TuLiP: A Software Toolbox for Receding Horizon Temporal Logic Planning

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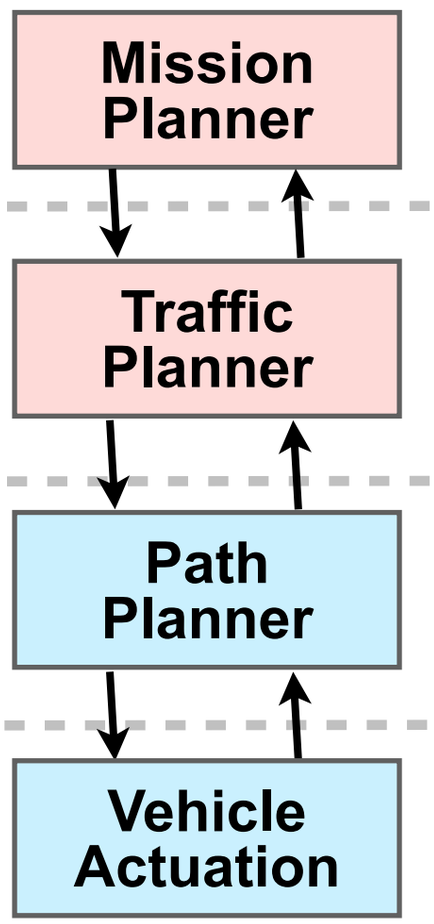
Outline

- Key Features of TuLiP
  - Embedded control software synthesis
  - Receding horizon temporal logic planning
- Computer Lab

TuLiP for receding horizon temporal logic planning

Alice's navigation stack



Different views

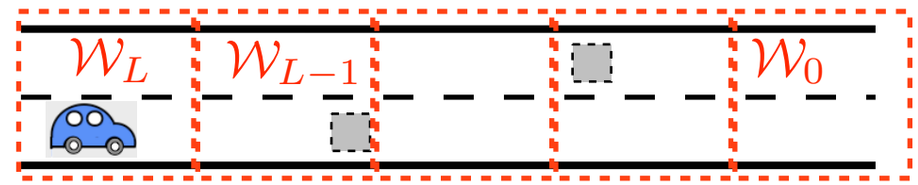
"long-horizon specification"

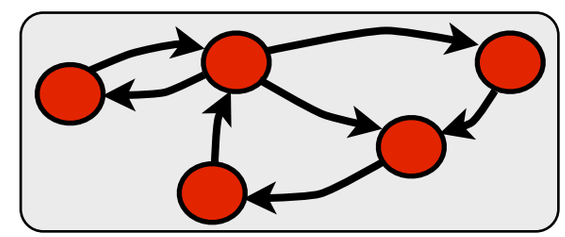
"short-horizon specification"

continuous dynamics& constraints

Multi-scale models

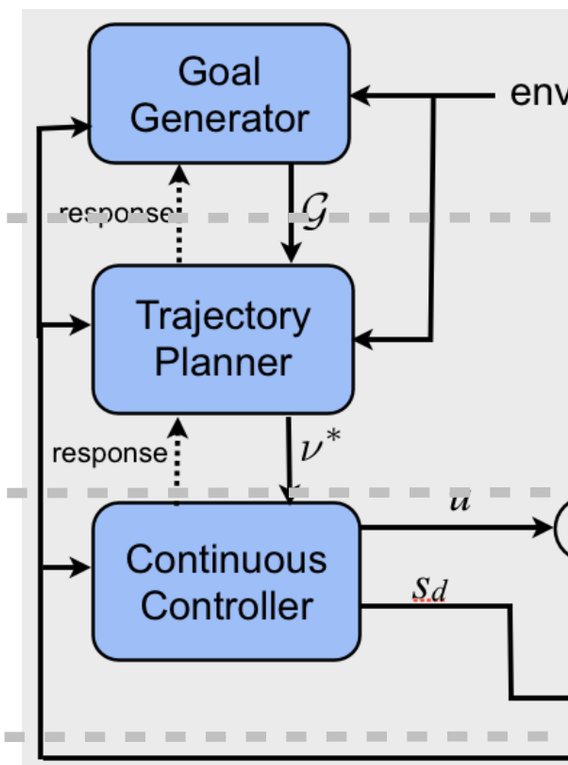
$\mathcal{W}_0 \prec \ldots \prec \mathcal{W}_{L-1} \prec \mathcal{W}_L$





$$\min \int_{t_0}^{T} L(x,u)dt$$
$$\text{s.t.} \quad \dot{x} = f(x,u)$$
$$g(x,u) \leq 0$$

Hierarchical control architecture



Inputs:

- Models for evolution of discrete/ continuous system states
- Assumptions on (discrete) environment state

Outputs:

- "Strategy" to be implemented in each layer

Digital design synthesis (middle layer)

- TuLiP interfaces to a game solver in JTLV to solve discrete, two-player, GR[1] games

