Geometric Mechanics
Some Recent Progress

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Control and Dynamical Systems,
Caltech

Oberwolfach, July 21, 2008

Collaborators: as we go along

Reduction Theory

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  - ★ Cotangent bundle (in, eg, Foundations of Mechanics)
  - ★ Poisson reduction (various + JM and Ratiu, 1986)
  - ★ Lagrangian reduction (Scheurle, JM Cendra, Ratiu).
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- Lets talk about ***reduction by stages*** to be specific.

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- ***Hamiltonian reduction by stages*** ...

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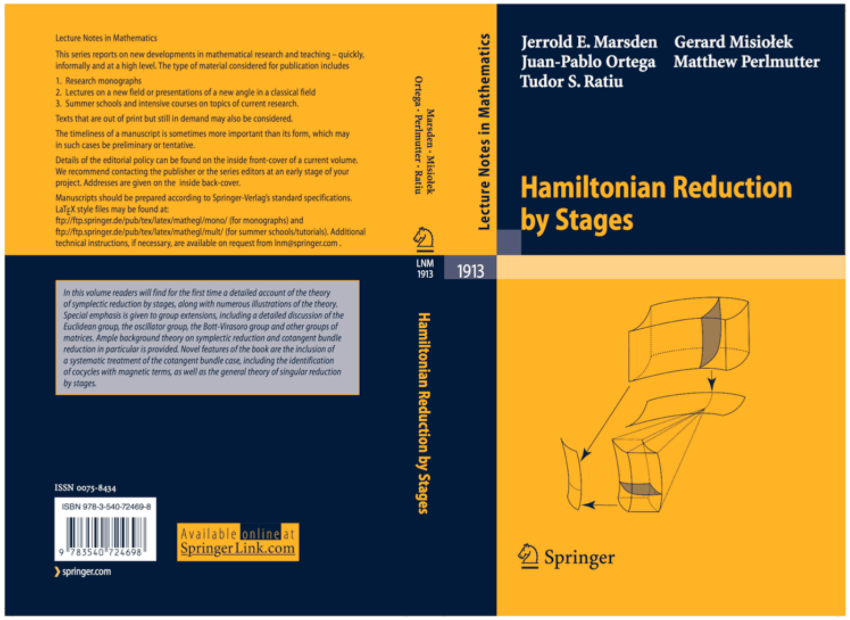


Lagrangian Reduction by Stages

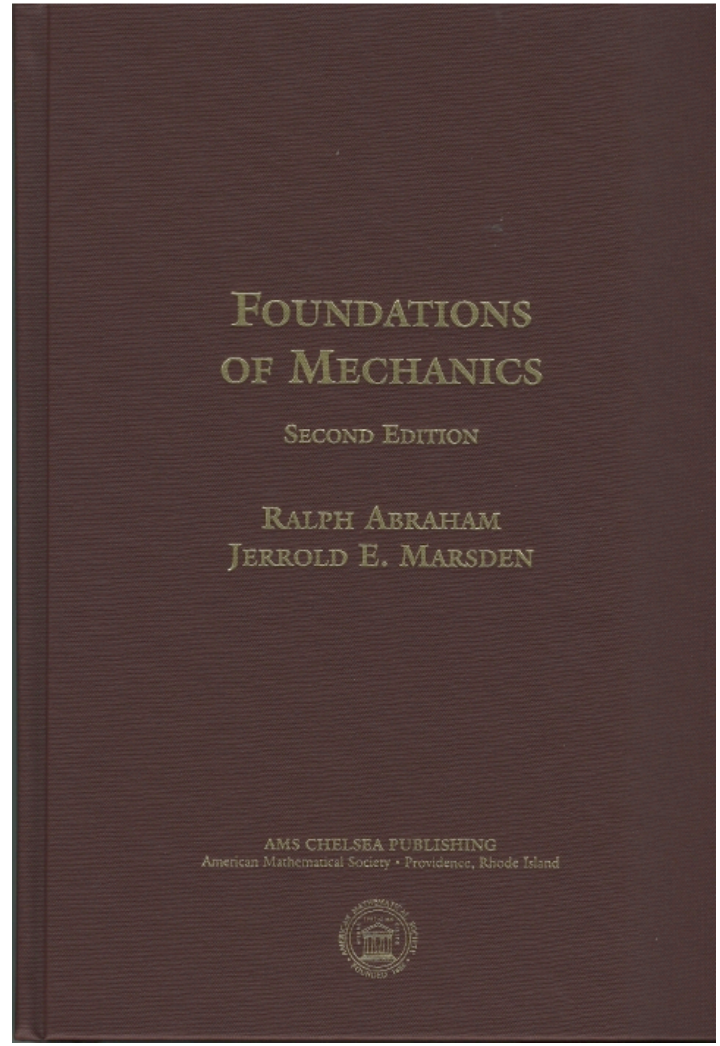
Memoirs of the American Mathematical Society
152, no 722, July 2001, 108 pp.

(Recieved by the AMS, April, 1999).

Hernán Cendra

Jerrold E. Marsden

Tudor S. Ratiu



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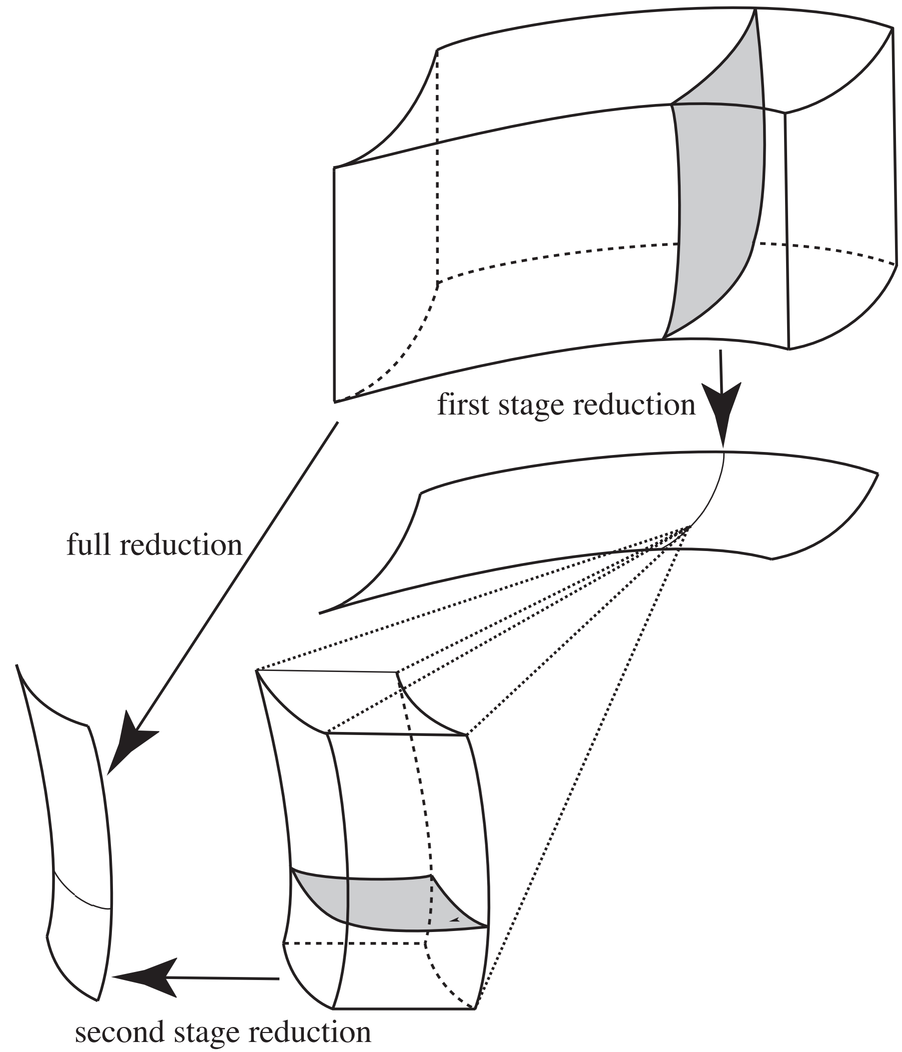

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- The first group is the particle relabeling group, while the second one is the Euclidean group. The big group is the synthesis of the two—the full symmetry group of the problem.
- The “hand calculation” of the resulting Hamiltonian and variational structure is difficult—Shashikanth et al and Borisov and Mamaev. Reduction by stages gives this (and more general) and how they are related.

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- Different sort of reduction (model reduction) in work of Tomohiro Yanao---radii of gyration in molecular systems. Geometric mechanics methods critical !
- Work of Cendra, Etchechoury, JM on Dirac-Gotay-Nester constraints in the Dirac Structure Context. Confused?

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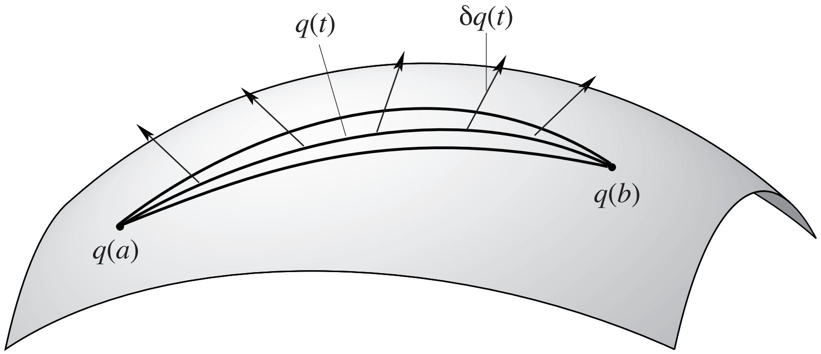

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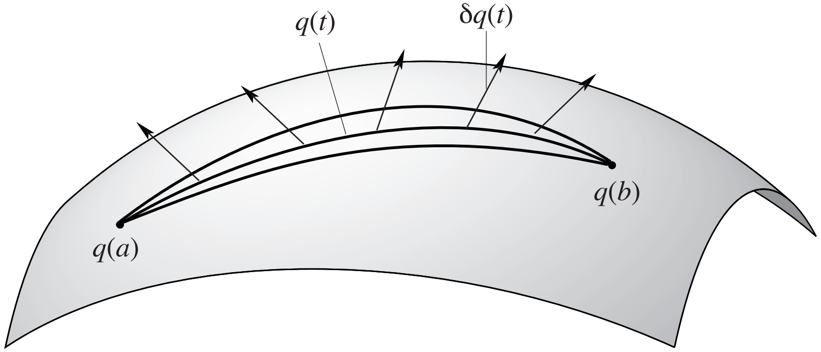

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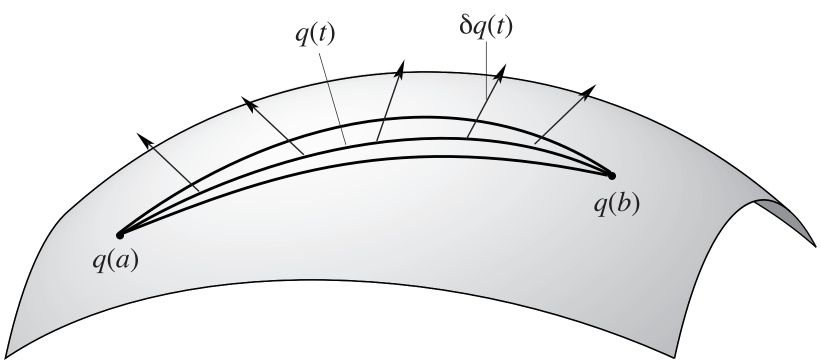

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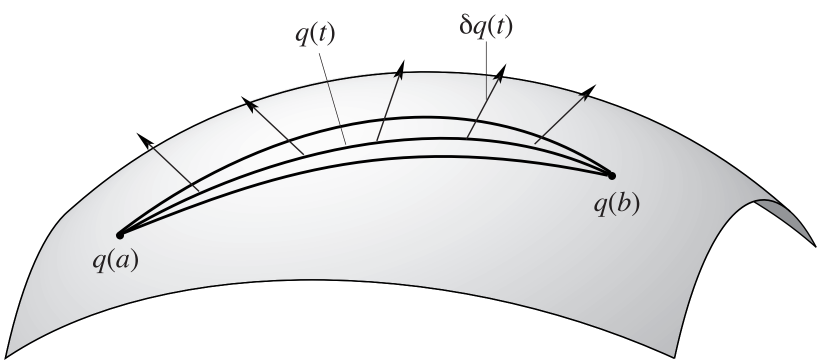

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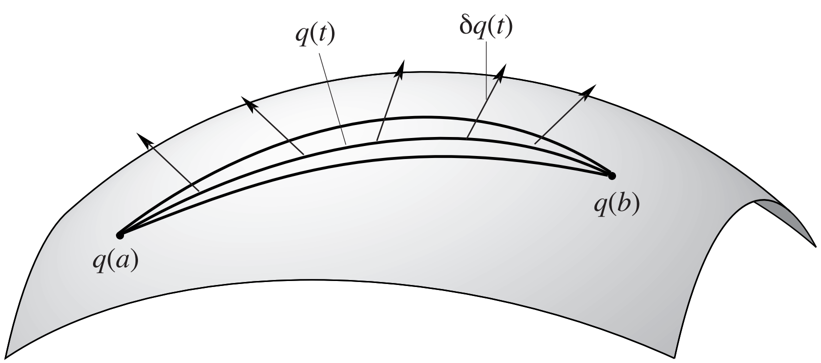

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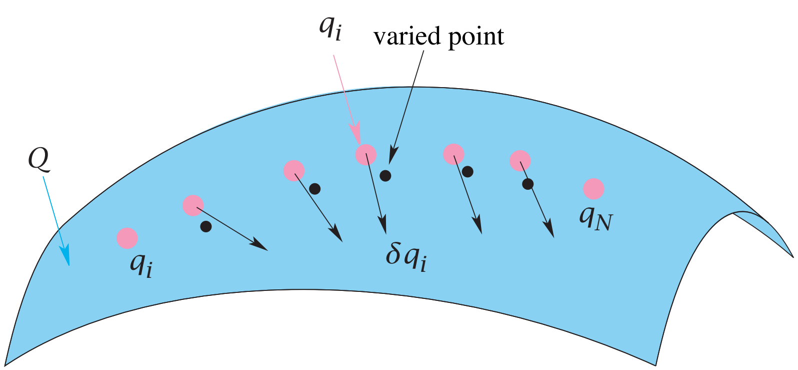

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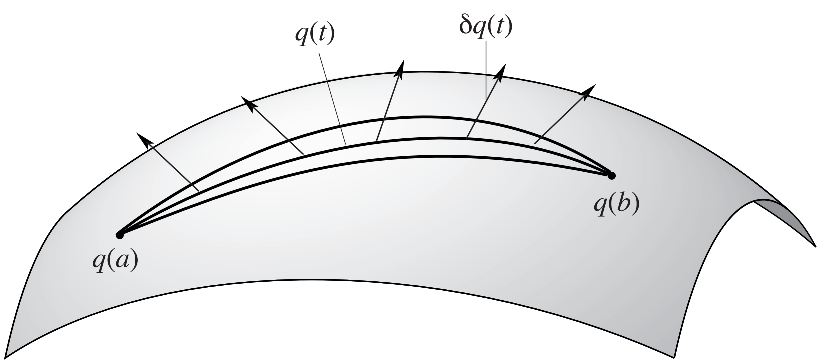

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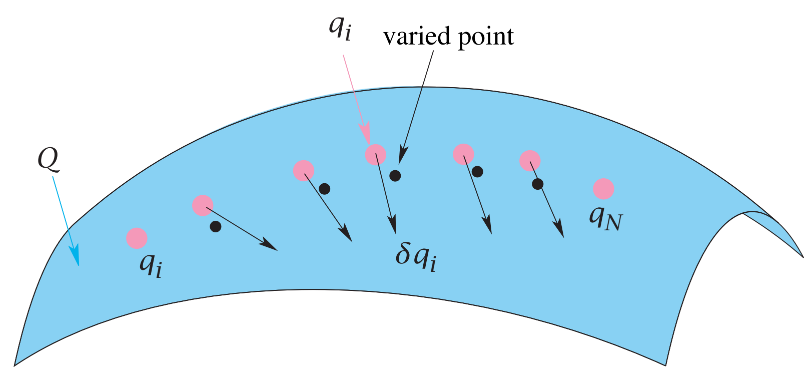


Add forces to this formalism, such as control forces

Simple Example

- Let M be a positive definite symmetric n by n matrix and V be a given potential. Choose the Lagrangian:

$$L(q, \dot{q}) = \frac{1}{2}\dot{q}^T M \dot{q} - V(q).$$

- Discrete Lagrangian chosen by using the *rectangle rule* on the action integral together with a naive finite difference approximation of the velocity (more sophisticated quadrature rules give higher order accurate algorithms, such as SPARK).

$$L_d(q_0, q_1, h) = h\left[\frac{1}{2}\left(\frac{q_1 - q_0}{h}\right)^T M \left(\frac{q_1 - q_0}{h}\right) - V(q_0)\right]$$

- Resulting DEL equations (the algorithm):

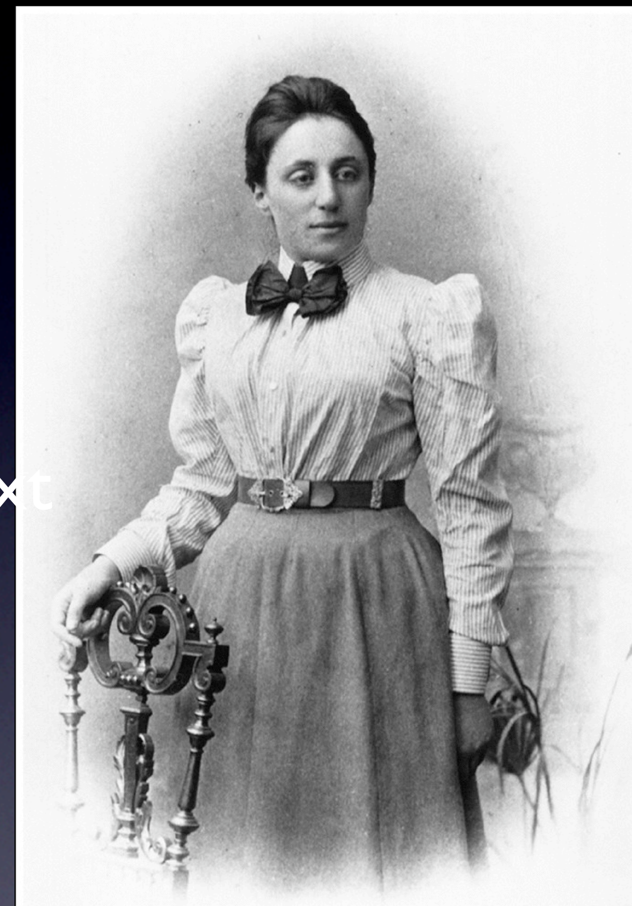
$$M\left(\frac{q_{k+1} - 2q_k + q_{k-1}}{h^2}\right) = -\nabla V(q_k)$$

Noether's theorem

A nice thing about the variational formulation is that such algorithms are naturally symplectic and results such as Noether's theorem remain valid.

Text

The proofs of these things are basically the same as in the continuous theory.



