AEROSPACE APPLICATIONS OF CONTROL

CDS 101 Seminar
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United Technologies
• $26.6 billion (2000)
• 153,800 employees
• 3 business groups...

- Aerospace
- Building Systems
- Power Solutions
Seminar Objectives

Emphasize the importance of modeling, feedback, uncertainty

- Provide three examples of modeling
- Illustrate the relationship among modeling, uncertainty and feedback
- Provide an example of dynamic analysis.

1. Jet Engine Control
2. Electric Generator Transmission Control
3. Combustion analysis & control
Jet Engine Control Objectives

Track commanded thrust while maintaining constraints

Thermal efficiency increases with burner temperature
- nominal temperatures near melting point of parts
- temperature overshoots rapidly degrade turbine life

Pressure and flow constraints on compressor stages, to avoid stall, surge, flutter.

Speed constraints on fan & spool (to avoid structural failure).
Structural constraints on fan & spool speeds

But...

- Fan and compressor efficiency best near stall, surge, and flutter boundaries.
- Thermal efficiencies highest with increased burner temperatures.
- Structure designed to minimize weight.
F-135 Engine Control

VTOL requirement means demanding control problem

JSF capable of vertical T/L
- 3DOF nozzle
- Roll posts
- Lift fan

Challenges:
- Split Thrust has 10x bandwidth
- Precise modeling requirements
- Multivariable, nonlinear
Block Diagram – Jet Engine Control

Input & Output Signals

- Temperatures (T3, T6…)
- Pressures (P23, …)
- Spool speeds (N1, N2)

• Fuel flow (wf)
• Stator Vane Angle (SVV),
• Nozzle area (A8)

- Airspeed,
- Altitude,
- Ambient temperature

Track commanded thrust while respecting constraints
Jet Engine Dynamic Modeling

Nonlinear modeling

Nonlinear Plant Model

- Component Maps (fans, compressors)
- Dynamic elements (control volumes, inertias)
- Thermal Capacitances,

\[ w \]

- Thrust
- Metal temperatures
- Flows

\[ u \]

- Temperatures (T3, T6…)
- Pressures (P23, …)
- Spool speeds (N1, N2)

\[ v \]

- Airspeed,
- Altitude,
- Ambient Temperature

- Fuel flow (wf)
- Stator Vane Angle (SVV),
- Nozzle area (A8)

K

Steady state

Constant N1

Pressure Rise

Corrected Airflow

Nonlinear models embodied in code, validated in engine or component tests.
Jet Engine Control-oriented model

F100 example

- Main burner fuel flow (wf)
- Nozzle Jet area (A8)
- Inlet guide vane angle (CVV)

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= C_1x + D_1u \\
z &= C_2x + D_2u
\end{align*}
\]

- Fan speed (N1)
- Compressor speed (N2)
- Compressor discharge pressure
- Interturbine volume pressure
- Augmentor pressure
- Fan discharge temperature
- Duct temperature
- Compressor discharge temperature
- Burner exit temperature 1 (fast)
- Burner exit temperature 2 (slow)
- Burner exit total temperature
- Fan inlet temperature 1 (fast)
- Fan inlet temperature 2 (slow)
- Fan turbine exit temperature
- Duct exit temperature 1
- Duct exit temperature 2

- Engine thrust
- Surge Margin
- Spool speeds

- N1 (low compressor speed)
- N2 (high compressor speed)
- P16 (high compressor outlet pressure)
- P18 (engine pressure ratio)
- P23 (low compressor pressure ratio)
- LPEMN (low compressor mach)
Jet Engine Control Design

*Model reduction, multivariable analysis, design, constraints*

- **Steps...**
  - Model reduction
  - Multivariable Analysis
    - Input / output pairing
  - Design & Analysis
    - Uncertainty
    - Customer specifications
  - Linear Simulation
  - Constraint strategy (min/max)
  - Nonlinear Simulation
  - Engine test

Control design concurrent to engine design
Jet Engine Control Design

**Constraint Strategy**

- **Inputs paired with outputs.**
- **Several controllers run in parallel.**
- **Smallest value wins (hybrid system).**
Can we close the loop on thrust?