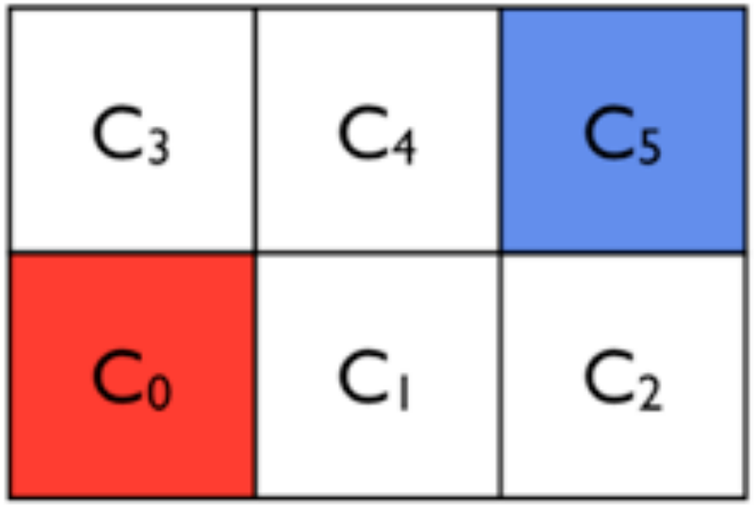
JTLV:
Java-based framework for developing formal verification algorithms

$$\varphi = \Big(\underbrace{\psi^e_{init}}_{\substack{\text{assumptions on}\\ \text{initial condition}}} \wedge \underbrace{\Box\psi^e_s \wedge \bigwedge_{i\in I_f} \Box\Diamond\psi^e_{f,i}}_{\substack{\text{assumptions on}\\ \text{environment}}} \Big) \implies \underbrace{\Big(\psi^s_{init} \wedge \Box\psi^s_s \wedge \bigwedge_{i\in I_g} \Box\Diamond\psi^s_{g,i} \Big)}_{\substack{\text{desired}\\ \text{behavior}}}$$

- prob = **generateJTLVInput**(env_vars, sys_disc_vars, spec, disc_props, disc_dynamics, smv_file, spc_file)
- realizability = **checkRealizability**(smv_file, spc_file, aut_file, heap_size)
- realizability = **computeStrategy**(smv_file, spc_file, aut_file, heap_size)
- aut = **Automaton**(aut_file)

Example: discrete synthesis using TuLiP

System model: Robot can move to the cells which share a face with the current cell.



Desired Properties

- Visit the blue cell infinitely often
- Eventually go to the red cell when a PARK signal is received

Assumption

- Infinitely often, PARK signal is not received

$$\varphi = \Box\Diamond(\neg park) \implies (\Box\Diamond(s \in C_5) \wedge \boxed{\Box(park \implies \Diamond(s \in C_0))})$$

The specification is not a GR[1] formula

- Introduce an auxiliary variable *X0reach* that starts with True
- $\Box(\bigcirc X0reach = ((s \in C_0 \vee X0reach) \wedge \neg park))$
- $\Box\Diamond X0reach$

Link to the tutorial: http://goo.gl/uOH8Q

smv and spc files for robot_discrete_simple.py

spc

```
LTLSPEC
s.X0reach & []<>(!e.park)
;

LTLSPEC
[]<>((s.cellID = 5)) & []<>(s.X0reach) & [](next(s.X0reach) = ((((s.cellID = 0)) | s.X0reach) & !e.park)) &
-- all initial system states
      ((s.cellID=0 | s.cellID=1 | s.cellID=2 | s.cellID=3 | s.cellID=4 | s.cellID=5)) &
-- transition relations for continuous dynamics
      []((s.cellID = 0) -> next((s.cellID = 0) | (s.cellID = 1) | (s.cellID = 3))) &
      []((s.cellID = 1) -> next((s.cellID = 0) | (s.cellID = 1) | (s.cellID = 2) | (s.cellID = 4))) &
      []((s.cellID = 2) -> next((s.cellID = 1) | (s.cellID = 2) | (s.cellID = 5))) &
      []((s.cellID = 3) -> next((s.cellID = 0) | (s.cellID = 3) | (s.cellID = 4))) &
      []((s.cellID = 4) -> next((s.cellID = 1) | (s.cellID = 3) | (s.cellID = 4) | (s.cellID = 5))) &
      []((s.cellID = 5) -> next((s.cellID = 2) | (s.cellID = 4) | (s.cellID = 5))) &
-- possible values of discrete system variables
      [](next((s.cellID=0 | s.cellID=1 | s.cellID=2 | s.cellID=3 | s.cellID=4 | s.cellID=5)))
;
```

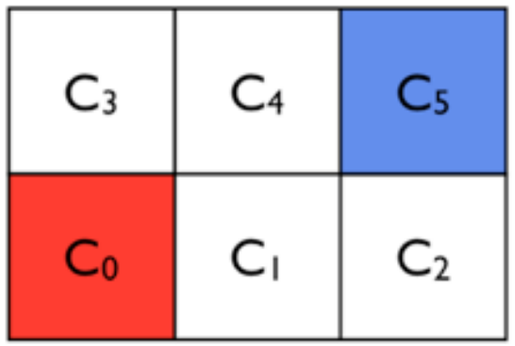
smv

```
MODULE main
      VAR
            e : env();
            s : sys();

MODULE env
      VAR
            park : boolean;

MODULE sys
      VAR
            cellID : {0, 1, 2, 3, 4, 5};
            X0reach : boolean;
```

$$\varphi = \Box\Diamond(\neg park) \quad \Longrightarrow \quad (\Box\Diamond(s \in C_5) \wedge \Box(park \Longrightarrow \Diamond(s \in C_0)))$$

