CALIFORNIA INSTITUTE OF TECHNOLOGY
Computing and Mathematical Sciences

ACM/EE 116

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Fall 2011

Problem Set #1

Issued: 27 Sep 2011
Due: 4 Oct 2011

Note: In the upper corner of the *second* page of your problem set, please put the number of hours that you spent on this problem set (including reading & office hours).

1. (G&S 1.1.2) Let A and B belong to some σ-field $\mathcal{F}$. Show that $\mathcal{F}$ contains the sets $A \cup B$, $A \cap B$, $A \backslash B$ and $A \triangle B$ (these symbols are defined in Table 1.1 and Appendix I of G&S).

2. (G&S 1.3.3) Six cups and saucers come in pairs: there are two cups and saucers that are red, two white, and two blue. If the cups are placed randomly onto the saucers (one each), find the probability that no cup is upon a saucer of the same color.

3. (G&S 1.4.1) Prove that $\mathbb{P}(A|B) = \mathbb{P}(B|A)P(A)/P(B)$ whenever $\mathbb{P}(A)\mathbb{P}(B) \neq 0$. Show that if $\mathbb{P}(A|B) > \mathbb{P}(A)$ then $\mathbb{P}(B|A) > \mathbb{P}(B)$.

4. (G&S 1.5.5) Show that the conditional independence of A and B given C neither implies, nor is implied by, the independence of A and B. For which events C is it the case that, for all A and B, the events A and B are independent if and only if they are conditionally independent given C?

5. (G&S 1.5.7a–c) Jane has three children, each of which is equally likely to be a boy or girl independently of the others. Define the events:

$$\begin{aligned} A &= \{\text{all of the children are of the same sex}\} \\ B &= \{\text{there is at most one boy}\} \\ C &= \{\text{the family includes a boy and a girl}\} \end{aligned}$$

   (a) Show that A is independent of B and B is independent of C.
   (b) Is A independent of C?
   (c) Do your answers for (a) and (b) remain the same if boys and girls are not equally likely?

6. (G&S 2.7.1) Each toss of a coin results in a head with probability p. The coin is tossed until the first head appears. Let X be the total number of tosses. What is $\mathbb{P}(X > m)$? Find the distribution function of the random variable X.

7. (G&S 2.7.6) Buses arrive at ten minute intervals starting at noon. A man arrives at the bus stop a random number X minutes after noon, where X has the distribution function

$$\mathbb{P}(X \le x) = \begin{cases} 0 & x < 0, \\ x/60 & 0 \le x \le 60, \\ 1 & x > 60. \end{cases}$$

   What is the probability that he waits less than five minutes for a bus?

Optional exercises: For students interested in a more analytical (and more challenging) set of problems, the following exercises may be substituted for the problems above (if you do more than the required number of problems, we'll drop problems with the lowest scores):

8. G&S Section 1.8, Exercise 3 instead of Problem 1 or 2
9. G&S Section 1.8, Exercise 14 instead of Problem 3
10. G&S Section 2.2, Exercise 2 instead of Problem 6 or 7
11. G&S Section 2.7, Exercise 13 instead of Problem 6 or 7