1 (CDS 101 only)

Choose any five of the feedback systems listed below. For each system you choose, answer the following questions:

(a) (CDS 101) Draw a block diagram for the system, with the plant and controller separately identified (you can use words to describe the contents of each block). For the *plant*, describe a plausible model of the system and give the "state", "inputs" and "outputs", and the "dynamics" of the model. You may give your answer in words, but please be as precise as possible. [10]

(b) (CDS 101) Provide a performance specification for the system, using the signals from your block diagram in part (a). Your performance specification should make sense for the problem, but should be in terms of features of the step response and or frequency response of the system. [5]

(c) (CDS 101) Describe the main sources of uncertainty for the system and indicate whether or not the feedback system compensates for these uncertainties. You should list at least two sources of uncertainty for each system. Be as explicit as possible. [5]

Systems to choose from:

1.1 Thermostat controlled heating of a room.

1.2 Human being balancing on a tightrope.

1.3 Autopilot control of heading and speed in an airplane.

1.4 Lateral (x) control of the Caltech ducted fan

1.5 Congestion control of the Internet using TCP Reno.

1.6 Insect flight during periods of straight motion.

1.7 Segway human transportation system.

1.8 Pupil control system for tracking a moving object with your eyes.

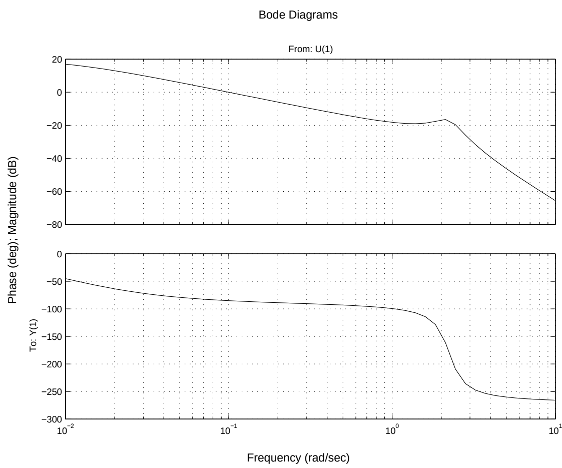
1.9 Population control for a rabbit/fox ecosystem.

1.10 Primary mirror control system for the Keck telescope.

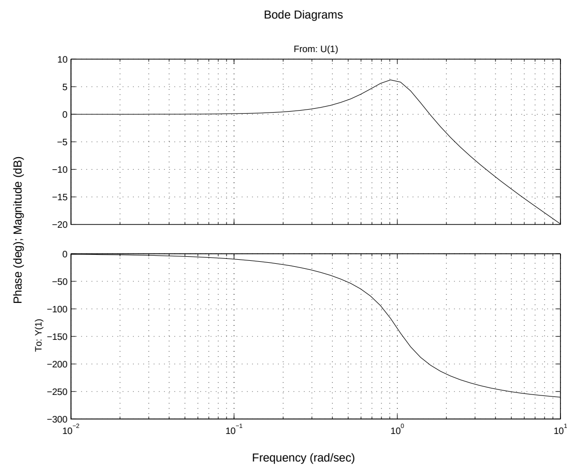
Please make sure to identify the number of the system when responding to the three questions above. Your answers will be graded based on your grasp of control concepts and not how well you know the details of the particular system (so it is OK if you make up some plausible description of the system).

Problem 2 (CDS 101 and CDS 110)

In this problem, you will answer a series questions about each of the systems illustrated in the Bode plots below. You should *summarize* your answer to this question by creating a matrix with the rows corresponding to each system and the columns corresponding the the answers for questions a–d. You should also include the analysis you used to obtain your answers (separate from the summary chart). If you use MATLAB, you should still give a description of the key features that justify your answers.



2.1



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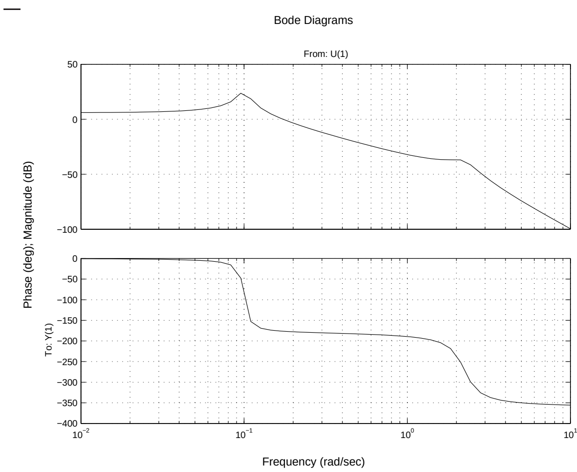

