CALIFORNIA INSTITUTE OF TECHNOLOGY
Computing and Mathematical Sciences

ACM/EE 116

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Fall 2011 Due: 15 Nov 2011

Note: In the upper corner of the *second* page of your problem set, please put the number of hours that you spent on this problem set (including reading & office hours).

1. (G&S 8.7.2) Let $\{Z_n\}$ be a sequence of uncorrelated real-value variables with zero means and unit variances. Suppose that $\{Y_n\}$ satisfies $Y_n = \alpha Y_{n-1} + Z_n$, $-\infty < n < \infty$ for some real α with $|\alpha| < 1$. Show that Y has autocovariance function $c(m) = \alpha^{|m|}/(1-\alpha^2)$.

   (Note: this was an optional problem on Problem Set #6; OK to turn in your same solution if you did that optional problem, but you might try Problem 6 below instead.)

2. (G&S 9.2.1, modified) Let X be a (weakly) stationary sequence with zero mean and autocovariance function $c(m)$.

   (a) Find the best linear predictor $\widehat{X}_{n+1}$ of X_{n+1} given X_n.

   (b) Find the best linear predictor $\widetilde{X}_{n+1}$ of X_{n+1} given X_n and X_{n-1}.

   (c) Consider the random process given by $X_n = (Y_n + Y_{n-1})/2$ where $\{Y_n\}$ are i.i.d. and $Y_n \sim N(0,1)$. Compute the mean square error for the predictors $\widehat{X}$ and $\widetilde{X}$.

3. (G&S 9.7.6a) Let N be a Poisson process with intensity λ and let $\alpha > 0$. Define $X(t) = N(t+\alpha) - N(t)$ for $t \geq 0$. Show that X is strongly stationary and find its mean, covariance function and spectral density function.

4. (G&S 10.1.3) Find an expression for the mass function of $N(t)$ in a renewal process whose interarrival times are Poisson distributed with parameter λ.

5. (G&S 10.6.13) Let $m(t)$ be the mean number of living individuals living at time t in an age-dependent branching process with exponential lifetimes, parameter λ, and mean family size $\nu > 1$. Prove that $m(t) = Ie^{(\nu-1)\lambda t}$, where I is the number of initial members.

Optional exercises: The following exercises may be substituted for the problems above (if you do more than the required number of problems, we'll drop problems with the lowest scores):

6. G&S Section 9.1, Exercise 1 instead of Problem 1.

7. G&S Section 9.7, Exercise 2 instead of Problem 3

8. G&S Section 10.1, Exercise 4 instead of Problem 4.

9. G&S Section 10.6, Exercise 2 instead of Problem 5.