Receding Horizon Temporal Logic Planning

Automatic Synthesis of Planners and Controllers for Dynamical Systems

Nok Wongpiromsarn, Ufuk Topcu, and Richard M. Murray

California Institute of Technology

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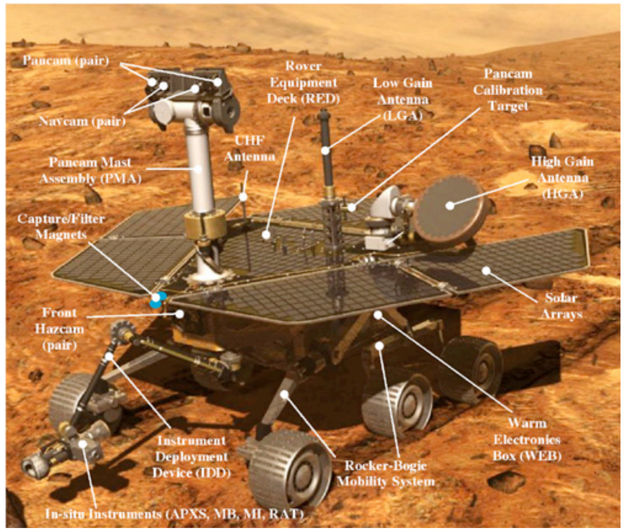
Motivation

- In many applications, dynamical systems need to perform complex tasks and interact with (potentially adversarial) environments.
- These systems are generally designed by hand based on their desired properties (specs). Changes in the specs potentially result in redesigning and reimplementing a large portion of the systems.
- Ideal: Automatically synthesize dynamical systems which are guaranteed, by construction, to satisfy their desired properties.



Outline

Methods from Computer Science and Control

Computer Science

- discrete system, finite domain
- a polynomial-time algorithm to construct finite state automata from their temporal logic specifications.
- automatic synthesis of digital designs.
- a large class of properties are ensured even in the presence of adversary.

Control

- continuous system, infinite domain
- optimal control, model predictive control, robust analysis, etc
- automatic synthesis of continuous controllers.
- safety and stability properties are ensured even in the presence of disturbances and modeling errors.

Hybrid System Theory

- Deal with systems with both discrete and continuous components.
- Properties of interest are typically limited to stability and safety.
- A wider class of properties needs to be considered
  - guarantee: eventually accomplish task 1, 2 and 3 in any order.
  - response: if the system fails, then eventually perform task 1, or perform tasks 1, 2 and 3 infinitely often in any order.

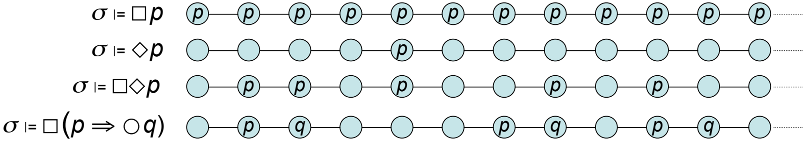
Describing Desired Properties: Linear Temporal Logic

- Built up from
  - atomic propositions: statements on system variables which have unique truth values
  - logical connectives: $\neg$, $\vee$, $\wedge$, $\Longrightarrow$
  - temporal operators: next ($\bigcirc$), always ($\Box$), eventually ($\Diamond$), until ($\mathcal{U}$)
- Syntax

$$\varphi ::= \pi \;\mid\; \neg\varphi \;\mid\; \varphi \vee \varphi \;\mid\; \bigcirc\varphi \;\mid\; \varphi\,\mathcal{U}\,\varphi$$

  - $\varphi \wedge \psi = \neg(\neg\varphi \vee \neg\psi)$
  - $\varphi \Longrightarrow \psi = \neg\varphi \vee \psi$
  - $\Diamond\varphi = \mathit{True}\,\mathcal{U}\,\varphi$
  - $\Box\varphi = \neg\Diamond\neg\varphi$
- An LTL formula is interpreted over an infinite sequence of states.
  - Given an execution $\sigma = v_0 v_1 v_2 \ldots$ and an LTL formula φ, we say that φ *holds at position* $i \geq 0$ of σ iff φ holds for the remainder of the execution σ starting at position i.
  - $\Box\varphi$ holds at position i iff φ holds at every position in σ starting at position i
  - $\Diamond\varphi$ holds at position i iff φ holds at some position $j \geq i$ in σ.
  - $\bigcirc\varphi$ holds at position i iff φ holds at position $i+1$ in σ.
- An execution $\sigma = v_0 v_1 v_2 \ldots$ satisfies φ iff $v_0 \models \varphi$.
- The system is said to be *correct* with respect to its specification φ if all its executions satisfy φ

Linear Temporal Logic



- Examples of LTL specifications
  - Safety: $\Box \text{dist}(x, obs) > \delta$
  - Guarantee: $\Diamond(ckpt_1 \wedge \Diamond(ckpt_2 \wedge \Diamond ckpt_3))$
  - Response: $\Box((\text{system fails}) \implies \Diamond\ (\text{perform task 1}))$, $\Box\Diamond(\text{perform tasks 1}) \ \wedge \Box\Diamond\ (\text{perform task 2})$

