Lecture 9
Extensions and Open Problems

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Caltech/AFRL Short Course

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Outline:

- Review key concepts from the course
- Talks about active extensions and research directions (Caltech centric)
- Discussion open issues and challenges

Some Important Trends in Control in the Last Decade

(Online) Optimization-based control

- Increased use of online optimization (MPC/RHC)
- Use knowledge of (current) constraints & environment to allow performance and adaptability

Layering and architectures

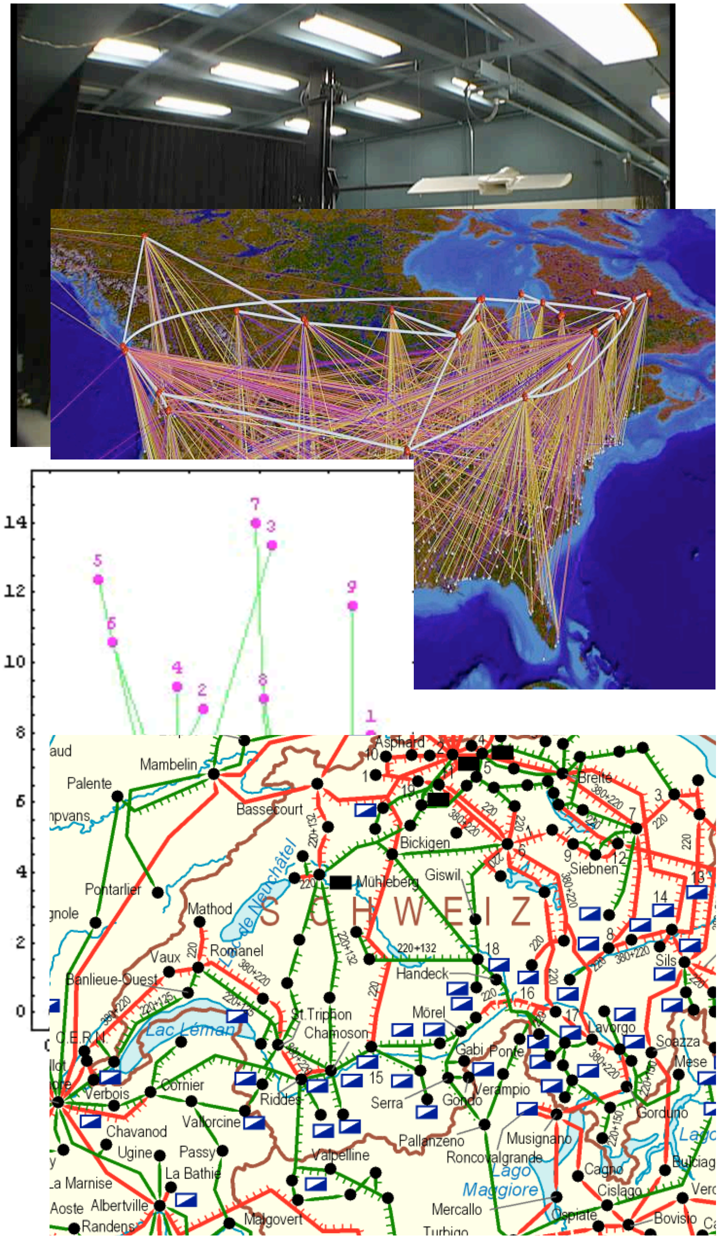
- Command & control at multiple levels of abstraction
- Modularity in product families via layers

Formal methods for analysis, design and synthesis

- Combinations of continuous and discrete systems
- Formal methods from computer science, adapted for hybrid systems (mixed continuous & discrete states)

Components → Systems → Enterprise

- Movement of control techniques from “inner loop” to “outer loop” to entire enterprise (eg, supply chains)
- Use of *systematic* modeling, analysis and synthesis techniques at all levels
- Integration of “software” with “controls” (Internet of things, cyber-physical systems, etc)



Problem Formulation: Controls + CS + Comms

Subsystem/agent dynamics - continuous

$$\dot{x}^i = f^i(x^i, y^{\sim i}, u^i) \qquad x^i \in \mathbb{R}^n, u^i \in \mathbb{R}^m$$
$$y^i = h^i(x^i) \qquad y^i \in \mathbb{R}^q$$

Agent mode (or "role") - discrete

- $\alpha \in \mathcal{A}$ encodes internal state + relationship to current task
- Transition $\alpha' = r(x, \alpha)$

Communications graph $\mathcal{G}$

- Encodes the system information flow
- Neighbor set $\mathcal{N}^i(x, \alpha)$

Communications channel

- Communicated information can be lost, delayed, reordered; rate constraints

$$y^i_j[k] = \gamma y^i(t_k - \tau_j) \qquad t_{k+1} - t_k > T_r$$

- γ = binary random process (packet loss)

Task

- Encode task as finite horizon optimal control + temporal logic (assume coupled)

$$J = \int_0^T L(x, \alpha, u)\, dt + V(x(T), \alpha(T)),$$

$$(\varphi_{init} \wedge \Box\varphi_e) \implies (\Box\varphi_s \wedge \Diamond\varphi_g)$$

Strategy

- Control action for individual agents

$$u^i = \gamma(x, \alpha) \qquad \{g^i_j(x, \alpha) : r^i_j(x, \alpha)\}$$

$$\alpha^{i\,\prime} = \begin{cases} r^i_j(x, \alpha) & g(x, \alpha) = \text{true} \\ \text{unchanged} & \text{otherwise.} \end{cases}$$

