

---

---

# Feedback Systems

---

*An Introduction for Scientists and Engineers*

Karl Johan Åström  
Richard M. Murray

Version v2.10c (March 4, 2010)

This is the electronic edition of *Feedback Systems* and is available from <http://www.cds.caltech.edu/~murray/amwiki>. Hardcover editions may be purchased from Princeton University Press, <http://press.princeton.edu/titles/8701.html>.

This manuscript is for personal use only and may not be reproduced, in whole or in part, without written consent from the publisher (see <http://press.princeton.edu/permissions.html>).

PRINCETON UNIVERSITY PRESS  
PRINCETON AND OXFORD

Copyright © 2008 by Princeton University Press

Published by Princeton University Press  
41 William Street, Princeton, New Jersey 08540

In the United Kingdom: Princeton University Press  
6 Oxford Street, Woodstock, Oxfordshire OX20 1TW

All Rights Reserved

Library of Congress Cataloging-in-Publication Data

Åström, Karl J. (Karl Johan), 1934-

Feedback systems : an introduction for scientists and engineers / Karl Johan

Åström and Richard M. Murray

p. cm.

Includes bibliographical references and index.

ISBN-13: 978-0-691-13576-2 (alk. paper)

ISBN-10: 0-691-13576-2 (alk. paper)

1. Feedback control systems. I. Murray, Richard M., 1963-. II. Title.

TJ216.A78 2008

629.8'3-dc22

2007061033

British Library Cataloging-in-Publication Data is available

This book has been composed in L<sup>A</sup>T<sub>E</sub>X

The publisher would like to acknowledge the authors of this volume for providing the camera-ready copy from which this book was printed.

Printed on acid-free paper. ∞

press.princeton.edu

Printed in the United States of America

10 9 8 7 6 5 4 3

This version of *Feedback Systems* is the electronic edition of the text. Revision history:

- Version 2.10c (4 Mar 2010): third printing, with corrections
- Version 2.10b (22 Feb 2009): second printing, with corrections
- Version 2.10a (2 Dec 2008): electronic edition. Corrected all errata listed on companion web site (multiple changes)
- Version 2.9d (30 Jan 2008): first printing

A full list of changes made in each revision is available on the companion web site:

<http://www.cds.caltech.edu/~murray/amwiki>



---

---

# Contents

<b>Preface</b>	<b>ix</b>
<b>Chapter 1. Introduction</b>	<b>1</b>
1.1 What Is Feedback?	1
1.2 What Is Control?	3
1.3 Feedback Examples	5
1.4 Feedback Properties	17
1.5 Simple Forms of Feedback	23
1.6 Further Reading	25
Exercises	25
<b>Chapter 2. System Modeling</b>	<b>27</b>
2.1 Modeling Concepts	27
2.2 State Space Models	34
2.3 Modeling Methodology	44
2.4 Modeling Examples	51
2.5 Further Reading	61
Exercises	61
<b>Chapter 3. Examples</b>	<b>65</b>
3.1 Cruise Control	65
3.2 Bicycle Dynamics	69
3.3 Operational Amplifier Circuits	71
3.4 Computing Systems and Networks	75
3.5 Atomic Force Microscopy	81
3.6 Drug Administration	85
3.7 Population Dynamics	89
Exercises	91
<b>Chapter 4. Dynamic Behavior</b>	<b>95</b>
4.1 Solving Differential Equations	95
4.2 Qualitative Analysis	98
4.3 Stability	102
4.4 Lyapunov Stability Analysis	110
4.5 Parametric and Nonlocal Behavior	120

4.6	Further Reading	126
	Exercises	126
<b>Chapter 5. Linear Systems</b>		<b>131</b>
5.1	Basic Definitions	131
5.2	The Matrix Exponential	136
5.3	Input/Output Response	145
5.4	Linearization	158
5.5	Further Reading	163
	Exercises	164
<b>Chapter 6. State Feedback</b>		<b>167</b>
6.1	Reachability	167
6.2	Stabilization by State Feedback	175
6.3	State Feedback Design	183
6.4	Integral Action	195
6.5	Further Reading	197
	Exercises	198
<b>Chapter 7. Output Feedback</b>		<b>201</b>
7.1	Observability	201
7.2	State Estimation	206
7.3	Control Using Estimated State	211
7.4	Kalman Filtering	215
7.5	A General Controller Structure	219
7.6	Further Reading	226
	Exercises	226
<b>Chapter 8. Transfer Functions</b>		<b>229</b>
8.1	Frequency Domain Modeling	229
8.2	Derivation of the Transfer Function	231
8.3	Block Diagrams and Transfer Functions	242
8.4	The Bode Plot	250
8.5	Laplace Transforms	259
8.6	Further Reading	262
	Exercises	262
<b>Chapter 9. Frequency Domain Analysis</b>		<b>267</b>
9.1	The Loop Transfer Function	267
9.2	The Nyquist Criterion	270
9.3	Stability Margins	278
9.4	Bode's Relations and Minimum Phase Systems	283
9.5	Generalized Notions of Gain and Phase	285
9.6	Further Reading	290