Synthesizing Finite State Automata

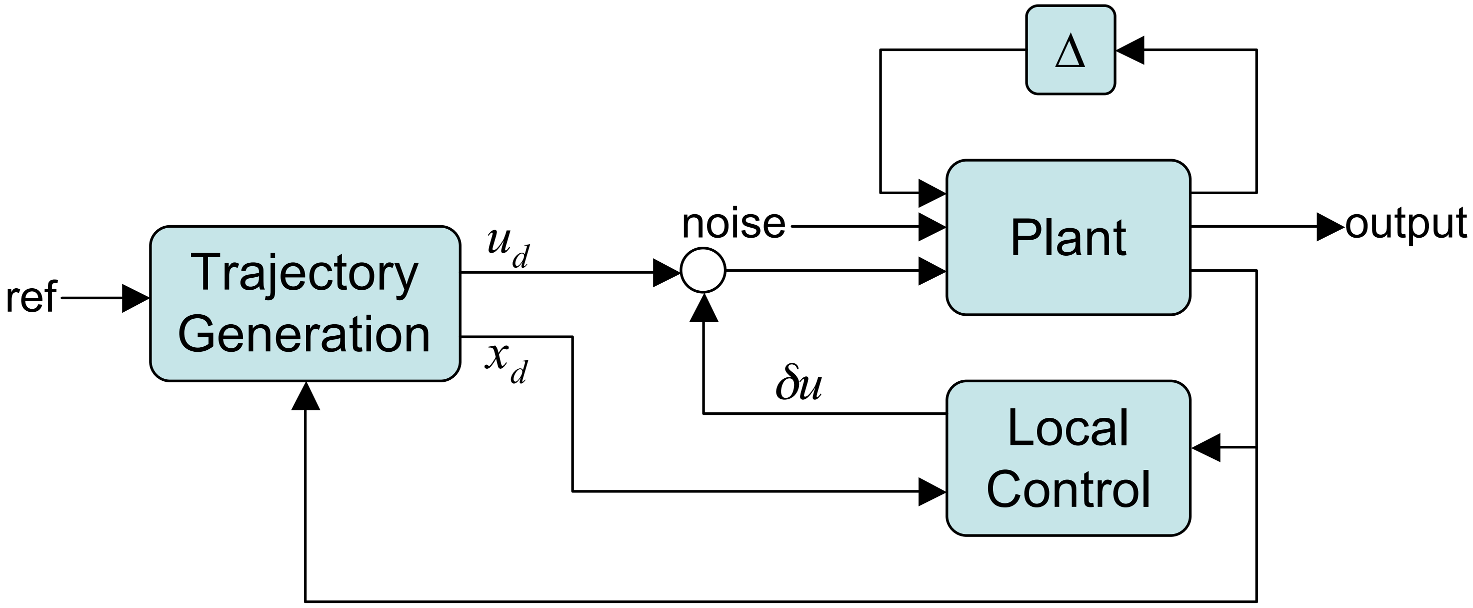
- For a system to be correct, its specification φ must be satisfied regardless of what the environment does.
- A two-player game between the system and the environment.
  - The environment attempts to falsify φ
  - The system attempts to satisfy φ
- φ is *realizable* if the system can satisfy φ no matter what the environment does.
- Interested in a specification of the form

$$\varphi = (\varphi_e \Longrightarrow \varphi_s)$$

  - φ_e is an LTL formula which characterizes the initial states of the system and the assumptions of the environment
  - φ_s is an LTL formula which describes the correct behavior of the system
- If φ is realizable, the synthesis algorithm generates a finite state automaton which satisfies φ no matter what the environment does. Otherwise, it provides an initial state starting from which φ cannot be satisfied.
- Limitation: state explosion

Designing Continuous Controllers

- Optimization-based design and robust analysis
- Receding Horizon Control
  - Solve finite time optimization over T seconds and implement first ΔT seconds.
  - Software: NTG, OTG, MPT, ...



$$u_{[t,t+\Delta T]} = \arg\min \overbrace{\int_t^{t+T} L(x(\tau), u(\tau))d\tau}^{\text{finite horizon optimization}} + \overbrace{V(x(t+T))}^{\text{terminal cost}}$$

$$\text{s.t. } \dot{x} = f(x,u), \quad x_0 = x(t), \quad (x_f = x_d(t+T))$$

$$g(x,u) \leq 0$$

Problem Formulation

- Consider a system with a set of variables $V = S \cup E$ where S represents the set of *controlled* variables and E represents the set of *environment* variables.
- The controlled state evolves according to the discrete-time piecewise-affine dynamics:

$$\begin{aligned} s[t+1] &= A_k s[t] + B_k u[t] + C_k \text{ if } (s[t], u[t]) \in \Omega_k \\ u[t] &\in U \end{aligned}$$

- Given φ, an LTL specification built from a set of atomic propositions Π. Assume that φ can be expressed without the next ($\bigcirc$) operator.
- Interested in designing a controller for the system which ensures that any execution satisfies φ.

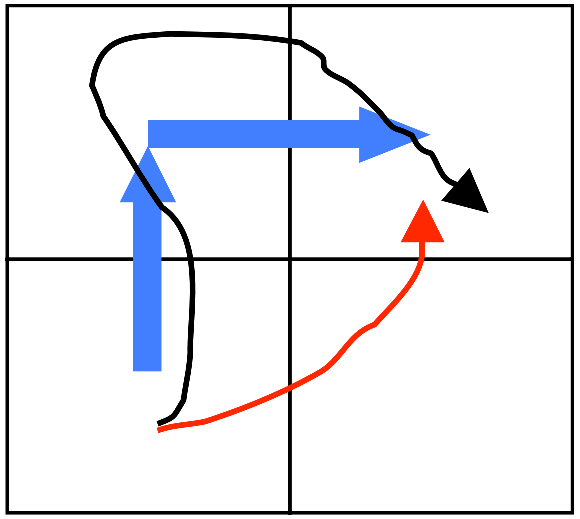
Hierarchical Approach

- The system must satisfy its specification φ regardless of what the environment does.
- In designing a controller, both the adversarial nature of the environment and the dynamics of the system need to be taken into account.
- Want to separate the concern of the environment from the concern of the dynamics.
- Hierarchical approach
  - Based on an abstract model of the system, a discrete planner computes a discrete plan satisfying φ regardless of what the environment does.
  - Based on the system dynamics, a continuous controller continuously *implements* the abstract plan.

Abstraction of System Dynamics

A finite transition system $\mathbb{D} = \{\mathcal{V}, \rightarrow\}$

- Serve as an abstract model of the system
- Partition the state space such that it is proposition preserving.
  - For any atomic proposition π in φ and any points v_1 and v_2 in the same cell, if v_1 satisfies π, then v_2 satisfies π.
  - Denote the discrete domain by $\mathcal{V} = \mathcal{S} \times \mathcal{E}$.
- Simulation abstraction
  - Ensure that the continuous execution preserves the correctness of the discrete plan.
  - Enforced by restricting the transition relations $\rightarrow$ of $\mathbb{D}$ to those satisfying the *reachability* relation



Reachability

- A discrete state $\mathcal{S}_j$ is reachable from a discrete state $\mathcal{S}_i$, written $\mathcal{S}_i \rightsquigarrow \mathcal{S}_j$, only if starting from any point s_{init} in cell $\mathcal{S}_i$, there exists a control law $u \in U$ which takes the system to a point s_{final} in cell $\mathcal{S}_j$ while always staying in cells $\mathcal{S}_i$ and $\mathcal{S}_j$.
- In general, for two discrete states $\mathcal{S}_i$ and $\mathcal{S}_j$, verifying the reachability relation $\mathcal{S}_i \rightsquigarrow \mathcal{S}_j$ is hard.
- Resort to a heuristic based on the optimal control problem.
  - allow *reachability* to be established by solving a multiparametric programming problem which can be solved automatically using the Multi-Parametric Toolbox (MPT).