***Decentralized* strategy**

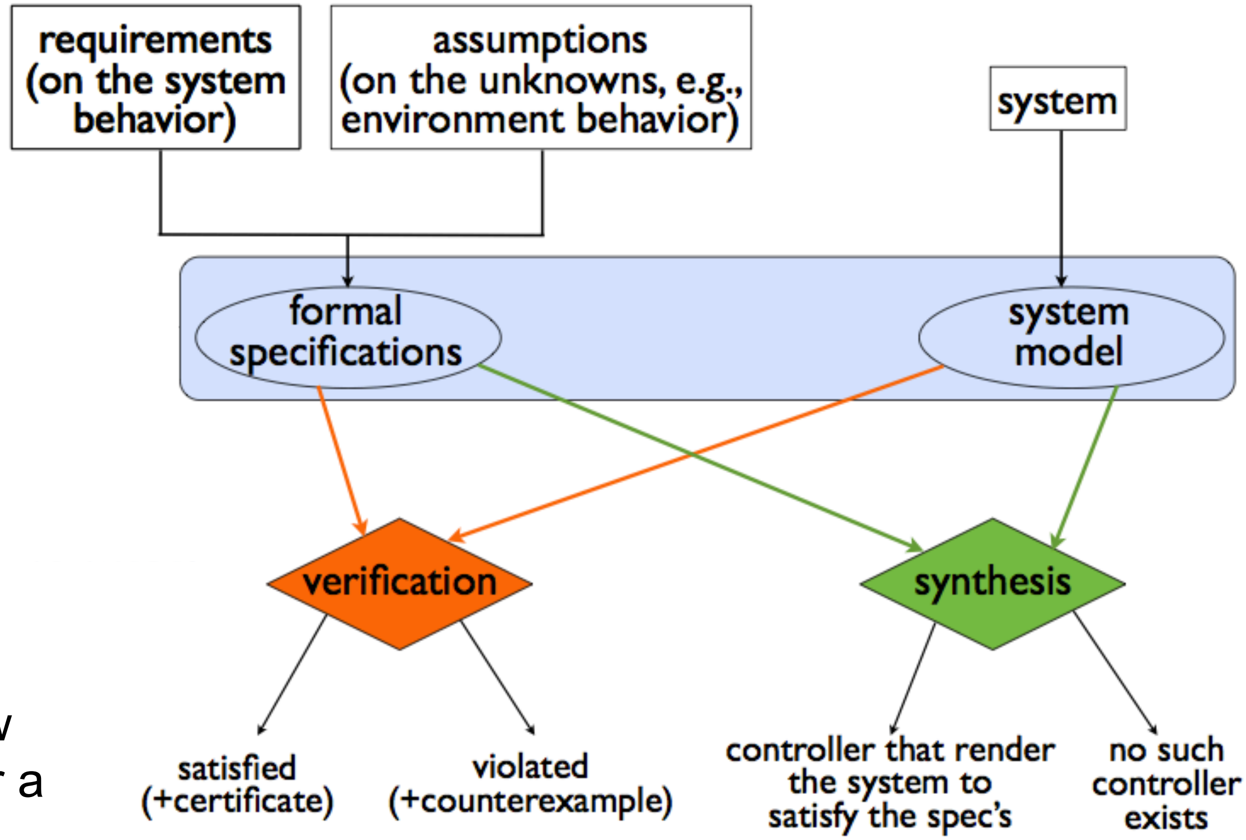
$$u^i(x, \alpha) = u^i(x^i, \alpha^i, y^{-i}, \alpha^{-i})$$
$$y^{-i} = \{y^{j_1}, \ldots, y^{j_{m_i}}\}$$
$$j_k \in \mathcal{N}^i \qquad m_i = |\mathcal{N}^i|$$

- Similar structure for role update

Formal Methods for System Verification & Synthesis

Specification using LTL

- Linear temporal logic (LTL) is a math'l language for describing linear-time prop's
- Provides a particularly useful set of operators for constructing LT properties without specifying sets



Methods for verifying an LTL specification

- *Theorem proving*: use formal logical manipulations to show that a property is satisfied for a given system model
- *Model checking*: explicitly check all possible executions of a system model and verify that each of them satisfies the formal specification

Methods for *synthesis* of correct-by-construction control protocols

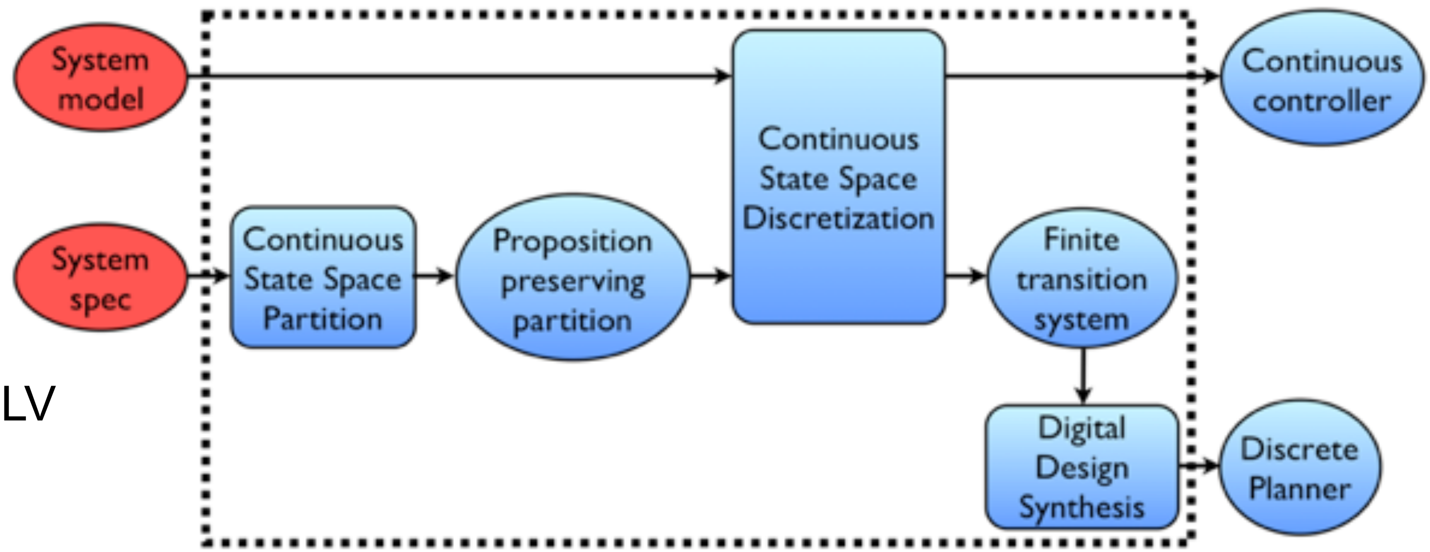
- Build on results in logic synthesis and (recent) results in GR(1) synthesis
- Key challenges: dynamics, uncertainty, complexity