Emmy Noether (1882–1935)

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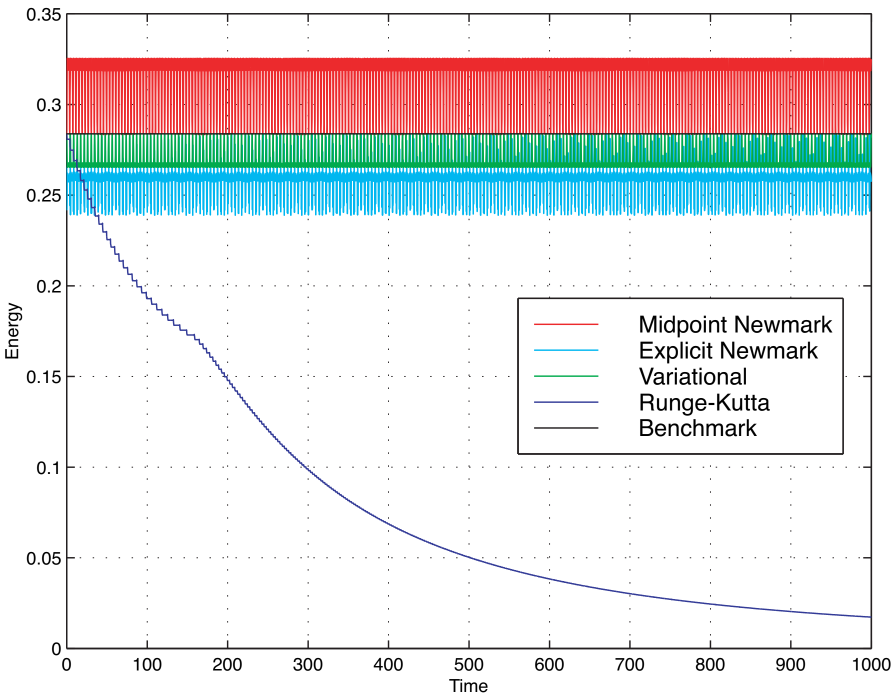

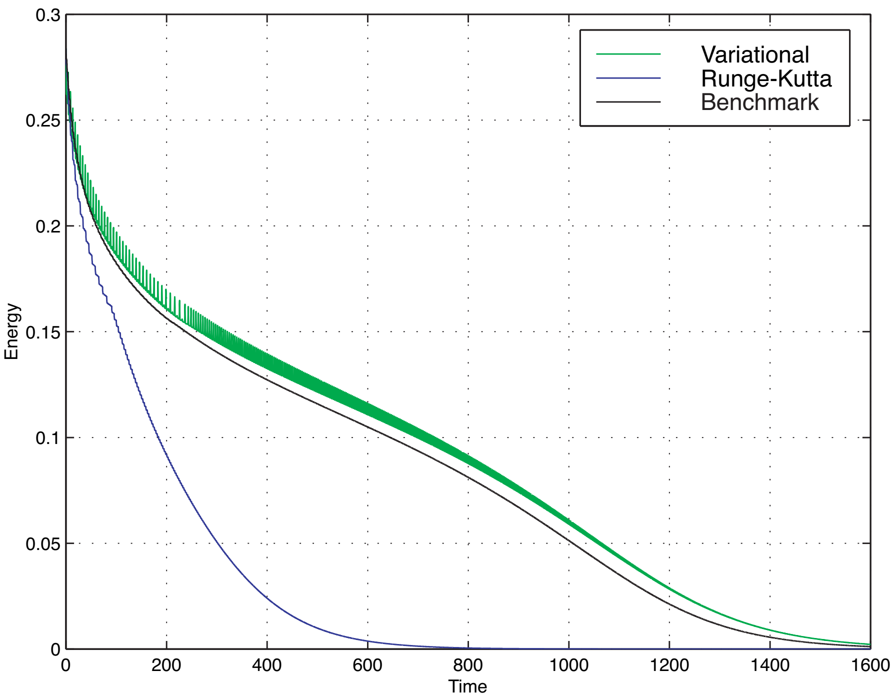

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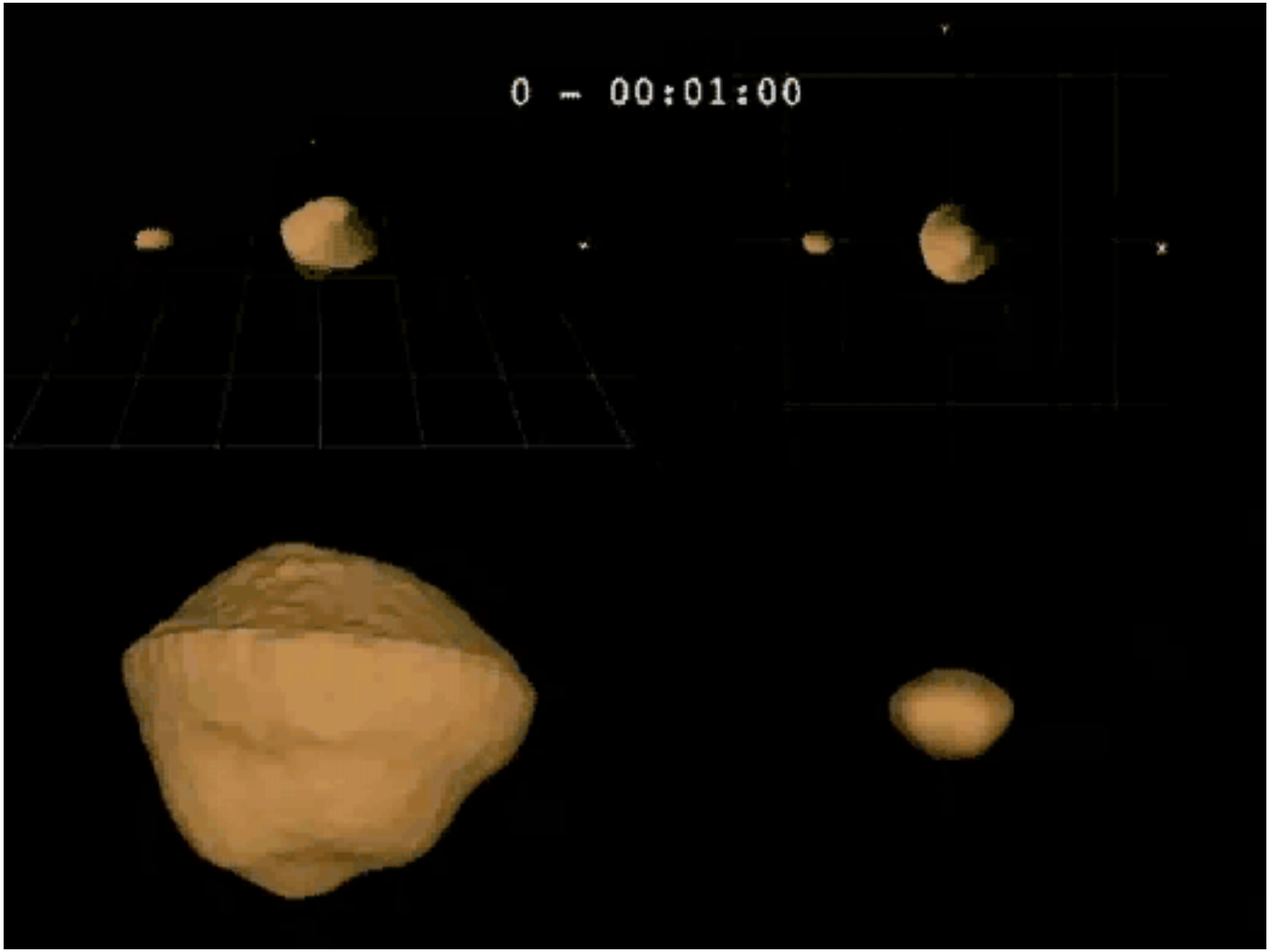

The Outer Solar System 250,000 years into the future

Runge-Kutta Method (RK4)

Störmer-Verlet method

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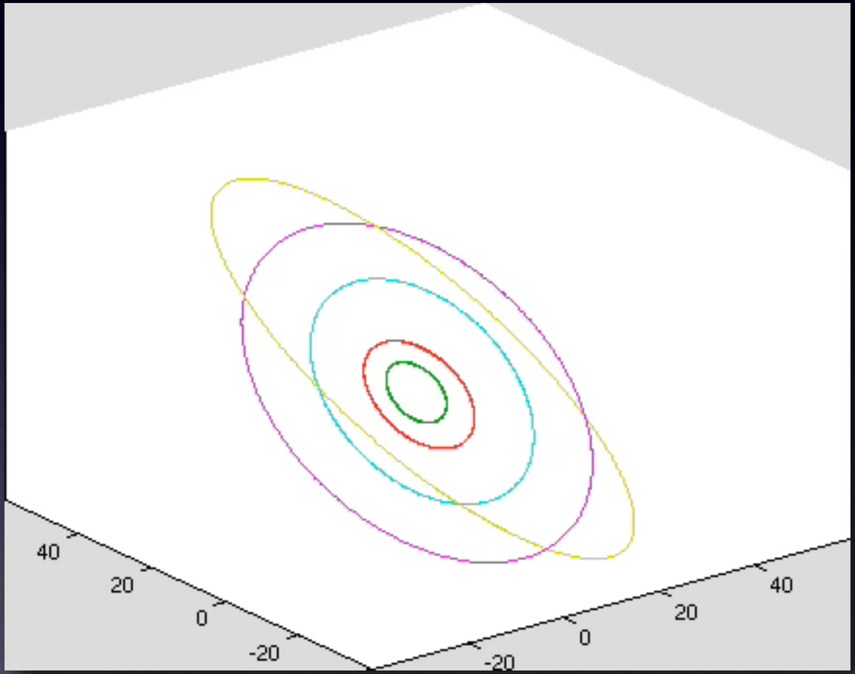

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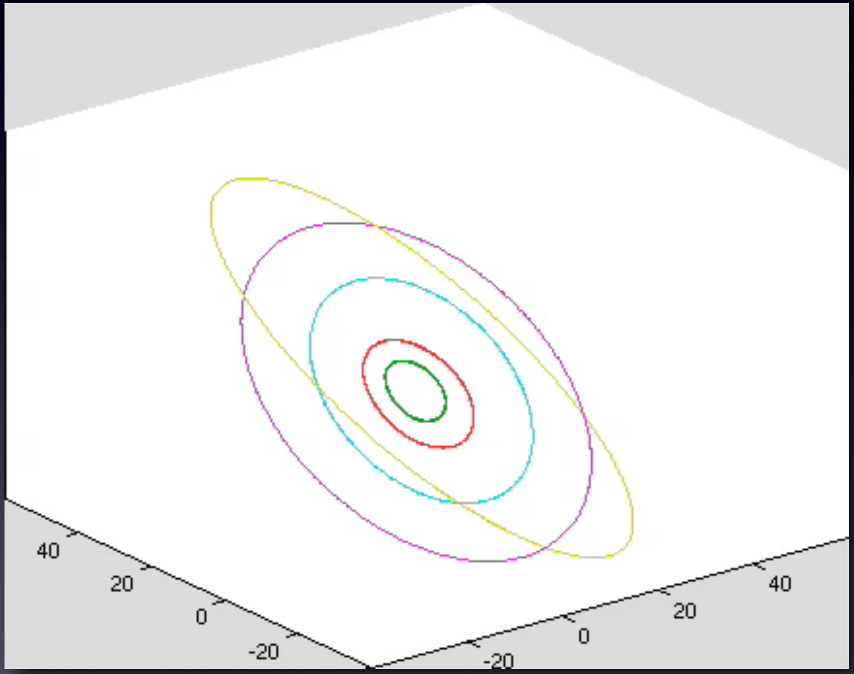

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Molecular Systems

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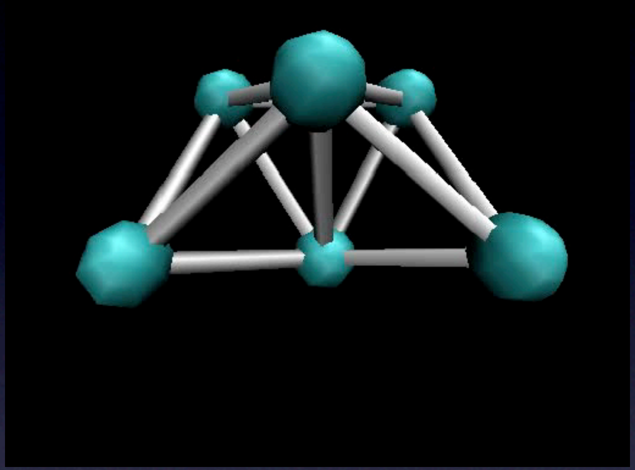

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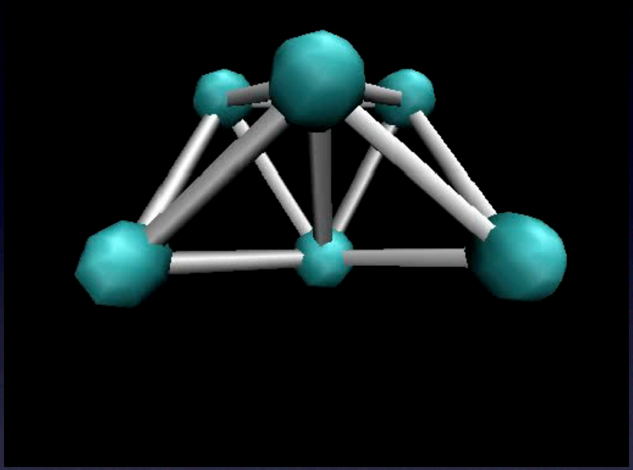

A "gauge theory"—one separates the shape dynamics from the orientation dynamics. The geometry is nontrivial ! (Yanao, Koon, Kevrekides, JM)

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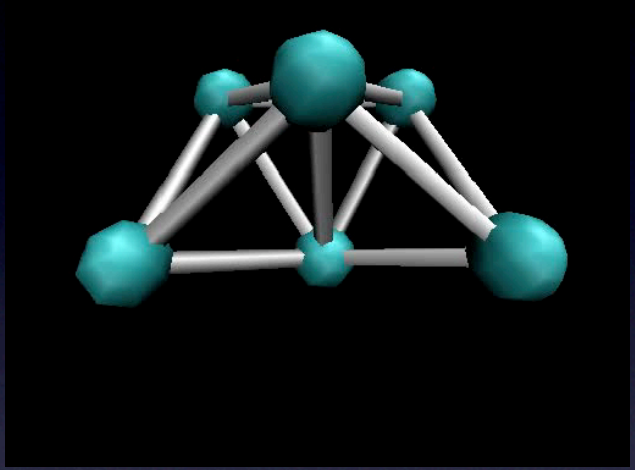

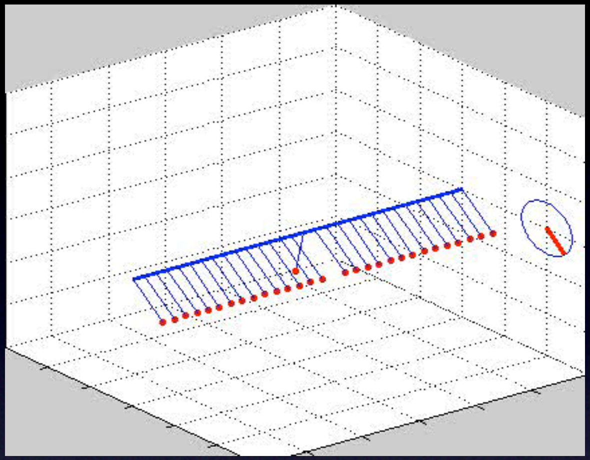

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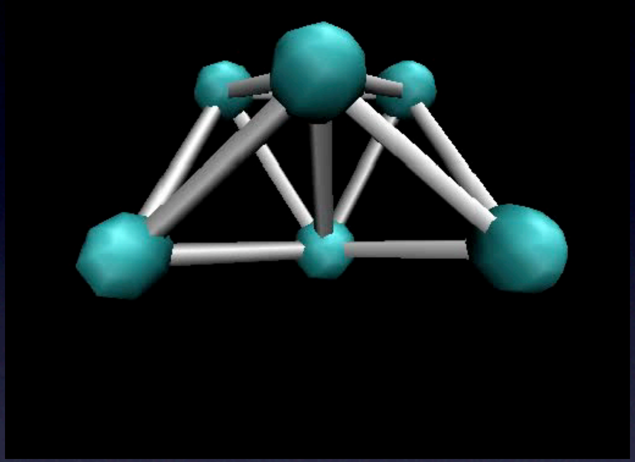

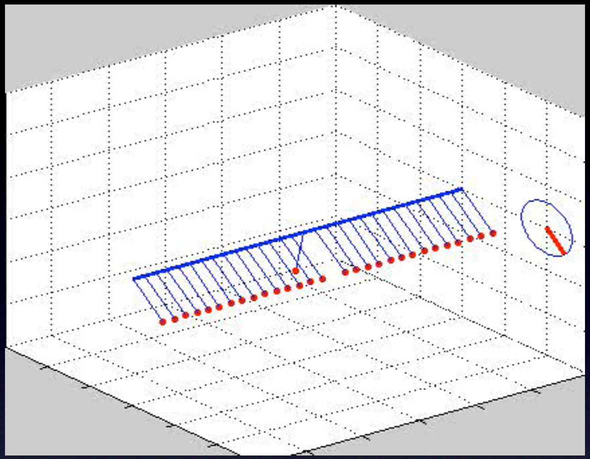

Unstructured perturbations require high energy to induce flipping

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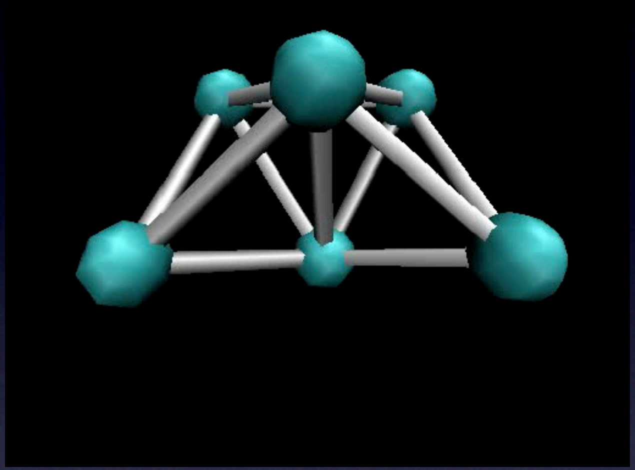

