Feedback Systems

An Introduction for Scientists and Engineers
SECOND EDITION

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Contents

Preface to the Second Edition

The second edition of *Feedback Systems* contains a variety of changes that are based on feedback on the first edition, particularly in its use for introductory courses in control. One of the primary comments from users of the text was that the use of control tools for design purposes occured only after several chapters of analytical tools, leaving the instructor having to try to convince students that the techniques would soon be useful. In our own teaching, we find that we often use design examples in the first few weeks of the class and use this to motivate the various techniques that follow. This approach has been particularly useful in engineering courses, where students are often eager to apply the tools to examples as part of gaining insight into the methods. We also found that universities that have a laboratory component attached to their controls class need to introduce some basic design techniques early, so that students can be implementing control laws in the laboratory in the early weeks of the course.

To help emphasize this more design-oriented flow, we have rearranged the material in the first third of the book. Chapter 3 in the original text, which introduced a number of examples in some detail, has been moved to an appendix, where it can be assigned as needed when specific examples arise. In its place, we have put a new chapter on “Feedback Principles” that illustrates some simple design principles and tools that can be used to show students what types of problems can be solved using feedback. This new chapter uses simple models, simulations and elementary analysis techniques, so that it should be accessible to students from a variety of engineering and scientific backgrounds. For courses in which students have already been exposed to the basic ideas of feedback, perhaps in an earlier discipline-specific course, this new chapter can easily be skipped without any loss of continuity.

In addition to this relatively large change in the first portion of the book, we have also taken the opportunity to make other smaller changes based on the feedback we have received from early adopters of the text.

Add a summary of changes here **RMM**

- Overview material on control logic
- Change disturbance and noise signals to v and w, respectively.

We are indebted to numerous individuals who have taught out of the text and sent us feedback on changes that would better serve their needs. In addition to the many individuals listed in the preface to the first edition, we would like to also thank Constantine Caramanis, Clancy Rowley and André Tits for their feedback and insights.

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Add more on : assume familiarity with linear algebra and ODEs. Encourage students to look up unfamiliar terms. **RMM**

Preface to the First Edition

This book provides an introduction to the basic principles and tools for the design and analysis of feedback systems. It is intended to serve a diverse audience of scientists and engineers who are interested in understanding and utilizing feedback in physical, biological, information and social systems. We have attempted to keep the mathematical prerequisites to a minimum while being careful not to sacrifice rigor in the process. We have also attempted to make use of examples from a variety of disciplines, illustrating the generality of many of the tools while at the same time showing how they can be applied in specific application domains.

A major goal of this book is to present a concise and insightful view of the current knowledge in feedback and control systems. The field of control started by teaching everything that was known at the time and, as new knowledge was acquired, additional courses were developed to cover new techniques. A consequence of this evolution is that introductory courses have remained the same for many years, and it is often necessary to take many individual courses in order to obtain a good perspective on the field. In developing this book, we have attempted to condense the current knowledge by emphasizing fundamental concepts. We believe that it is important to understand why feedback is useful, to know the language and basic mathematics of control and to grasp the key paradigms that have been developed over the past half century. It is also important to be able to solve simple feedback problems using back-of-the-envelope techniques, to recognize fundamental limitations and difficult control problems and to have a feel for available design methods.

This book was originally developed for use in an experimental course at Caltech involving students from a wide set of backgrounds. The course was offered to undergraduates at the junior and senior levels in traditional engineering disciplines, as well as first- and second-year graduate students in engineering and science. This latter group included graduate students in biology, computer science and physics. Over the course of several years, the text has been classroom tested at Caltech and at Lund University, and the feedback from many students and colleagues has been incorporated to help improve the readability and accessibility of the material.

Because of its intended audience, this book is organized in a slightly unusual fashion compared to many other books on feedback and control. In particular, we introduce a number of concepts in the text that are normally reserved for second-year courses on control and hence often not available to students who are not control systems majors. This has been done at the expense of certain traditional topics, which we felt that the astute student could learn independently and are often explored through the exercises. Examples of topics that we have included are nonlinear dynamics, Lyapunov stability analysis, the matrix exponential, reachability and observability, and fundamental limits of performance and robustness. Topics that we have deemphasized include root locus techniques, lead/lag compensation and detailed rules for generating Bode and Nyquist plots by hand.