Temporal Logic Planning (TuLiP) toolbox

http://tulip-control.sourceforge.net

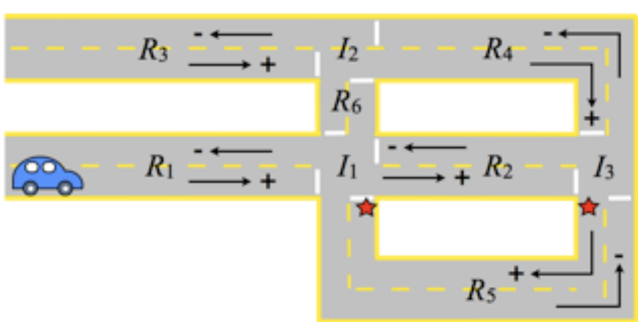
Python Toolbox

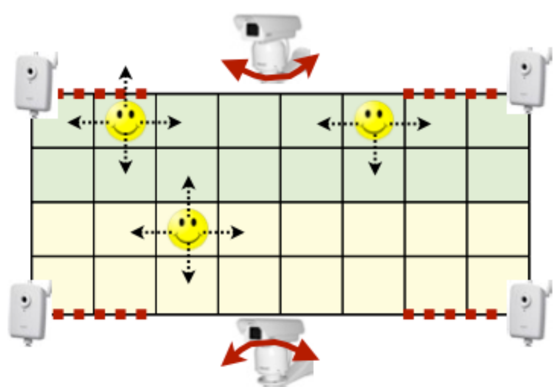
- GR(1), LTL specs
- Nonlin dynamics
- Supports discret-ization via MPT
- Control protocol designed using JTLV
- Receding horizon compatible

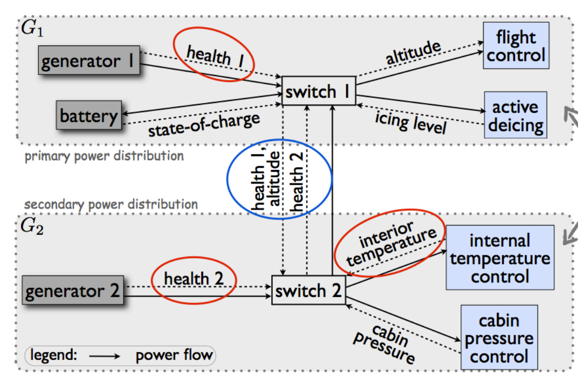


Applications of TuLiP in the last year

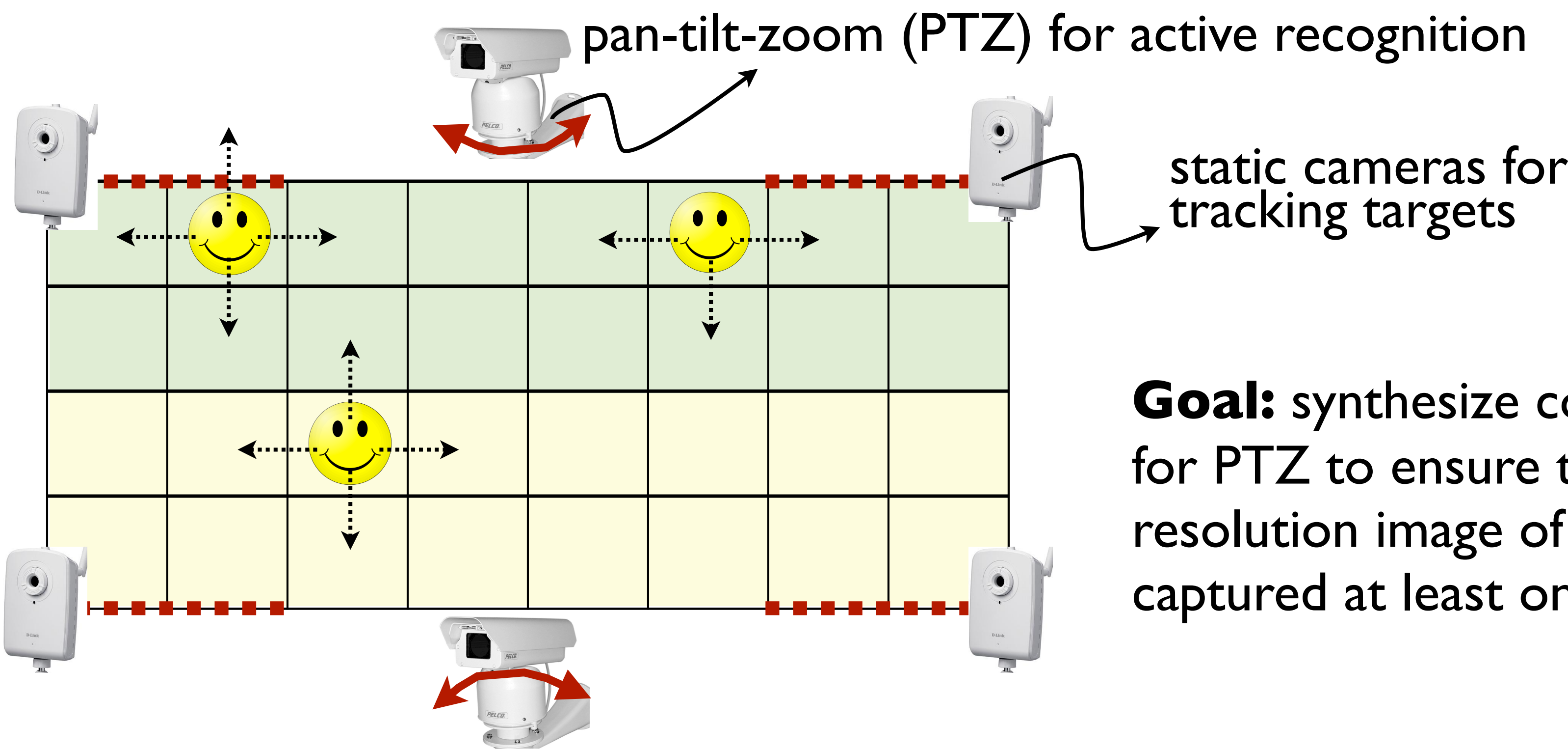
- Autonomous vehicles - traffic planner (intersections and roads, with other vehicles)
- Distributed camera networks - cooperating cameras to track people in region
- Electric power transfer - fault-tolerant control of generator + switches + loads