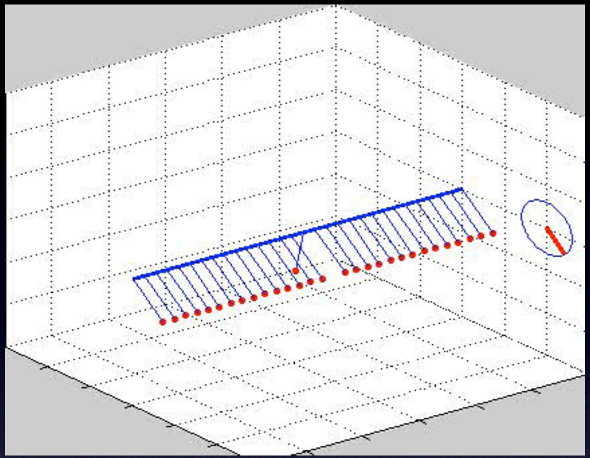

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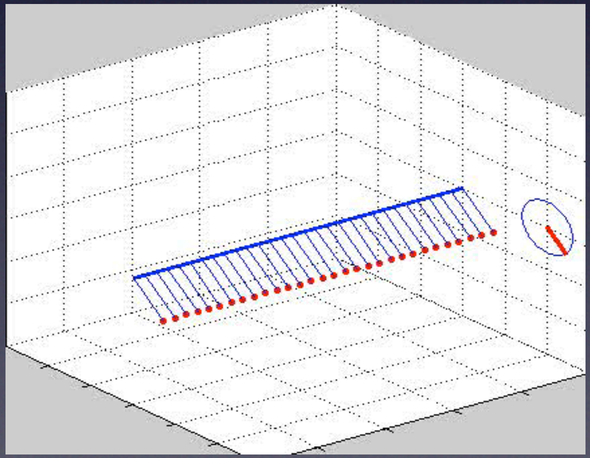

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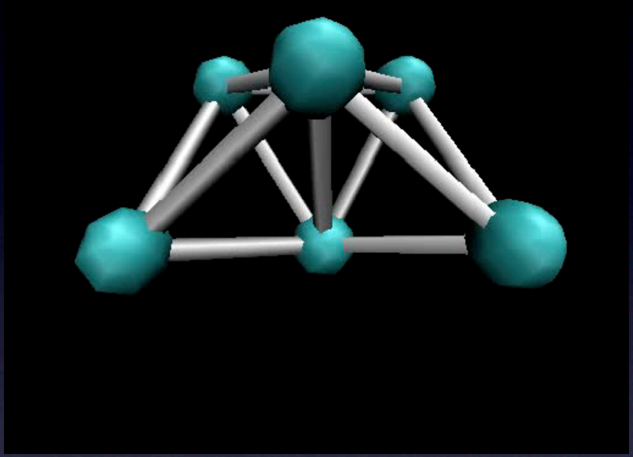

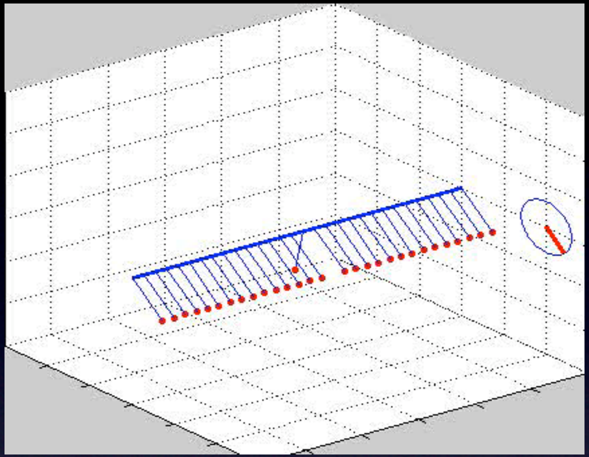

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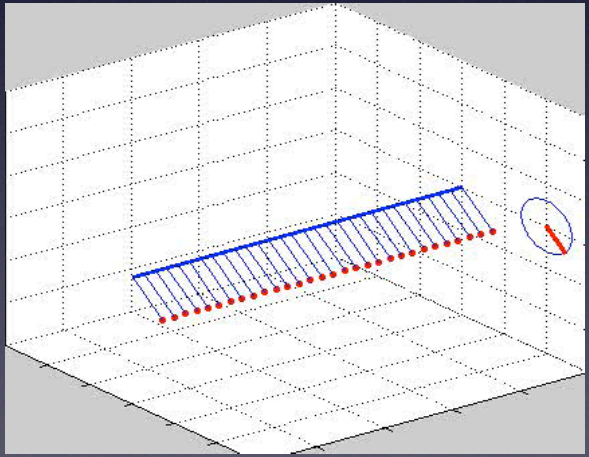

Structured perturbations induce flipping with low energy

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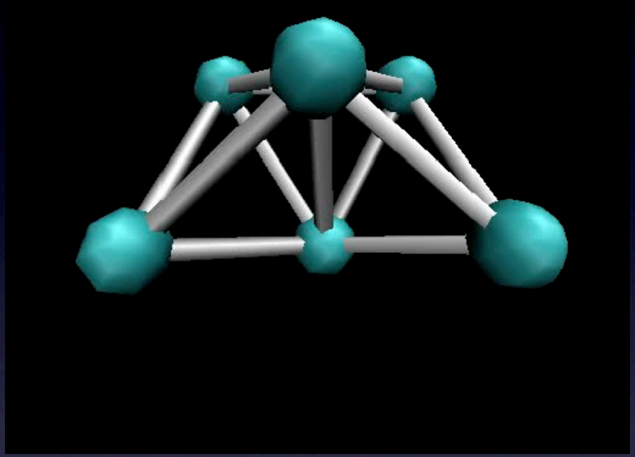

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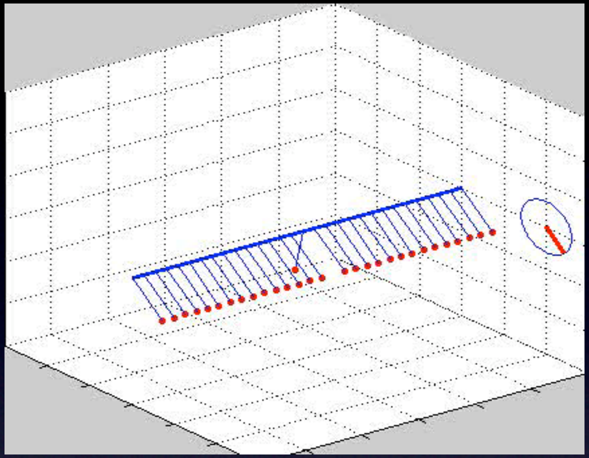

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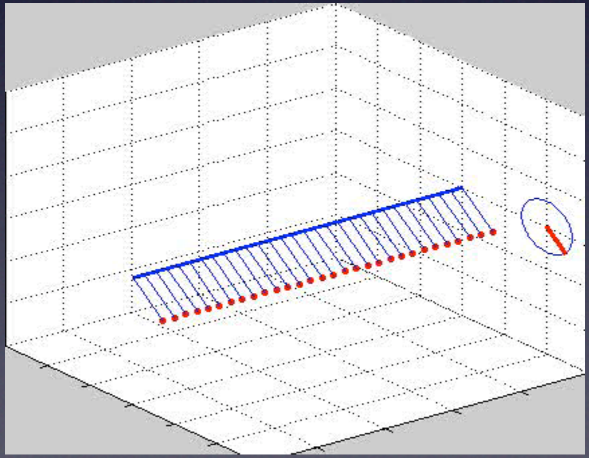

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Philip DuToit, Igor Mezic, JM

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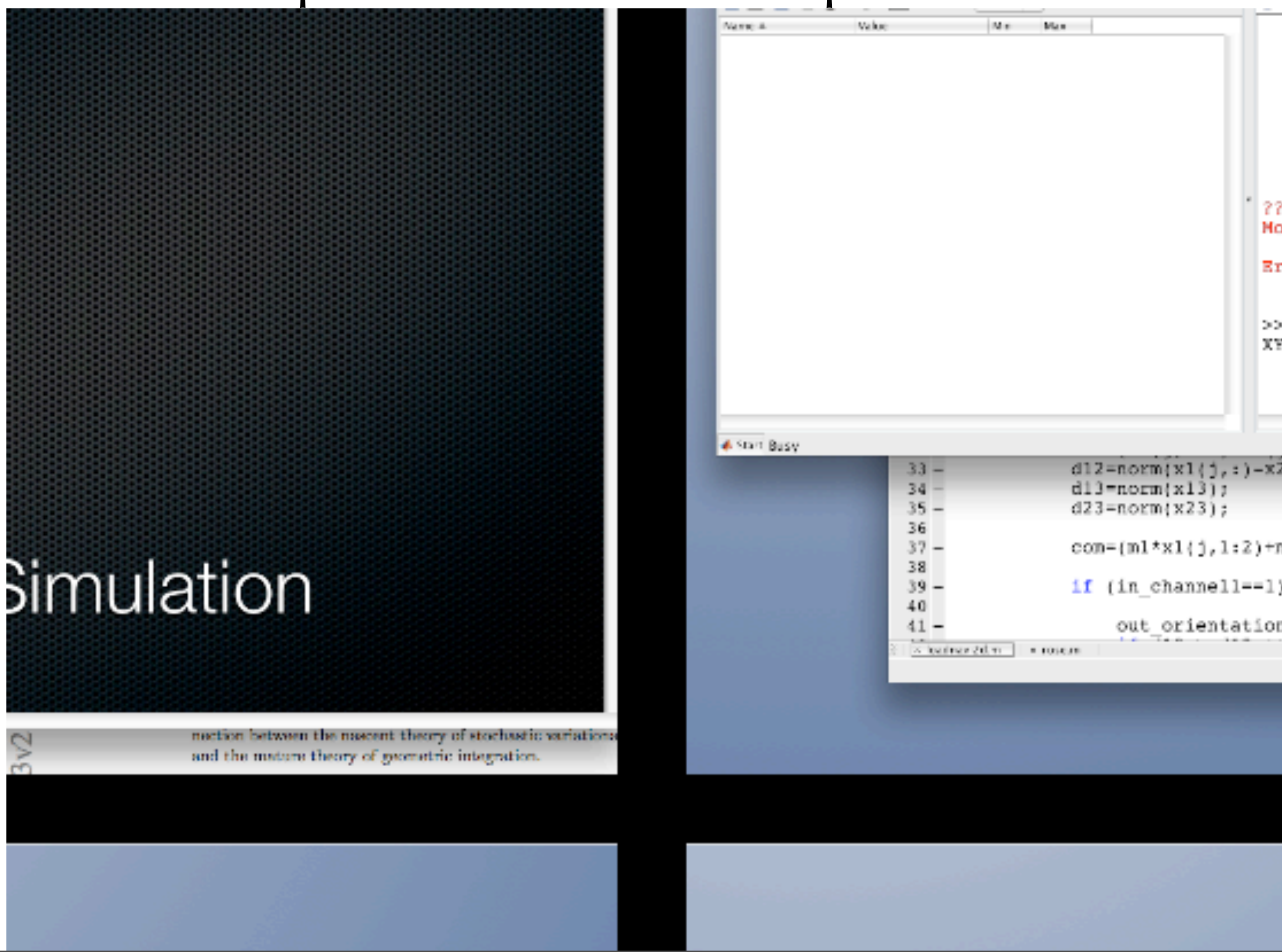
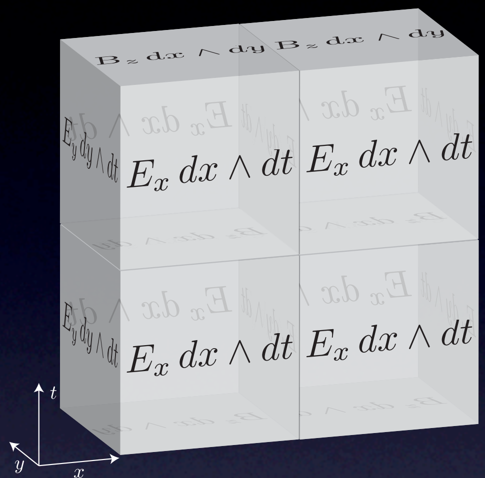
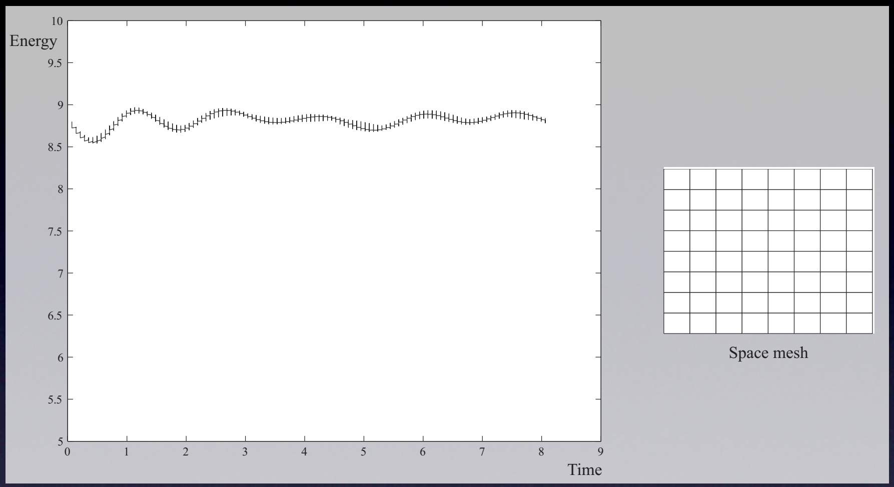
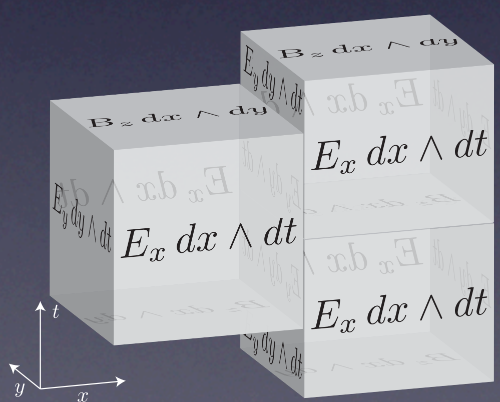


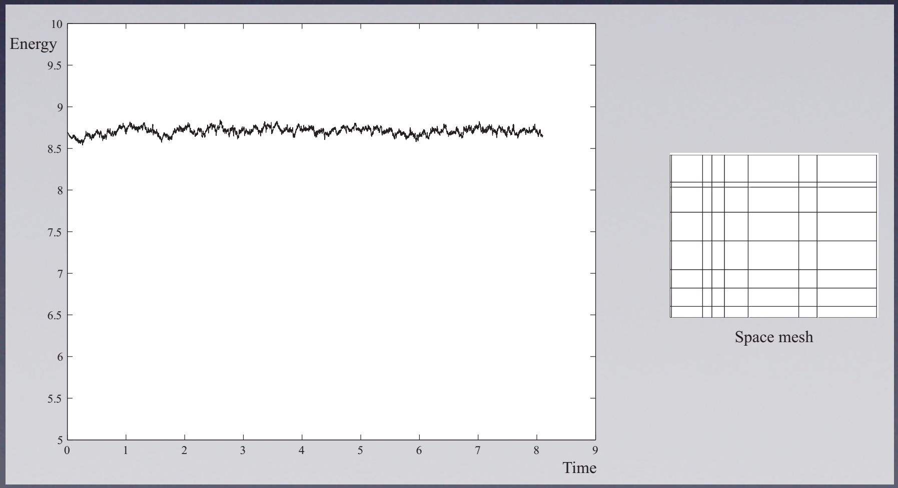
Discrete Mechanics also Applies to Field Theories, such as E and M





standard "synchronized" method





our new "asynchronous" method

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- Discrete Dirac structures and Dirac integrators are surely interesting; Melvin Leok and student are looking into this.
- Reduction for discrete mechanics is also troublesome; perhaps the Dirac setting helps there too.

Discrete Mechanics and Optimal Control (DMOC)

Sina Ober-Blöbaum, Oliver Junge, JM

Discrete Mechanics and Optimal Control (DMOC)

- DMOC recasts the problem in discrete time as

$$\text{Minimize } J_d(q_d, u_d) = \sum_{k=0}^{N-1} F_d(q_k, q_{k+1}, u_k, u_{k+1}),$$

subject to boundary conditions (prescribed or periodic), and the appropriate forced DEL equations of motion.

- This formulation is an equality constrained, nonlinear optimization problem that can be solved with sequential quadratic programming (SQP).

Sina Ober-Blöbaum, Oliver Junge, JM

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- Some examples...

Falling Cats, Divers, Swimming

James Martin, Eva Kanso

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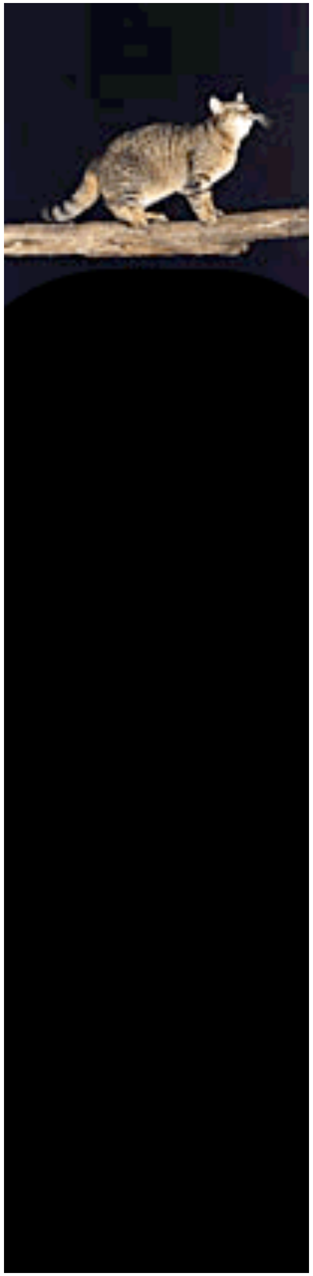

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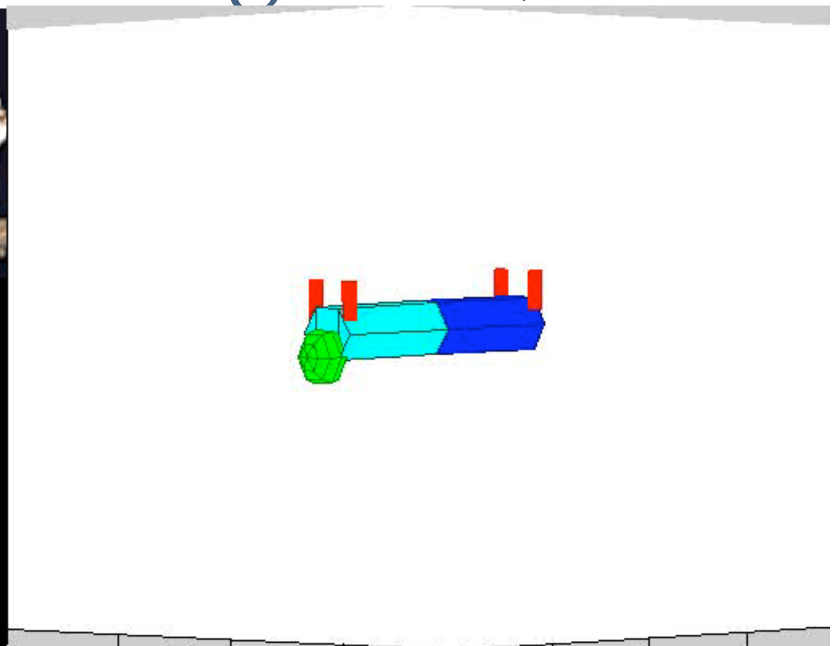