Simplified Robustness Analysis

\[
\frac{E(j\omega)}{R_{N1}(j\omega)} = \frac{1}{1 + CP}
\]

\[
\frac{E}{R} = \frac{1}{1 + CP}
\]

\[
\frac{E}{R} = \frac{PC(M - \hat{M})}{1 + PC\hat{M}}
\]

\[
\frac{E(j\omega)}{R_{N1}(j\omega)} = \frac{PC(N - \hat{N})}{1 + PC\hat{N}}
\]
Example 2: Continuously Variable Transmissions

Introduction

CVTs Used to couple variable speed spool to constant frequency generator.

Current designs use planetary gear with hydraulic actuation to “add” or “subtract” speed.

HS owns 92% of the world’s market in airborne electrical generation.
Belt – Type CVT

Controls objectives

Used to couple variable speed spool to constant frequency generator

Control problem:
• Regulate generator speed
• minimize slip, maximize belt life
• Define control structure
  • Sensors,
  • actuators,
  • input/output variables
• Dynamic analysis of mechanism
Belt Type CVT Modeling

Non-holonomic rotational, nonlinear slip…

\[ r_1 \dot{q}_1 = r_2 \dot{q}_2 \quad \text{Non-holonomic constraint} \quad \gamma(r_1) = \frac{\dot{q}_2}{\dot{q}_1} = \frac{\omega_2}{\omega_1} \]

\[ m_1(r_1) \ddot{r}_1 - \frac{1}{2} m'(r_1) \dot{r}_1^2 + b_5 \ddot{r}_1 = c_1(F_1 - \frac{\theta_1}{\theta_2} F_2) \]

\[ (J_1 + \gamma(r_1)^2 J_2) \ddot{\omega}_1 + J_2 \gamma'(r_1) \gamma(r_1) \omega_1 + b_3 \omega_1 + b_4 = \tau_1 - \frac{g_3 \gamma(r_1)}{g_4} \tau_2 \]

Radial dynamics

Rotational dynamics

No-slip conditions

\[ \mu F_1 \geq \frac{\tau_1}{\tau_2} \cos \nu \]

\[ \mu F_2 \geq \frac{\tau_2}{\tau_2} \cos \nu \]
CVT Anti-Slip Control

Models enable dynamic analysis, trade studies, parameter selection

\[ \mu F_1 \geq \frac{\tau_1 \cos \nu}{\tau_2} \]
\[ \mu F_2 \geq \frac{\tau_2 \cos \nu}{\tau_2} \]

\[ \frac{\tau_2}{\tau_L} = \frac{1}{1 + \frac{b_2}{k_2} + s^2 \frac{J}{k_2}} \]

\[ \frac{d_2}{\tau_L} = \frac{1}{1 + \frac{b_2}{k_2} + s^2 \frac{J}{k_2}} \]

Main result: Bandwidth of hydraulic actuation system \((F_1, F_2)\) must exceed bandwidth of \(\tau_2 / \tau_L\) to prevent slip.

Zero means overshoot
CVT Speed Control

Models enable dynamic analysis, trade studies, parameter selection

- Generator speed can be regulated using engine spool, generator speeds as measurements.
- Model uncertainty in spring / damper limits performance.
- Higher performance achievable if \( \omega_2 \) measured (robustness to uncertainty).
Example 3: Combustion dynamics & control

Performance limitations in aero engines

- Inlet separation
  - Separation of flow from surface
  - Possible use of flow control to modify

- Distortion
  - Major cause of compressor disturbances

- Rotating stall and surge
  - Control using BV, AI, IGVs demonstrated
  - Increase pressure ratio $\Rightarrow$ reduce stages

- Flutter and high cycle fatigue
  - Aeromechanical instability
  - Active Control a possibility

- Combustion instabilities
  - Large oscillations cannot be tolerated
  - Active control demonstrated

- Jet noise and shear layer instabilities
  - Government regulations driving new ideas
Combustion Instabilities Will Occur

**Combustion Instabilities Limit Minimum Achievable NOx Emissions**

- **Goals:**
  - NOx/CO limits
  - RMS pressure limits
- **Wide range of operating conditions**
  - 50 - 100% power
  - -40 to 120 F ambient temp.
- **Instabilities inevitable**
  - combustion delay
  - convective delay
- **Passive design solution may be possible**
- **AIC can enable product**

**Product Need**

“Stability boundary” defined as maximum allowable pressure fluctuation level
Combustors Experience Instabilities

Data obtained in single nozzle rig environment showing abrupt growth of oscillations as equivalence ratio is leaned out to obtain emissions benefit
Feedback Perspective on System Dynamics

• *What is feedback?* System coupling of a special type where inputs and outputs are dependent.