$$\Box(\bigcirc X0reach = ((s \in C_0 \vee X0reach) \wedge \neg park))$$

$$\Box\Diamond X0reach$$



Results for robot_discrete_simple.py

aut file (partial)

```
Specification is realizable...
==== Building an implementation =========
-----------------------------------------
State 0 with rank 0 -> <park:1, cellID:0, X0reach:0>
      With successors : 1, 2
State 1 with rank 0 -> <park:0, cellID:1, X0reach:0>
      With successors : 3, 4
State 2 with rank 0 -> <park:1, cellID:1, X0reach:0>
      With successors : 3, 4
State 3 with rank 0 -> <park:0, cellID:4, X0reach:0>
      With successors : 5, 4
State 4 with rank 0 -> <park:1, cellID:4, X0reach:0>
      With successors : 5, 4
State 5 with rank 0 -> <park:0, cellID:5, X0reach:0>
      With successors : 6, 7
State 6 with rank 1 -> <park:0, cellID:4, X0reach:0>
      With successors : 8, 9
State 7 with rank 1 -> <park:1, cellID:4, X0reach:0>
      With successors : 8, 9
State 8 with rank 1 -> <park:0, cellID:1, X0reach:0>
      With successors : 10, 11
State 9 with rank 1 -> <park:1, cellID:1, X0reach:0>
      With successors : 10, 11
State 10 with rank 1 -> <park:0, cellID:0, X0reach:0>
      With successors : 12, 13
State 11 with rank 1 -> <park:1, cellID:0, X0reach:0>
      With successors : 10, 11
State 12 with rank 1 -> <park:0, cellID:0, X0reach:1>
      With successors : 22, 23
State 13 with rank 1 -> <park:1, cellID:0, X0reach:1>
```

Sample simulation from grsim with inputs:

- park signal
- initial value of *X0reach*

```
{'park': 0, 'cellID': 4, 'X0reach': 1}
{'park': 0, 'cellID': 4, 'X0reach': 1}
{'park': 0, 'cellID': 5, 'X0reach': 1}
{'park': 1, 'cellID': 4, 'X0reach': 1}
{'park': 0, 'cellID': 4, 'X0reach': 0}
{'park': 0, 'cellID': 5, 'X0reach': 0}
{'park': 1, 'cellID': 4, 'X0reach': 0}
{'park': 0, 'cellID': 1, 'X0reach': 0}
{'park': 0, 'cellID': 0, 'X0reach': 0}
{'park': 1, 'cellID': 0, 'X0reach': 1}
{'park': 0, 'cellID': 0, 'X0reach': 0}
{'park': 0, 'cellID': 1, 'X0reach': 1}
{'park': 1, 'cellID': 4, 'X0reach': 1}
{'park': 0, 'cellID': 5, 'X0reach': 0}
{'park': 0, 'cellID': 4, 'X0reach': 0}
{'park': 1, 'cellID': 1, 'X0reach': 0}
{'park': 0, 'cellID': 0, 'X0reach': 0}
{'park': 0, 'cellID': 0, 'X0reach': 1}
{'park': 1, 'cellID': 0, 'X0reach': 1}
{'park': 0, 'cellID': 1, 'X0reach': 0}
{'park': 0, 'cellID': 4, 'X0reach': 0}
{'park': 1, 'cellID': 4, 'X0reach': 0}
{'park': 0, 'cellID': 5, 'X0reach': 0}
{'park': 0, 'cellID': 4, 'X0reach': 0}
```

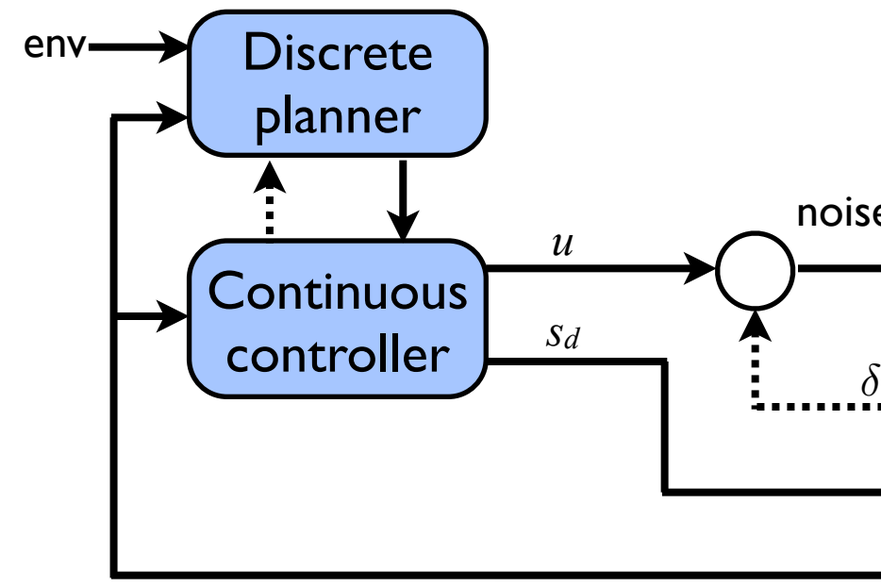
$$\varphi = \Box\Diamond(\neg park) \implies (\Box\Diamond(s \in C_5) \wedge \Box(park \implies \Diamond(s \in C_0)))$$

$$\Box(\bigcirc X0reach = ((s \in C_0 \vee X0reach) \wedge \neg park))$$

$$\Box\Diamond X0reach$$

Interface the discrete planner and continuous controller

Use the finite-state abstraction procedure to write the problem in terms of the inputs to digital design synthesis



