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# Feedback Systems

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*An Introduction for Scientists and Engineers*  
*SECOND EDITION*

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## Preface to the Second Edition

The second edition of *Feedback Systems* contains a variety of changes that are based on feedback on the first edition, particularly in its use for introductory courses in control. One of the primary comments from users of the text was that the use of control tools for design purposes occurred only after several chapters of analytical tools, leaving the instructor having to try to convince students that the techniques would soon be useful. In our own teaching, we find that we often use design examples in the first few weeks of the class and use this to motivate the various techniques that follow (particularly Chapters 2, 4 and 5). This approach has been particularly useful in engineering courses, where students are often eager to apply the tools to examples as part of gaining insight into the methods. We also found that universities that have a laboratory component attached to their controls class need to introduce some basic design techniques early, so that students can be implementing control laws in the laboratory in the early weeks of the course.

To help emphasize this more design-oriented flow, we have rearranged the material in the first third of the book. Chapter 3 in the original text, which introduced a number of examples in some detail, has been moved to an appendix, where it can be assigned as needed when specific examples arise. In its place, we have put a new chapter on “Feedback Principles” that illustrates some simple design principles and tools that can be used to show students what types of problems can be solved using feedback. This new chapter uses simple models, simulations and elementary analysis techniques, so that it should be accessible to students from a variety of engineering and scientific backgrounds. For courses in which students have already been exposed to the basic ideas of feedback, perhaps in an earlier discipline-specific course, this new chapter can easily be skipped without any loss of continuity.

In addition to this relatively large change in the first portion of the book, we have also taken the opportunity to make other smaller changes based on the feedback we have received from early adopters of the text.

[Add a summary of changes here](#)

**RMM**

We are indebted to numerous individuals who have taught out of the text and sent us feedback on changes that would better serve their needs. In addition to the many individuals listed in the preface to the first edition, we would like to also thank Constantine Caramanis and Clancy Rowley for their feedback and insights.

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PREFACE

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## Preface to the First Edition

This book provides an introduction to the basic principles and tools for the design and analysis of feedback systems. It is intended to serve a diverse audience of scientists and engineers who are interested in understanding and utilizing feedback in physical, biological, information and social systems. We have attempted to keep the mathematical prerequisites to a minimum while being careful not to sacrifice rigor in the process. We have also attempted to make use of examples from a variety of disciplines, illustrating the generality of many of the tools while at the same time showing how they can be applied in specific application domains.

A major goal of this book is to present a concise and insightful view of the current knowledge in feedback and control systems. The field of control started by teaching everything that was known at the time and, as new knowledge was acquired, additional courses were developed to cover new techniques. A consequence of this evolution is that introductory courses have remained the same for many years, and it is often necessary to take many individual courses in order to obtain a good perspective on the field. In developing this book, we have attempted to condense the current knowledge by emphasizing fundamental concepts. We believe that it is important to understand why feedback is useful, to know the language and basic mathematics of control and to grasp the key paradigms that have been developed over the past half century. It is also important to be able to solve simple feedback problems using back-of-the-envelope techniques, to recognize fundamental limitations and difficult control problems and to have a feel for available design methods.

This book was originally developed for use in an experimental course at Caltech involving students from a wide set of backgrounds. The course was offered to undergraduates at the junior and senior levels in traditional engineering disciplines, as well as first- and second-year graduate students in engineering and science. This latter group included graduate students in biology, computer science and physics. Over the course of several years, the text has been classroom tested at Caltech and at Lund University, and the feedback from many students and colleagues has been incorporated to help improve the readability and accessibility of the material.

Because of its intended audience, this book is organized in a slightly unusual fashion compared to many other books on feedback and control. In particular, we introduce a number of concepts in the text that are normally reserved for second-year courses on control and hence often not available to students who are not control systems majors. This has been done at the expense of certain traditional topics, which we felt that the astute student could learn independently and are often

explored through the exercises. Examples of topics that we have included are non-linear dynamics, Lyapunov stability analysis, the matrix exponential, reachability and observability, and fundamental limits of performance and robustness. Topics that we have deemphasized include root locus techniques, lead/lag compensation and detailed rules for generating Bode and Nyquist plots by hand.



Several features of the book are designed to facilitate its dual function as a basic engineering text and as an introduction for researchers in natural, information and social sciences. The bulk of the material is intended to be used regardless of the audience and covers the core principles and tools in the analysis and design of feedback systems. Advanced sections, marked by the “dangerous bend” symbol shown here, contain material that requires a slightly more technical background, of the sort that would be expected of senior undergraduates in engineering. A few sections are marked by two dangerous bend symbols and are intended for readers with more specialized backgrounds, identified at the beginning of the section. To limit the length of the text, several standard results and extensions are given in the exercises, with appropriate hints toward their solutions.

To further augment the printed material contained here, a companion web site has been developed and is available from the publisher’s web page:

<http://www.cds.caltech.edu/~murray/amwiki>

The web site contains a database of frequently asked questions, supplemental examples and exercises, and lecture material for courses based on this text. The material is organized by chapter and includes a summary of the major points in the text as well as links to external resources. The web site also contains the source code for many examples in the book, as well as utilities to implement the techniques described in the text. Most of the code was originally written using MATLAB M-files but was also tested with LabView MathScript to ensure compatibility with both packages. Many files can also be run using other scripting languages such as Octave, SciLab, SysQuake and Xmath.

The first half of the book focuses almost exclusively on state space control systems. We begin in Chapter 2 with a description of modeling of physical, biological and information systems using ordinary differential equations and difference equations. Chapter 3<sup>1</sup> presents a number of examples in some detail, primarily as a reference for problems that will be used throughout the text. Following this, Chapter 4 looks at the dynamic behavior of models, including definitions of stability and more complicated nonlinear behavior. We provide advanced sections in this chapter on Lyapunov stability analysis because we find that it is useful in a broad array of applications and is frequently a topic that is not introduced until later in one’s studies.

The remaining three chapters of the first half of the book focus on linear systems, beginning with a description of input/output behavior in Chapter 5. In Chapter 6, we formally introduce feedback systems by demonstrating how state space control laws can be designed. This is followed in Chapter 7 by material on output

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<sup>1</sup>Now Appendix A