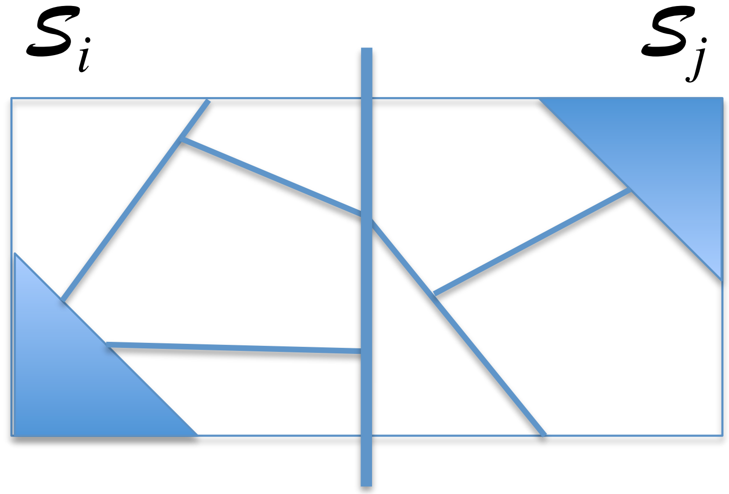
Establishing Reachability

Optimal Control Problem Formulation

Given two cells $\mathcal{S}_i, \mathcal{S}_j \subseteq dom(S)$, the set of admissible control inputs U, the matrices A_k and B_k, a horizon length $N \geq 0$ and the cost matrices $P_N, Q \succeq 0$ and $R \succ 0$, solve

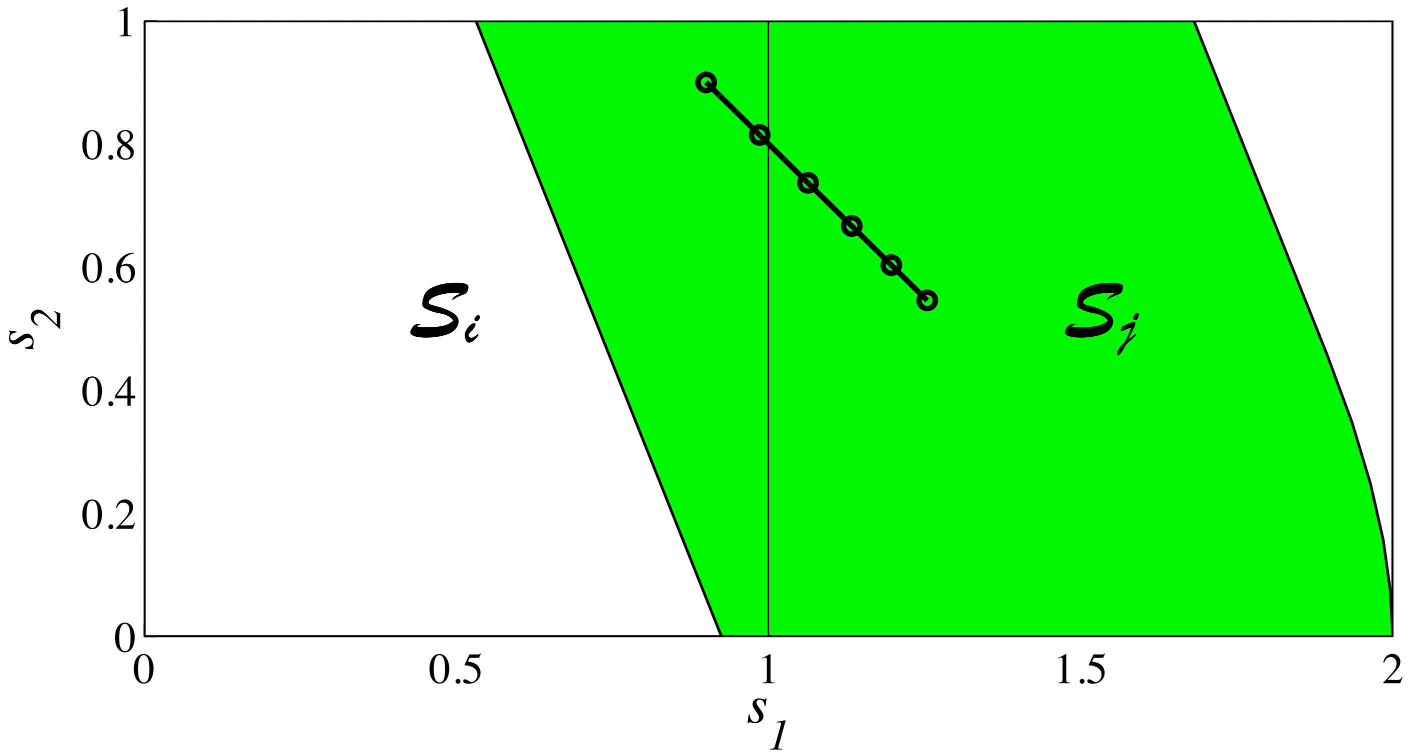
$$\begin{array}{ll} \min\limits_{u[0],\dots,u[N-1]} & \|P_N s[N]\|_2 + \sum\limits_{t=0}^{N-1} \|Qs[t]\|_2 + \|Ru[t]\|_2 \\ \text{s.t.} & s[N] \in \mathcal{S}_j, \quad s[0] \in dom(S) \\ & s[t+1] = A_k s[t] + B_k u[t] \text{ if } (s[t], u[t]) \in D_k \\ & s[t] \in \mathcal{S}_i \cup \mathcal{S}_j, \quad u[t] \in U \\ & \forall t \in \{0, \dots, N-1\}. \end{array} \tag{1}$$

- Solve for a set $\mathcal{P}_{i,j}$ such that for all $s[0] \in \mathcal{P}_{i,j}$, (1) is feasible
  - Can be regarded as a multiparametric programming problem
  - For polytopic sets $\mathcal{S}_i$, $\mathcal{S}_j$ and U, MPT computes the explicit solution $\mathcal{P}_{i,j} \subseteq \mathcal{S}_i \cup \mathcal{S}_j$



Example

$$
\begin{aligned}
s[t+1] &= \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.9048 \end{bmatrix} s[t] + \begin{bmatrix} 0.0048 \\ 0.0952 \end{bmatrix} u \\
u &\in [-\sqrt{0.5}, \sqrt{0.5}]
\end{aligned}
$$