Several features of the book are designed to facilitate its dual function as a basic engineering text and as an introduction for researchers in natural, information and social sciences. The bulk of the material is intended to be used regardless of the audience and covers the core principles and tools in the analysis and design of feedback systems. Advanced sections, marked by the "dangerous bend" symbol shown here, contain material that requires a slightly more technical background, of the sort that would be expected of senior undergraduates in engineering. A few sections are marked by two dangerous bend symbols and are intended for readers with more specialized backgrounds, identified at the beginning of the section. To limit the length of the text, several standard results and extensions are given in the exercises, with appropriate hints toward their solutions.

To further augment the printed material contained here, a companion web site has been developed and is available from the publisher's web page:

http://www.cds.caltech.edu/∼murray/amwiki

The web site contains a database of frequently asked questions, supplemental examples and exercises, and lecture material for courses based on this text. The material is organized by chapter and includes a summary of the major points in the text as well as links to external resources. The web site also contains the source code for many examples in the book, as well as utilities to implement the techniques described in the text. Most of the code was originally written using MATLAB M-files but was also tested with LabView MathScript to ensure compatibility with both packages. Many files can also be run using other scripting languages such as Octave, SciLab, SysQuake and Xmath.

The first half of the book focuses almost exclusively on state space control systems. We begin in Chapter 2 with a description of modeling of physical, biological and information systems using ordinary differential equations and difference equations. Chapter 3[1] presents a number of examples in some detail, primarily as a reference for problems that will be used throughout the text. Following this, Chapter 4 looks at the dynamic behavior of models, including definitions of stability and more complicated nonlinear behavior. We provide advanced sections in this chapter on Lyapunov stability analysis because we find that it is useful in a broad array of applications and is frequently a topic that is not introduced until later in one's studies.

The remaining three chapters of the first half of the book focus on linear systems, beginning with a description of input/output behavior in Chapter 5. In Chapter 6, we formally introduce feedback systems by demonstrating how state space control laws can be designed. This is followed in Chapter 7 by material on output

[1]Now Appendix **??**

feedback and estimators. Chapters 6 and 7 introduce the key concepts of reachability and observability, which give tremendous insight into the choice of actuators and sensors, whether for engineered or natural systems.

The second half of the book presents material that is often considered to be from the field of “classical control.” This includes the transfer function, introduced in Chapter 8, which is a fundamental tool for understanding feedback systems. Using transfer functions, one can begin to analyze the stability of feedback systems using frequency domain analysis, including the ability to reason about the closed loop behavior of a system from its open loop characteristics. This is the subject of Chapter 9, which revolves around the Nyquist stability criterion.

In Chapters 10 and 11, we again look at the design problem, focusing first on proportional-integral-derivative (PID) controllers and then on the more general process of loop shaping. PID control is by far the most common design technique in control systems and a useful tool for any student. The chapter on frequency domain design introduces many of the ideas of modern control theory, including the sensitivity function. In Chapter 12, we combine the results from the second half of the book to analyze some of the fundamental trade-offs between robustness and performance. This is also a key chapter illustrating the power of the techniques that have been developed and serving as an introduction for more advanced studies.

The book is designed for use in a 10- to 15-week course in feedback systems that provides many of the key concepts needed in a variety of disciplines. For a 10-week course, Chapters 1–2, 4–6 and 8–11 can each be covered in a week’s time, with the omission of some topics from the final chapters. A more leisurely course, spread out over 14–15 weeks, could cover the entire book, with 2 weeks on modeling (Chapters 2 and **??**)—particularly for students without much background in ordinary differential equations—and 2 weeks on robust performance (Chapter 12).