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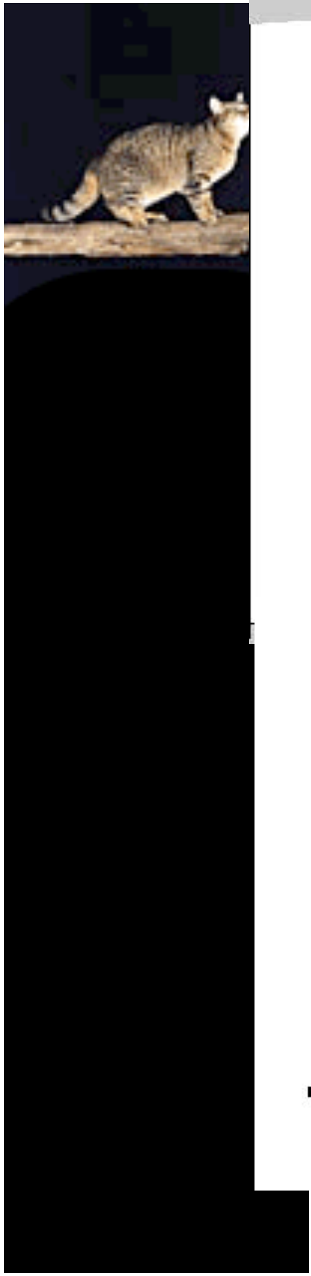

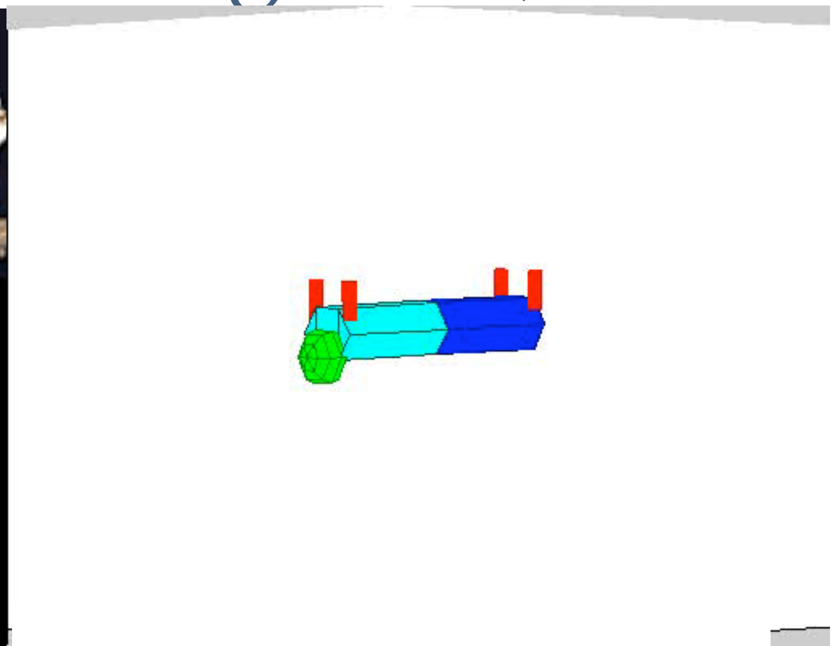

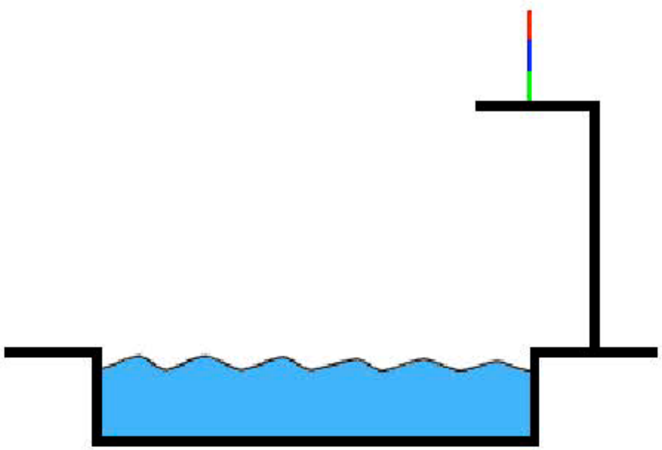

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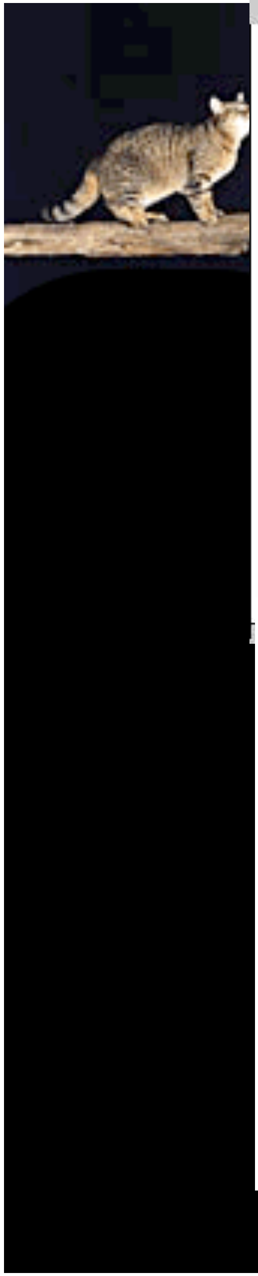

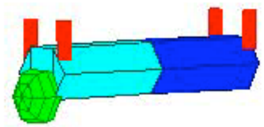

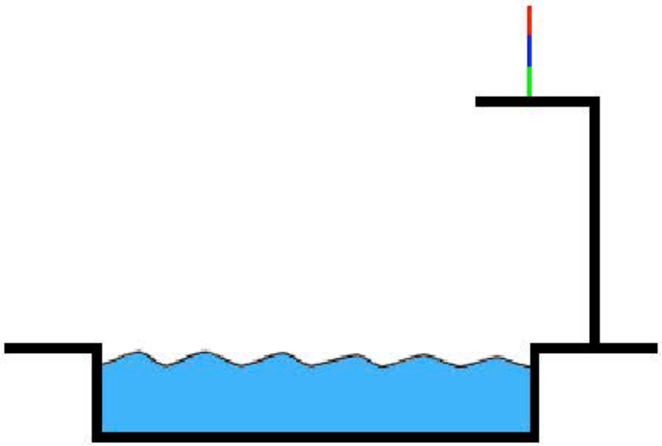

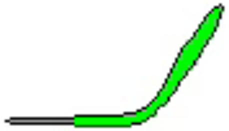

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Satellite Reorientation

Sigrid Leyendecker, Sina Ober-Blöbaum, Michael Ortiz, JM

Satellite Reorientation

Handling Constraints: variational discrete null space method

- Easy formulation of discrete Lagrangian with constant mass matrix.
- Symplectic-momentum conserving.
- Exact constraint fulfillment.
- Minimal dimension of resulting system.
- Condition number independent of time step.

Unlike

- Lagrange multiplier method: larger dimension of system than necessary and conditioning problems.
- Generalized coordinates: higher nonlinearity difficult for large multibody systems.

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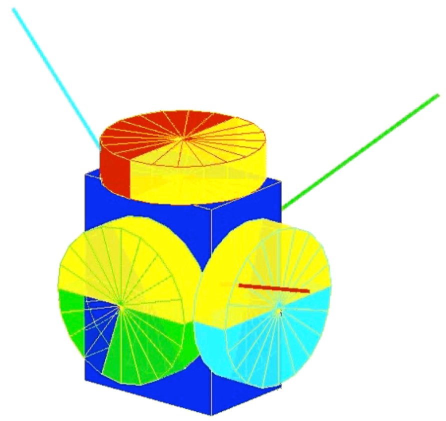

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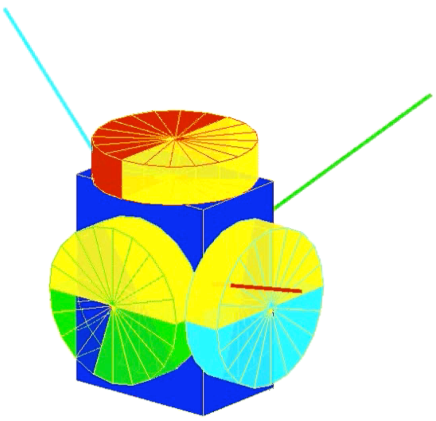

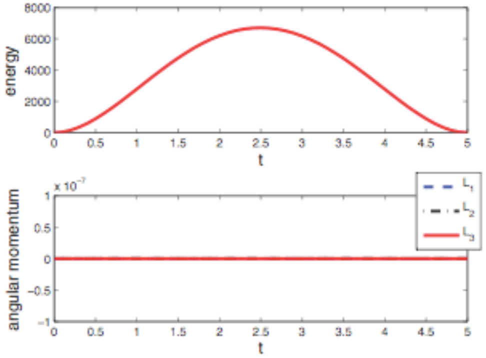

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Optimal Helicopter Flight

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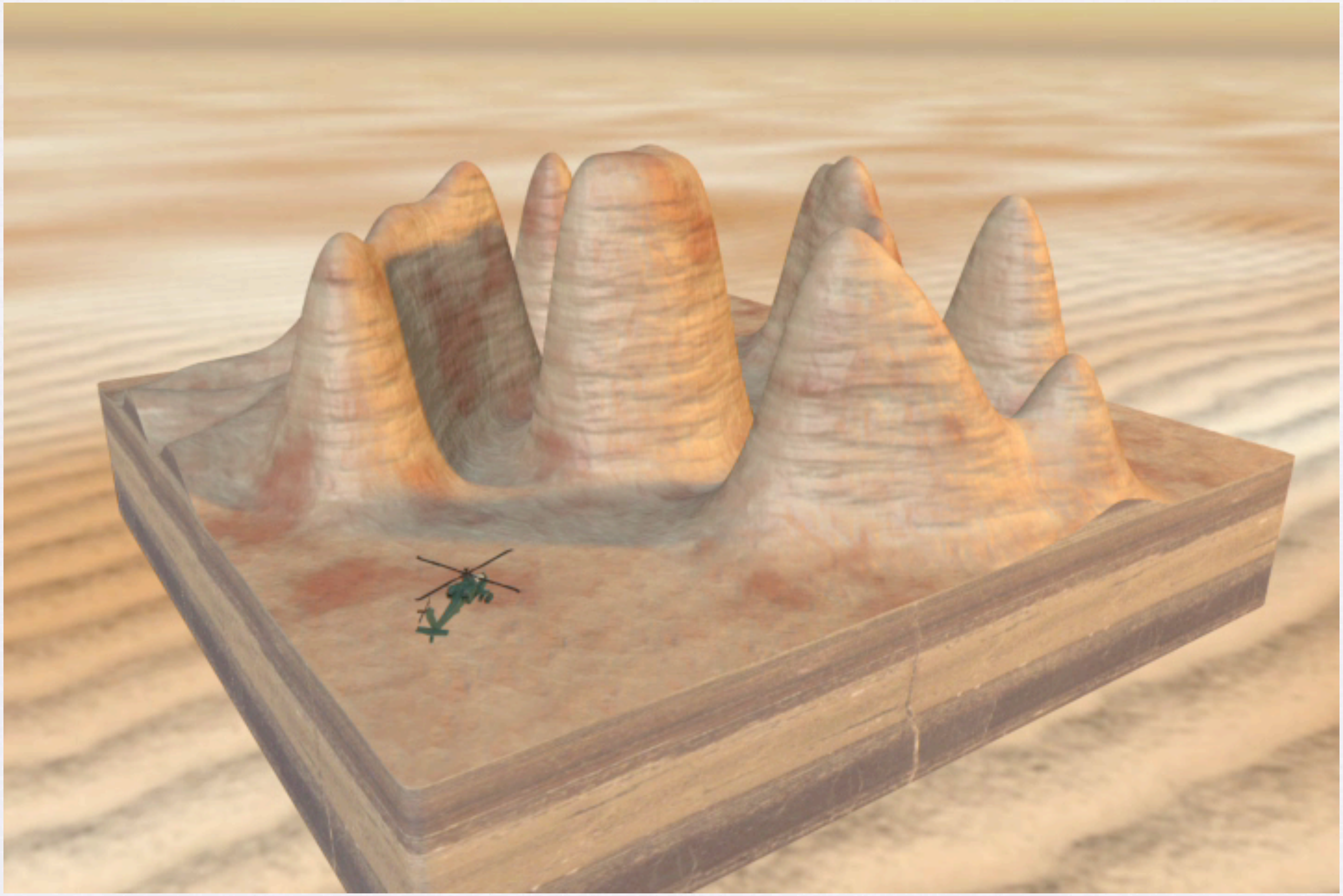

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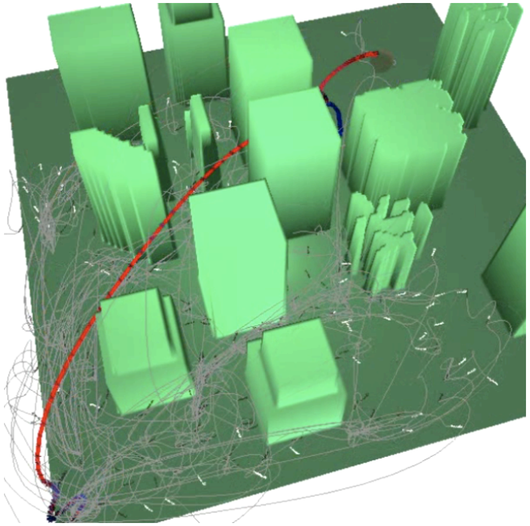

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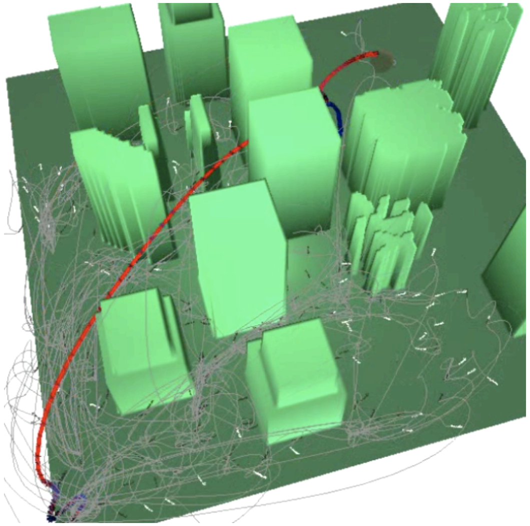

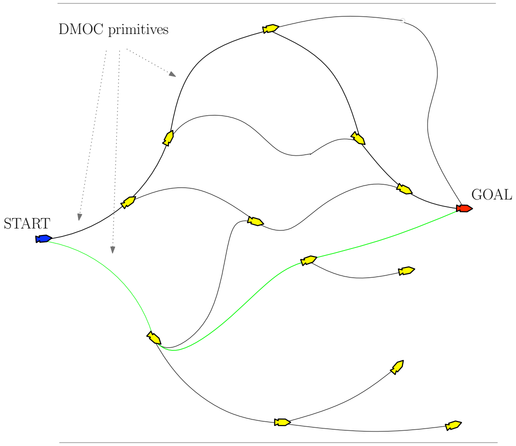

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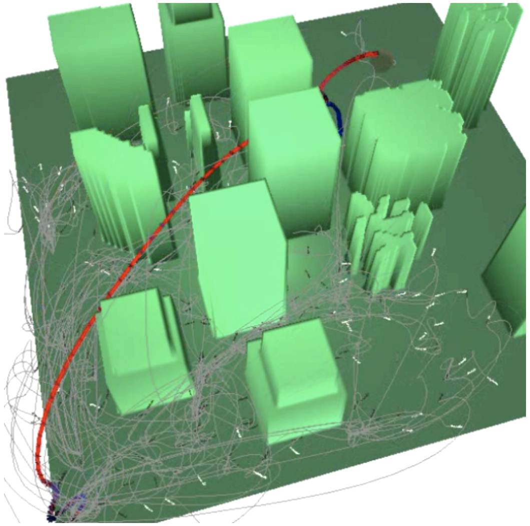

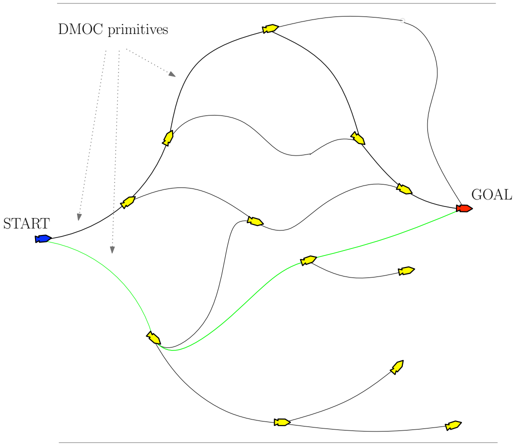

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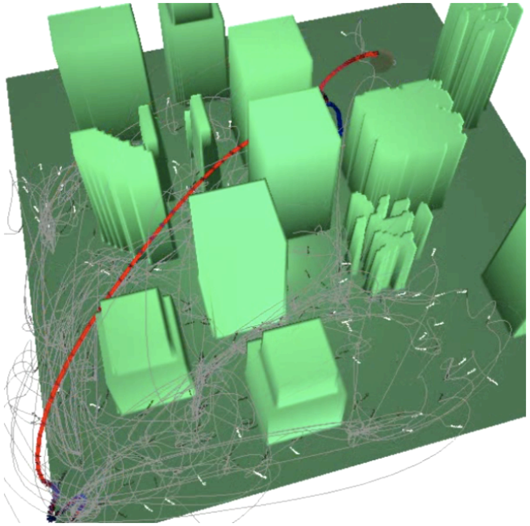

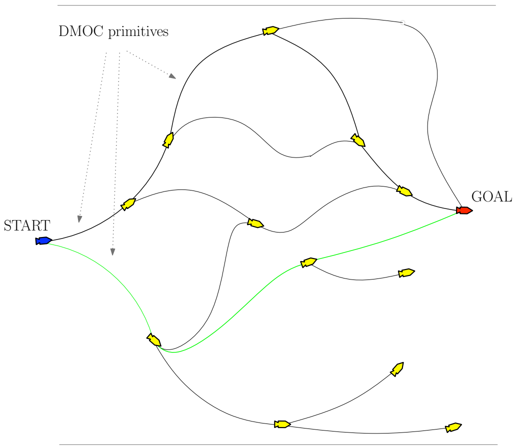

Future Directions: Combine with trend optimization techniques for charting efficient roadmaps. Make use of this technique in the surveillance problem.