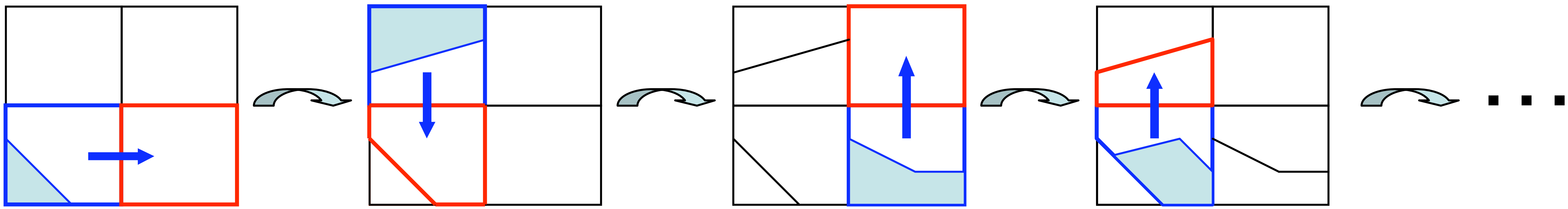
State Space Discretization

- The reachability relation between any discrete states in the original partition may not be established through the solution of the multiparametric problem
  - constraints on u
  - a specific choice of the finite horizon N
- Want to refine the partition to increase the number of valid discrete state transitions of the resulting finite transition system.

State Space Discretization

Discretization Algorithm

$\mathcal{S}' = \mathcal{S}$; $IJ = \{(i,j) \mid i,j \in \{1,\ldots,size(\mathcal{S})\}\}$;
while $(size(IJ) > 0)$
  Pick an $(i,j) \in IJ$;
  Solve for $\mathcal{P}_{i,j}$ such that for any $s[0] \in \mathcal{P}_{i,j}$, the optimal control problem (1) is feasible;
  if $(volume(\mathcal{S}_i \cap \mathcal{P}_{i,j}) > Vol_{min}$ **and** $volume(\mathcal{S}_i \setminus \mathcal{P}_{i,j}) > Vol_{min})$ **then**
    Replace $\mathcal{S}_i \in \mathcal{S}'$ with $\mathcal{S}_i \cap \mathcal{P}_{i,j}$ and add $\mathcal{S}_i \setminus \mathcal{P}_{i,j}$ to $\mathcal{S}'$;
    For each $k \in \{1,\ldots,size(\mathcal{S}')\}$, add $(i,k),(k,i),(size(\mathcal{S}'),k),(k,size(\mathcal{S}'))$ to IJ;
  else
    Remove (i,j) from IJ;
  endif
endwhile



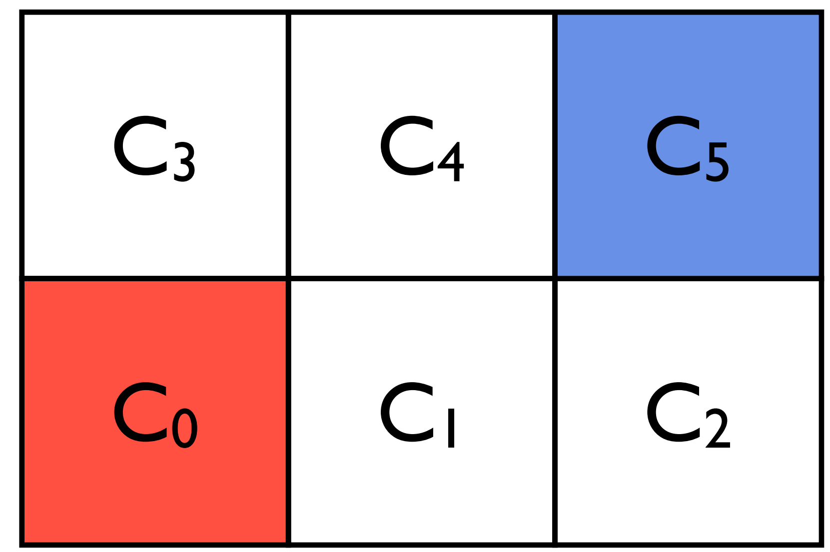
Synthesizing the Discrete Planner

- Construct a finite transition system $\mathbb{D} = \{\mathcal{V}, \rightarrow\}$
  - $\mathcal{V}$ is the set of all the discrete states corresponding to the partition of $dom(V)$ after applying the discretization algorithm.
  - For any two states $\mathcal{V}_i = (\mathcal{S}_{is}, \mathcal{E}_{is})$ and $\mathcal{V}_j = (\mathcal{S}_{js}, \mathcal{E}_{js})$, $\mathcal{V}_i \rightarrow \mathcal{V}_j$ only if $\mathcal{S}_{is} \rightsquigarrow \mathcal{S}_{js}$.
- Based on this abstract model $\mathbb{D}$, a discrete planner, represented by a finite state automaton, can be automatically synthesized to satisfy φ using a digital design synthesis tool.

Example

Desired Properties

- Visit the blue cell infinitely often.
- Eventually go to the red cell when a PARK signal is received.



Assumption

- Infinitely often, PARK signal is not received.

$$\varphi = \Box \Diamond (\neg park) \implies (\Box \Diamond (s \in C_5) \wedge \Box (park \implies \Diamond (s \in C_0)))$$

Correctness of the System

Theorem

Let $\sigma_d = \nu_0\nu_1\ldots$ be an infinite sequence of discrete states of $\mathbb{D}$ where for each natural number k, $\nu_k \rightarrow \nu_{k+1}$, $\nu_k = (\rho_k, \epsilon_k)$, ρ_k is the discrete controlled state and ϵ_k is the discrete environment state. If $\sigma_d \models_d \varphi$ then by applying a sequence of control laws, each corresponding to the solution of the optimal control problem (1) with $\mathcal{S}_i = \rho_k$ and $\mathcal{S}_j = \rho_{k+1}$, the infinite sequence of continuous states $\sigma = v_0v_1v_2\ldots$ satisfies φ.