Exercises	290
<b>Chapter 10. PID Control</b>	<b>293</b>
10.1 Basic Control Functions	293
10.2 Simple Controllers for Complex Systems	298
10.3 PID Tuning	302
10.4 Integrator Windup	306
10.5 Implementation	308
10.6 Further Reading	312
Exercises	313
<b>Chapter 11. Frequency Domain Design</b>	<b>315</b>
11.1 Sensitivity Functions	315
11.2 Feedforward Design	319
11.3 Performance Specifications	322
11.4 Feedback Design via Loop Shaping	326
11.5 Fundamental Limitations	331
11.6 Design Example	340
11.7 Further Reading	343
Exercises	344
<b>Chapter 12. Robust Performance</b>	<b>347</b>
12.1 Modeling Uncertainty	347
12.2 Stability in the Presence of Uncertainty	352
12.3 Performance in the Presence of Uncertainty	358
12.4 Robust Pole Placement	361
12.5 Design for Robust Performance	369
12.6 Further Reading	374
Exercises	374
<b>Bibliography</b>	<b>377</b>
<b>Index</b>	<b>387</b>





---

---

## Preface

This book provides an introduction to the basic principles and tools for the design and analysis of feedback systems. It is intended to serve a diverse audience of scientists and engineers who are interested in understanding and utilizing feedback in physical, biological, information and social systems. We have attempted to keep the mathematical prerequisites to a minimum while being careful not to sacrifice rigor in the process. We have also attempted to make use of examples from a variety of disciplines, illustrating the generality of many of the tools while at the same time showing how they can be applied in specific application domains.

A major goal of this book is to present a concise and insightful view of the current knowledge in feedback and control systems. The field of control started by teaching everything that was known at the time and, as new knowledge was acquired, additional courses were developed to cover new techniques. A consequence of this evolution is that introductory courses have remained the same for many years, and it is often necessary to take many individual courses in order to obtain a good perspective on the field. In developing this book, we have attempted to condense the current knowledge by emphasizing fundamental concepts. We believe that it is important to understand why feedback is useful, to know the language and basic mathematics of control and to grasp the key paradigms that have been developed over the past half century. It is also important to be able to solve simple feedback problems using back-of-the-envelope techniques, to recognize fundamental limitations and difficult control problems and to have a feel for available design methods.

This book was originally developed for use in an experimental course at Caltech involving students from a wide set of backgrounds. The course was offered to undergraduates at the junior and senior levels in traditional engineering disciplines, as well as first- and second-year graduate students in engineering and science. This latter group included graduate students in biology, computer science and physics. Over the course of several years, the text has been classroom tested at Caltech and at Lund University, and the feedback from many students and colleagues has been incorporated to help improve the readability and accessibility of the material.

Because of its intended audience, this book is organized in a slightly unusual fashion compared to many other books on feedback and control. In particular, we introduce a number of concepts in the text that are normally reserved for second-year courses on control and hence often not available to students who are not control systems majors. This has been done at the expense of certain traditional topics, which we felt that the astute student could learn independently and are often

explored through the exercises. Examples of topics that we have included are non-linear dynamics, Lyapunov stability analysis, the matrix exponential, reachability and observability, and fundamental limits of performance and robustness. Topics that we have deemphasized include root locus techniques, lead/lag compensation and detailed rules for generating Bode and Nyquist plots by hand.



Several features of the book are designed to facilitate its dual function as a basic engineering text and as an introduction for researchers in natural, information and social sciences. The bulk of the material is intended to be used regardless of the audience and covers the core principles and tools in the analysis and design of feedback systems. Advanced sections, marked by the “dangerous bend” symbol shown here, contain material that requires a slightly more technical background, of the sort that would be expected of senior undergraduates in engineering. A few sections are marked by two dangerous bend symbols and are intended for readers with more specialized backgrounds, identified at the beginning of the section. To limit the length of the text, several standard results and extensions are given in the exercises, with appropriate hints toward their solutions.

To further augment the printed material contained here, a companion web site has been developed and is available from the publisher’s web page:

<http://www.cds.caltech.edu/~murray/amwiki>

The web site contains a database of frequently asked questions, supplemental examples and exercises, and lecture material for courses based on this text. The material is organized by chapter and includes a summary of the major points in the text as well as links to external resources. The web site also contains the source code for many examples in the book, as well as utilities to implement the techniques described in the text. Most of the code was originally written using MATLAB M-files but was also tested with LabView MathScript to ensure compatibility with both packages. Many files can also be run using other scripting languages such as Octave, SciLab, SysQuake and Xmath.

The first half of the book focuses almost exclusively on state space control systems. We begin in Chapter 2 with a description of modeling of physical, biological and information systems using ordinary differential equations and difference equations. Chapter 3 presents a number of examples in some detail, primarily as a reference for problems that will be used throughout the text. Following this, Chapter 4 looks at the dynamic behavior of models, including definitions of stability and more complicated nonlinear behavior. We provide advanced sections in this chapter on Lyapunov stability analysis because we find that it is useful in a broad array of applications and is frequently a topic that is not introduced until later in one’s studies.

