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# Biomolecular Feedback Systems

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## Preface

This text is intended for researchers interested in the application of feedback and control to biomolecular systems. The material has been designed so that it can be used in parallel with the textbook *Feedback Systems* [1] as part of a course on biomolecular feedback and control systems, or as a stand-alone reference for readers who have had a basic course in feedback and control theory. The full text for this book, along with additional supplemental material, is available on a companion Web site:

<http://www.cds.caltech.edu/~murray/BFS>

The material in this book is intended to be useful to three overlapping audiences: graduate students in biology and bioengineering interested in understanding the role of feedback in natural and engineered biomolecular systems; advanced undergraduates and graduate students in engineering disciplines who are interested in the use of feedback in biological circuit design; and established researchers in the biological sciences who want to explore the potential application of principles and tools from control theory to biomolecular systems. We have written the text assuming some familiarity with basic concepts in feedback and control, but have tried to provide insights and specific results as needed, so that the material can be learned in parallel. We also assume some familiarity with cell biology, at the level of a first course for non-majors. The individual chapters in the text indicate the prerequisites in more detail, most of which are covered either in Åström and Murray [1] or in the supplemental information available from the companion Web site.

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