Feedback Systems:
An Introduction for Scientists and Engineers

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Preface

This book provides an introduction to the basic principles and tools for design and analysis of feedback systems. It is intended to serve a diverse audience of scientists and engineers who are interested in understanding and utilizing feedback in physical, biological, information, and economic systems. To this end, we have chosen to keep the mathematical prerequisites to a minimum while being careful not to sacrifice rigor in the process. Advanced sections, marked by the "dangerous bend" symbol shown to the right, contain material that is of a more advanced nature and can be skipped on first reading.

This book was originally developed for use in an experimental course at Caltech involving students from a wide variety of disciplines. The course consisted of undergraduates at the junior and senior level in traditional engineering disciplines, as well as first and second year graduate students in engineering and science. This included graduate students in biology, computer science and economics, requiring a broad approach that emphasized basic principles and did not focus on applications in any one given area.

A web site has been prepared as a companion to this text:

`http://www.cds.caltech.edu/~murray/amwiki`

The web site contains a database of frequently asked questions, supplemental examples and exercises, and lecture materials for a course based on this text. It also contains the source code for many examples in the book, as well as libraries to implement the techniques described in the text. Most of the code was originally written using MATLAB M-files, but was also tested with LabVIEW MathScript to ensure compatibility with both packages. Most files can also be run using other scripting languages such as Octave, SciLab and SysQuake. [Author's note: the web site is under construction as of this writing and some features described in the text may not yet be available.]

Because of its intended audience, this book is organized in a slightly unusual fashion compared to many other books on feedback and control. In particular, we introduce a number of concepts in the text that are normally reserved for second year courses on control (and hence often not available to students who are not control systems majors). This has been done at the expense of certain "traditional" topics, which we felt that the astute student could learn on their own (and are often explored through the exercises). Examples of topics that we have included are nonlinear dynamics, Lyapunov stability, reachability and observability, and fundamental limits of performance and robustness. Topics that we have de-emphasized include root locus techniques, lead/lag compensation (although this is essentially covered in Chapters 10 and 11), and detailed rules for generating Bode and Nyquist plots by hand.

The first half of the book focuses almost exclusively on so-called "state-space" control systems. We begin in Chapter 2 with a description of modeling of physical, biological and information systems using ordinary differential equations and difference equations. Chapter 3 presents a number of examples in some detail, primarily as a reference for problems that will be used throughout the text. Following this, Chapter 4 looks at the dynamic behavior of models, including definitions of stability and more complicated nonlinear behavior. We provide advanced sections in this chapter on Lyapunov stability, because we find that it is useful in a broad array of applications (and frequently a topic that is not introduced until much later in one's studies).

The remaining three chapters of the first half of the book focus on linear systems, beginning with a description of input/output behavior in Chapter 5. In Chapter 6, we formally introduce feedback systems by demonstrating how state space control laws can be designed. This is followed in Chapter 7 by material on output feedback and estimators. Chapters 6 and 7 introduce the key concepts of reachability and observability, which give tremendous insight into the choice of actuators and sensors, whether for engineered or natural systems.

The second half of the book presents material that is often considered to be from the field of "classical control." This includes the transfer function, introduced in Chapter 8, which is a fundamental tool for understanding feedback systems. Using transfer functions, one can begin to analyze the stability of feedback systems using loop analysis, which allows us to reason about the closed loop behavior (stability) of a system from its open loop characteristics. This is the subject of Chapter 9, which revolves around the Nyquist stability criterion.

In Chapters 10 and 11, we again look at the design problem, focusing first on proportional-integral-derivative (PID) controllers and then on the more general process of loop shaping. PID control is by far the most common design technique in control systems and a useful tool for any student. The chapter on loop shaping introduces many of the ideas of modern control theory, including the sensitivity function. In Chapter 12, we pull together the results from the second half of the book to analyze the fundamental tradeoffs between robustness and performance. This is also a key chapter illustrating the power of the techniques that have been developed.

The book is designed for use in a 10–15 week course in feedback systems that can serve to introduce many of the key concepts that are needed in a variety of disciplines. For a 10 week course, Chapters 1–6 and 8–11 can each be covered in a week's time, with some dropping of topics from the final chapters. A more leisurely course, spread out over 14–15 weeks, could cover the entire book, with two weeks on modeling (Chapters 2 and 3)—particularly for students without much background in ordinary differential equations—and two weeks on robust performance (Chapter 12).

In choosing the set of topics and ordering for the main text, we necessarily left out some tools which will cause many control systems experts to raise their eyebrows (or choose another textbook). Overall, we believe that the early focus on state space systems, including the concepts of reachability and observability, are of such importance to justify trimming other topics to make room for them. We also included some relatively advanced material on fundamental tradeoffs and limitations of performance, feeling that these provided such insight into the principles of feedback that they could not be left for later. Throughout the text, we have attempted to maintain a balanced set of examples that touch many disciplines, relying on the companion web site for more discipline specific examples and exercises. Additional notes covering some of the "missing" topics are available on the web.

One additional choice that we felt was very important was the decision not to rely on the use of Laplace transforms in the book. While this is by far the most common approach to teaching feedback systems in engineering, many students in the natural and information sciences may lack the necessary mathematical background. Since Laplace transforms are not required in any essential way, we have only made a few remarks to tie things together for students with that background. Of course, we make tremendous use of *transfer functions*, which we introduce through the notion of response to exponential inputs, an approach we feel is much more accessible to a broad array of scientists and engineers.

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