The remaining three chapters of the first half of the book focus on linear systems, beginning with a description of input/output behavior in Chapter 5. In Chapter 6, we formally introduce feedback systems by demonstrating how state space control laws can be designed. This is followed in Chapter 7 by material on output feedback and estimators. Chapters 6 and 7 introduce the key concepts of reachability

and observability, which give tremendous insight into the choice of actuators and sensors, whether for engineered or natural systems.

The second half of the book presents material that is often considered to be from the field of “classical control.” This includes the transfer function, introduced in Chapter 8, which is a fundamental tool for understanding feedback systems. Using transfer functions, one can begin to analyze the stability of feedback systems using frequency domain analysis, including the ability to reason about the closed loop behavior of a system from its open loop characteristics. This is the subject of Chapter 9, which revolves around the Nyquist stability criterion.

In Chapters 10 and 11, we again look at the design problem, focusing first on proportional-integral-derivative (PID) controllers and then on the more general process of loop shaping. PID control is by far the most common design technique in control systems and a useful tool for any student. The chapter on frequency domain design introduces many of the ideas of modern control theory, including the sensitivity function. In Chapter 12, we combine the results from the second half of the book to analyze some of the fundamental trade-offs between robustness and performance. This is also a key chapter illustrating the power of the techniques that have been developed and serving as an introduction for more advanced studies.

The book is designed for use in a 10- to 15-week course in feedback systems that provides many of the key concepts needed in a variety of disciplines. For a 10-week course, Chapters 1–2, 4–6 and 8–11 can each be covered in a week’s time, with the omission of some topics from the final chapters. A more leisurely course, spread out over 14–15 weeks, could cover the entire book, with 2 weeks on modeling (Chapters 2 and 3)—particularly for students without much background in ordinary differential equations—and 2 weeks on robust performance (Chapter 12).

The mathematical prerequisites for the book are modest and in keeping with our goal of providing an introduction that serves a broad audience. We assume familiarity with the basic tools of linear algebra, including matrices, vectors and eigenvalues. These are typically covered in a sophomore-level course on the subject, and the textbooks by Apostol [Apo69], Arnold [Arn87] and Strang [Str88] can serve as good references. Similarly, we assume basic knowledge of differential equations, including the concepts of homogeneous and particular solutions for linear ordinary differential equations in one variable. Apostol [Apo69] and Boyce and DiPrima [BD04] cover this material well. Finally, we also make use of complex numbers and functions and, in some of the advanced sections, more detailed concepts in complex variables that are typically covered in a junior-level engineering or physics course in mathematical methods. Apostol [Apo67] or Stewart [Ste02] can be used for the basic material, with Ahlfors [Ahl66], Marsden and Hoffman [MH98] or Saff and Snider [SS02] being good references for the more advanced material. We have chosen not to include appendices summarizing these various topics since there are a number of good books available.

One additional choice that we felt was important was the decision not to rely on a knowledge of Laplace transforms in the book. While their use is by far the most common approach to teaching feedback systems in engineering, many stu-

dents in the natural and information sciences may lack the necessary mathematical background. Since Laplace transforms are not required in any essential way, we have included them only in an advanced section intended to tie things together for students with that background. Of course, we make tremendous use of *transfer functions*, which we introduce through the notion of response to exponential inputs, an approach we feel is more accessible to a broad array of scientists and engineers. For classes in which students have already had Laplace transforms, it should be quite natural to build on this background in the appropriate sections of the text.

### **Acknowledgments**

The authors would like to thank the many people who helped during the preparation of this book. The idea for writing this book came in part from a report on future directions in control [Mur03] to which Stephen Boyd, Roger Brockett, John Doyle and Gunter Stein were major contributors. Kristi Morgansen and Hideo Mabuchi helped teach early versions of the course at Caltech on which much of the text is based, and Steve Waydo served as the head TA for the course taught at Caltech in 2003–2004 and provided numerous comments and corrections. Charlotta Johnson and Anton Cervin taught from early versions of the manuscript in Lund in 2003–2007 and gave very useful feedback. Other colleagues and students who provided feedback and advice include Leif Andersson, John Carson, K. Mani Chandy, Michel Charpentier, Domitilla Del Vecchio, Kate Galloway, Per Hagander, Toivo Henningsson Perby, Joseph Hellerstein, George Hines, Tore Hägglund, Cole Lepine, Anders Rantzer, Anders Robertsson, Dawn Tilbury and Francisco Zabala. The reviewers for Princeton University Press and Tom Robbins at NI Press also provided valuable comments that significantly improved the organization, layout and focus of the book. Our editor, Vickie Kearn, was a great source of encouragement and help throughout the publishing process. Finally, we would like to thank Caltech, Lund University and the University of California at Santa Barbara for providing many resources, stimulating colleagues and students, and pleasant working environments that greatly aided in the writing of this book.

Karl Johan Åström  
Lund, Sweden  
Santa Barbara, California

Richard M. Murray  
Pasadena, California