Global Strategies using DMOC Primitives

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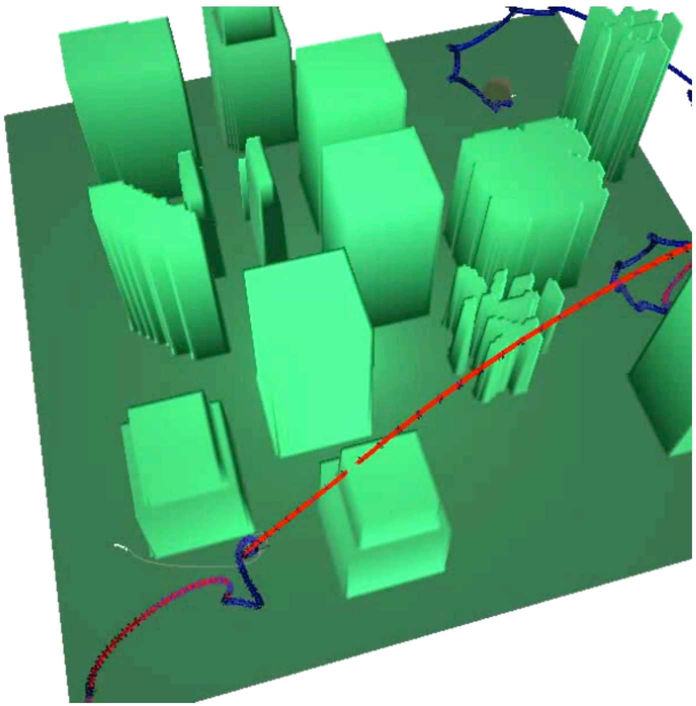

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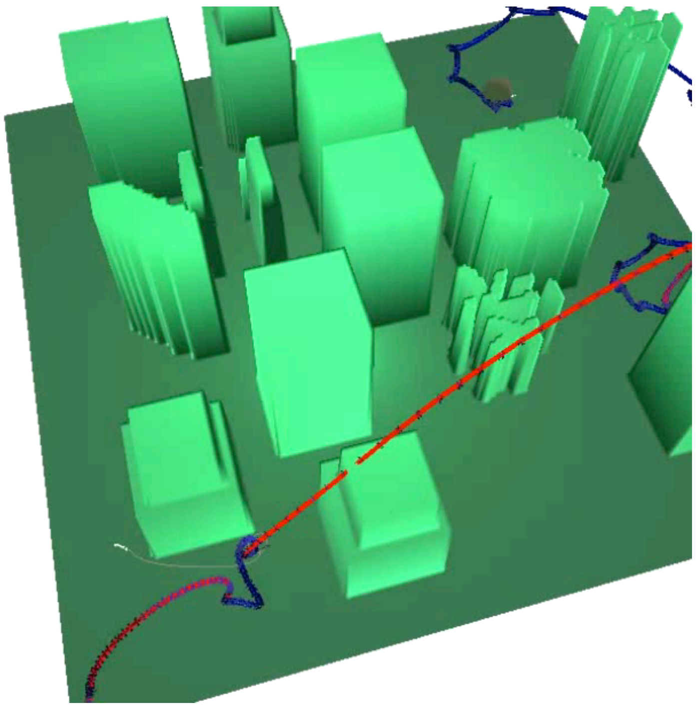

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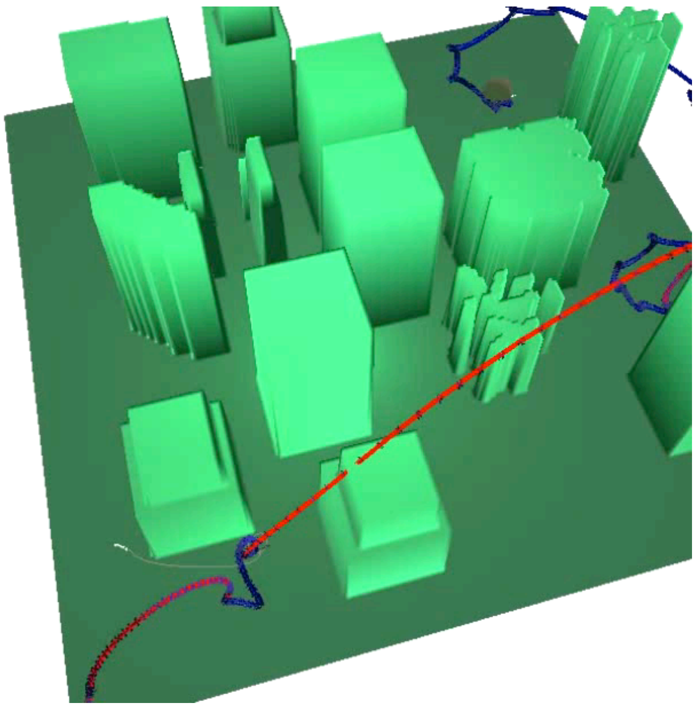
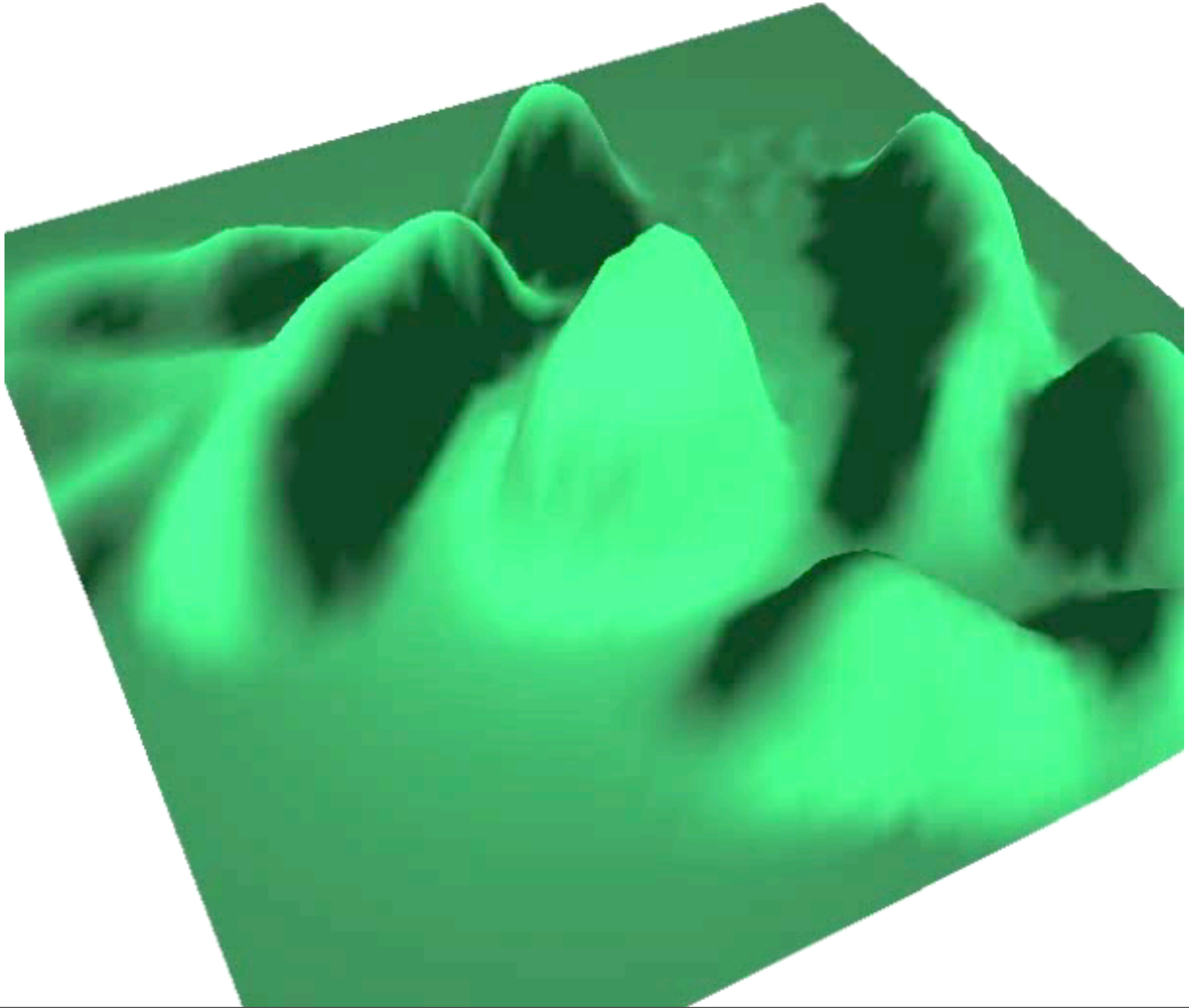


Roadmap, dynamic programming strategies for rapid search methods drawing from the DMOC primitives library for the component pieces. Dynamics is faithfully represented.

Another Examples



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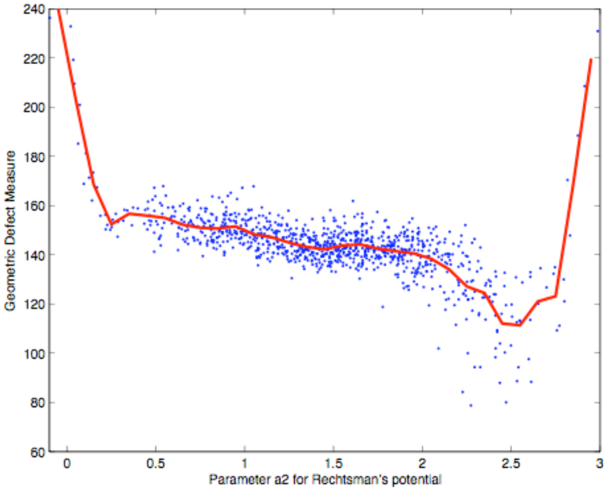

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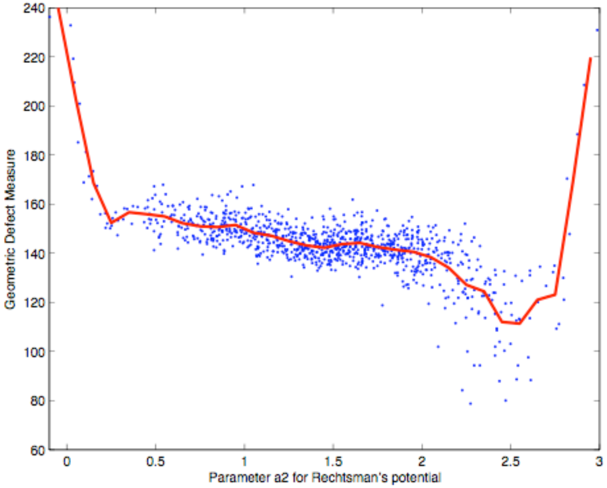

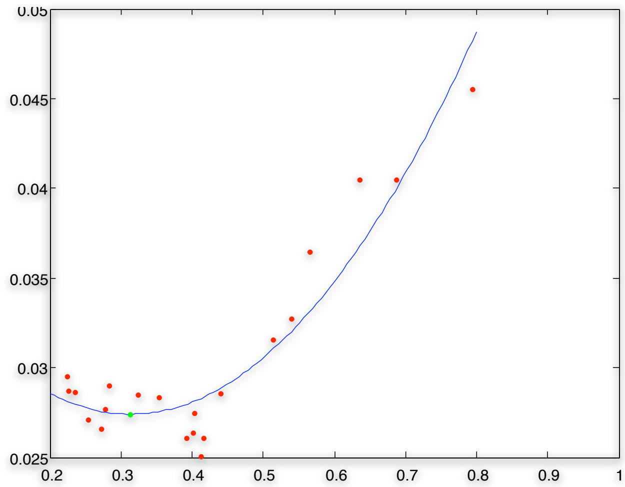

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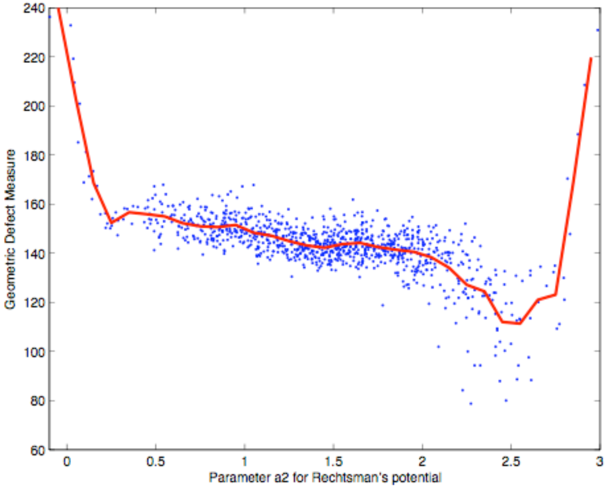

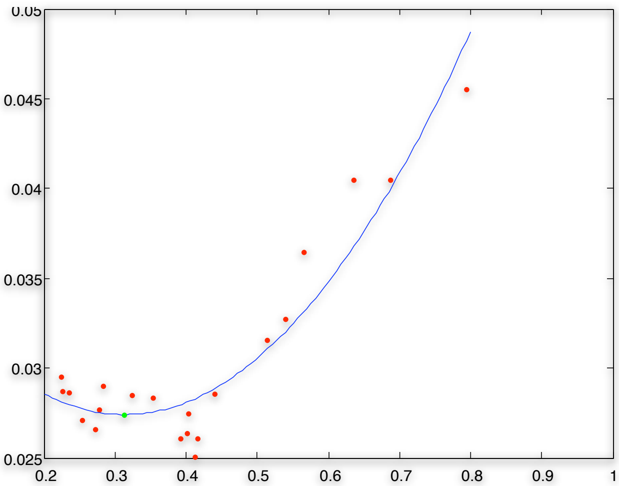

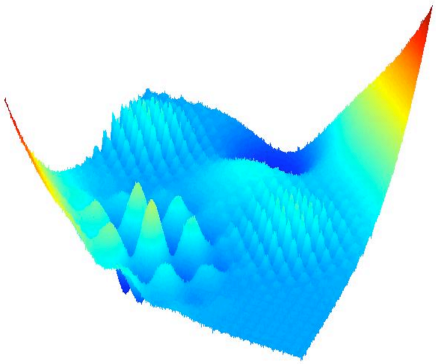

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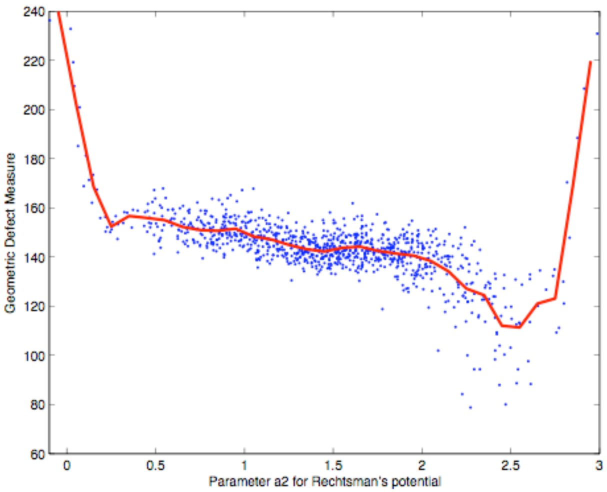
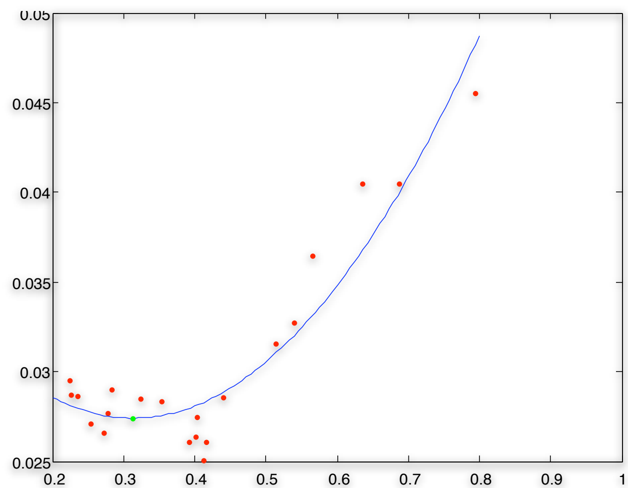
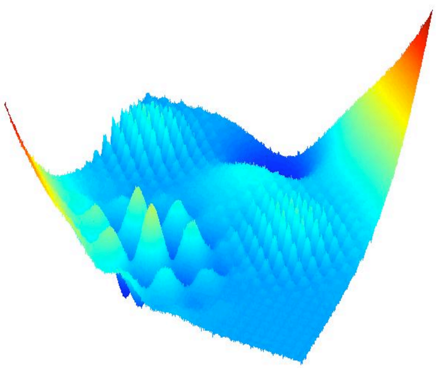
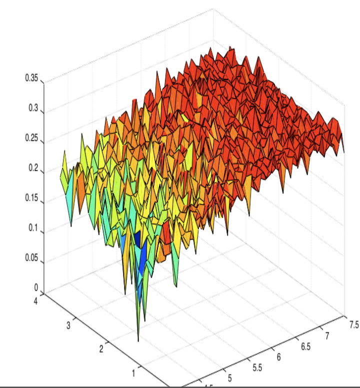
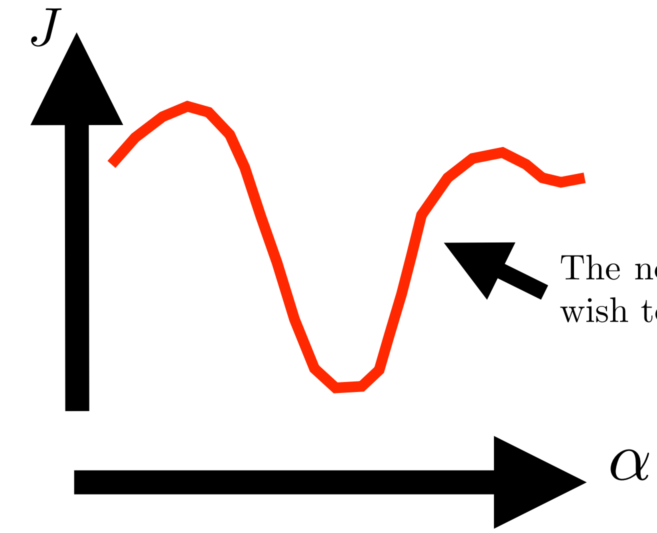








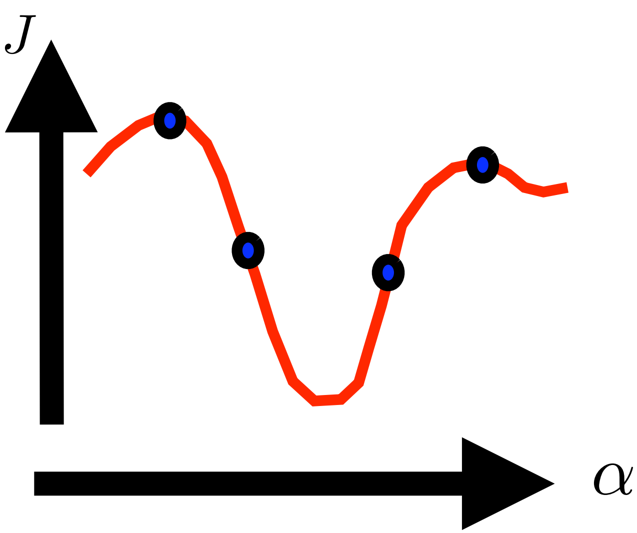
The Idea of MT Optimization



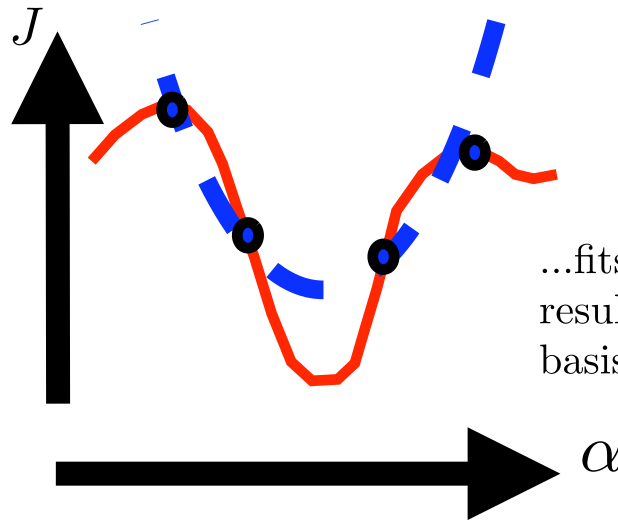
The nonlinear function we wish to locally minimize.