The mathematical prerequisites for the book are modest and in keeping with our goal of providing an introduction that serves a broad audience. We assume familiarity with the basic tools of linear algebra, including matrices, vectors and eigenvalues. These are typically covered in a sophomore-level course on the subject, and the textbooks by Apostol [7], Arnold [10] and Strang [112] can serve as good references. Similarly, we assume basic knowledge of differential equations, including the concepts of homogeneous and particular solutions for linear ordinary differential equations in one variable. Apostol [7] and Boyce and DiPrima [31] cover this material well. Finally, we also make use of complex numbers and functions and, in some of the advanced sections, more detailed concepts in complex variables that are typically covered in a junior-level engineering or physics course in mathematical methods. Apostol [6] or Stewart [111] can be used for the basic material, with Ahlfors [3], Marsden and Hoffman [84] or Saff and Snider [102] being good references for the more advanced material. We have chosen not to include appendices summarizing these various topics since there are a number of good books available.

One additional choice that we felt was important was the decision not to rely on a knowledge of Laplace transforms in the book. While their use is by far the most common approach to teaching feedback systems in engineering, many students in the natural and information sciences may lack the necessary mathematical background. Since Laplace transforms are not required in any essential way, we have included them only in an advanced section intended to tie things together for students with that background. Of course, we make tremendous use of *transfer functions*, which we introduce through the notion of response to exponential inputs, an approach we feel is more accessible to a broad array of scientists and engineers. For classes in which students have already had Laplace transforms, it should be quite natural to build on this background in the appropriate sections of the text.

Acknowledgments

The authors would like to thank the many people who helped during the preparation of this book. The idea for writing this book came in part from a report on future directions in control [92] to which Stephen Boyd, Roger Brockett, John Doyle and Gunter Stein were major contributors. Kristi Morgansen and Hideo Mabuchi helped teach early versions of the course at Caltech on which much of the text is based, and Steve Waydo served as the head TA for the course taught at Caltech in 2003–2004 and provided numerous comments and corrections. Charlotta Johnsson and Anton Cervin taught from early versions of the manuscript in Lund in 2003–2007 and gave very useful feedback. Other colleagues and students who provided feedback and advice include Leif Andersson, John Carson, K. Mani Chandy, Michel Charpentier, Domitilla Del Vecchio, Kate Galloway, Per Hagander, Toivo Henningsson Perby, Joseph Hellerstein, George Hines, Tore Hägglund, Cole Lepine, Anders Rantzer, Anders Robertsson, Dawn Tilbury and Francisco Zabala. The reviewers for Princeton University Press and Tom Robbins at NI Press also provided valuable comments that significantly improved the organization, layout and focus of the book. Our editor, Vickie Kearn, was a great source of encouragement and help throughout the publishing process. Finally, we would like to thank Caltech, Lund University and the University of California at Santa Barbara for providing many resources, stimulating colleagues and students, and pleasant working environments that greatly aided in the writing of this book.

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Chapter One
Introduction

> *Feedback is a central feature of life. The process of feedback governs how we grow, respond to stress and challenge, and regulate factors such as body temperature, blood pressure and cholesterol level. The mechanisms operate at every level, from the interaction of proteins in cells to the interaction of organisms in complex ecologies.*
>
> M. B. Hoagland and B. Dodson, *The Way Life Works*, 1995 [61].

In this chapter we provide an introduction to the basic concept of *feedback* and the related engineering discipline of *control*. We focus on both historical and current examples, with the intention of providing the context for current tools in feedback and control. Much of the material in this chapter is adapted from [92], and the authors gratefully acknowledge the contributions of Roger Brockett and Gunter Stein to portions of this chapter.

Further Reading

The material in this section draws heavily from the report of the Panel on Future Directions on Control, Dynamics and Systems [92]. Several additional papers and reports have highlighted the successes of control [95] and new vistas in control [33, 77, 120]. The early development of control is described by Mayr [86] and in the books by Bennett [20, 21], which cover the period 1800–1955. A fascinating examination of some of the early history of control in the United States has been written by Mindell [89]. A popular book that describes many control concepts across a wide range of disciplines is *Out of Control* by Kelly [74]. There are many textbooks available that describe control systems in the context of specific disciplines. For engineers, the textbooks by Franklin, Powell and Emami-Naeini [49], Dorf and Bishop [39], Kuo and Golnaraghi [79] and Seborg, Edgar and Mellichamp [104] are widely used. More mathematically oriented treatments of control theory include Sontag [108] and Lewis [80]. The book by Hellerstein et al. [60] provides a description of the use of feedback control in computing systems. A number of books look at the role of dynamics and feedback in biological systems, including Milhorn [88] (now out of print), J. D. Murray [91] and Ellner and Guckenheimer [45]. The book by Fradkov [47] and the tutorial article by Bechhoefer [18] cover many specific topics of interest to the physics community.