Control Protocols for Smart Camera Networks



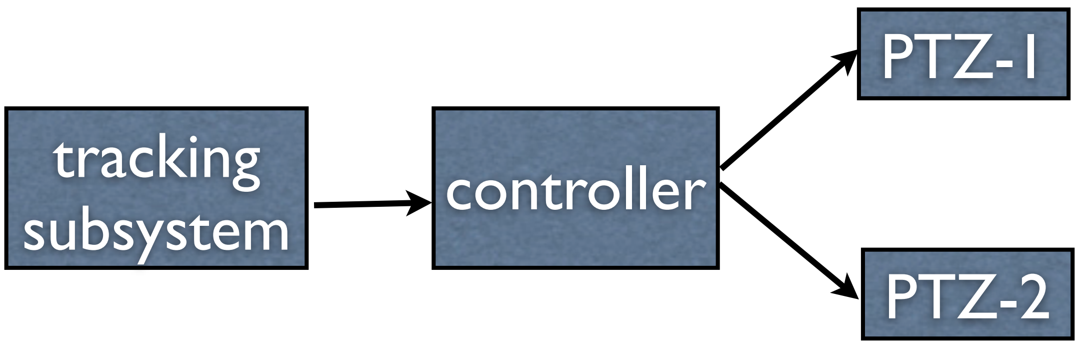
Goal: synthesize control protocols for PTZ to ensure that one high resolution image of each target is captured at least once

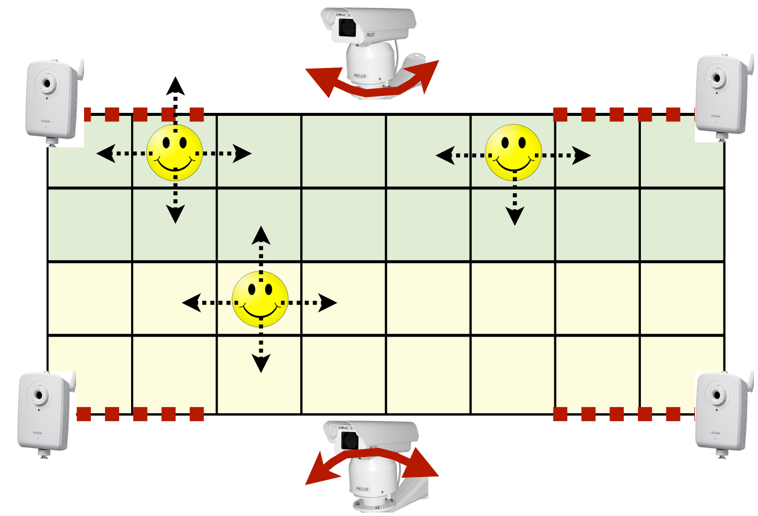
System:
- region of view of PTZs
- governed by finite state automata

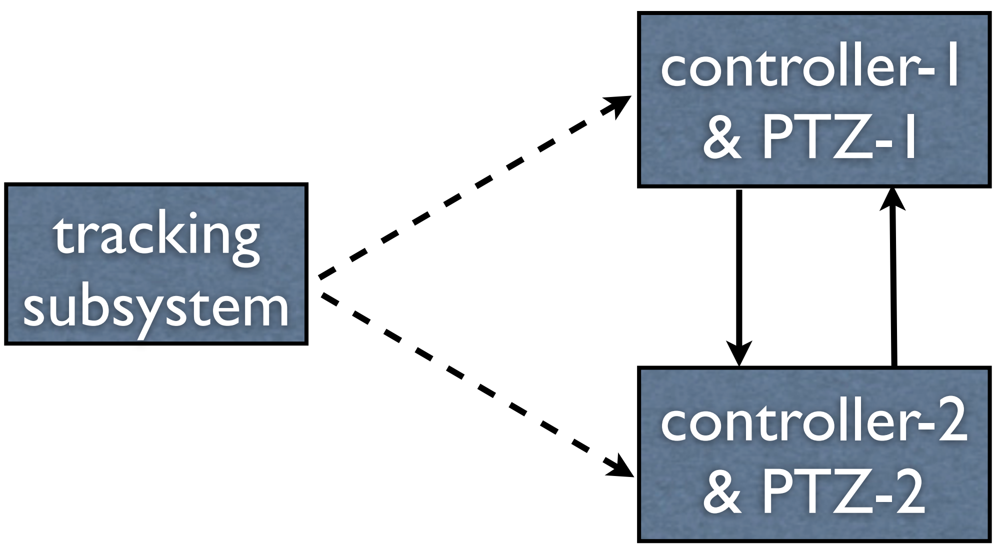
Requirement:
- Zoom-in the corner cells infinitely often.

Environment specifications:
- At most N targets at a time.
- Every target remains at least T time steps and eventually leaves.
- Can only enter/exit through doors.
- Can only move to neighbors.

Centralized vs. decentralized control architecture







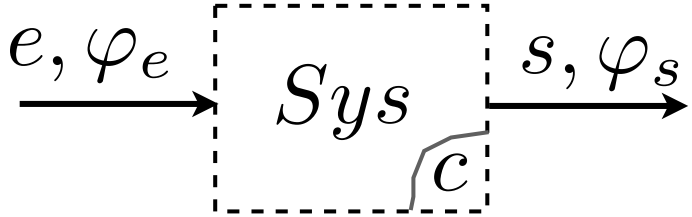
How to design control protocols that can be