Evaluations of J given α are assumed to be expensive, and thus gradient based methods would be inefficient.

Good for situations in which J is noisy



Trend optimization samples a small number of points,...



...fits a trend to the results using prescribed basis functions,...

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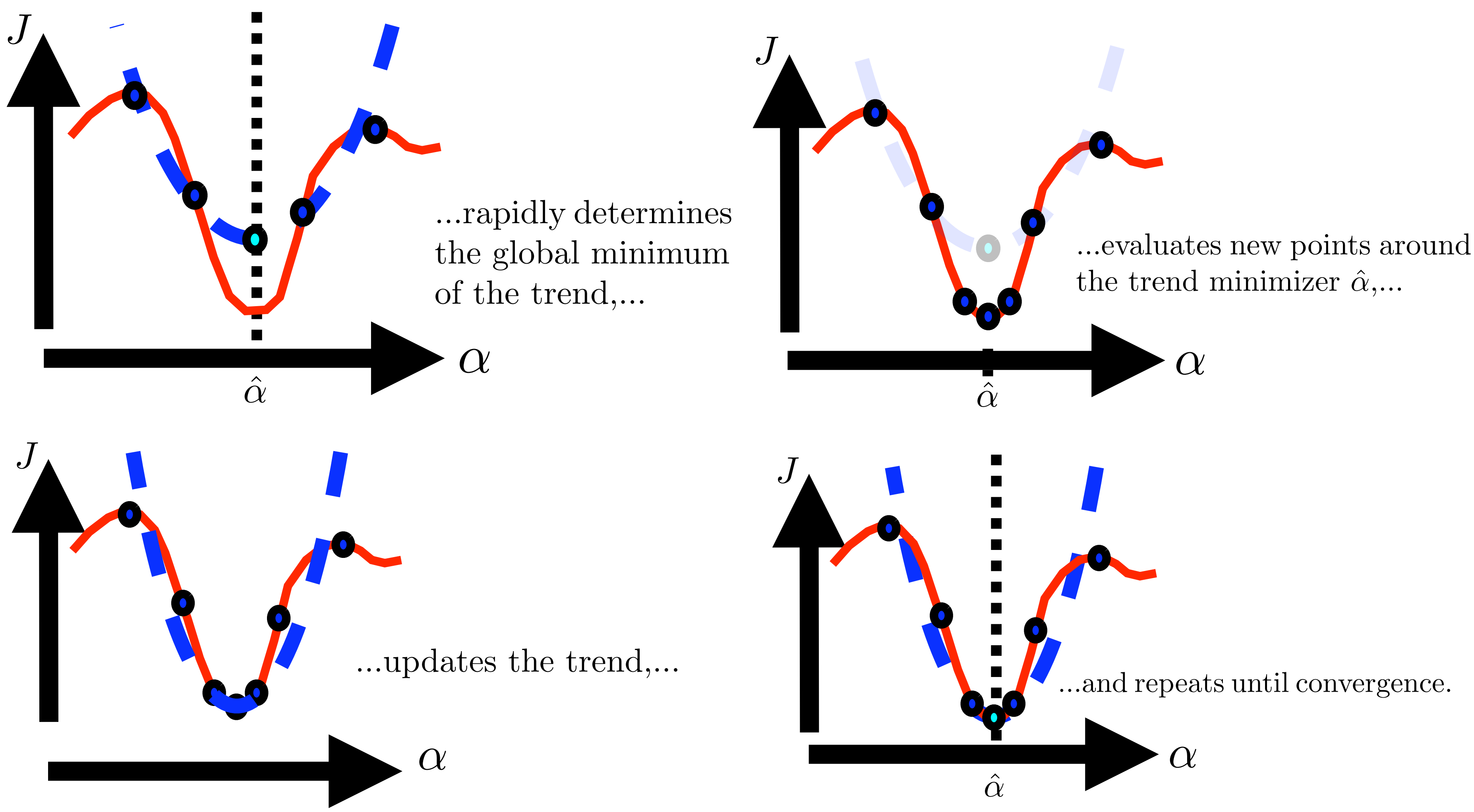

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We plan to use this tool in surveillance

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- DMOC and inner-outer loop strategy—more on DMOC shortly
- What is the best place for the knees?

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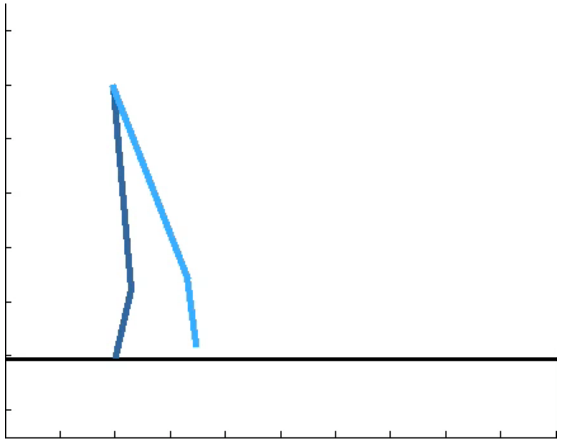

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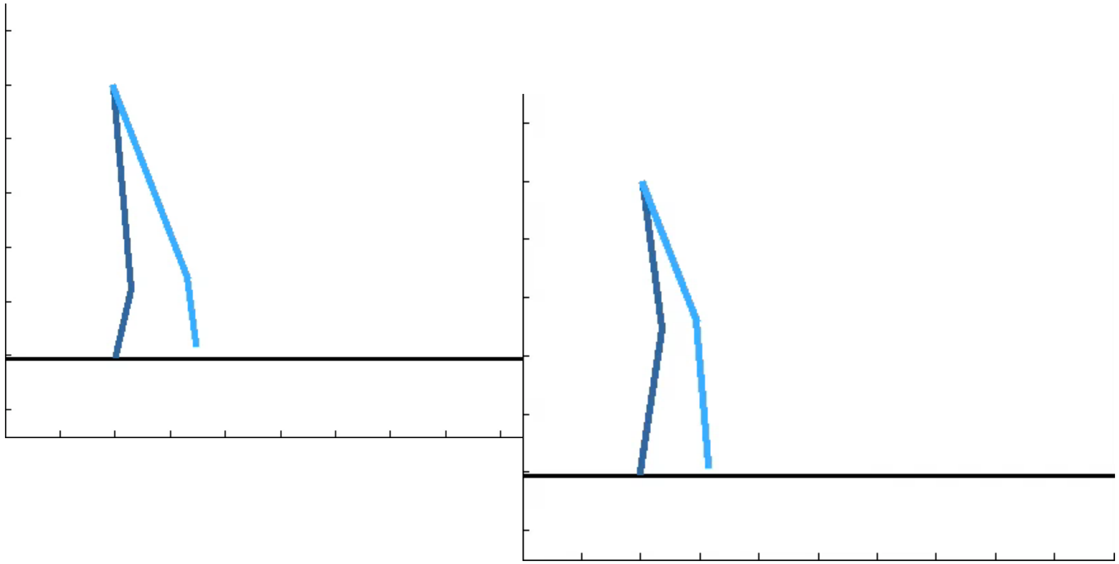

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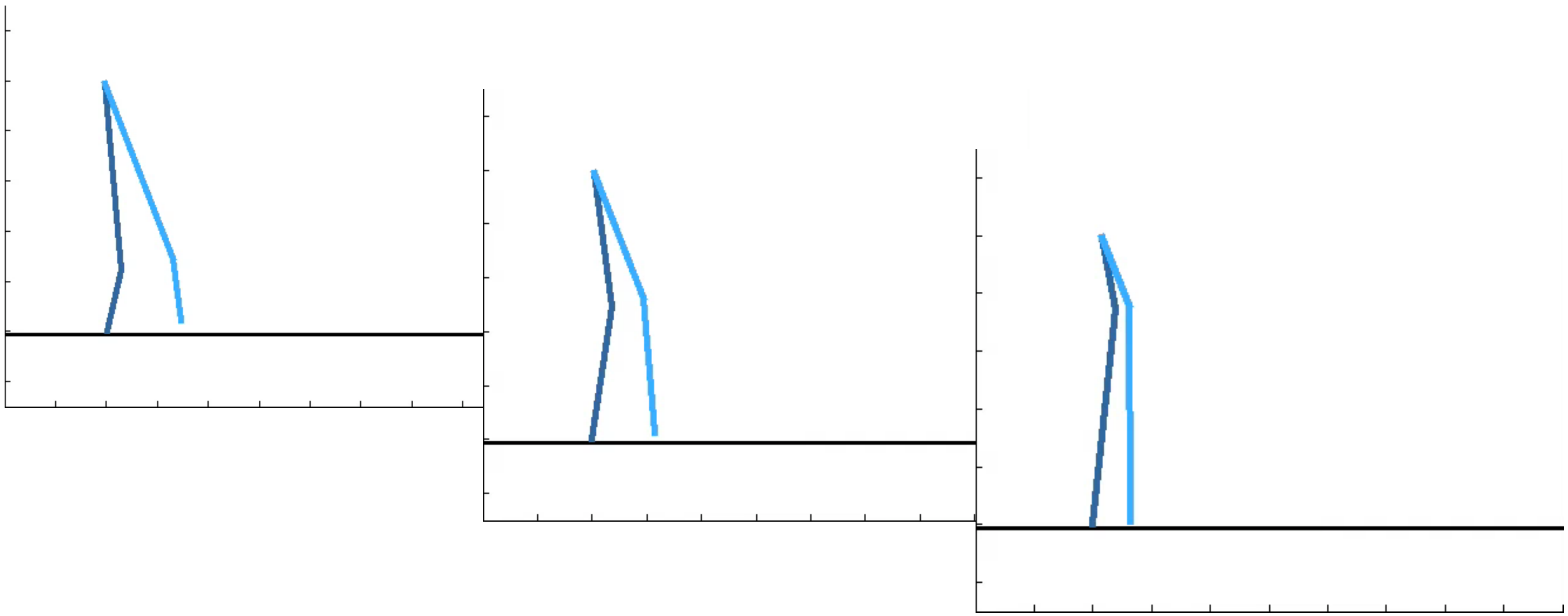
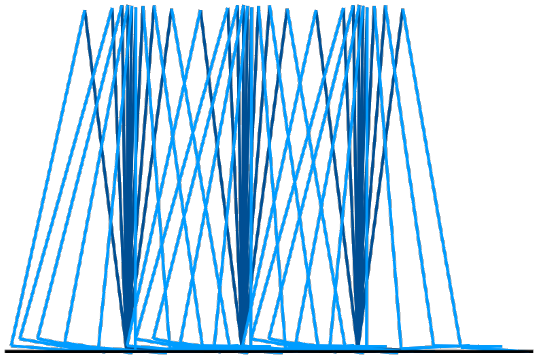


Design of Dynamics

Example Systems to Optimize:

Bipedal Robots



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- Outer loop efficiently searches for optimal design parameters (trend optimization)
- Scheme yields optimal mechanical system design for specified tasks

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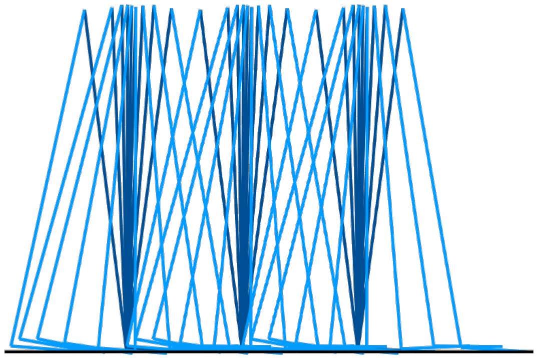

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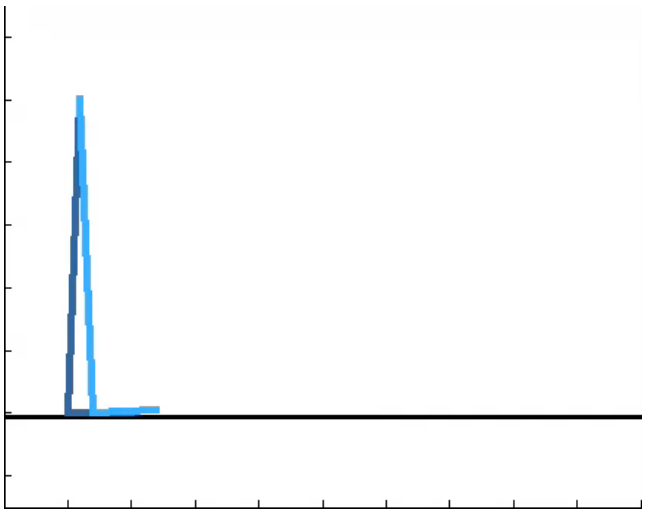

Design of Dynamics

Inner/Outer Loop Architecture

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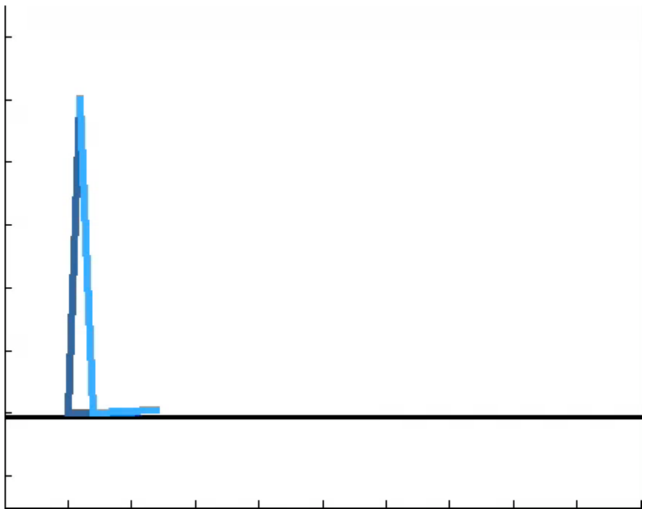

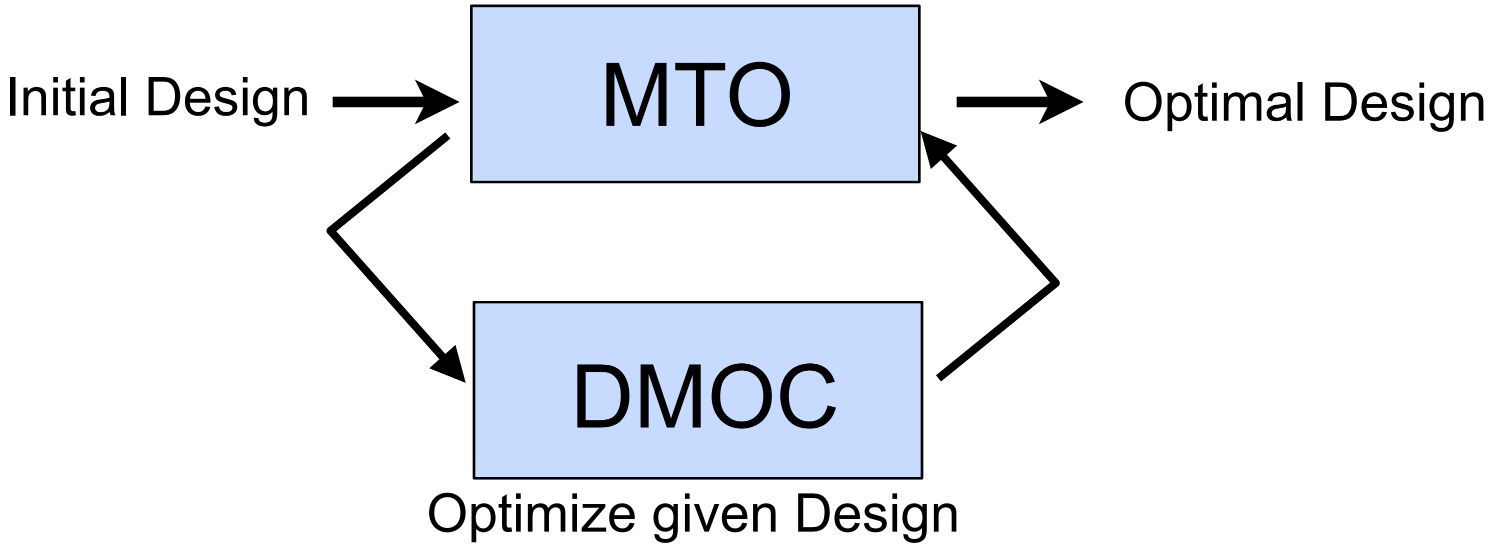

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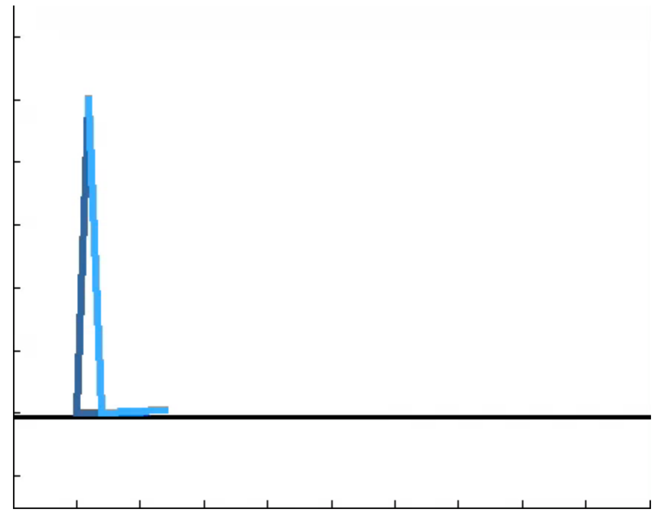