Main steps:

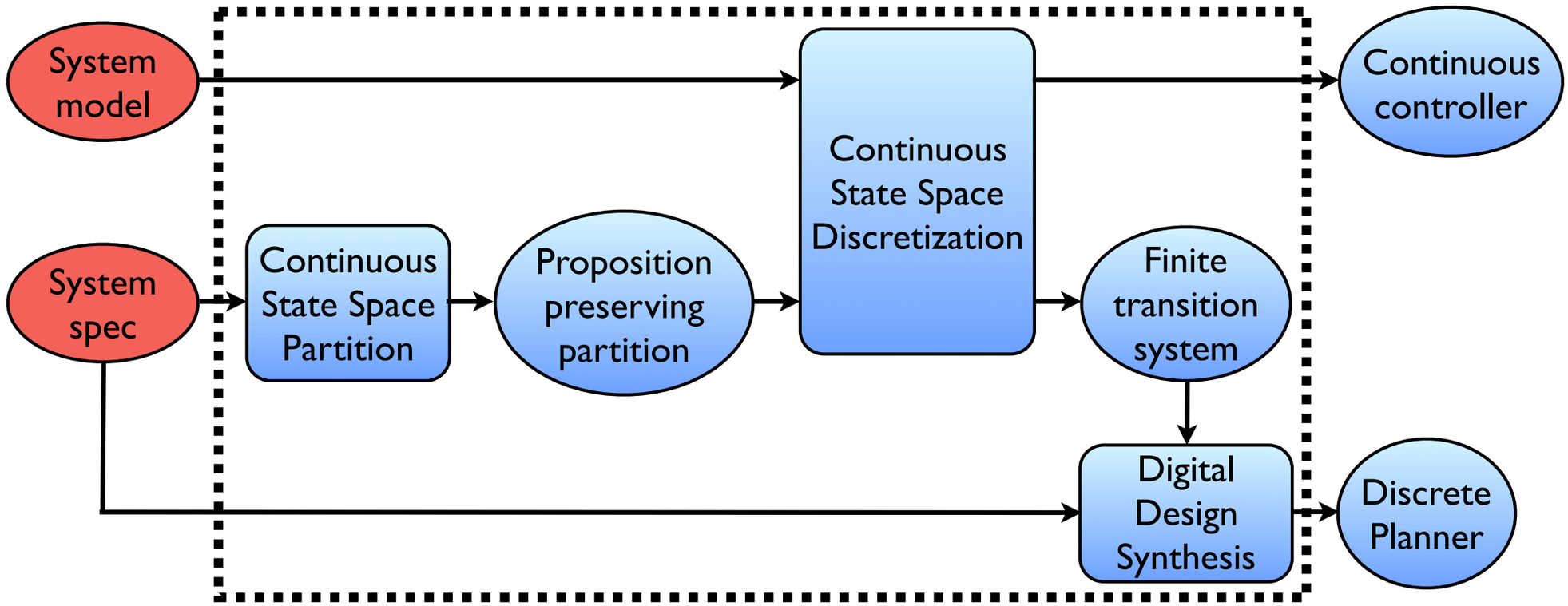
- Partition the continuous state space
  - Propositions shall preserve their meanings
- Establish transitions between the cells in the partition
- Call the routines for digital design synthesis

Continuous dynamics:

$$\begin{aligned} s(t+1) &= As(t) + Bu(t) + Ed(\\ u(t) &\in U \\ d(t) &\in D \end{aligned}$$

$$s \in \mathbb{R}^n, U \subseteq \mathbb{R}^m, D \subseteq \mathbb{R}^p$$

Workflow: discrete planner + continuous controller

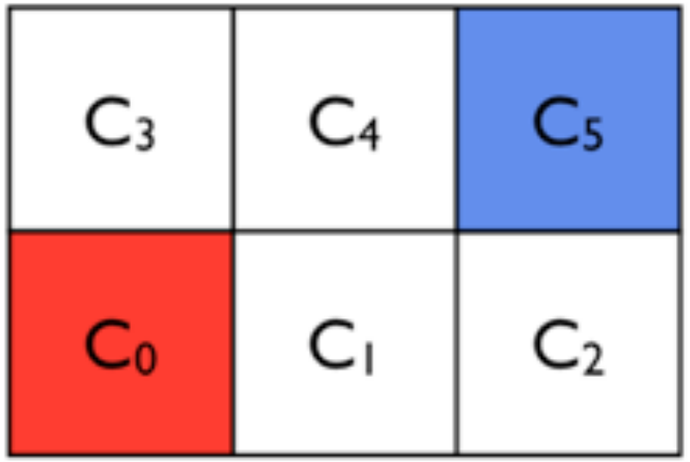


- Generate a proposition preserving partition of the continuous state space
  - cont_partition = **prop2part2**(state_space, cont_props)
- Discretize the continuous state space based on the evolution of the continuous state
  - disc_dynamics = **discretizeM**(cont_partition, ssys, N=10)
- Digital design synthesis: generateJTLVInput, checkRealizability, computeStrategy, Automaton,...

