

# Optimization-Based Control

Richard M. Murray  
Control and Dynamical Systems  
California Institute of Technology

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# Bibliography

- [AF06] M. Athans and P. L. Falb. *Optimal Control: An Introduction to the Theory and Its Applications*. Dover, 2006. Originally published in 1963.
- [ÅM08] K. J. Åström and R. M. Murray. *Feedback Systems: An Introduction for Scientists and Engineers*. Princeton University Press, 2008. Available at <http://fbsbook.org>.
- [ÅM21] K. J. Åström and R. M. Murray. *Feedback Systems: An Introduction for Scientists and Engineers*. Princeton University Press, second edition, 2021. Available at <http://fbsbook.org>.
- [Åst06] K. J. Åström. *Introduction to Stochastic Control Theory*. Dover, New York, 2006. Originally published by Academic Press, New York, 1970.
- [BBM17] F. Borrelli, A. Bemporad, and M. Morari. *Predictive Control for Linear and Hybrid Systems*. Cambridge University Press, 2017.
- [BBvB<sup>+</sup>01] K. Beck, M. Beedle, A. van Bennekum, A. Cockburn, W. Cunningham, M. Fowler, J. Grenning, J. Highsmith, A. Hunt, R. Jeffries, Jon Kern, Brian Marick, Robert C. Martin, Steve Mallor, Ken Shwaber, and Jeff Sutherland. The Agile Manifesto. Technical report, The Agile Alliance, 2001. Available at <http://agilemanifesto.org>.
- [BdTH<sup>+</sup>07] J. W. Burdick, N. du Toit, A. Howard, C. Looman, J. Ma, R. M. Murray, and T. Wongpiromsarn. Sensing, navigation and reasoning technologies for the darpa urban challenge. Technical report, California Institute of Technology, 2007. Available from <https://apps.dtic.mil/sti/citations/ADA475619>.
- [BH75] A. E. Bryson, Jr. and Y.-C. Ho. *Applied Optimal Control: Optimization, Estimation, and Control*. Wiley, New York, 1975.
- [Bro81] R. W. Brockett. Control theory and singular Riemannian geometry. In *New Directions in Applied Mathematics*, pages 11–27. Springer-Verlag, New York, 1981.
- [Bry99] A. E. Bryson. *Dynamic optimization*. Addison Wesley, 1999.
- [CHHR22] N. Correll, B. Hayes, C. Heckman, and A. Roncone. *Introduction to Autonomous Robots: Mechanisms, Sensors, Actuators, and Algorithms*. MIT Press, Cambridge, MA, 1st edition, 2022.
- [dB78] C. de Boor. *A Practical Guide to Splines*. Springer-Verlag, 1978.
- [Dra55] C. S. Draper. Flight control. *Journal Royal Aeronautical Society*, 59(July):451–477, 1955. 45th Wilber Wright Memorial Lecture.
- [FGM<sup>+</sup>21] S. Fuller, B. Greiner, J. Moore, R. Murray, R. van Paassen, and R. Yorke. The python control systems library (python-control). In *Proc. IEEE Control and Decision Conference*, 2021.

- [FLMR92] M. Fliess, J. Levine, P. Martin, and P. Rouchon. On differentially flat nonlinear systems. *Comptes Rendus des Séances de l'Académie des Sciences*, 315:619–624, 1992. Serie I.
- [FLMR95] M. Fliess, J. Levine, P. Martin, and P. Rouchon. Flatness and defect of nonlinear systems: Introductory theory and examples. *International Journal of Control*, 61(6):1327–1361, 1995.
- [Fri04] B. Friedland. *Control System Design: An Introduction to State Space Methods*. Dover, New York, 2004.
- [GMSW] P. E. Gill, W. Murray, M. A. Saunders, and M. Wright. *User's Guide for NPSOL 5.0: A Fortran Package for Nonlinear Programming*. Systems Optimization Laboratory, Stanford University, Stanford, CA 94305.
- [GS01] G. R. Grimmett and D. R. Stirzaker. *Probability and Random Processes*. Oxford University Press, third edition, 2001.
- [HO01] J. Hauser and H. Osinga. On the geometry of optimal control: The inverted pendulum example. In *American Control Conference*, 2001.
- [HP87] C. Hargraves and S. Paris. Direct trajectory optimization using nonlinear programming and collocation. *AIAA J. Guidance and Control*, 10:338–342, 1987.
- [Isi89] A. Isidori. *Nonlinear Control Systems*. Springer-Verlag, 2nd edition, 1989.
- [Jad01] A. Jadbabaie. *Nonlinear Receding Horizon Control: A Control Lyapunov Function Approach*. PhD thesis, California Institute of Technology, Control and Dynamical Systems, 2001.
- [JSK99] M. Jankovic, R. Sepulchre, and P. V. Kokotović. CLF based designs with robustness to dynamic input uncertainties. *Systems Control Letters*, 37:45–54, 1999.
- [JYH01] A. Jadbabaie, J. Yu, and J. Hauser. Unconstrained receding horizon control of nonlinear systems. *IEEE Transactions on Automatic Control*, 46(5):776–783, 2001.
- [Kal64] R. E. Kalman. When is a linear control system optimal? *J. Basic Engrg. Trans. ASME Ser. D*, 86:51–60, 1964.
- [KKK95] M. Krstić, I. Kanellakopoulos, and P. Kokotović. *Nonlinear and Adaptive Control Design*. Wiley, 1995.
- [KKM91] I. Kanellakopoulos, P. V. Kokotovic, and A. S. Morse. Systematic design of adaptive controllers for feedback linearizable systems. *IEEE Transactions on Automatic Control*, 36(11):1241–1253, 1991.
- [KV86] P. R. Kumar and P. Varaiya. *Stochastic Systems: Estimation, Identification, and Adaptive Control*. Prentice Hall, Inc., 1986.
- [LAMK17] Sikang Liu, Nikolay Atanasov, Kartik Mohta, and Vijay Kumar. Search-based motion planning for quadrotors using linear quadratic minimum time control. In *2017 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pages 2872–2879, 2017.
- [Lév10] Jean Lévine. On necessary and sufficient conditions for differential flatness. *Applicable Algebra in Engineering, Communication and Computing*, 22(1):47–90, 2010.
- [Lib10] D. Liberzon. Calculus of variations and optimal control theory: A concise introduction. Online notes, 2010. Retrieved, 16 Jan 2022.