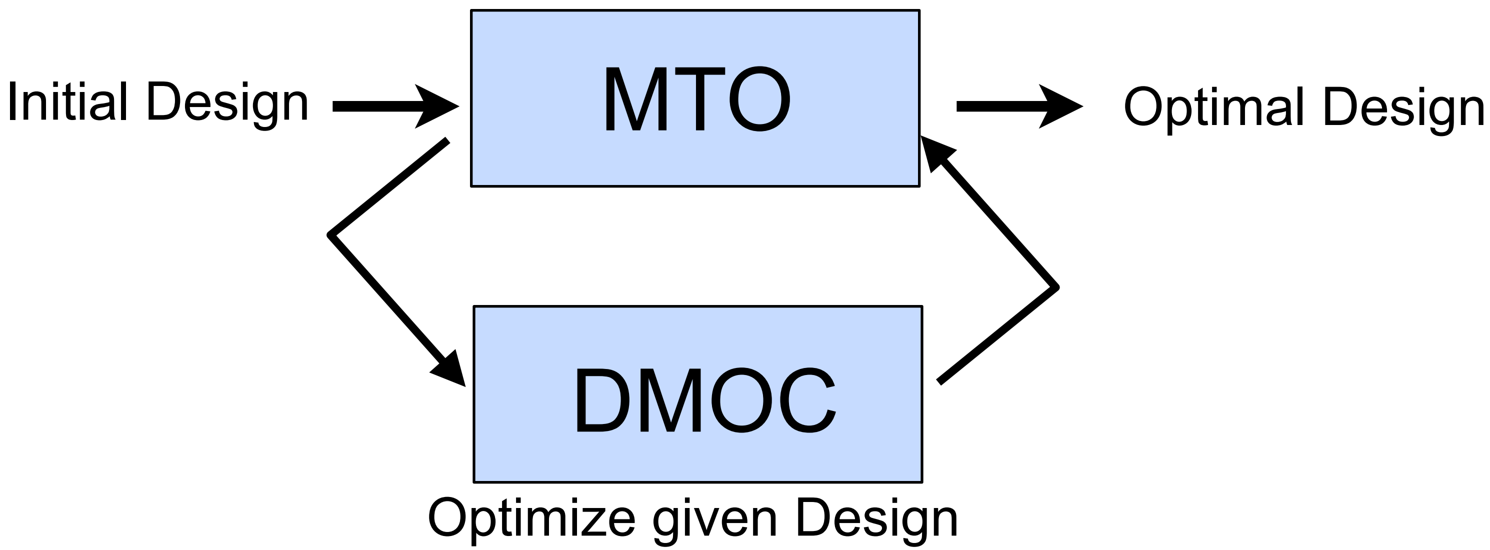

Future: Stochastic DMOC

Trend Optimization’s minimizer

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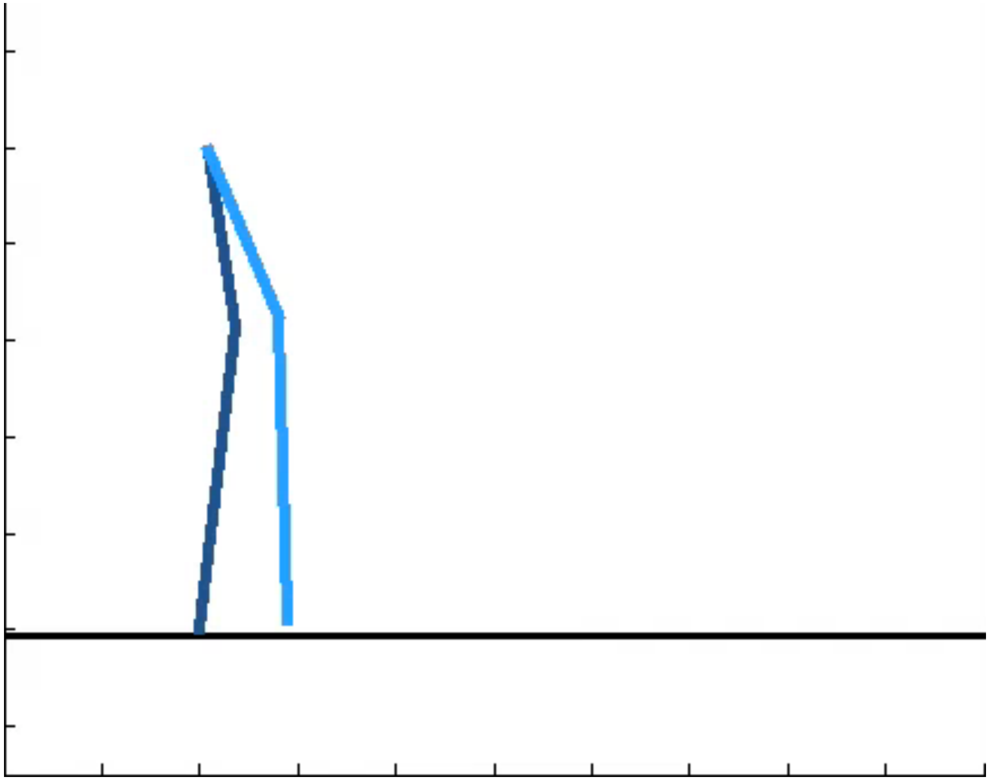

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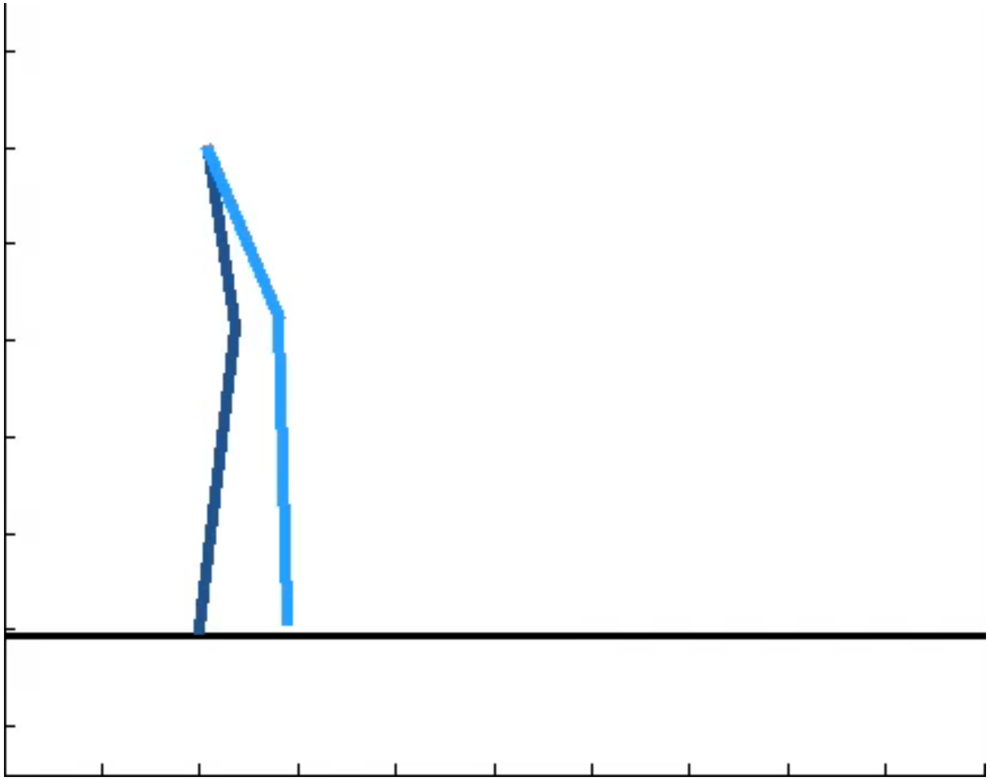
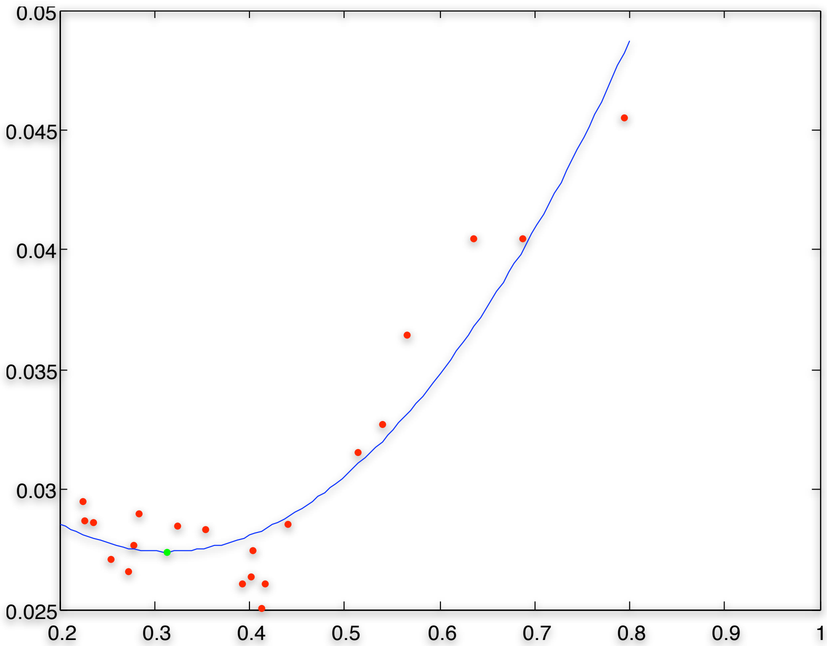
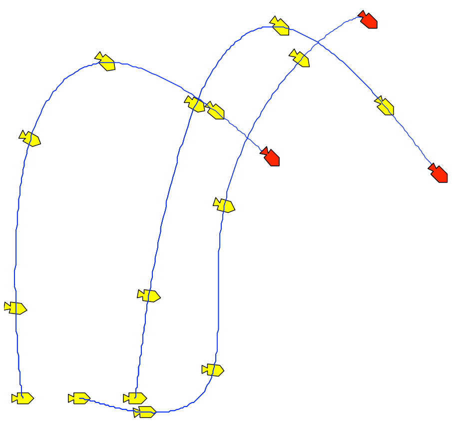


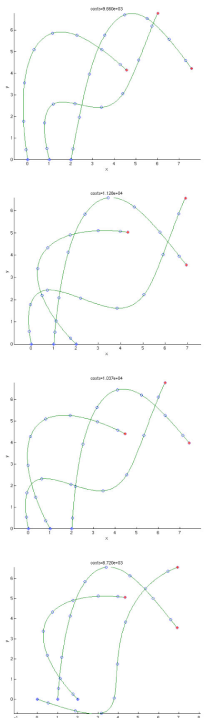


Global solution for optimal control problems

Formation of hovercraft

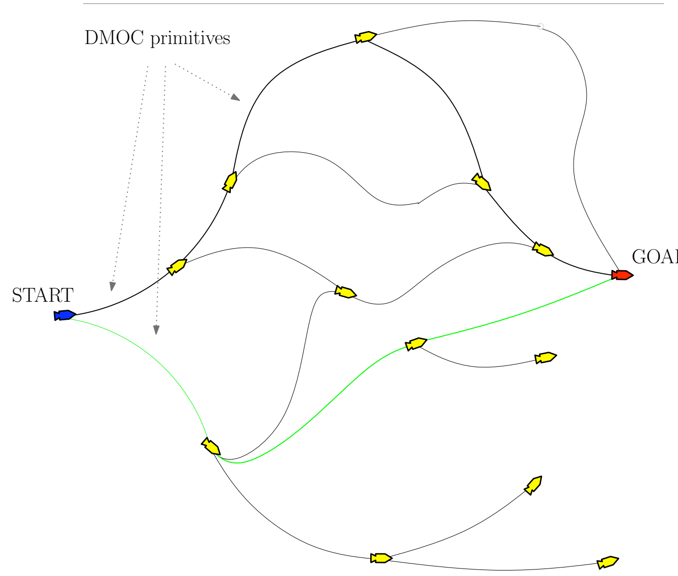
- relative arrangement on target manifold
- minimize control effort
- many local minima



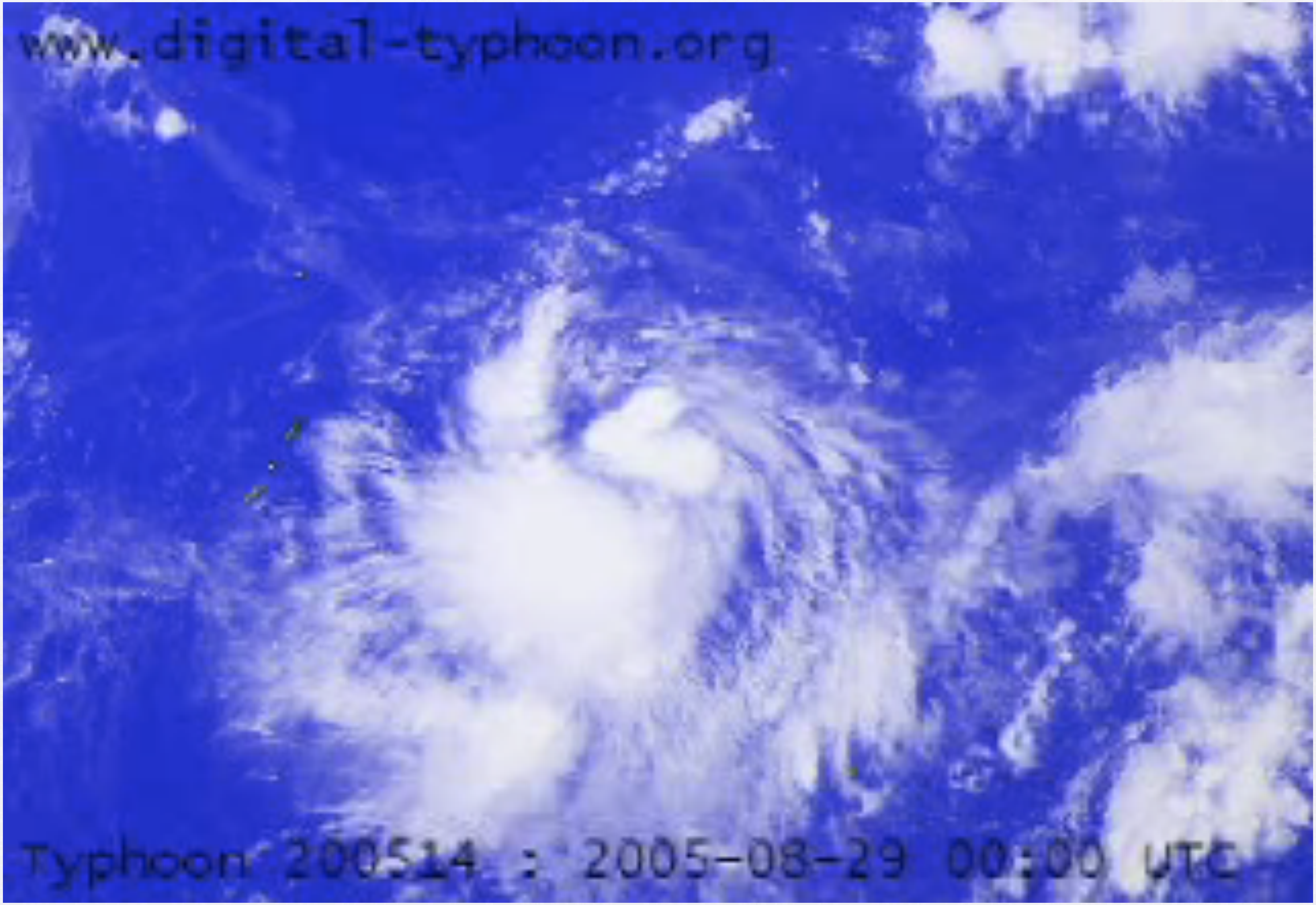


Sampling-based Roadmap

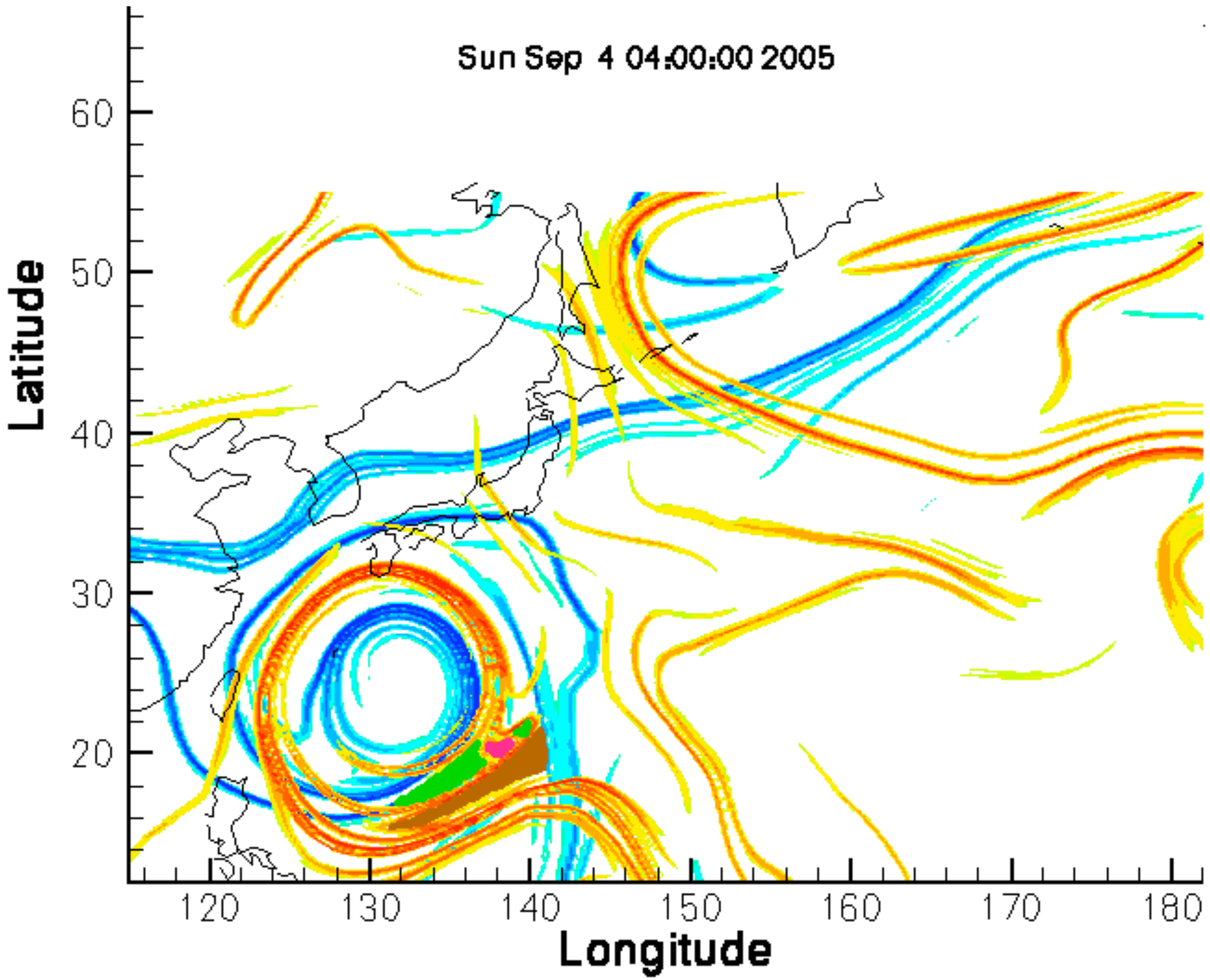
- graph of *DMOC primitives*
- dynamic programming
- global state space exploration
- near globally optimal solution



Hurricane Nabi (Philip DuToit)



LCS for Hurricane Nabi



NCEP/NCAR Reanalysis Data at the 850mb pressure level.

