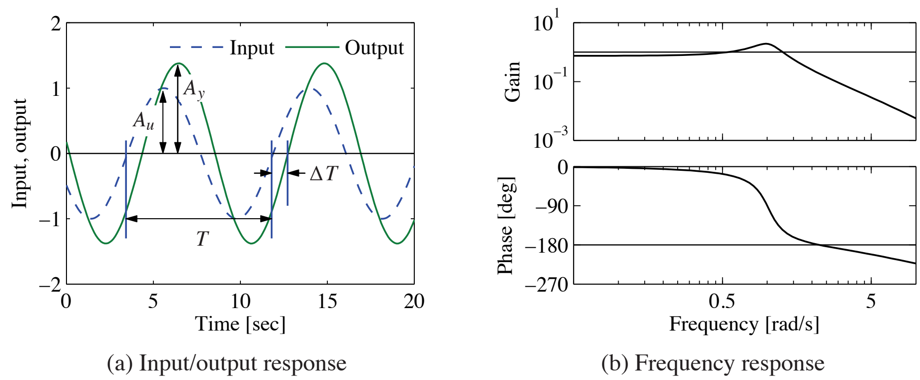

Figure 5.11: Response of a linear system to a sinusoid. (a) A sinusoidal input of magnitude A_u (dashed) gives a sinusoidal output of magnitude A_y (solid), delayed by ΔT seconds. (b) Frequency response, showing gain and phase. The gain is given by the ratio of the output amplitude to the input amplitude, $M = A_y/A_u$. The phase lag is given by $\theta = -2\pi\,\Delta T/T$; it is negative for the case shown because the output lags the input.

A convenient way to view the frequency response is to plot how the gain and phase in equation (5.24) depend on ω (through $s = i\omega$). Figure 5.11b shows an example of this type of representation.

Example 5.8 Active band-pass filter

Consider the op amp circuit shown in Figure 5.12a. We can derive the dynamics of the system by writing the *nodal equations*, which state that the sum of the currents at any node must be zero. Assuming that $v_- = v_+ = 0$, as we did in Section 3.3, we have

$$0 = \frac{v_1 - v_2}{R_1} - C_1\frac{dv_2}{dt}, \qquad 0 = C_1\frac{dv_2}{dt} + \frac{v_3}{R_2} + C_2\frac{dv_3}{dt}.$$

Choosing v_2 and v_3 as our states and using these equations, we obtain

$$\frac{dv_2}{dt} = \frac{v_1 - v_2}{R_1C_1}, \qquad \frac{dv_3}{dt} = \frac{-v_3}{R_2C_2} - \frac{v_1 - v_2}{R_1C_2}.$$

Rewriting these in linear state space form, we obtain

$$\frac{dx}{dt} = \begin{pmatrix} -\dfrac{1}{R_1C_1} & 0 \\ \dfrac{1}{R_1C_2} & -\dfrac{1}{R_2C_2} \end{pmatrix} x + \begin{pmatrix} \dfrac{1}{R_1C_1} \\ \dfrac{-1}{R_1C_2} \end{pmatrix} u, \qquad y = \begin{pmatrix} 0 & 1 \end{pmatrix} x, \qquad (5.25)$$

where $x = (v_2, v_3)$, $u = v_1$ and $y = v_3$.

The frequency response for the system can be computed using equation (5.24):

$$Me^{j\theta} = C(sI - A)^{-1}B + D = -\frac{R_2}{R_1}\frac{R_1C_1s}{(1 + R_1C_1s)(1 + R_2C_2s)}, \qquad s = i\omega.$$

The magnitude and phase are plotted in Figure 5.12b for $R_1 = 100\ \Omega$, $R_2 = 5\ \mathrm{k}\Omega$ and $C_1 = C_2 = 100\ \mu$F. We see that the circuit passes through signals with