CALIFORNIA INSTITUTE OF TECHNOLOGY
Computing and Mathematical Sciences

ACM/EE 116

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Fall 2011 — Due: **1 Dec 2011**

Note: In the upper corner of the *second* page of your problem set, please put the number of hours that you spent on this problem set (including reading & office hours).

1. (Gubner, 11.27) Let $W(t)$ be a Wiener process with $\mathbb{E}(W^2(t)) = \sigma^2 t$. Put $Y(t) = e^{W(t)}$. Find the correlation function $R_Y(t_1, t_2) = \mathbb{E}(Y(t_1)Y(t_2))$ for $t_2 > t_1$.

2. (G&S, 13.3.3) Write down Bartlett's equation in the case of a Wiener process D having drift m and instantaneous variance 1, and solve it subject to the boundary condition $D(0) = 0$. Use the solution to compute the mean and variance of $D(t)$.

3. (G&S, 13.9.2) Let $W_t = W(t)$ denote a standard Wiener process. Write down the SDE obtained via Itō's formula for the process $Y_t = W_t^4$ and compute the expected value of Y_t.

4. (G&S, 13.12.5) Let $D = \{D(t) : t \geq 0\}$ be a diffusion process with instantaneous mean $a(x,t) = \alpha x$ and instantaneous variance $b(x,t) = \beta x$ where $\alpha, \beta > 0$. Let $D(0) = d$. Show that the moment generating function of $D(t)$ is

$$M(t, \theta) = \exp\left(\frac{2\alpha d \theta e^{\alpha t}}{\beta\theta(1 - e^{\alpha t}) + 2\alpha}.\right)$$

   (You don't have to derive this from scratch unless you want to; it is sufficient to show that this satisfies Bartlett's equation.) Find the mean and variance of $D(t)$.