• *Where does it occur?* Most physically oscillatory (resonant) systems contain feedback - combustion dynamics is a prime example as shown. All control systems (sensor-actuator-controller or passive realization) contain explicit feedback loops.

• *How is it used?* To change dynamics and to cope with uncertainty.

• *Why use it and when should it be applied?* When dynamics are not favorable.
Stability analysis of combustion models using Nyquist criterion

\[ \frac{d^2 \eta}{dt^2} + \alpha \frac{d \eta}{dt} + \sigma^2 \eta = F(t) \]  

Acoustic mode driven by heat release

\[ F(t) = K \frac{d}{dt} \eta(t - \tau) \]  

Pressure-sensitive heat release

Bode plots for \( \tau = 0, 4\text{ms}, 5\text{ms} \)

-1 inside \( \Rightarrow \) unstable
Example: Combustion Dynamics - Controlled and Uncontrolled

experiment in UTRC 4MW Single Nozzle Rig demonstrates 6X reduction in pressure amplitude, decrease in NO\textsubscript{X} & CO emissions

Control: a fraction of main fuel modulated by a valve driven by phase-shifted signal from a pressure transducer

Control theory provides methods for enforcing desirable behavior
**Combustion Dynamics & Control: Model Calibration and Use in Evaluation of System Modifications**

**Data Analysis**

Key parameters extracted from experiment (forced response tests) - trend in equivalence ratio (time delay) drives dynamical behavior.

**Analysis allows calibration of model from data to enable quantitative studies**

**Calibration**

- System level model captures experimental data quantitatively.

**Evaluation of Mitigation Strategies**

- Evaluate passive design changes (resonators) for size, placement, prediction of performance.
- Evaluate active control for actuation requirements (bandwidth) and prediction of performance.
Case study: sector combustor controlled with on/off valves

Experimental setup

UTRC 8 MW three-nozzle sector rig controlled with on/off valves

Results of closed-loop experiments

Can harmonic balance explain the observed behavior?
Model-based analysis explains **peak-splitting**

*phase-shifting control excites the side bands*

**Experimental results in sector rig**

- **Simulation**

  - Linear control theory indicates that peak-splitting will occur in the case of large delay in actuation path, high combustor damping, and limited actuation bandwidth
  - Random-input describing function analysis allows to extend the results to nonlinear case
Fundamental limits can be studied in nonlinear models with noise using Random Input Describing Functions

Conservation Principle: are under logarithm of sensitivity function is preserved => peak splitting will occur

Pressure \((j \omega)\) = \(G_o(j \omega)\)

Noise \((j \omega)\) = \(1 + N(A, \sigma)G_c(j \omega) G_o(j \omega)\)

\(= G_o(j \omega) S(j \omega), \text{ for } S(j \omega) = 1/(1+ N(A, \sigma) G_c(j \omega) G_o(j \omega))\)

\(\sigma\) - STD of Gaussian component of valve command

\(A\) - amplitude of limit cycle in valve command

\(N(\sigma)\) - Random Input Describing Function

\(S(j \omega)\) is a nonlinear analog of sensitivity function

Peak splitting can occur in nonlinear systems, even limit cycling!
Combustion Dynamics & Control: Role of Dynamic Analysis in Modeling/Design Cycle

Observed Unacceptable Time Response Behavior

Effect of Parameter Variation on Stability Boundary

Evaluation of Design Options
- Evaluation of model sensitivities
- Development of experimental protocols and model calibration
- Evaluation of paths to mitigate undesirable behavior

Alter system dynamics to obtain acceptable behavior

System Level Model Showing Feedback Coupling

Model description capturing system dynamics

Enabling effective use of dynamic model

Parametric analysis of system model

Evaluation of Design Options
Conclusions

Aerospace Applications of Control

- Control (feedback) analysis is useful beyond control system design.

- Modeling plays a central role.
  - Nominal plant
  - Uncertainty, disturbance signals,

- Modeling is done for a well-defined purpose.
Backup

Aerospace Applications of Control
Nyquist criterion: translate closed loop properties into open loop properties

Observation: closed loop stability is equivalent to all poles of the closed loop transfer function lie in the open left half of the complex plane

Now use complex variable theory on the relevant transfer function - specifically use the so called principle of the argument

Principle of the argument: \# of poles - \# of zeros of a rational function inside $\Gamma = $ winding number about the origin of the map $G(\Gamma)$

Graphically this is a mapping result (and a fancy way to count!)
Modeling and Analysis - Nyquist Criterion (3)

System shown is stable iff the zeros of $1 + PC$ are in the open left half plane.