Systems that combine continuous feedback with logic and sequencing called hybrid system [], their theory is outside the scope of this book. It is, however, very common that practical control systems combine feedback control with logic sequencing and selectors, many examples are given in [13].

KJ: We should add reference on hybrid systems

Chapter Two
System Modeling

... I asked Fermi whether he was not impressed by the agreement between our calculated numbers and his measured numbers. He replied, "How many arbitrary parameters did you use for your calculations?" I thought for a moment about our cut-off procedures and said, "Four." He said, "I remember my friend Johnny von Neumann used to say, with four parameters I can fit an elephant, and with five I can make him wiggle his trunk."

Freeman Dyson on describing the predictions of his model for meson-proton scattering to Enrico Fermi in 1953 [43].

A model is a precise representation of a system's dynamics used to answer questions via analysis and simulation. The model we choose depends on the questions we wish to answer, and so there may be multiple models for a single dynamical system, with different levels of fidelity depending on the phenomena of interest. In this chapter we provide an introduction to the concept of modeling and present some basic material on two specific methods commonly used in feedback and control systems: differential equations and difference equations.

Further Reading

Modeling is ubiquitous in engineering and science and has a long history in applied mathematics. For example, the Fourier series was introduced by Fourier when he modeled heat conduction in solids [46]. Models of dynamics have been developed in many different fields, including mechanics [9, 53], heat conduction [36], fluids [26], vehicles [1, 27, 44], robotics [93, 109], circuits [57], power systems [78], acoustics [22] and micromechanical systems [105]. Control theory requires modeling from many different domains, and most control theory texts contain several chapters on modeling using ordinary differential equations and difference equations (see, for example, [49]). A classic book on the modeling of physical systems, especially mechanical, electrical and thermofluid systems, is Cannon [35]. The book by Aris [8] is highly original and has a detailed discussion of the use of dimension-free variables. Two of the authors' favorite books on modeling of biological systems are J. D. Murray [91] and Wilson [119].

Chapter Three
Feedback Principles

Feedback - it is the fundamental principle that underlies all self-regulating systems, not only machines but also the processes of life and the tides of human affairs.

A. Tustin, *Feedback* 1952, [116].

This chapter presents examples that illustrate fundamental properties of feedback: disturbance attenuation, command signal following, robustness and shaping of behavior. Simple methods for analysis and design of low order systems are introduced. After reading this chapter, readers should have some insight inot the power of feedback and they should be able to design simple feedback systems and perform laboratory experiments.

Further Reading

The books by Bennett [20, 21] and Mindel [89, 90] give interesting perspective on the development of control. Much of the material touched upon in this chapter is classical control see [66], [37] and [115].The notion of controllers with two degrees of freedom was introduced by Horowitz [62]. The analysis will be elaborated in the rest of the book. Transfer functions and other descriptions of dynamics are discussed in Chapters 5 and 8, methods to investigate stability in Chapter 9. The simple method to find parameters of controllers based on matching of coefficients of the closed loop characteristic polynomial is developed further in Chapters **??**, **??** and 12. Feedforward control is discussed in Section **??**.

Chapter Four
Dynamic Behavior

It Don't Mean a Thing If It Ain't Got That Swing.
Duke Ellington (1899–1974)

In this chapter we present a broad discussion of the behavior of dynamical systems focused on systems modeled by nonlinear differential equations. This allows us to consider equilibrium points, stability, limit cycles and other key concepts in understanding dynamic behavior. We also introduce some methods for analyzing the global behavior of solutions.