Receding Horizon Temporal Logic Planning

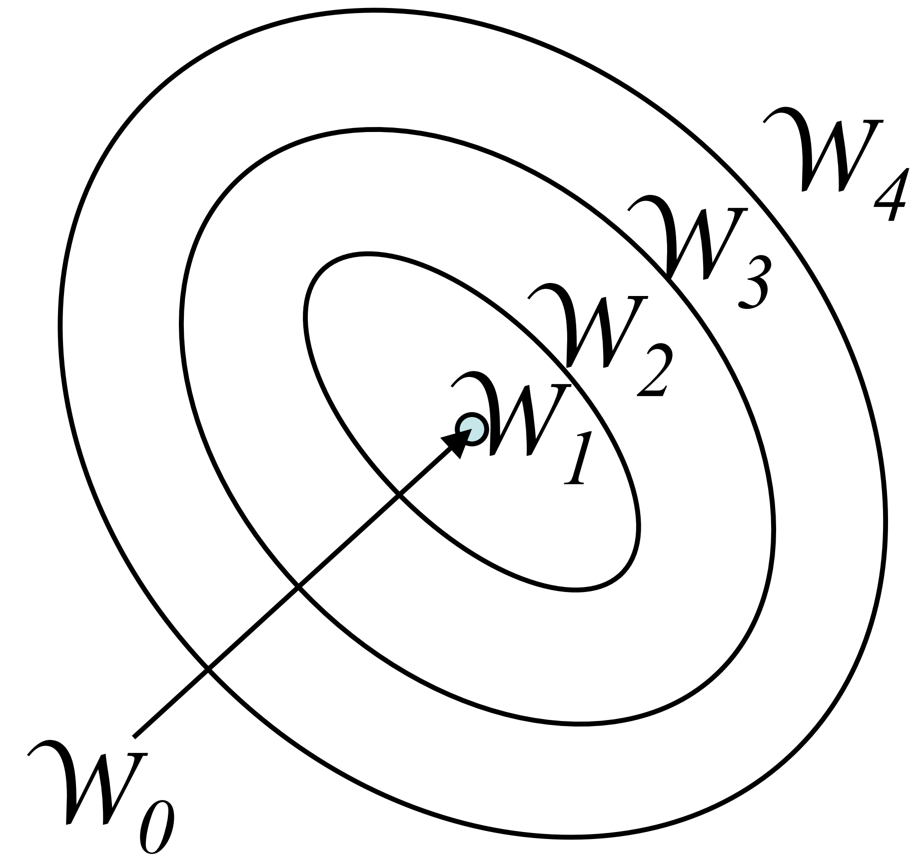
- Digital design synthesis tools suffer from the state explosion problem.
- Under certain conditions, it is not necessary to plan for the whole execution in one shot.
- Consider a subclass of *Generalized Reactivity(1)* formula of the form

$$(\varphi_{init} \wedge \Box\varphi_e) \implies (\Box\varphi_s \wedge \Diamond\varphi_g)$$

  - φ_{init} and φ_g are propositional formulas of variables from V
  - φ_e and φ_s are boolean combinations of propositional formulas of variables from V and expressions of the form $\bigcirc\psi$

Receding Horizon Temporal Logic Planning

- Assume certain structure of the system: Suppose there exists a collection of disjoint subsets $\mathcal{W}_0, \ldots, \mathcal{W}_p$ of $\mathcal{V}$ such that
  - (a) $\mathcal{W}_0 \cup \mathcal{W}_2 \cup \ldots \cup \mathcal{W}_p = \mathcal{V}$
  - (b) φ_g is satisfied for any state in a cell in $\mathcal{W}_0$
  - (c) $(\{\mathcal{W}_0, \ldots, \mathcal{W}_p\}, \preceq_{\varphi_g})$ is a partially ordered set



$$\mathcal{W}_0 \preceq_{\varphi_g} \mathcal{W}_1 \preceq_{\varphi_g} \ldots \preceq_{\varphi_g} \mathcal{W}_4$$

Synthesizing Subautomata

- Recall that the specification of the system is given by

$$(\varphi_{init} \wedge \Box\varphi_e) \implies (\Box\varphi_s \wedge \Diamond\varphi_g)$$

- Suppose there exists a propositional formula Φ and for each $i \in \{1, \dots, p\}$, there exists $g_i \in \{1, \dots, p\}$ and a subset $\mathcal{D}_i$ of $dom(S)$ satisfying
  (a) $\varphi_{init} \implies \Phi$ is a tautology
  (b) $\mathcal{W}_{g_i} \preceq_{\varphi_g} \mathcal{W}_i$ and for each $i \neq 0$, $\mathcal{W}_{g_i} \prec_{\varphi_g} \mathcal{W}_i$

  such that

$$\Psi_i = ((v \in \mathcal{W}_i) \wedge \Phi \wedge \Box\varphi_e) \implies (\Box\varphi_s \wedge \Diamond(v \in \mathcal{W}_{g_i}) \wedge \Box\Phi)$$

  is realizable with the domain of S restricted to $\mathcal{D}_i$.
- An automaton $\mathcal{A}_i$ satisfying Ψ_i can be synthesized using a digital design synthesis tool.

