Optimization-Based Control

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Preface

These notes serve as a supplement to *Feedback Systems* by Åström and Murray [ÅM21] and expand on some of the topics introduced there. They are motivated by the increasing role of online optimization in feedback systems. This is a change from the traditional use of optimization in control theory for *offline* design of control laws and state estimators. Fueled by Moore's law and improvements in real-time algorithms, it is now possible to perform estimation and control design algorithms online, allowing the system to better account for nonlinearities and to adapt to changes in the underlying dynamics of the controlled process. This changes the way that we think about estimation and control since it allows much greater flexibility in the design process and more modularity and flexibility in the overall system.

Our goal in this supplement is to introduce the essential formalisms and tools required to design optimization-based controllers. Key topics include real-time trajectory generation using differential flatness, the maximum principle, dynamic programming, receding horizon optimal control, stochastic processes, Kalman filtering, moving horizon estimation, and (distributed) sensor fusion. While these topics might normally constitute separate textbooks, in this set of notes we attempt to present them in a compact way that allows them to be used in engineering design. We also briefly survey additional advanced topics through the text, with pointers to further information for interested readers.

The various mathematical techniques that are described have been implemented in the Python Control Systems Library (python-control), which is available online at `http://python-control.org`. Most chapters have a section describing the relevant python-control module that implements the topics of the chapter, and exercises are included to allow students to explore various concepts using python-control to carry out numerical calculations. Files associated with these exercises can be downloaded from the relevant portion of the *Feedback Systems* website:

`https://fbswiki.org/OBC`

This supplement has been used in a second quarter controls course at Caltech, taken by a mixture of advanced undergraduates and beginning graduate students with interest in a variety of application areas. The first half of the 10 week course focuses on trajectory generation and optimal control, ending with receding horizon control. In the second half of the course, we introduce stochastic processes and derive the Kalman filter and its various extensions, including the information filter and sensor fusion. The prerequisites for the course are based on the material covered in *Feedback Systems*, including basic knowledge in Lyapunov stability theory and observers. If needed, these topics can be inserted at the appropriate point in covering the material in this supplement.

The notation and conventions in the book follow those used in the main text. Because the notes may not be used for a standalone class, we have attempted to write each as a standalone reference for advanced topics that are introduced in *Feedback Systems*. To this end, each chapter starts with a short description of the prerequisites for the chapter and citations to the relevant literature. Advanced sections, marked by the "dangerous bend" symbol shown in the margin, contain material that requires a slightly more technical background, of the sort that would be expected of graduate students in engineering. Additional information is available on the *Feedback Systems* website:

https://fbswiki.org

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