Further Reading

The field of dynamical systems has a rich literature that characterizes the possible features of dynamical systems and describes how parametric changes in the dynamics can lead to topological changes in behavior. Readable introductions to dynamical systems are given by Strogatz [113] and the highly illustrated text by Abraham and Shaw [2]. More technical treatments include Andronov, Vitt and Khaikin [5], Guckenheimer and Holmes [56] and Wiggins [118]. For students with a strong interest in mechanics, the texts by Arnold [10] and Marsden and Ratiu [85] provide an elegant approach using tools from differential geometry. Finally, good treatments of dynamical systems methods in biology are given by Wilson [119] and Ellner and Guckenheimer [45]. There is a large literature on Lyapunov stability theory, including the classic texts by Malkin [83], Hahn [59] and Krasovski [76]. We highly recommend the comprehensive treatment by Khalil [75].

Chapter Five
Linear Systems

Few physical elements display truly linear characteristics. For example the relation between force on a spring and displacement of the spring is always nonlinear to some degree. The relation between current through a resistor and voltage drop across it also deviates from a straight-line relation. However, if in each case the relation is reasonably *linear, then it will be found that the system behavior will be very close to that obtained by assuming an ideal, linear physical element, and the analytical simplification is so enormous that we make linear assumptions wherever we can possibly do so in good conscience.*

Robert H. Cannon, *Dynamics of Physical Systems*, 1967 [35].

In Chapters 2–4 we considered the construction and analysis of differential equation models for dynamical systems. In this chapter we specialize our results to the case of linear, time-invariant input/output systems. Two central concepts are the matrix exponential and the convolution equation, through which we can completely characterize the behavior of a linear system. We also describe some properties of the input/output response and show how to approximate a nonlinear system by a linear one.

Further Reading

The majority of the material in this chapter is classical and can be found in most books on dynamics and control theory, including early works on control such as James, Nichols and Phillips [66] and more recent textbooks such as Dorf and Bishop [39], Franklin, Powell and Emami-Naeini [49] and Ogata [98]. An excellent presentation of linear systems based on the matrix exponential is given in the book by Brockett [32], a more comprehensive treatment is given by Rugh [101] and an elegant mathematical treatment is given in Sontag [108]. Material on feedback linearization can be found in books on nonlinear control theory such as Isidori [64] and Khalil [75]. The idea of characterizing dynamics by considering the responses to step inputs is due to Heaviside, he also introduced an operator calculus to analyze linear systems. The unit step is therefore also called the *Heaviside step function*. Analysis of linear systems was simplified significantly, but Heaviside's work was heavily criticized because of lack of mathematical rigor, as described in the biography by Nahin [94]. The difficulties were cleared up later by the mathematician Laurent Schwartz who developed *distribution theory* in the late 1940s. In engineering, linear systems have traditionally been analyzed using Laplace transforms as described in Gardner and Barnes [51]. Use of the matrix exponential started with developments of control theory in the 1960s, strongly stimulated by a textbook by Zadeh and Desoer [122]. Use of matrix techniques expanded rapidly when the powerful methods of numeric linear algebra were packaged in programs like LabVIEW, MATLAB and Mathematica.

We should reference Gantmacher someplace; perhaps here? (from KJA, Jul 08) **RMM**

Chapter Six
State Feedback

Intuitively, the state may be regarded as a kind of information storage or memory or accumulation of past causes. We must, of course, demand that the set of internal states Σ be sufficiently rich to carry all information about the past history of Σ to predict the effect of the past upon the future. We do not insist, however, that the state is the least *such information although this is often a convenient assumption.*

R. E. Kalman, P. L. Falb and M. A. Arbib, *Topics in Mathematical System Theory*, 1969 [72].

This chapter describes how the feedback of a system's state can be used to shape the local behavior of a system. The concept of reachability is introduced and used to investigate how to design the dynamics of a system through assignment of its eigenvalues. In particular, it will be shown that under certain conditions it is possible to assign the system eigenvalues arbitrarily by appropriate feedback of the system state.