Example: incorporating continuous dynamics

System model: $\dot{x} = u_x, \dot{y} = u_y$ where $u_x, u_y \in [-1, 1]$

difference from the previous example



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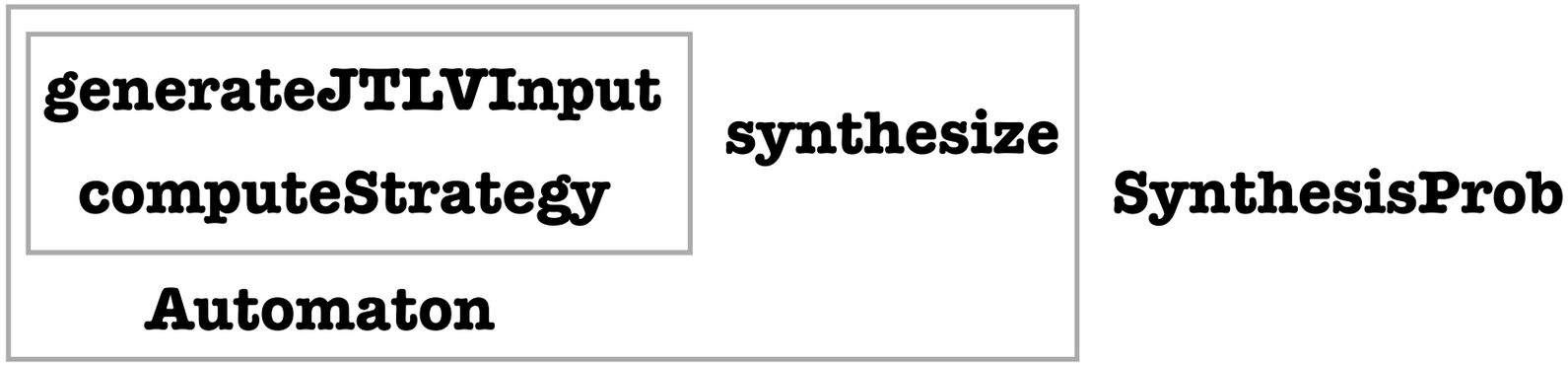
Link to the tutorial: http://goo.gl/6FsM0

Defining a synthesis problem: SynthesisProb class

Combine the three steps of digital design synthesis:



Fields of SynthesisProb:

- *env_vars*
- *sys_vars*
- *spec*
- *disc_cont_var*
- *disc_dynamics*

Useful methods:

- ***checkRealizability****(heap_size='-Xmx128m', pick_sys_init=True, verbose=0)*:
  checks whether the problem is realizable
- ***getCounterExamples****(recompute=False, heap_size='-Xmx128m', pick_sys_init=True, verbose=0)*
  returns the set of initial states from which the system cannot satisfy the spec
- ***synthesizePlannerAut****(heap_size='-Xmx128m', priority_kind=3, init_option=1, verbose=0)*
  synthesizes the planner that ensures system correctness

