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# Optimization-Based Control

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## Index

- algebraic Riccati equation, 2-11
- bang-bang control, 2-9
- cost function, 2-1
- costate variables, 2-6
- differential flatness, 1-8
- error system, 1-3
- expectation, 4-4
- extended Kalman filter, 5-4
- extremum, 2-4
- feasible trajectory, 2-4
- feedforward, 1-4
- final cost, 2-5
- finite horizon, 2-5
- gain scheduling, 1-4
- Hamiltonian, 2-5
- Harrier AV-8B aircraft, 2-14
- infinite horizon, 2-5
- information filter, 6-6
- information matrix, 6-6
- innovations process, 5-2
- integral cost, 2-5
- Kalman filter
  - recursive form, 5-2
- Lagrange multipliers, 2-3
- linear quadratic, 2-5
- linearization, 1-3
- matrix differential equation, 2-10
- mean, 4-4
- noise intensity, 4-10
- optimal control problem, 2-4
- optimal value, 2-1
- optimization, 2-1
- Ornstein-Uhlenbeck process, 4-9

**Note:** Under construction! The indexing for the text has not yet been done and so this index contains an incomplete and unedited set of terms.