Further Reading

The importance of state models and state feedback was discussed in the seminal paper by Kalman [68], where the state feedback gain was obtained by solving an optimization problem that minimized a quadratic loss function. The notions of reachability and observability (Chapter 7) are also due to Kalman [70] (see also [52, 73]). Kalman defines controllability and reachability as the ability to reach the origin and an arbitrary state, respectively [72]. We note that in most textbooks the term "controllability" is used instead of "reachability," but we prefer the latter term because it is more descriptive of the fundamental property of being able to reach arbitrary states. Most undergraduate textbooks on control contain material on state space systems, including, for example, Franklin, Powell and Emami-Naeini [49] and Ogata [98]. Friedland's textbook [50] covers the material in the previous, current and next chapter in considerable detail, including the topic of optimal control.

Chapter Seven
Output Feedback

One may separate the problem of physical realization into two stages: computation of the "best approximation" $\hat{x}(t_1)$ of the state from knowledge of $y(t)$ for $t \leq t_1$ and computation of $u(t_1)$ given $\hat{x}(t_1)$.

R. E. Kalman, "Contributions to the Theory of Optimal Control," 1960 [68].

In this chapter we show how to use output feedback to modify the dynamics of the system through the use of observers. We introduce the concept of observability and show that if a system is observable, it is possible to recover the state from measurements of the inputs and outputs to the system. We then show how to design a controller with feedback from the observer state. An important concept is the separation principle quoted above, which is also proved. The structure of the controllers derived in this chapter is quite general and is obtained by many other design methods.

Further Reading

The notion of observability is due to Kalman [70] and, combined with the dual notion of reachability, it was a major stepping stone toward establishing state space control theory beginning in the 1960s. The observer first appeared as the Kalman filter, in the paper by Kalman [69] on the discrete-time case and Kalman and Bucy [71] on the continuous-time case. Kalman also conjectured that the controller for output feedback could be obtained by combining a state feedback with an observer; see the quote in the beginning of this chapter. This result was formally proved by Josep and Tou [67] and Gunckel and Franklin [58]. The combined result is known as the linear quadratic Gaussian control theory; a compact treatment is given in the books by Anderson and Moore [4] and Åström [12]. Much later it was shown that solutions to robust control problems also had a similar structure but with different ways of computing observer and state feedback gains [42]. The general controller structure discussed in Section **??**, which combines feedback and feedforward, was described by Horowitz in 1963 [62]. The particular form in Figure **??** appeared in [14], which also treats digital implementation of the controller. The hypothesis that motion control in humans is based on a combination of feedback and feedforward was proposed by Ito in 1970 [65].

Chapter Eight
Transfer Functions

The typical regulator system can frequently be described, in essentials, by differential equations of no more than perhaps the second, third or fourth order. . . . In contrast, the order of the set of differential equations describing the typical negative feedback amplifier used in telephony is likely to be very much greater. As a matter of idle curiosity, I once counted to find out what the order of the set of equations in an amplifier I had just designed would have been, if I had worked with the differential equations directly. It turned out to be 55.

Hendrik Bode, 1960 [30].

This chapter introduces the concept of the *transfer function*, which is a compact description of the input/output relation for a linear system. Combining transfer functions with block diagrams gives a powerful method for dealing with complex linear systems. The relationship between transfer functions and other descriptions of system dynamics is also discussed.

Further Reading

The idea of characterizing a linear system by its steady-state response to sinusoids was introduced by Fourier in his investigation of heat conduction in solids [46]. Much later, it was used by the electrical engineer Steinmetz who introduced the $i\omega$ method for analyzing electrical circuits. Transfer functions were introduced via the Laplace transform by Gardner Barnes [51], who also used them to calculate the response of linear systems. The Laplace transform was very important in the early phase of control because it made it possible to find transients via tables (see, e.g., [66]). Combined with block diagrams, transfer functions and Laplace transforms provided powerful techniques for dealing with complex systems. Calculation of responses based on Laplace transforms is less important today, when responses of linear systems can easily be generated using computers. There are many excellent books on the use of Laplace transforms and transfer functions for modeling and analysis of linear input/output systems. Traditional texts on control such as [39], [49] and [98] are representative examples. Pole/zero cancellation was one of the mysteries of early control theory. It is clear that common factors can be canceled in a rational function, but cancellations have system theoretical consequences that were not clearly understood until Kalman's decomposition of a linear system was introduced [73]. In the following chapters, we will use transfer functions extensively to analyze stability and to describe model uncertainty.