- synthesized
- implemented

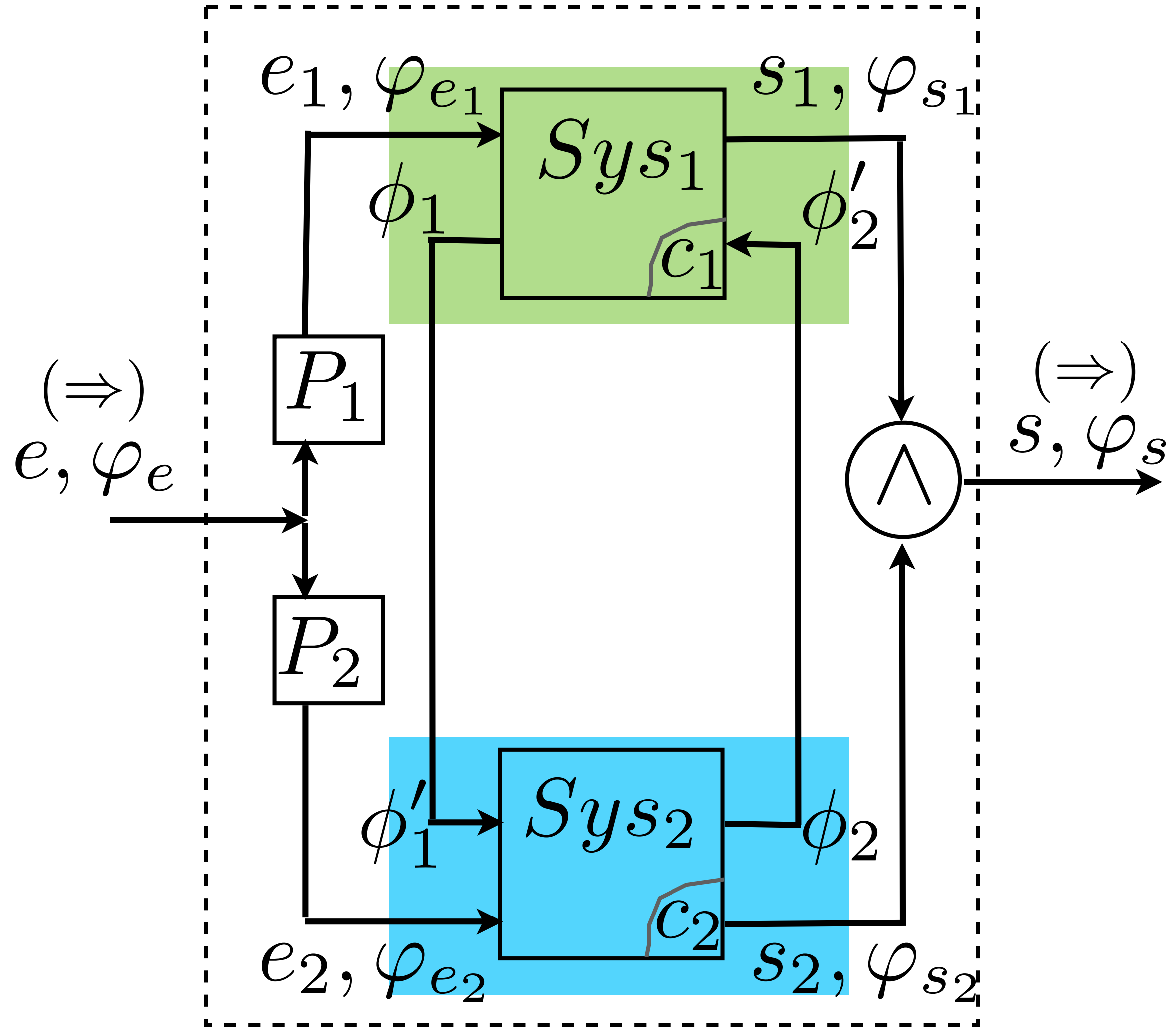
in a decentralized way?

What information exchange & interface models are needed?

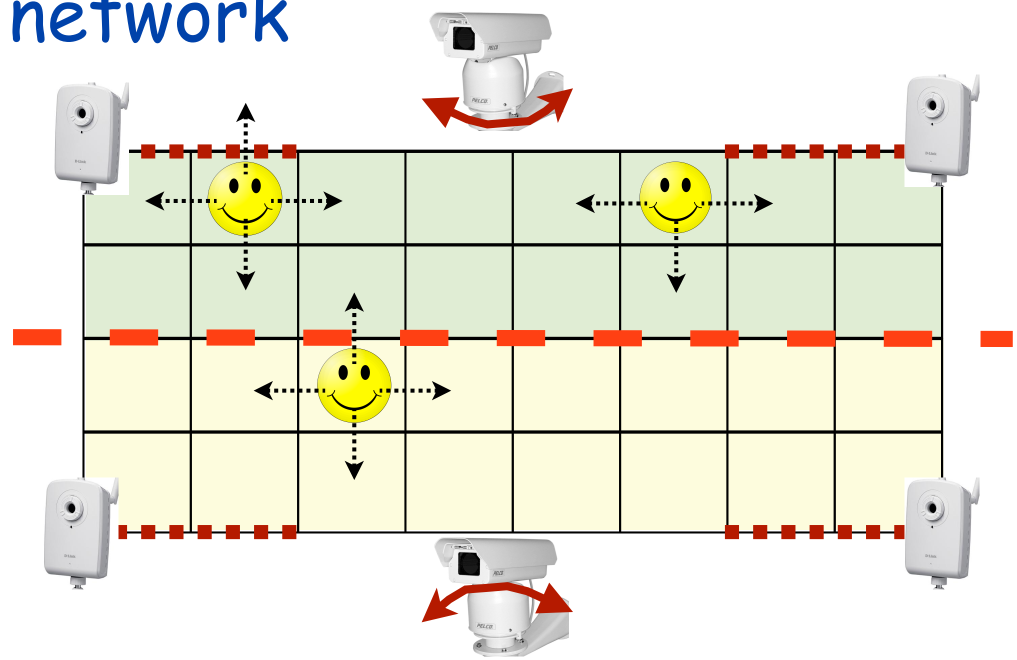
Central



Compositional



Application to smart camera network



Interface model:

- Restrict the number of un-zoomed people passing between the regions

$$in^l_{i,j} \doteq \quad (x^{(i)} = c_0) \wedge \left(\bigvee_{k\in\{4,5,6\}} \bigcirc(x^{(i)} = c_k)\right) \wedge \bigcirc(\neg isZoomed^{(i)}) \wedge (x^{(j)} = c_0) \wedge \left(\bigvee_{k\in\{4,5,6\}} \bigcirc(x^{(j)} = c_k)\right) \wedge \bigcirc(\neg isZoomed^{(j)})$$

$$\varphi^l_{e,refine} \doteq \Box \left(\bigwedge_{i,j\in\{1,2,3\},i\neq j} \neg in^l_{i,j} \right)$$