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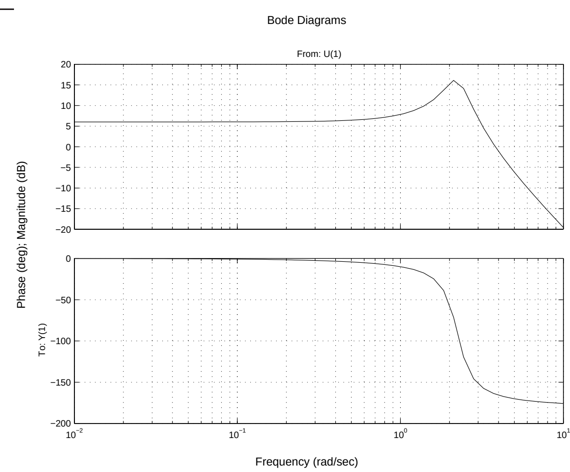
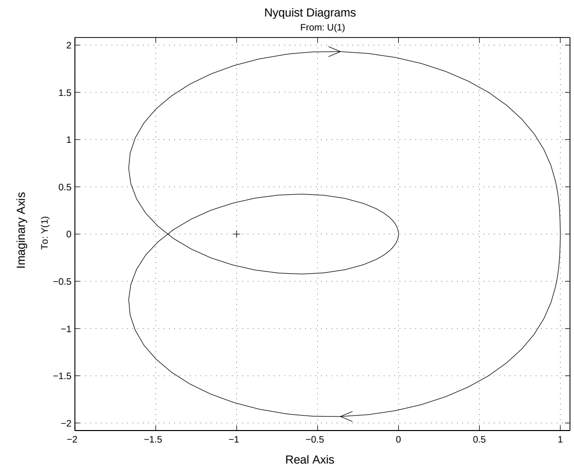


2.4

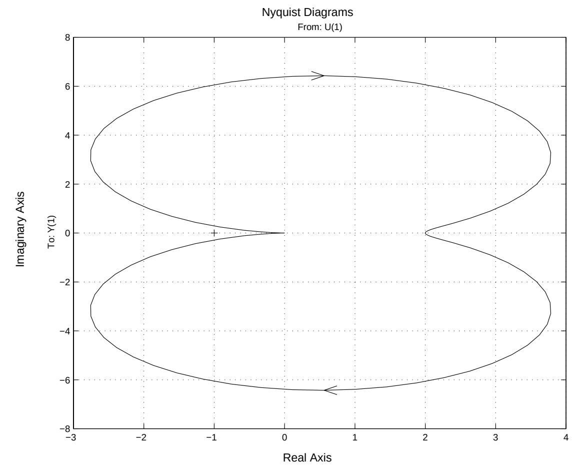
For each of the Bode plots above, answer the following:

(a) Assuming that the open loop systems have no right half plane poles, is the closed loop system (assuming unity feedback) asymptotically stable or unstable? [5]

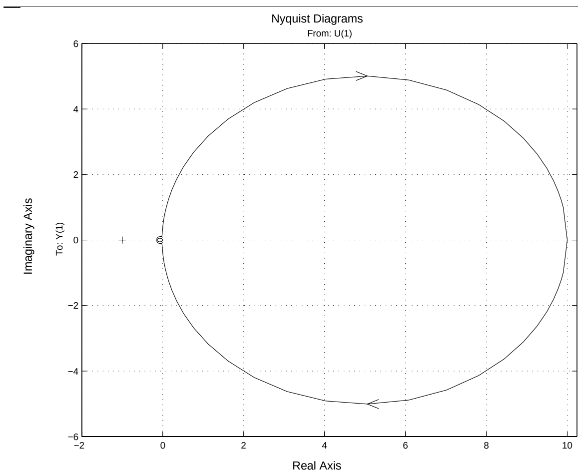
(b) Which of the following Nyquist plots corresponds to the Bode plots? Assume that all open loop poles have non-positive real part (they might have zero real part). [5]



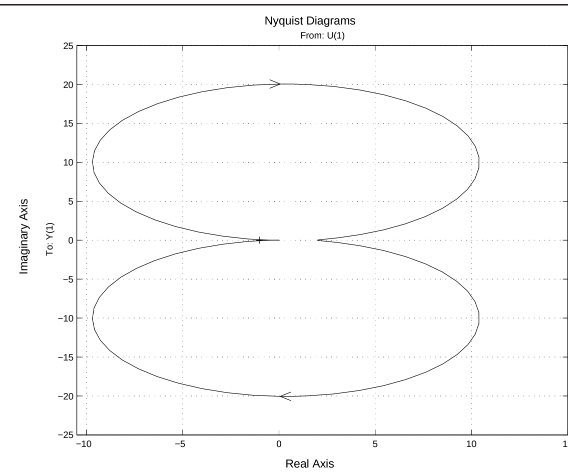
(N1)