Link to an example: http://goo.gl/fNZxS

Receding Horizon Framework for LTL Specifications

Idea: Reduce the synthesis problem to a set of smaller problems of short horizon

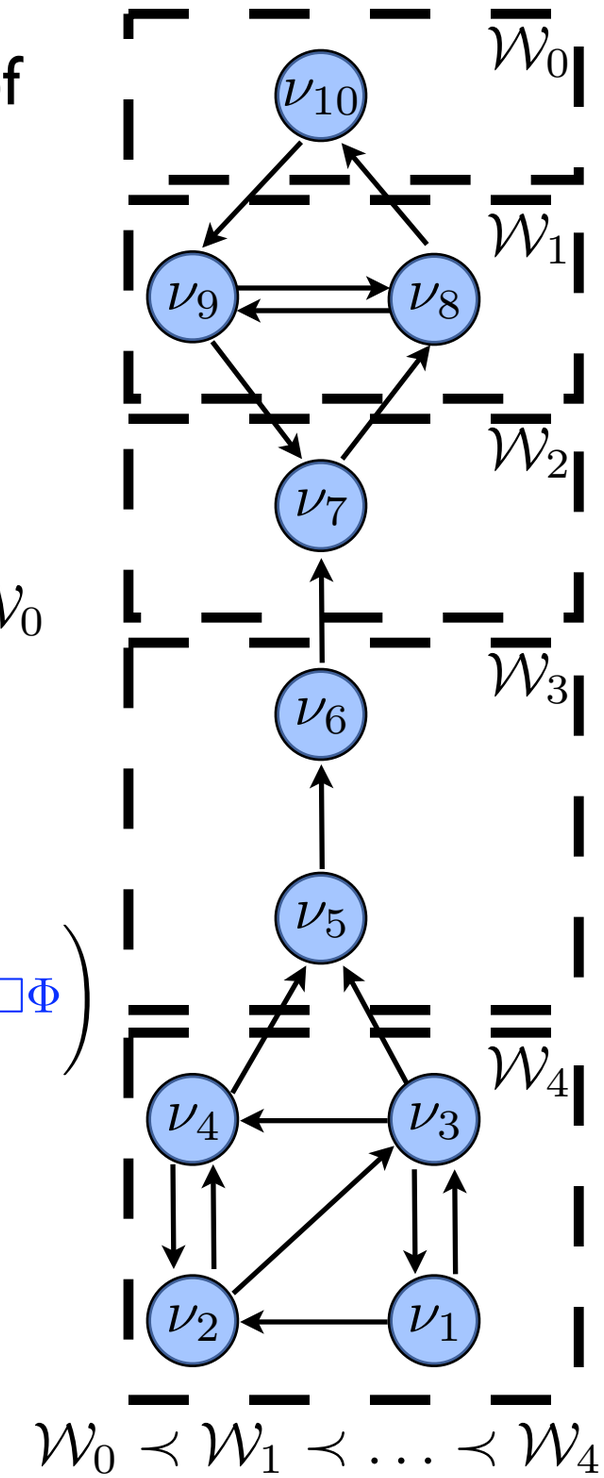
- Consider a specification of the form

$$\varphi = \left(\psi_{init} \wedge \underbrace{\Box}_{\text{always}} \psi_e^e \wedge \bigwedge_{i \in I_f} \underbrace{\Box \diamond}_{\text{Infinitely often}} \psi_{f,i}^e \right) \implies \left(\bigwedge_{i \in I_s} \underbrace{\Box}_{\text{always}} \psi_{s,i} \wedge \overbrace{\Box \diamond}^{\text{Infinitely often}} \psi_g \right)$$

- Organize cells into a partially ordered set $\mathcal{P} = (\{\mathcal{W}_j\}, \preceq_{\psi_g})$ where $\mathcal{W}_0$ is the set of "goal states," i.e., all cells in $\mathcal{W}_0$ satisfy ψ_g
- Assume that for each *j*, there exist a proposition Φ and a mapping $\mathcal{F}$ such that the following short-horizon specification is realizable

$$\Psi_j = \left((\varsigma \in \mathcal{W}_j) \wedge \Phi \wedge \Box \psi_e^e \wedge \bigwedge_{k \in I_f} \Box \diamond \psi_{f,k}^e \right) \implies \left(\bigwedge_{k \in I_s} \Box \psi_{s,k} \wedge \Box \diamond (\varsigma \in \mathcal{F}(\mathcal{W}_j)) \wedge \Box \Phi \right)$$

  - Φ describes receding horizon invariants
  - $\mathcal{F}(\mathcal{W}_j) \prec \mathcal{W}_j, \forall j \neq 0$ defines intermediate goal for starting in $\mathcal{W}_j$
  - Partial order condition guarantees that we move closer to goal



$$\mathcal{F}(\mathcal{W}_4) = \mathcal{W}_2, \mathcal{F}(\mathcal{W}_3) = \mathcal{W}_1, \mathcal{F}(\mathcal{W}_2) = \mathcal{W}_0, \mathcal{F}(\mathcal{W}_1) = \mathcal{W}_0, \mathcal{F}(\mathcal{W}_0) = \mathcal{W}_0$$

Key Elements

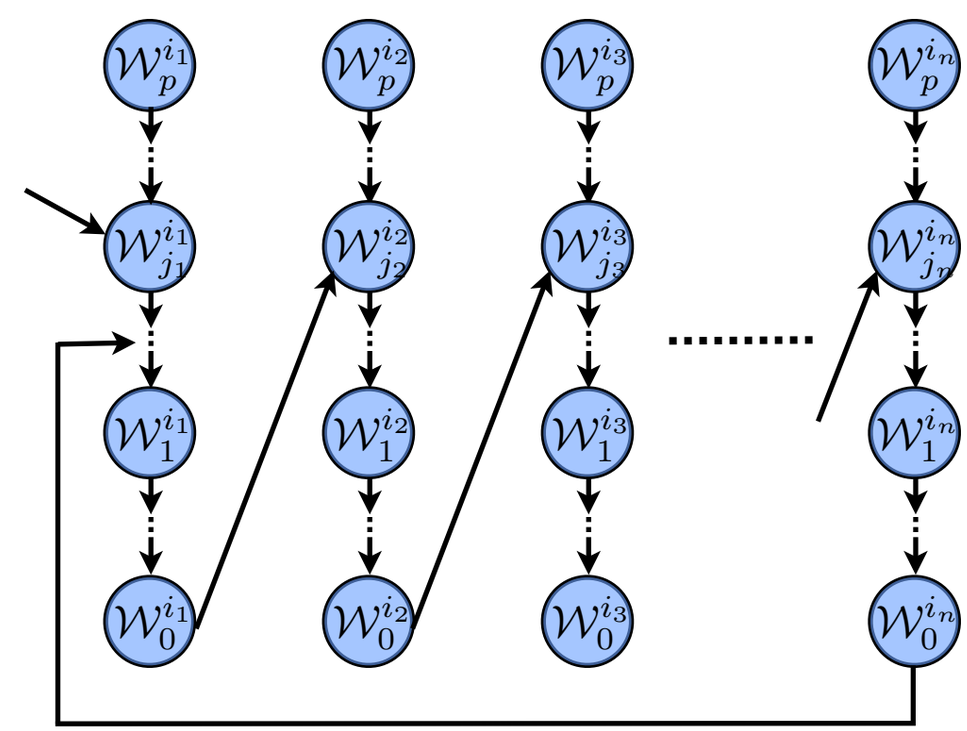
Original specification:

$$\varphi = \left(\psi_{init} \wedge \Box \psi_e^e \wedge \bigwedge_{i \in I_f} \Box \diamond \psi_{f,i}^e \right) \implies \left(\bigwedge_{i \in I_s} \Box \psi_{s,i} \wedge \bigwedge_{i \in I_g} \Box \diamond \psi_{g,i} \right)$$