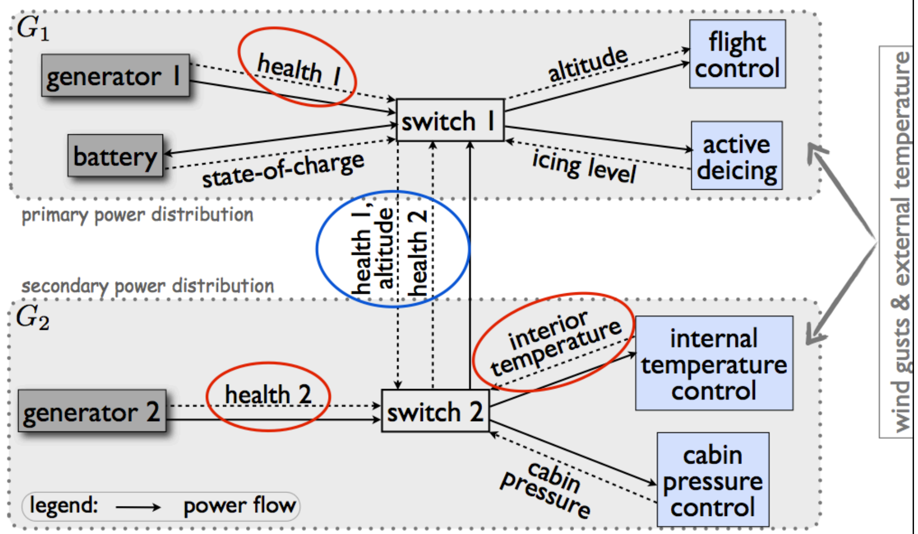
Controllers exchange information:

- IsZoomed (Boelean) indicates whether a crossing person has been already zoomed-in.
- StepsInZone: number of steps a crossing person has spent in the area.

Example: Electrical Power Management for Aircraft

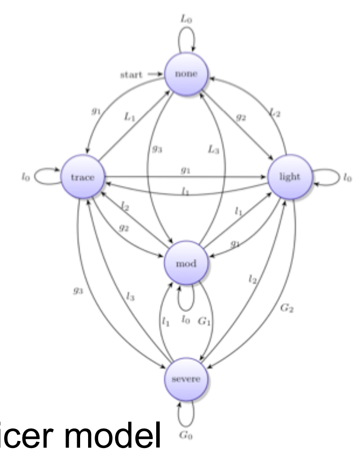
Power management of three VMS subsystems

- Flight control (actuation) - highest priority
- Active de-icing - elevation dependent demand
- Environmental control - slower timescale

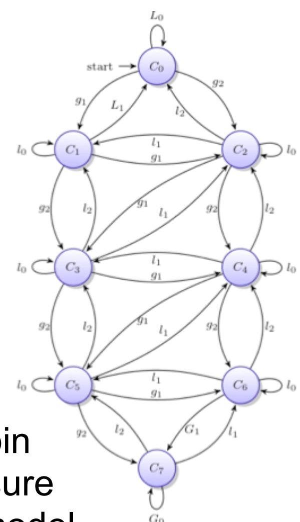


Specifications

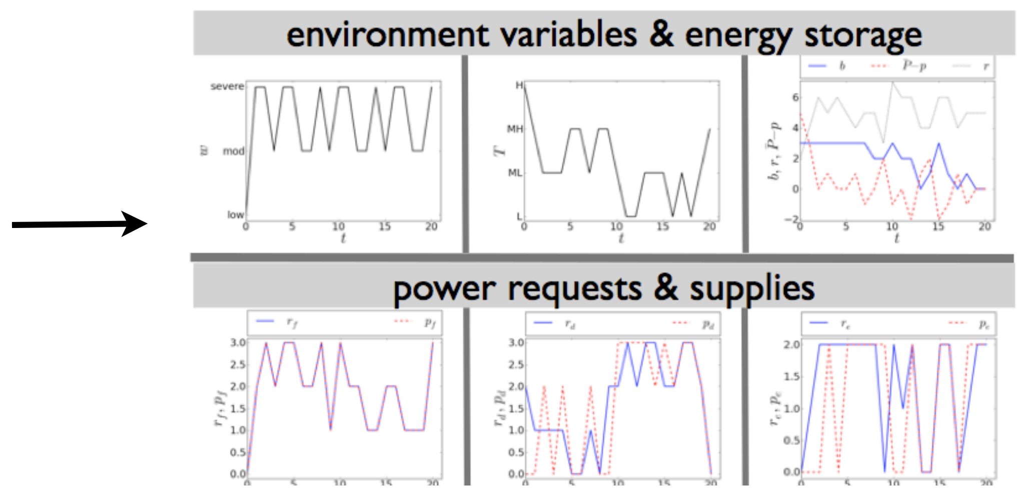
- Constraint on maximum total power
- Prioritization: actuation, de-icing, environment
- Safety: ice accumulation, altitude change
- Performance: desired altitude and environmental conditions
- External environment: wind gusts, outside temperature, generator health



De-icer model



Cabin pressure level model



Huan, Topcu, Murray
CDC 2012 (s)

Reactive Protocols for Electric Power Distribution

Problem setup

- Primary distribution: guarantee power buses are correctly powered
- Synthesize control protocol for allowable combinations of faults/failures