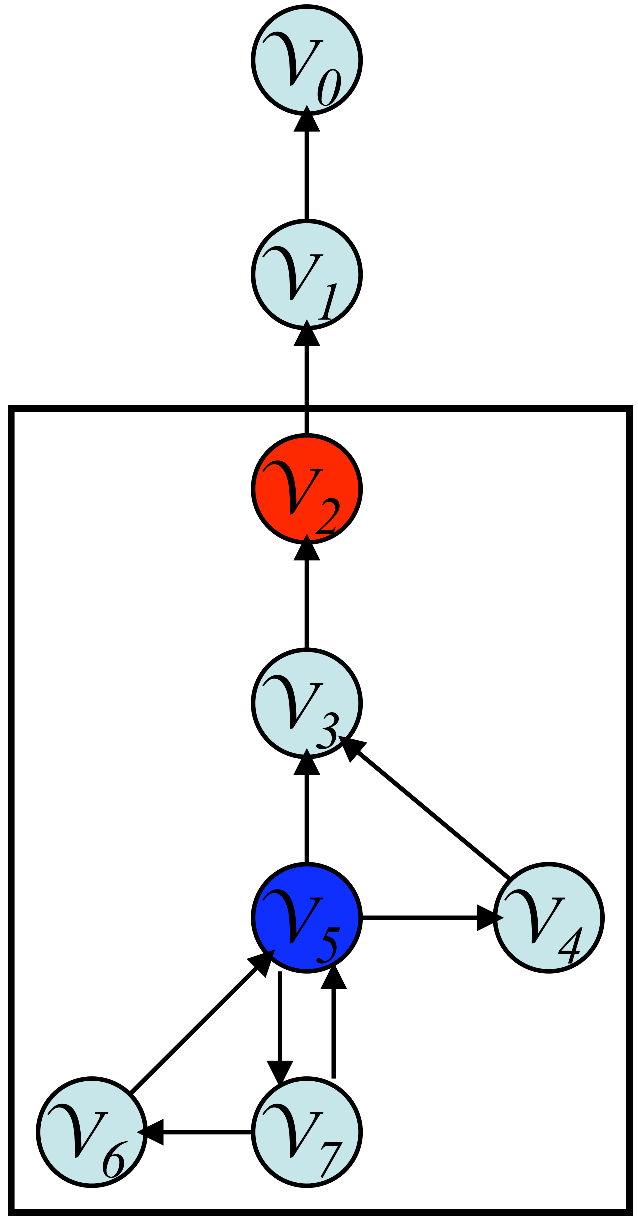
Receding Horizon Strategy

Receding Horizon Strategy

Starting from the state v_0, pick an automaton $\mathcal{A}_i$ such that v_0 is in a cell in $\mathcal{W}_i$ and execute $\mathcal{A}_i$ until the system reaches the state in a cell in $\mathcal{W}_j \prec_{\varphi_g} \mathcal{W}_i$, at which point, switch to the automaton $\mathcal{A}_j$. Keep iterating this process until $\mathcal{A}_0$ is executed.

Theorem

Suppose for each $i \in \{1, \ldots, p\}$, Ψ_i is realizable. Then the receding horizon strategy ensures the correctness of the system.



$\mathcal{W}_i = \{\mathcal{V}_i\}, \forall i$

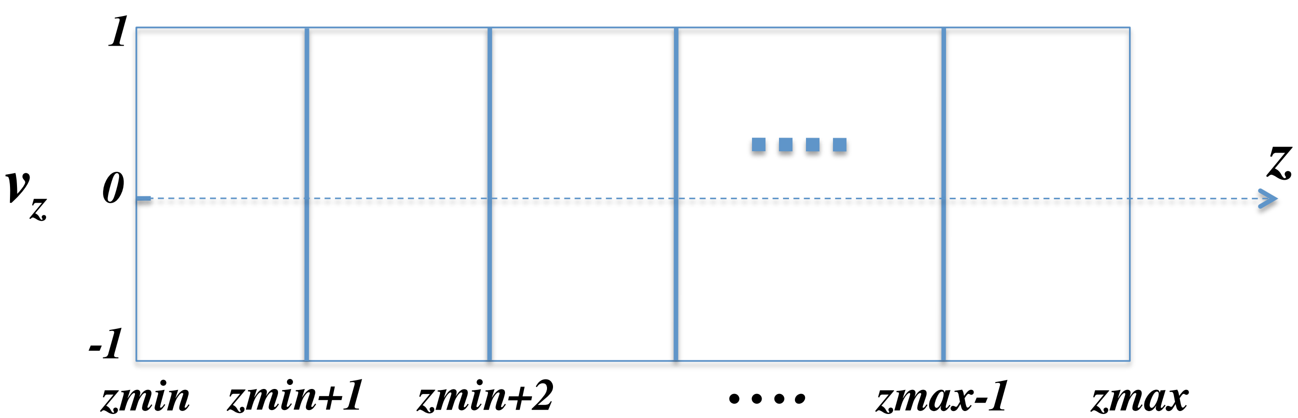
System Model

- Discretized nondimensional dynamics of a point-mass omnidirectional vehicle

$$\begin{bmatrix} z[t+1] \\ v_z[t+1] \end{bmatrix} = \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.9048 \end{bmatrix} \begin{bmatrix} z[t] \\ v_z[t] \end{bmatrix} + \begin{bmatrix} 0.0048 \\ 0.0952 \end{bmatrix} q_z$$
$$|q_z| \leq \sqrt{0.5}$$

  where z represents either x or y and v_z represents the rate of change in z.

- Domain: $C = C_x \times C_y$,
  $C_x = [xmin, xmax] \times [-1, 1]$,
  $C_y = [ymin, ymax] \times [-1, 1]$.
- Partition C_z as



$$C_z = \bigcup_{i \in \{zmin+1, \ldots, zmax\}} C_{z,i} \text{ where } C_{z,i} = [i-1, i] \times [-1, 1]$$

- Consider a straight road of length L with 2 lanes, each of width 1: $x_{min} = 0$, $x_{max} = L$, $y_{min} = 0$, $y_{max} = 2$.

System Specification: Desired Properties

- No collision:

$$\square(O_{i,j} \implies \neg(x \in C_{x,i} \wedge y \in C_{y,j}))$$

- The vehicle stays in the right lane unless there is an obstacle blocking the lane:

$$\square((\neg O_{i,1} \wedge x \in C_{x,i}) \implies (y \in C_{y,1}))$$

- Eventually the vehicle gets to the end of the road:

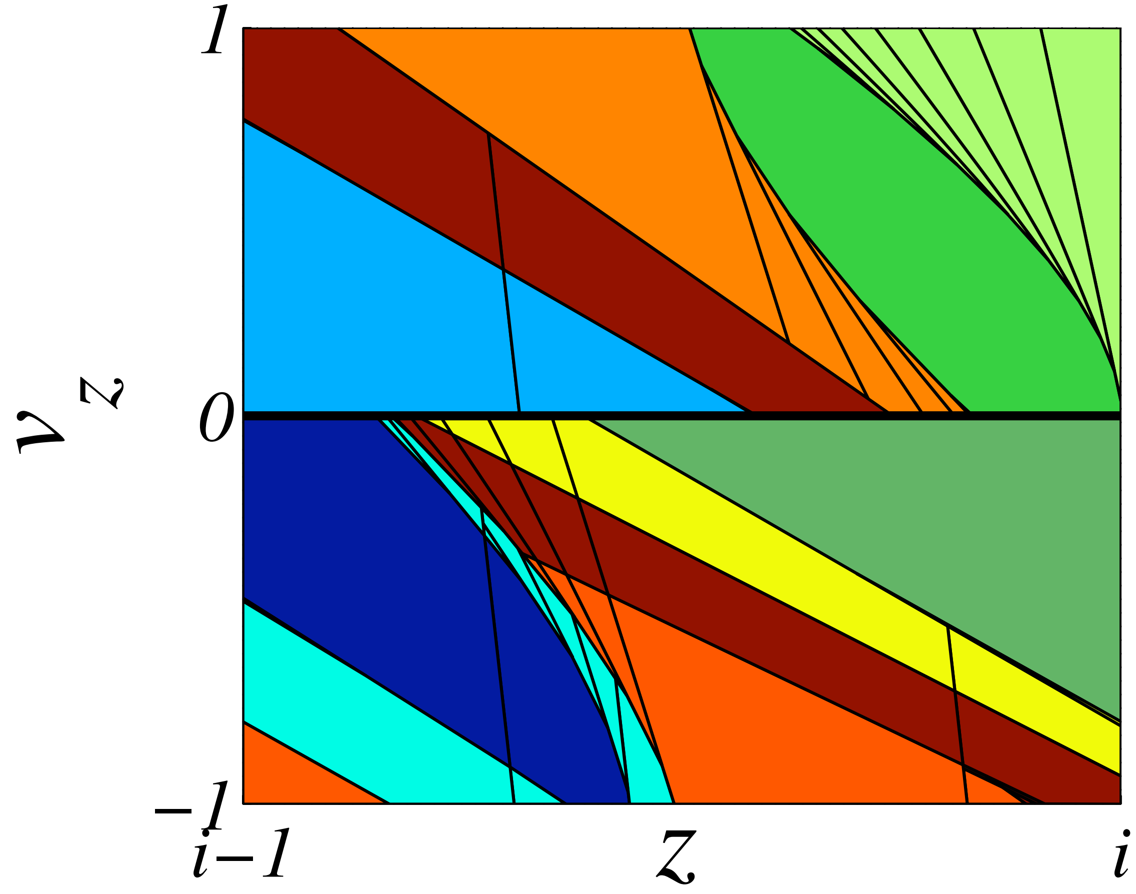
$$\diamond(x \in C_{x,L})$$

System Specification: Assumptions

- At the initial configuration, the vehicle is at least d_{obs} away from any obstacle and the vehicle starts in the right lane.
$$\left(x \in \bigcup_{k=i-d_{obs}}^{i+d_{obs}} C_{x,k} \implies (\neg O_{i,1} \wedge \neg O_{i,2})\right) \wedge y \in C_{y,1}$$
- An obstacle is always detected before the vehicle gets too close to it. That is, there is a lower bound d_{popup} on the distance from the vehicle for which obstacle is allowed to instantly pop up.
$$\square\left(\left(x \in \bigcup_{j=i-d_{popup}}^{i+d_{popup}} C_{x,j} \wedge \neg O_{i,k}\right) \implies \bigcirc(\neg O_{i,k})\right)$$
- Sensing range is limited. That is, the vehicle cannot detect an obstacle that is away from it farther than d_{sr}.
$$\square\left(x \in C_{x,i} \implies \bigwedge_{j>i+d_{sr}} (\neg O_{j,1} \wedge \neg O_{j,2})\right)$$
- The road is not blocked: $\square(\neg O_{i,1} \vee \neg O_{i,2})$
- An obstacle on the right lane does not disappear. $\square(O_{i,1} \implies \bigcirc(O_{i,1}))$

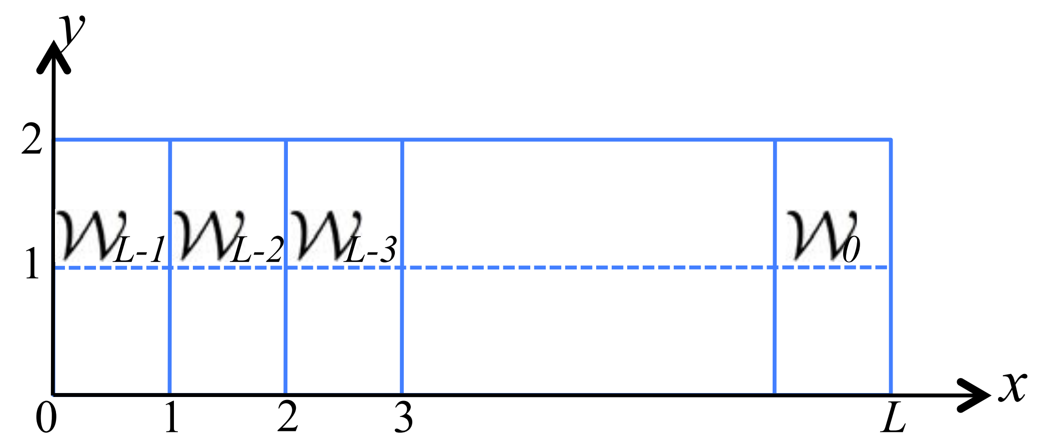
State Space Discretization

- Apply the discretization algorithm for the x and y components separately for computational efficiency.
- With horizon length $N = 10$ and $Vol_{min} = 0.1$, get 11 cells $\{C^1_{z,i}, C^2_{z,i}, \ldots, C^{11}_{z,i}\}$ for each $C_{z,i}$.



Receding Horizon Formulation

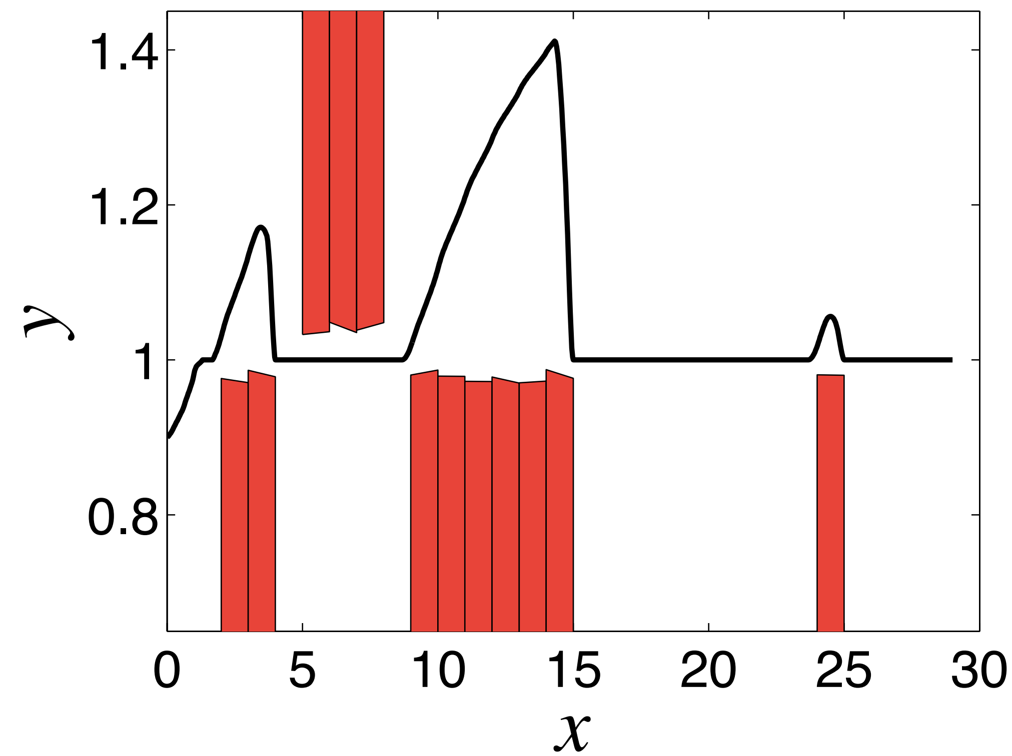
- Partial order structure: $\mathcal{W}_i = \{(\nu_x, \nu_y, O_{1,1}, \ldots, O_{L,2}) \mid \nu_x \in \mathcal{C}_{x,L-i}\}$