(N2)



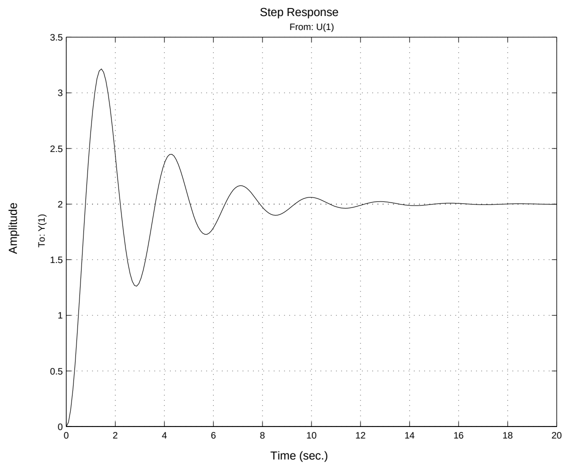
(N3)



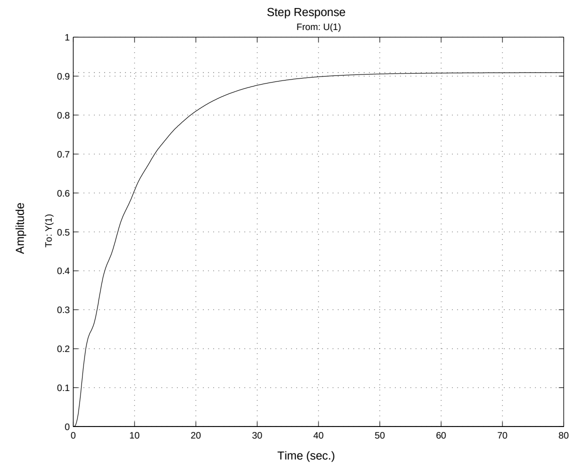
(N4)

Please note that not all features of the plots above are completely discernible (a limitation of Nyquist plots), so you may need to use multiple features to sort things out. Don't forget to describe how you obtained your answer.

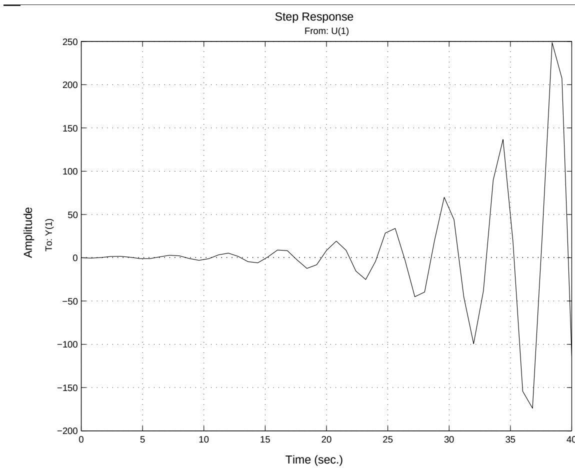
(c) (CDS 110 only) Which of the following sets of unit step responses are possible for the given frequency response? Note that the scales are different on each plot. [5]



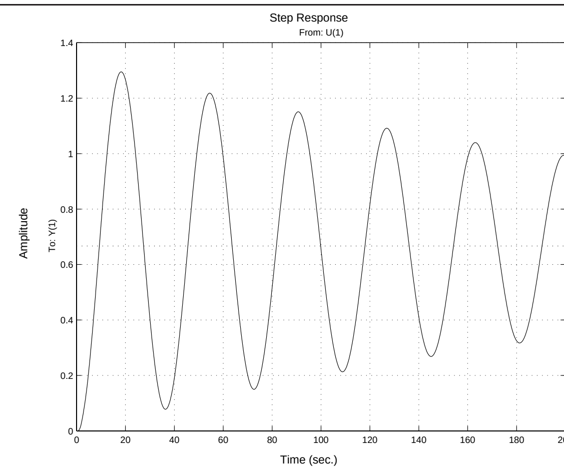
(S1)



(S2)



(S3)