Specifications

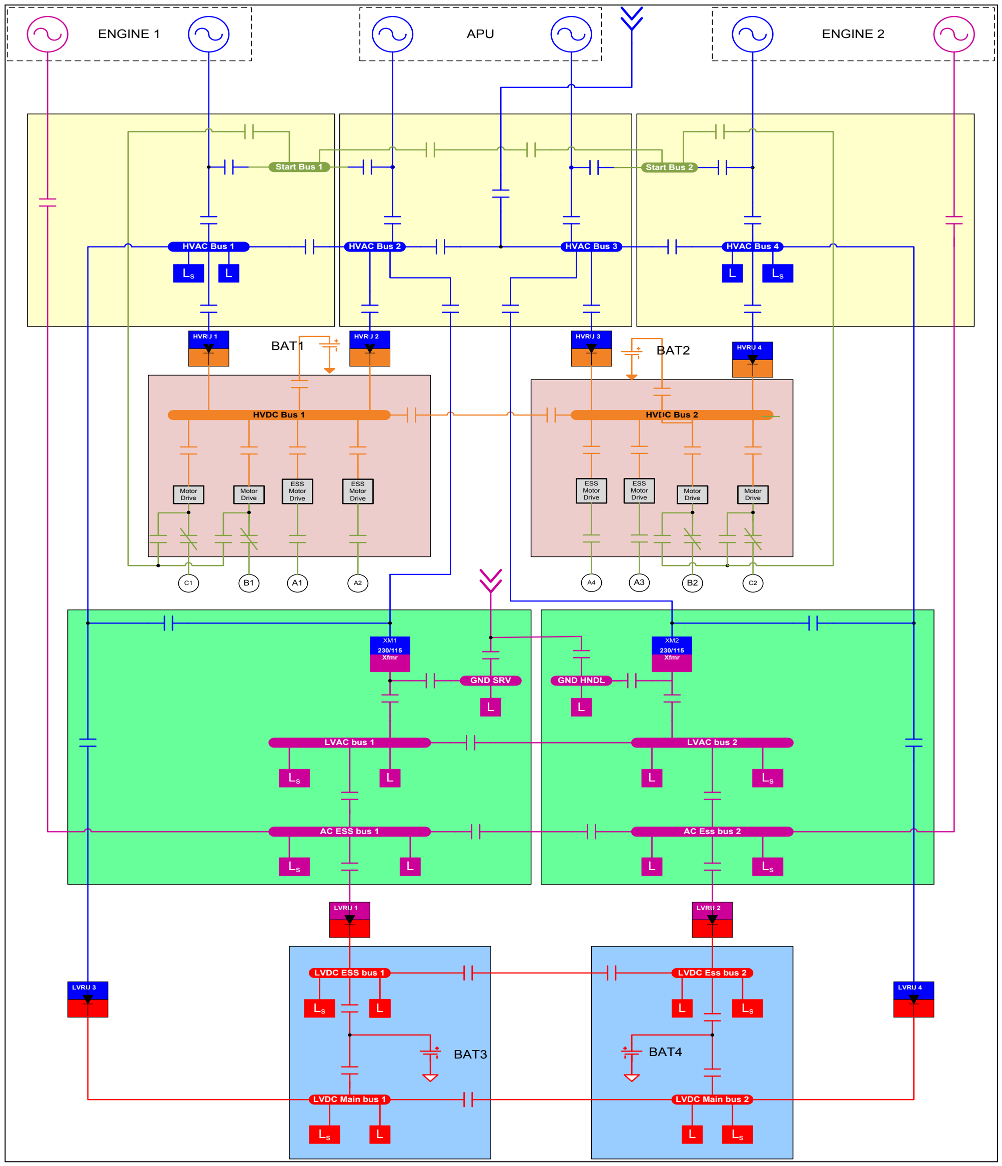
- Buses never unpowered for more than 50 ms
- Non-paralleling of AC sources
- Priority of generators
- Probability of failure: maintain reliability level

Results to date

- Synthesis for simplified (4 contactor) case, but with temporal constraints

Open questions

- Scaling (multiple clocks), optimal, modular, hierarchical, ...



Open (Research) Issues

Optimality: "language-constrained, optimal trajectory generation"

$$(\varphi_{init} \wedge \Box\varphi_e) \implies (\Box\varphi_s \wedge \Diamond\varphi_g) \qquad J = \int_0^T L(x, \alpha, u)\, dt + V(x(T), \alpha(T)),$$

Partial order computation and hierarchical structure

- How do we determine the partial order for RHTLP and link to "supervisory" levels?

Verification and synthesis with (hard) real-time constraints

- How do we incorporate time in our specifications, verification and synthesis tools?
- Note: time automata and timed temporal logic formulas available...

Contract-based design: automate search interfaces for distributed synthesis

- How do we decompose a larger problem into smaller pieces?
- Especially important for large scale projects with multiple teams/companies

Uncertainty and robustness

- How to specify uncertainty for transition systems, robustness for controllers, specs
- New methods for describing robustness by Tabuada et al: look at how much the specifications must be enlarged to capture new behaviors based on uncertainty

Many other directions: incremental, probabilistic, performance metrics, ...

- Identify problems where knowledge of dynamics, uncertainty and feedback matter

Wolff, Topcu, Murray
RSS 2012 (s)

Optimal Synthesis with Weighted Average Costs

Problem Setting

- Deterministic weighted transition system TS
- LTL specification ϕ
- $J(\sigma) := \limsup_{n \to \infty} \sum_{i=0}^{n} c(\sigma_i, \sigma_{i+1})$
- Problem: Compute run σ that minimizes J over all σ and satisfies ϕ.

Main Results

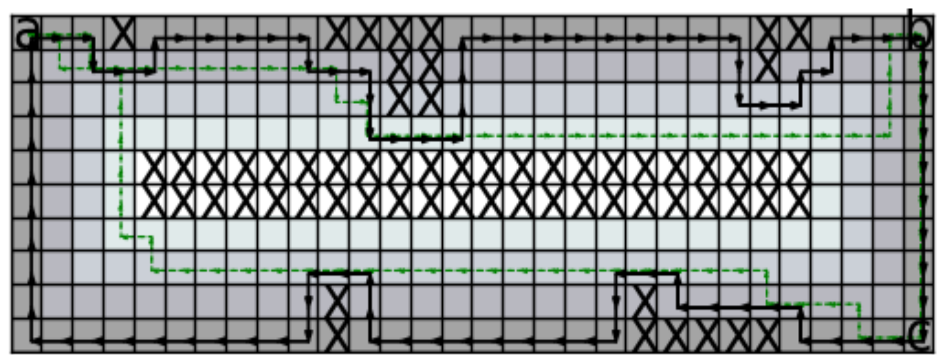
- Reduce problem to finding optimal cycle in product automaton P.
- Dynamic programming recurrence computes optimal cost cycle on P = (V,E). Fk(v) is minimum cost walk of length k between vertices s, v in V.

$$F_k(v) = \min_{(u,v)\in E} \left[F_{k-1}(u) + c(u,v) \right]$$

- Complexity: O(na(mn +n2log(n)) for 0-1 weights, where na is the number of accepting states.

Example

- Costs lower near boundary
- ϕ = [] <> a && [] <> b && [] -x
- Optimal (black) and feasible using DFS (green)



(shading represents cost)

Questions

- Nondeterministic transition system?
- Reactive environments?
- Multi-objective?
- Discounted cost function?