Chapter Nine
Frequency Domain Analysis

Mr. Black proposed a negative feedback repeater and proved by tests that it possessed the advantages which he had predicted for it. In particular, its gain was constant to a high degree, and it was linear enough so that spurious signals caused by the interaction of the various channels could be kept within permissible limits. For best results the feedback factor $\mu\beta$ had to be numerically much larger than unity. The possibility of stability with a feedback factor larger than unity was puzzling.

Harry Nyquist, "The Regeneration Theory," 1956 [97].

In this chapter we study how the stability and robustness of closed loop systems can be determined by investigating how sinusoidal signals of different frequencies propagate around the feedback loop. This technique allows us to reason about the closed loop behavior of a system through the frequency domain properties of the open loop transfer function. The Nyquist stability theorem is a key result that provides a way to analyze stability and introduce measures of degrees of stability.

Further Reading

Nyquist's original paper giving his now famous stability criterion was published in the *Bell Systems Technical Journal* in 1932 [96]. More accessible versions are found in the book [19], which also includes other interesting early papers on control. Nyquist's paper is also reprinted in an IEEE collection of seminal papers on control [16]. Nyquist used $+1$ as the critical point, but Bode changed it to -1, which is now the standard notation. Interesting perspectives on early developments are given by Black [25], Bode [30] and Bennett [21]. Nyquist did a direct calculation based on his insight into the propagation of sinusoidal signals through systems; he did not use results from the theory of complex functions. The idea that a short proof can be given by using the principle of variation of the argument is presented in the delightful book by MacColl [81]. Bode made extensive use of complex function theory in his book [29], which laid the foundation for frequency response analysis where the notion of minimum phase was treated in detail. A good source for complex function theory is the classic by Ahlfors [3]. Frequency response analysis was a key element in the emergence of control theory as described in the early texts by James et al. [66], Brown and Campbell [34] and Oldenburger [99], and it became one of the cornerstones of early control theory. Frequency response methods underwent a resurgence when robust control emerged in the 1980s, as will be discussed in Chapter 12.

Chapter Ten
PID Control

Based on a survey of over eleven thousand controllers in the refining, chemicals and pulp and paper industries, 97% of regulatory controllers utilize PID feedback.

L. Desborough and R. Miller, 2002 [38].

This chapter treats the basic properties of proportional-integral-derivative (PID) control and the methods for choosing the parameters of the controllers. We also analyze the effects of actuator saturation and time delay, two important features of many feedback systems, and describe methods for compensating for these effects. Finally, we will discuss the implementation of PID controllers as an example of how to implement feedback control systems using analog or digital computation.

Further Reading

The history of PID control is very rich and stretches back to the beginning of the foundation of control theory. Very readable treatments are given by Bennett [20, 21] and Mindel [89]. The Ziegler–Nichols rules for tuning PID controllers, first presented in 1942 [125], were developed based on extensive experiments with pneumatic simulators and Vannevar Bush's differential analyzer at MIT. An interesting view of the development of the Ziegler–Nichols rules is given in an interview with Ziegler [28]. An industrial perspective on PID control is given in [23], [106] and [121] and in the paper [38] cited in the beginning of this chapter. A comprehensive presentation of PID control is given in [13]. Interactive learning tools for PID control can be downloaded from http://www.calerga.com/contrib.

Chapter Eleven
Frequency Domain Design

Sensitivity improvements in one frequency range must be paid for with sensitivity deteriorations in another frequency range, and the price is higher if the plant is open-loop unstable. This applies to every controller, no matter how it was designed.

Gunter Stein in the inaugural IEEE Bode Lecture, 1989 [110].

In this chapter we continue to explore the use of frequency domain techniques with a focus on the design of feedback systems. We begin with a more thorough description of the performance specifications for control systems and then introduce the concept of "loop shaping" as a mechanism for designing controllers in the frequency domain. We also introduce some fundamental limitations to performance for systems with time delays and right half-plane poles and zeros.