But examine $1 + PC$:

\[
1 + PC = 1 + \frac{n_p n_c}{d_p d_c} = \frac{d_p d_c + n_p n_c}{d_p d_c}
\]

Now map the RHP under $1 + PC$ and count encirclements of the origin.

Or

Map the RHP under $PC$ and count encirclements of $-1$. 

Open-loop TF

\[
\frac{\hat{u}}{\hat{d}} = PC = \frac{n_p n_c}{d_p d_c}
\]

Closed-loop TF

\[
\frac{\hat{u}}{\hat{d}} = \frac{PC}{1 + PC} = \frac{n_p n_c}{d_p d_c + n_p n_c}
\]
Nyquist Criterion: closed loop feedback system is stable \( \iff \) the number of encirclements of \(-1\) is equal to the number of open loop poles

Special case: If the open loop system is stable then the closed loop system will be stable \( \iff \) the Nyquist plot of the open loop system does not encircle or touch \(-1\)
Consider making the applied force a function of the velocity and choosing the function to add damping:

\[ F = -k_1 \dot{x} \]

**Approach I: Compute.** Form the “closed loop system” and use computation to determine the closed loop eigenvalues (poles) and plot them as a function of the gain.

Results for all gain (positive and negative) shown by the “root locus” plot.
Approach II: Use Nyquist theorem. Form the block diagram of the "closed loop system" and use open loop properties to determine the closed loop stability as a function of the gain.

Nyquist plot shows no encirclements of -1 point (for positive gain) so system is closed loop stable (but for increasing gain plot comes close and so there is decreasing margin) - for negative gain there are always two encirclements (unstable)
Approach II: A bit of realism - suppose that the sensor for the rate (tachometer) has a small delay - what will be the effect on the stability of the closed loop system?

\[ M\ddot{x}(t) + B\dot{x}(t) + Kx(t) = -k_1 x(t - \tau) \]

\[ \frac{s}{Ms^2 + Bs + K} \]

\[ -k_1 e^{-s\tau} \]

For small delays the Nyquist plot is unchanged but for large delays (or large frequencies) the plot is significantly different and the presence of a delay may alter stability - easily seen in the frequency domain!

Effect of delay: subtracts \( \omega \tau \) from the phase response of oscillator
Synthesis for Linear Systems - Nominal Stability

- Nominal stability of the closed loop system can be easily ascertained using the Nyquist plot as shown,

- Typical values for gain margin are 6 dB and for phase margin 45-60 degrees

Gain margin: amplification that can be applied to Nyquist curve until instability

Phase margin: angle that Nyquist curve can be rotated until instability
Performance typically refers to tracking where the output (y) is required to (asymptotically) follow the input (u) or disturbance rejection where the output (y) is made to be insensitive to the disturbance (d) - both over a range of frequencies.

Both problems are the same (single degree of freedom feedback structure) and require that the sensitivity function S be small over the range of frequencies where performance is required.

\[
S = \frac{1}{1 + PC}
\]

complementary sensitivity \[
T = \frac{PC}{1 + PC}
\]

so performance (tracking or disturbance rejection requires):

\[|S(i\omega)| \ll 1\]

over the range of frequencies for which tracking and disturbance rejection is desired.
**Problem:** consider the perturbed system (for example only shown perturbed at output - analysis could be done for other cases). What conditions on the open loop ensure that the closed loop will be robustly stable where robustly means for all $\Delta$ that is norm bounded (say by $\delta$)?

**Solution:** first redraw the block diagram to illustrate what the uncertainty “sees”

\[
z = \Delta p, \ y = p + z, \ p = PCe, \ e = u - y
\]

\[
p = \frac{PC}{1 + PC} (u - z) = T(u - z)
\]
Solution (continued):

• if the uncertainty is unknown (phase) then the $T\Delta$ loop is stable iff the “loop gain” is strictly $< 1$,

• recognize that the “loop gain” is the complementary sensitivity function and then this requires that the loop “roll off” at high frequencies

Typical shape for uncertainty (percentage of output) is small at low frequency and increasingly uncertain (especially phase) at high frequency

Robust stability then implies that the complementary sensitivity function “roll off” or to preserve robustness performance is sacrificed

$$|T(i\omega)| \leq \frac{1}{|\Delta(i\omega)|} \forall \omega$$