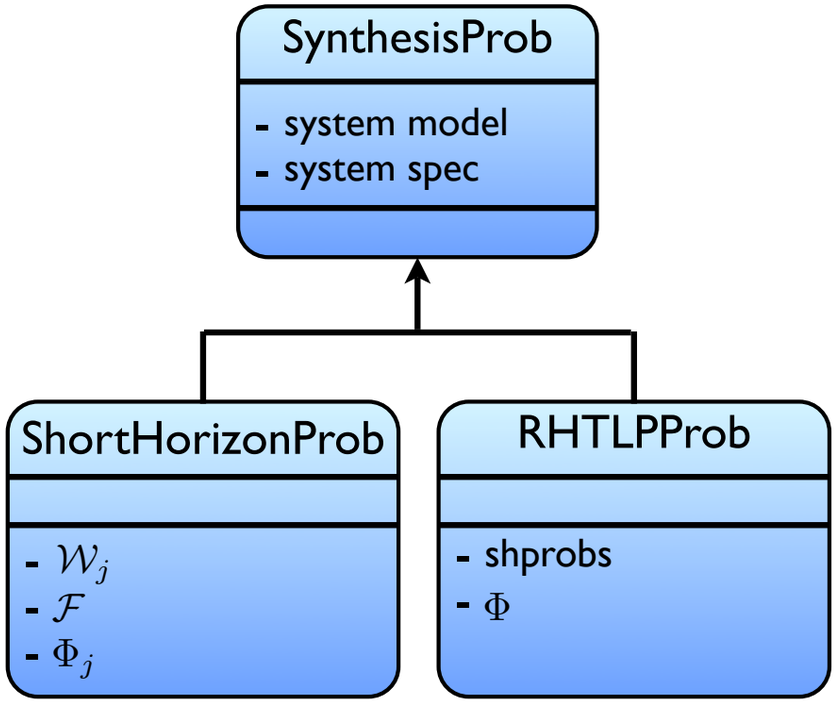
Short horizon specification:

$$\Psi_j^i = \left((\varsigma \in \mathcal{W}_j^i) \wedge \Phi \wedge \Box \psi_e^e \wedge \bigwedge_{k \in I_f} \Box \diamond \psi_{f,k}^e \right) \implies \left(\bigwedge_{k \in I_s} \Box \psi_{s,k} \wedge \Box \diamond (\varsigma \in \mathcal{F}^i(\mathcal{W}_j^i)) \wedge \Box \Phi \right)$$

- Partially ordered set $\mathcal{P}^i = (\{\mathcal{W}_0^i, \ldots, \mathcal{W}_p^i\}, \preceq_{\psi_{g,i}})$
  - $\mathcal{W}_0^i \cup \mathcal{W}_1^i \cup \ldots \cup \mathcal{W}_p^i = \mathcal{V}$
  - $\mathcal{W}_0^i$ is the set of "goal states," i.e., all cells in $\mathcal{W}_0^i$ satisfy $\psi_{g,i}$
  - $\mathcal{W}_0^i \prec_{\psi_{g,i}} \mathcal{W}_j^i, \forall j \neq 0$
- Receding horizon invariant Φ
  - $\psi_{init} \implies \Phi$ is a tautology
- Mapping $\mathcal{F}^i : \{\mathcal{W}_0^i, \ldots, \mathcal{W}_p^i\} \to \{\mathcal{W}_0^i, \ldots, \mathcal{W}_p^i\}$
  - $\mathcal{F}^i(\mathcal{W}_j^i) \prec_{\psi_{g,i}} \mathcal{W}_j^i, \forall j \neq 0$



Receding Horizon Temporal Logic Planning Problem



ShortHorizonProb

- A class for defining a short horizon problem
- Useful methods
  - *computeLocalPhi*(): automatically compute Φ that makes this short horizon problem realizable.

RHTLPProb

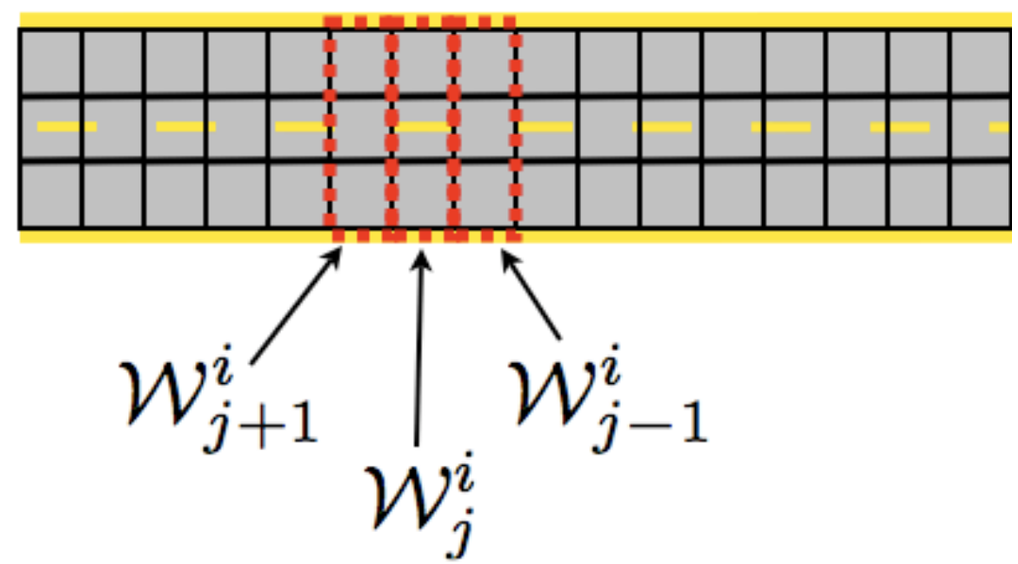
- A class for defining a receding horizon temporal logic planning problem
- Contains a collection of short-horizon problems
- Useful methods
  - *computePhi*(): automatically compute Φ for this receding horizon temporal logic planning problem if one exists.
  - *validate*(): check whether all the sufficient conditions for doing receding horizon temporal logic planning are satisfied