Further Reading

Design by loop shaping was a key element in the early development of control, and systematic design methods were developed; see James, Nichols and Phillips [66], Chestnut and Mayer [37], Truxal [115]† and Thaler [114]. Loop shaping is also treated in standard textbooks such as Franklin, Powell and Emami-Naeini [49], Dorf and Bishop [39], Kuo and Golnaraghi [79] and Ogata [98]. Systems with two degrees of freedom were developed by Horowitz [62], who also discussed the limitations of poles and zeros in the right half-plane. Fundamental results on limitations are given in Bode [29]; more recent presentations are found in Goodwin, Graebe and Salgado [54]. The treatment in Section **??** is based on [11]. Much of the early work was based on the loop transfer function; the importance of the sensitivity functions appeared in connection with the development in the 1980s that resulted in H_∞ design methods. A compact presentation is given in the texts by Doyle, Francis and Tannenbaum [41] and Zhou, Doyle and Glover [124]. Loop shaping was integrated with the robust control theory in McFarlane and Glover [87] and Vinnicombe [117]. Comprehensive treatments of control system design are given in Maciejowski [82] and Goodwin, Graebe and Salgado [54].

RMM: check spelling

Chapter Twelve
Robust Performance

However, by building an amplifier whose gain is deliberately made, say 40 decibels higher than necessary (10000 fold excess on energy basis), and then feeding the output back on the input in such a way as to throw away that excess gain, it has been found possible to effect extraordinary improvement in constancy of amplification and freedom from non-linearity.

Harold S. Black, "Stabilized Feedback Amplifiers," 1934 [24].

This chapter focuses on the analysis of robustness of feedback systems, a vast topic for which we provide only an introduction to some of the key concepts. We consider the stability and performance of systems whose process dynamics are uncertain and derive fundamental limits for robust stability and performance. To do this we develop ways to describe uncertainty, both in the form of parameter variations and in the form of neglected dynamics. We also briefly mention some methods for designing controllers to achieve robust performance.

Further Reading

The topic of robust control is a large one, with many articles and textbooks devoted to the subject. Robustness was a central issue in classical control as described in Bode's classical book [29]. Robustness was deemphasized in the euphoria of the development of design methods based on optimization. The strong robustness of controllers based on state feedback, shown by Anderson and Moore [4], contributed to the optimism. The poor robustness of output feedback was pointed out by Rosenbrock [100], Horowitz [63] and Doyle [40] and resulted in a renewed interest in robustness. A major step forward was the development of design methods where robustness was explicitly taken into account, such as the seminal work of Zames [123]. Robust control was originally developed using powerful results from the theory of complex variables, which gave controllers of high order. A major breakthrough was made by Doyle, Glover, Khargonekar and Francis [42], who showed that the solution to the problem could be obtained using Riccati equations and that a controller of low order could be found. This paper led to an extensive treatment of H_∞ control, including books by Francis [48], McFarlane and Glover [87], Doyle, Francis and Tannenbaum [41], Green and Limebeer [55], Zhou, Doyle and Glover [124], Skogestand and Postlethwaite [107] and Vinnicombe [117]. A major advantage of the theory is that it combines much of the intuition from servomechanism theory with sound numerical algorithms based on numerical linear algebra and optimization. The results have been extended to nonlinear systems by treating the design problem as a game where the disturbances are generated by an adversary, as described in the book by Basar and Bernhard [17]. Gain scheduling and adaptation are discussed in the book by Åström and Wittenmark [15].

Chapter Thirteen
Implementation

In this chapter we briefly describe some of the issues involved in implementing control systems, focusing on computer-controlled systems.

Changes to make: RMM

- Add chapter quote
- Goal: what feedback systems *look* like: eng, bio, markets
- Simplify somewhat; currently too elaborate
- Add a discussion of how to discretize controllers

Further Reading

The early history of control systems is described very nicely in the book by Mindell [89].

There are many excellent textbooks on computer control systems that provide important details for modern implementation of control laws. The text by Åström and Wittenmark [14] focuses on implementation of digital systems, including most of the topics described in Section **??**.

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