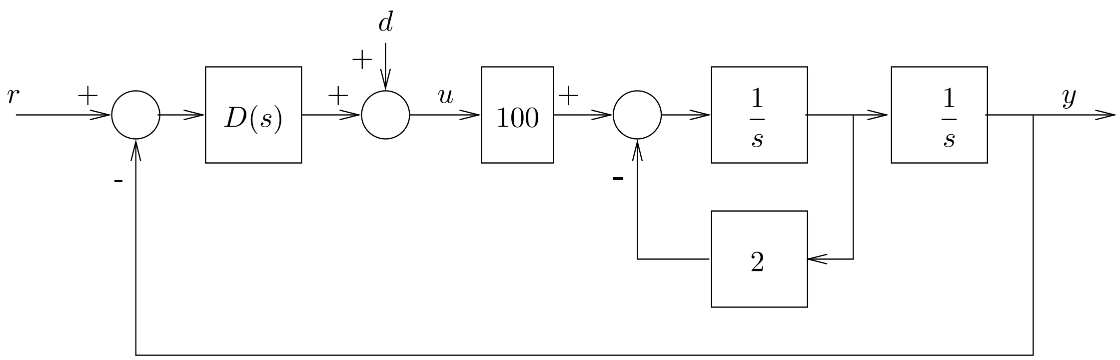


(S4)

(d) (CDS 110 only) What is the maximum and minimum amount of additional positive loop gain for which each system is stable? [5]

Problem 3 (CDS 101 and CDS 110)

Consider the block diagram given below:



(a) Compute the transfer function $G(s)$ that takes d to y in the diagram above. [5]

(b) Sketch the frequency response of the open loop system with $D(s) = 1$ [5]

(c) Design a compensator that satisfies the following specifications: [10]

- Steady-state error to a unit step input is less than 1/150.
- The unit step response has an overshoot of less than 25%.
- The bandwidth for the compensated system is no less than that of the uncompensated system.

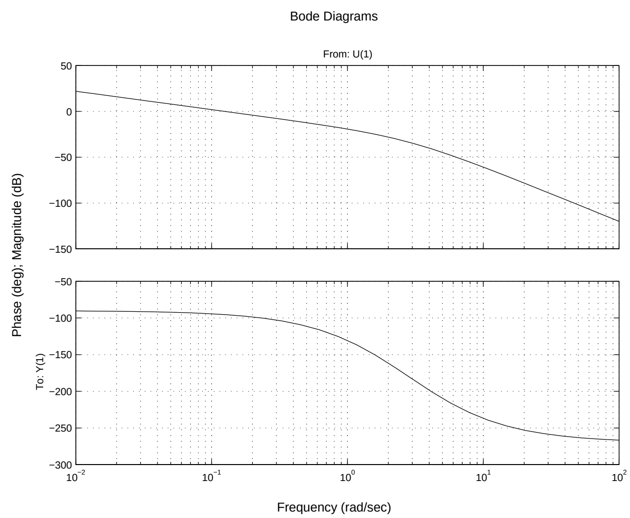
Your computations do not have to be exact, but you should include enough detail to indicate how and why the specifications are satisfied.

Problem 4 (CDS 101 and CDS 110)

Consider a unity feedback control system with plant and controller dynamics given by

$$P(s) = \frac{1}{(s+2)(s+4)} \qquad C(s) = \frac{K}{s}.$$

A Bode plot for the open loop transfer function with $K = 1$ is given below:

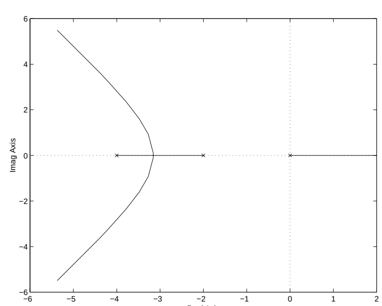


(a) Compute the closed loop transfer function between the reference input, $r(t)$, and the plant output $y(t)$. [5]

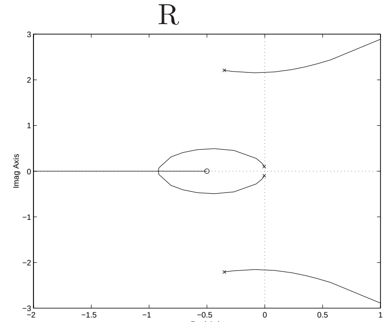
(b) Using either the Nyquist criterion, direct computation, or a root locus plot, determine the largest gain, $K_{\max}$, for which the closed loop system is stable. [5]

(c) (CDS 110 only) One of the three root locus plots below corresponds to the system pictured above. Figure out which one and describe why. [5]

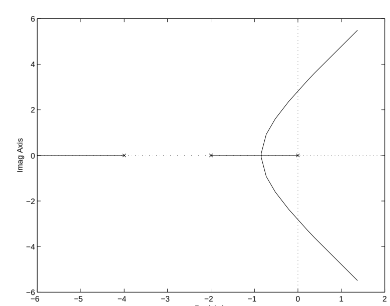
R



(R1)



(R2)



(R3)

(d) (CDS 110 only) Sketch the response of the system to a unit step input for some $0 < K < K_{\max}$. [5]