RHTLP example

Autonomous vehicle navigating an urban environment

Traffic rules

- No collision
- Stay in the travel lane unless there is an obstacle blocking the lane



Progress requirement

- Reach the end of the road

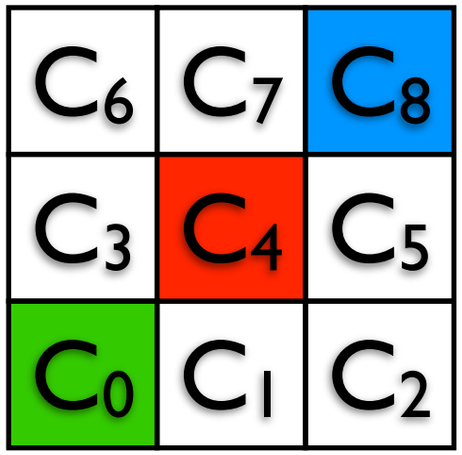
Assumptions

- Obstacle may not block a road
- Obstacle is detected before the vehicle gets too close to it
- Limited sensing range
- Obstacle does not disappear when the vehicle is in its vicinity

Link to an example: http://goo.gl/oyHvg

Computer exercise 1

Synthesize a reactive planner with the following specifications



Desired Properties

- Visit the blue cell (C_8) infinitely often
- Eventually go to the green cell (C_0) when a PARK signal is received
- Avoid an obstacle (red cell) which can be one of the C_1, C_4, C_7 cells and can move arbitrarily

Assumption

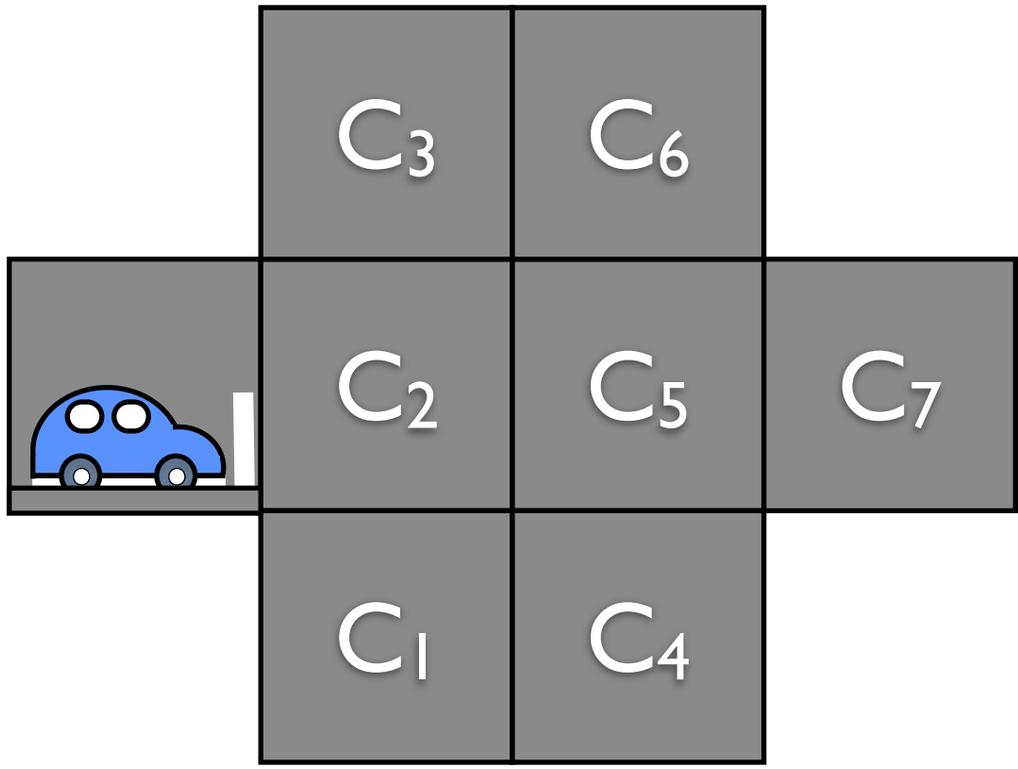
- Infinitely often, PARK signal is not received
- The obstacle always moves to an adjacent cell

Constraints (or discrete dynamics)

- The robot can only move to an adjacent cell, i.e., a cell that shares an edge with the current cell

Computer exercise 2

Synthesize intersection logic for the car with the following specification



Desired Properties

- Eventually go to C_6
- If there is a car at one of the C_3, C_4, C_7 cells at initial state, need to wait until it disappears before going through the intersection
- Go through the intersection only when C_2 and C_5 are clear
- No collision with other cars

Assumption

- ?? (find a set of “non-trivial” assumptions that render the problem realizable)

Constraint

- The robot can only move forward to an adjacent cell, i.e., a cell that shares an edge with the current cell