- [LM67] E. B. Lee and L. Markus. *Foundations of Optimal Control Theory*. Robert E. Krieger Publishing Company, 1967.
- [LS95] F. L. Lewis and V. L. Syrmos. *Optimal Control*. Wiley, second edition, 1995.
- [Lue97] D. G. Luenberger. *Optimization by Vector Space Methods*. Wiley, New York, 1997.
- [LVS12] F. L. Lewis, D. L. Vrabie, and V. L. Syrmos. *Optimal Control*. John Wiley & Sons, Ltd, 2012.
- [MA73] P. J. Moylan and B. D. O. Anderson. Nonlinear regulator theory and an inverse optimal control problem. *IEEE Trans. on Automatic Control*, 18(5):460–454, 1973.
- [MDP94] P. Martin, S. Devasia, and B. Paden. A different look at output tracking—Control of a VTOL aircraft. *Automatica*, 32(1):101–107, 1994.
- [MFHM05] M. B. Milam, R. Franz, J. E. Hauser, and R. M. Murray. Receding horizon control of a vectored thrust flight experiment. *IEE Proceedings on Control Theory and Applications*, 152(3):340–348, 2005.
- [MHJ<sup>+</sup>03] R. M. Murray, J. Hauser, A. Jadbabaie, M. B. Milam, N. Petit, W. B. Dunbar, and R. Franz. Online control customization via optimization-based control. In T. Samad and G. Balas, editors, *Software-Enabled Control: Information Technology for Dynamical Systems*. IEEE Press, 2003.
- [Mil03] M. B. Milam. *Real-Time Optimal Trajectory Generation for Constrained Dynamical Systems*. PhD thesis, California Institute of Technology, 2003.
- [MM99] M. B. Milam and R. M. Murray. A testbed for nonlinear flight control techniques: The Caltech ducted fan. In *Proc. IEEE International Conference on Control and Applications*, 1999.
- [MMM00] M. B. Milam, K. Mushambi, and R. M. Murray. A computational approach to real-time trajectory generation for constrained mechanical systems. In *Proc. IEEE Control and Decision Conference*, 2000.
- [MRRS00] D. Q. Mayne, J. B. Rawlings, C. V. Rao, and P. O. M. Scokaert. Constrained model predictive control: Stability and optimality. *Automatica*, 36(6):789–814, 2000.
- [Mur96] R. M. Murray. Trajectory generation for a towed cable flight control system. In *Proc. IFAC World Congress*, 1996.
- [Mur97] R. M. Murray. Nonlinear control of mechanical systems: A Lagrangian perspective. *Annual Reviews in Control*, 21:31–45, 1997.
- [PBGM62] L. S. Pontryagin, V. G. Boltyanskii, R. V. Gamkrelidze, and E. F. Mishchenko. *The Mathematical Theory of Optimal Processes*. Wiley-Interscience, 1962. (translated from Russian).
- [PND99] J. A. Primbs, V. Nevistić, and J. C. Doyle. Nonlinear optimal control: A control Lyapunov function and receding horizon perspective. *Asian Journal of Control*, 1(1):1–11, 1999.
- [QB97] S. J. Qin and T. A. Badgwell. An overview of industrial model predictive control technology. In J.C. Kantor, C.E. Garcia, and B. Carnahan, editors, *Fifth International Conference on Chemical Process Control*, pages 232–256, 1997.
- [RM98] M. Rathinam and R. Murray. Configuration flatness of Lagrangian systems underactuated by one control. *SIAM Journal of Control and Optimization*, 36(1):164–179, 1998.

- [RMD17] J.B. Rawlings, D.Q. Mayne, and M. Diehl. *Model Predictive Control: Theory, Computation, and Design*. Nob Hill Publishing, 2017.
- [Rug90] W. J. Rugh. Analytical framework for gain scheduling. In *Proc. American Control Conference*, pages 1688–1694, 1990.
- [SC92] R. Shishko and R. G. Chamberlain. NASA systems engineering handbook. Technical report, National Aeronautics and Space Administration, 1992.
- [Sey94] H. Seywald. Trajectory optimization based on differential inclusion. *J. Guidance, Control and Dynamics*, 17(3):480–487, 1994.
- [Sha90] J. S. Shamma. Analysis of gain scheduled control for nonlinear plants. *IEEE Transactions on Automatic Control*, 35(12):898–907, 1990.
- [SJK97] R. Sepulchre, M. Jankovic, and P. V. Kokotović. *Constructive Nonlinear Control*. Springer, London, 1997.
- [Son83] E. D. Sontag. A Lyapunov-like characterization of asymptotic controllability. *SIAM Journal of Control and Optimization*, 21:462–471, 1983.
- [vNM98] M. J. van Nieuwstadt and R. M. Murray. Rapid hover to forward flight transitions for a thrust vectored aircraft. *Journal of Guidance, Control, and Dynamics*, 21(1):93–100, 1998.
- [vNRM98] M. van Nieuwstadt, M. Rathinam, and R. M. Murray. Differential flatness and absolute equivalence. *SIAM Journal of Control and Optimization*, 36(4):1225–1239, 1998.

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