Wolff, Topcu, Murray
CDC 2012 (s)

Markov Decision Processes with LTL Specifications

Problem Setting

- Markov decision process (MDP) system model, with uncertainty in transitions (disturbances, failures)
- LTL specification ϕ (probably GR(1))
- Problem: Maximize probability of MDP satisfying ϕ over uncertainty set:

$$\max_{\pi \in \Pi} \min_{\tau \in \mathcal{T}} \mathbb{P}^{\pi,\tau}(s_0 \models \varphi)$$

Main Results

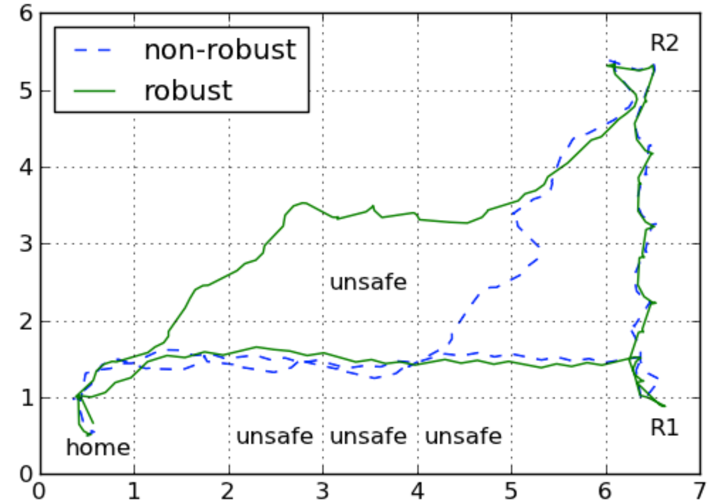
- Transform P = MDP x LTL to stochastic shortest path (SSP) form
- Compute satisfaction probabilities on SSP with robust dynamic program'g

$$(TJ)(s) \quad := \quad \min_{a \in A(s)} \left[c(s,a) + \max_{p \in \mathcal{P}_s^a} p^T J \right]$$

- Project policy π back to MDP
- Complexity: O(n2m log(1/ε)2) for ε-suboptimal policy

Example

- Differential drive robot (x,y,theta)
- Transition probabilities estimated (MC)
- ϕ = <> (R1 && <> R2) && [] –unsafe && <> [] home



Questions

- Simpler fragments of temporal logic?
- Tradeoffs between costs and probability of success?
- Principled abstraction of MDPs from continuous systems?

Technical Challenges and Risks

1. Writing LTL (or other temporal logic) specifications is not a job for mortals

- Easy to make mistakes when writing LTL and hard to interpret complex formulas
- Possible approach: domain specific tools that provide engineer-friendly interface

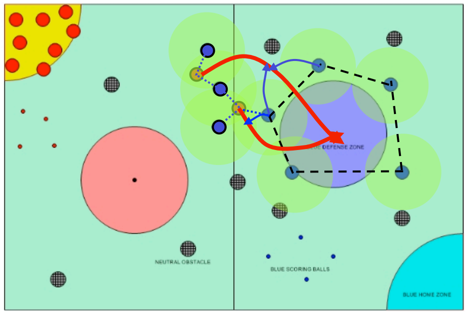
2. Model checking and logic synthesis tools won't work on large problems

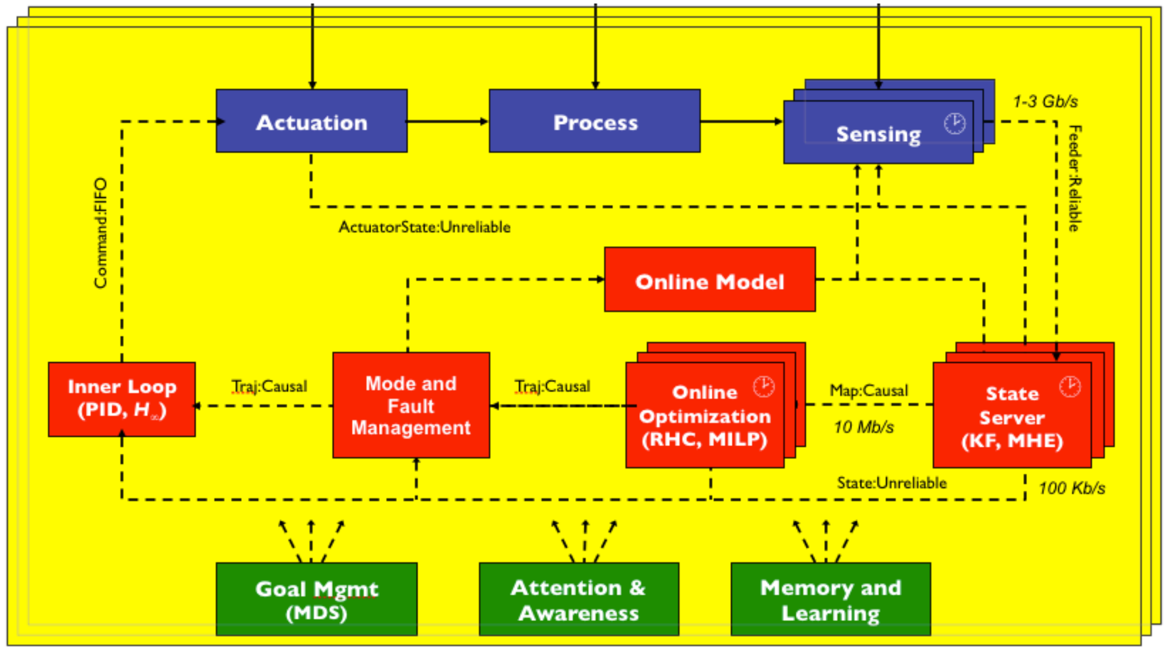
- Combinatorial explosion in discrete states for modest engineering systems will make it impossible to apply model checking/synthesis to "raw" problem
- Approaches: abstraction layers and modularity via interfaces
  - Vertical layering: apply tools to different layers and enforce bisimulation
  - Horizontal contracts: define formal subsystem interfaces & reason about them

3. Expertise in modeling and specification not yet developed

- Engineers in domains in which these tools are needed don't yet have experience developing models that ignore the right sets of things
  - Compare to reduced order models for aircraft (aerodynamic, aeroelastic) and agreed on specifications (bandwidth, response time, stability margins, etc)
  - Particularly worried about dynamics, uncertainty, interconnection
  - How do we convince FAA to allow use of these tools?
- Approach (?): explore application domains, moving from modest to complex problems, and develop expertise, tools, tool chains, processes, ...

Specification, Design and Verification of NCS





Specification

- How do we describe correct behavior?

Design

- What tools can we use to design protocols to implement that behavior?

Verification

- How do we know if it is actually correct?

$$J = \int_0^T L(x, \alpha, u)\, dt + V(x(T), \alpha(T)),$$

$$(\varphi_{init} \ \wedge \ \Box\varphi_e) \implies (\Box\varphi_s \ \wedge \ \Diamond\varphi_g)$$