

CALIFORNIA INSTITUTE OF TECHNOLOGY
Computing and Mathematical Sciences

ACM/EE 116

R. M. Murray, L. M. Eaton
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Problem Set #3

Issued: 11 Oct 2011
Due: 18 Oct 2011

Note: In the upper corner of the *second* page of your problem set, please put the number of hours that you spent on this problem set (including reading & office hours).

- (Gubner 4.1.7) A certain computer is equipped with a hard drive whose lifetime, measured in months, is $X \sim \exp(\lambda)$. The lifetime of the monitor (also measured in months) is $Y \sim \exp(\mu)$. Assume the lifetimes are independent.
 - Find the probability that the monitor fails during the first 2 months.
 - Find the probability that both the hard drive and the monitor fail during the first year.
 - Find the probability that either the hard drive or the monitor fails during the first year.
- (G&S 4.1.2) Find the density function of $Y = aX$, where $a > 0$ in terms of the density function of X . Use this to show that continuous random variables X and $-X$ have the same distribution function if and only if $f_X(x) = f_X(-x)$ for all $x \in \mathbb{R}$.
- (G&S 4.4.3) Let X have a uniform distribution on $[0, 1]$. For what function $g : \mathbb{R} \rightarrow \mathbb{R}$ does $Y = g(X)$ have the exponential distribution with parameter 1?
- (G&S 4.5.4) Let X and Y be independent continuous random variables. Show that

$$\mathbb{E}(g(X)h(Y)) = \mathbb{E}(g(X))\mathbb{E}(h(Y)),$$

whenever the expectations exist. If X and Y have the exponential distribution with parameter 1, find $\mathbb{E}\{\exp(\frac{1}{2}(X + Y))\}$.

- (Gubner 5.1.17) The received voltage in a digital communication system is $Z = X + Y$, where $X \sim \text{Bernoulli}(p)$ is a random message, and $Y \sim N(0, 1)$ is a Gaussian noise voltage. Assume X and Y are independent. Find the conditional distribution function $F_{Z|X}(z|i)$ for $i = 0, 1$, the distribution function $F_Z(z)$, and the density $f_Z(z)$.

Optional exercises: For students interested in a more analytical (and more challenging) set of problems, the following exercises may be substituted for the problems above (if you do more than the required number of problems, we'll drop problems with the lowest scores):

- G&S Section 4.4, Exercise 2 instead of Problem 3
- G&S Section 4.6, Exercise 2 instead of Problem 5