- An invariant Φ is determined by reasoning about the valid discrete transitions and the desired properties.
  - To ensure the progress property $\Diamond \varphi_g$, the vehicle cannot be in a state with no vaild transition to other state.
  - To ensure no collision, the vehicle cannot collide with an obstacle at the initial state.
  - etc
- With $d_{popup} = 1$ and the horizon length 2, the specifications for the subautomata are realizable.
- If $d_{obs} > 1$ and the possible initial states of the system are restricted to those with valid transition to other state, then $\varphi_{init} \implies \Phi$ is a tautology.

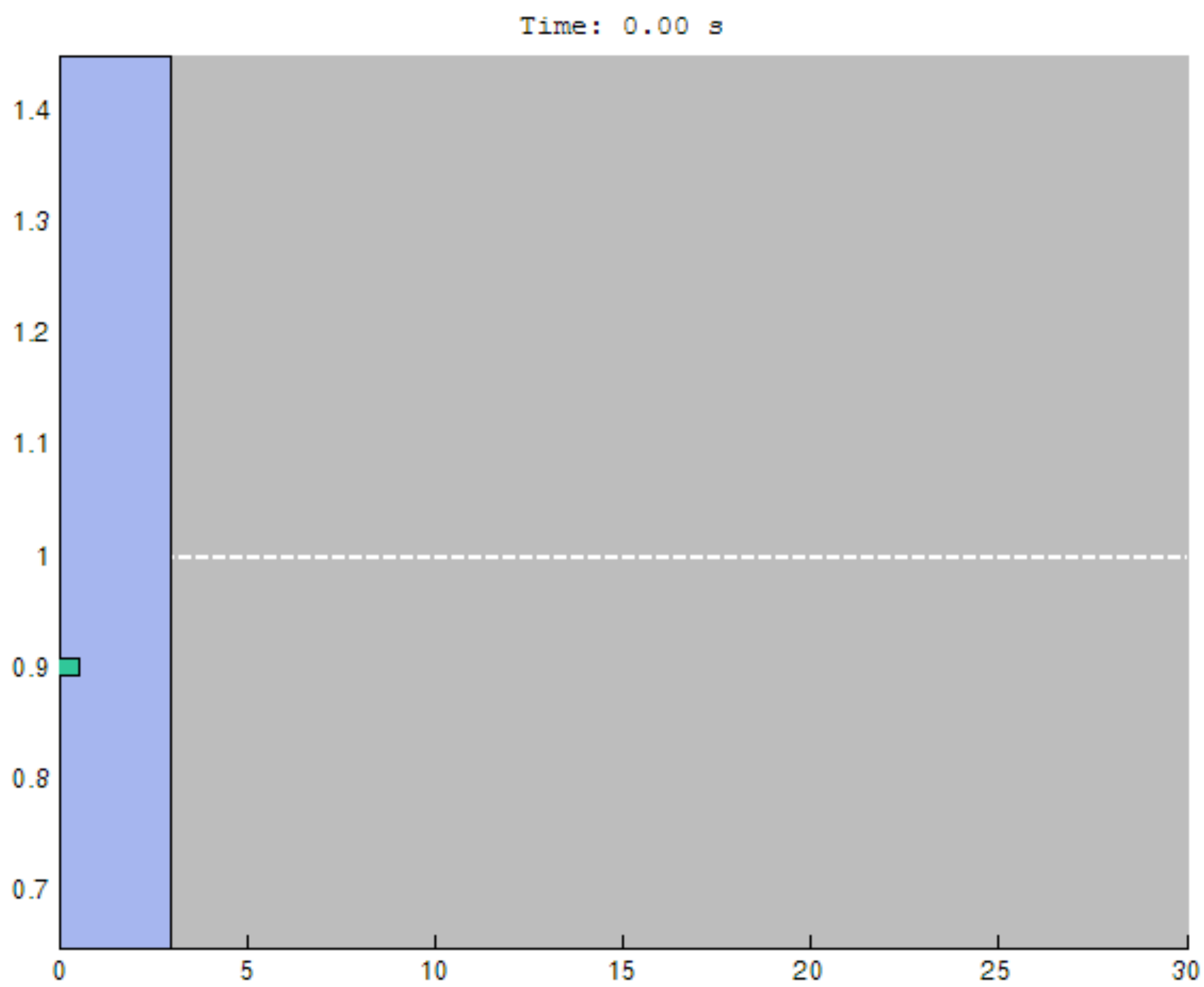
Results

- The synthesis was performed on a Pentium 4, 3.4 GHz computer with 4 Gb of memory.
- Computation time: 1230 seconds
- The resulting automaton contains 2845 nodes.



- During the synthesis process, 96796 nodes were generated.
  - This particular computer crashes when approximately 97500 nodes are generated.
  - Without the receding horizon strategy, problems with the road of length greater than 3 cannot be solved.

Results



Conclusions

- Described how off-the-shelf tools from computer science and control can be integrated to allow automatic synthesis of complex dynamical systems.
  - The resulting systems are guaranteed, by construction, to satisfy the desired properties expressed in linear temporal logic even in the presence of adversary.
- Described a receding horizon scheme
  - Address the main limitation of the synthesis algorithm: the state explosion problem.
  - Allow more complex problems to be solved.
- The example illustrated that without the receding horizon scheme, the synthesis problem can be extremely computationally challenging.

Acknowledgments

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- Knot Pipatsrisawat
- Multi-Parametric Toolbox (MPT): http://control.ee.ethz.ch/~mpt/
- Temporal Logic Verifier (TLV): http://www.cs.nyu.edu/acsys/tlv/index.html
- TLV Module for Synthesis: http://www.wisdom.weizmann.ac.il/~saar/synthesis/