Celestial Invariant Manifolds
(Koon, Lo, JM, Ross)

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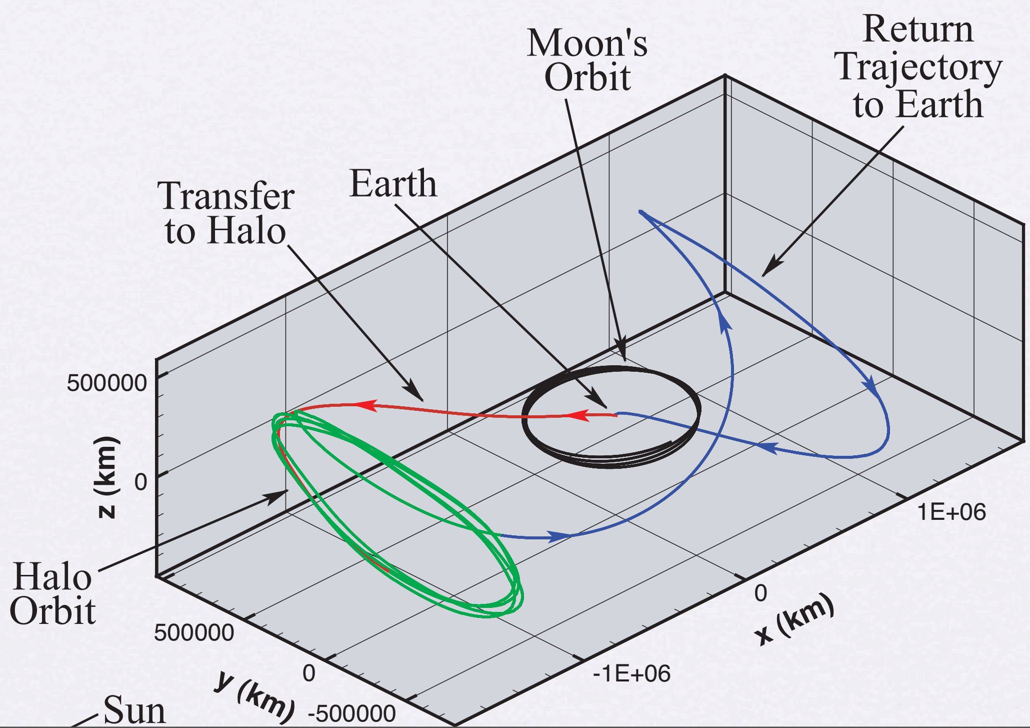

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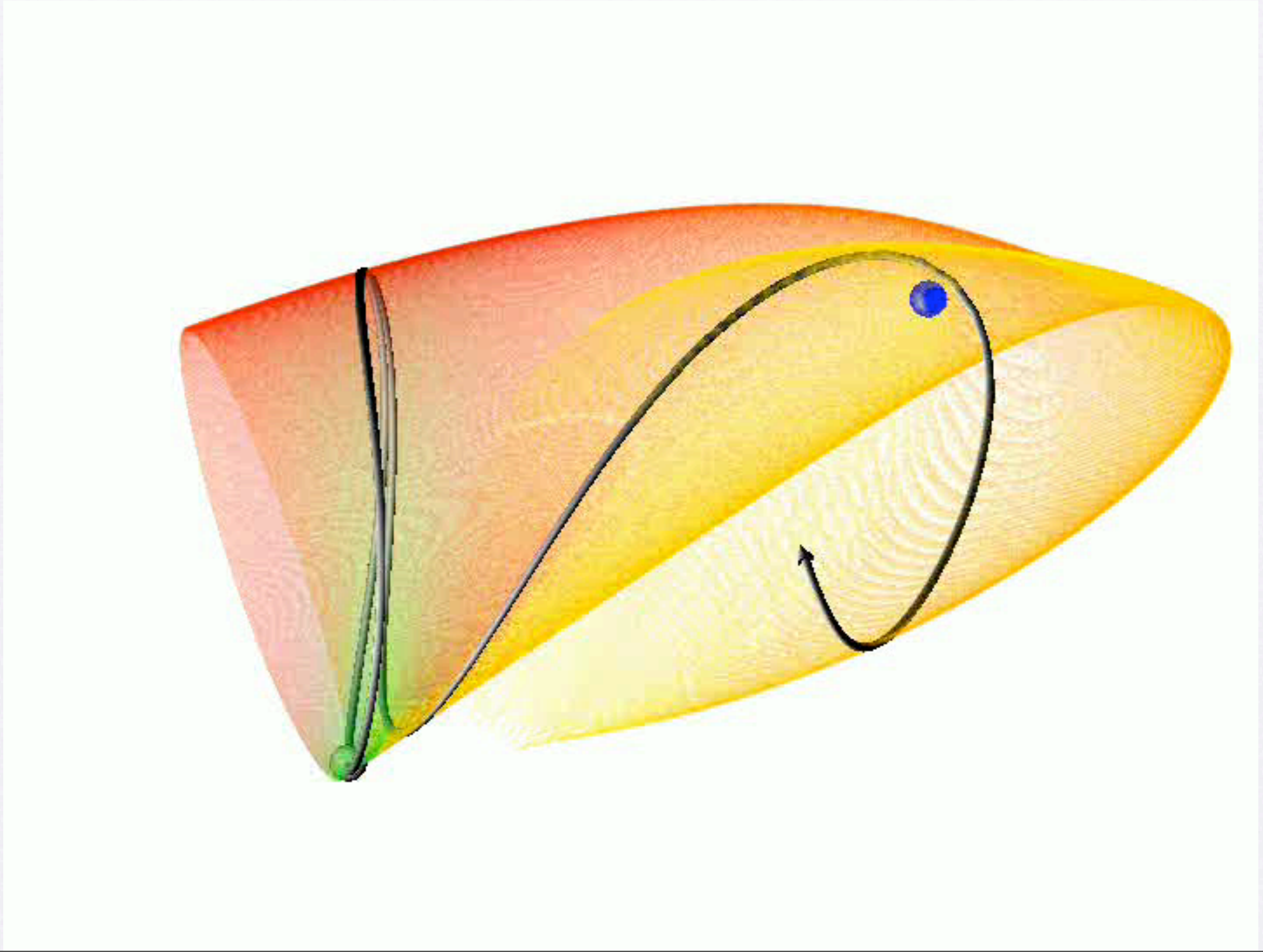

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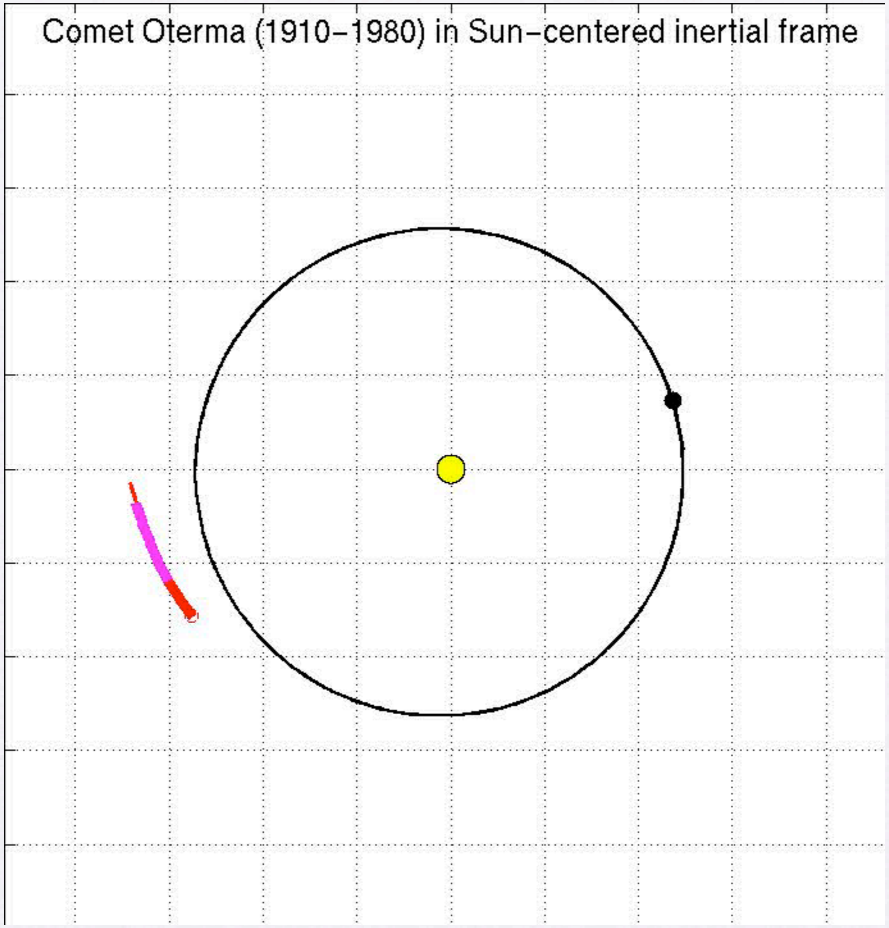

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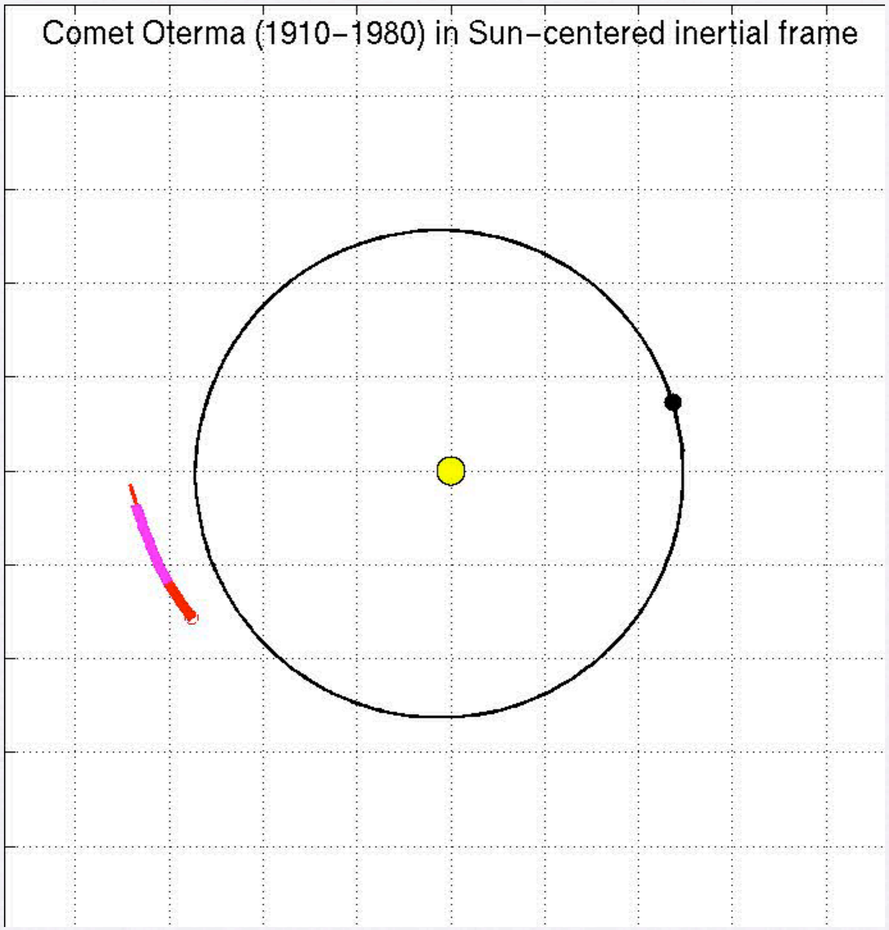

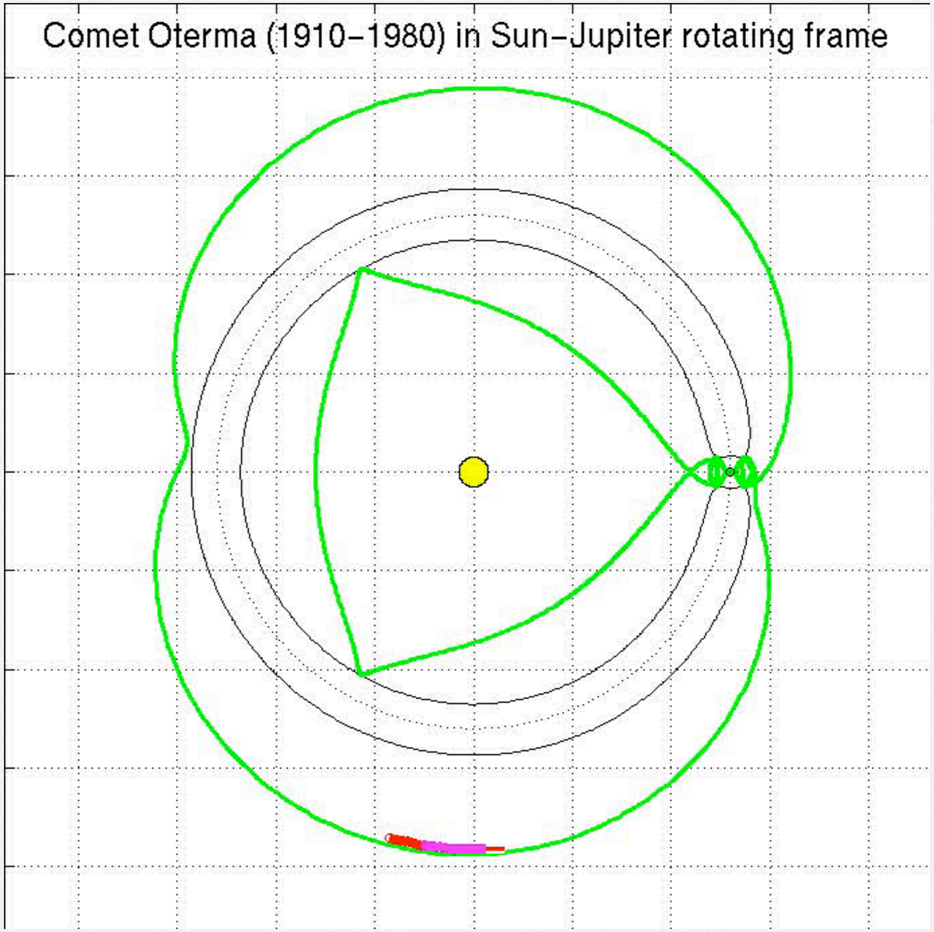

Here DMOC and LCS come together (Ashley Moore, Evan Gawlick)

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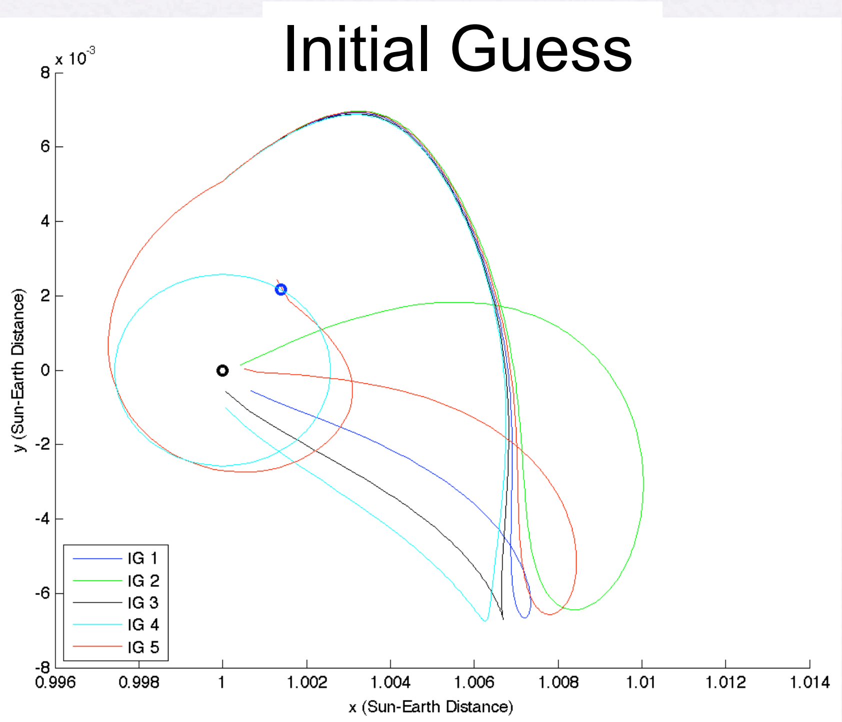
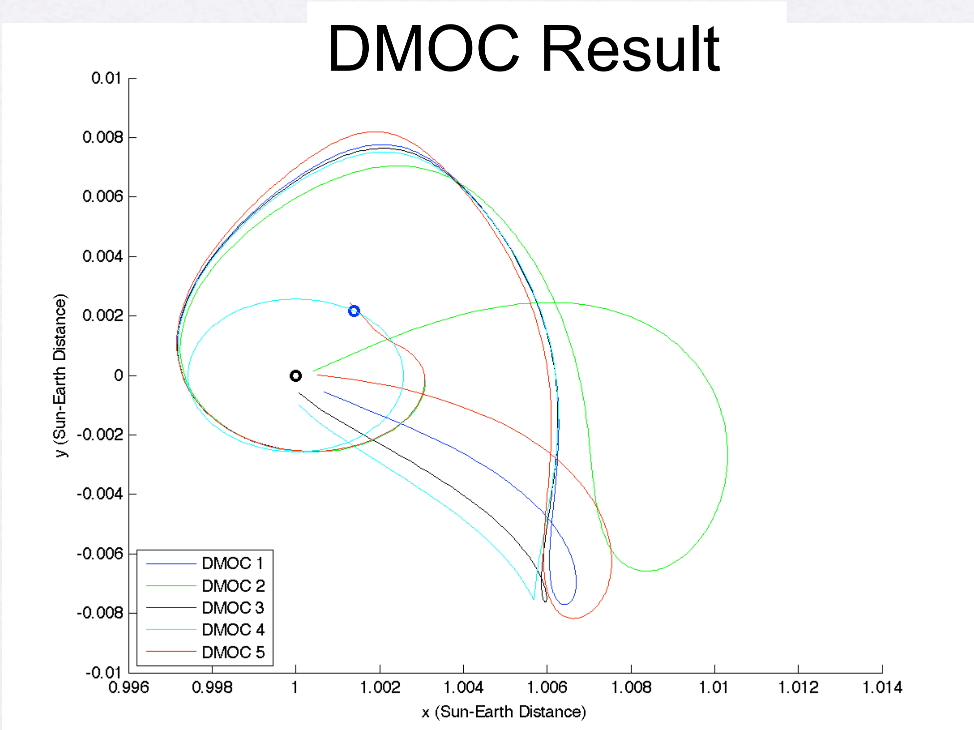
Delta V (m/s)		
	Initial Guess	DMOC
case 1	175.8273	0.2331
case 2	178.5763	0.4452
case 3	172.7951	0.0672
case 4	171.3516	0.0902
case 5	177.8498	0.4386





Coherent Structures Everywhere

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- Jupiter's red spot

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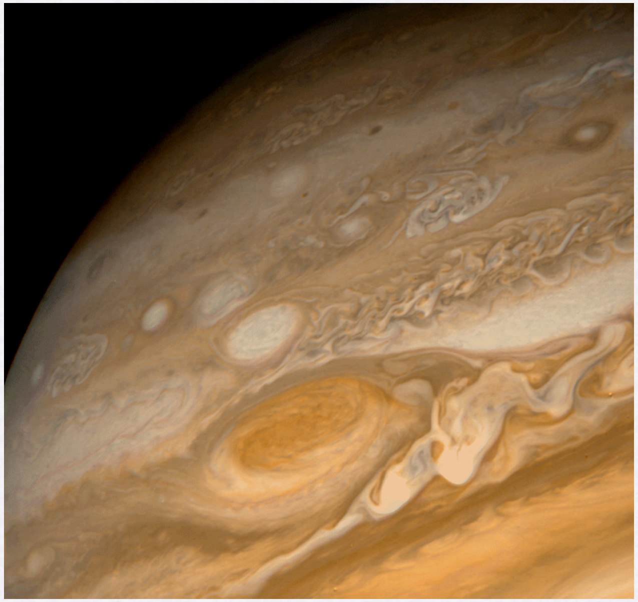

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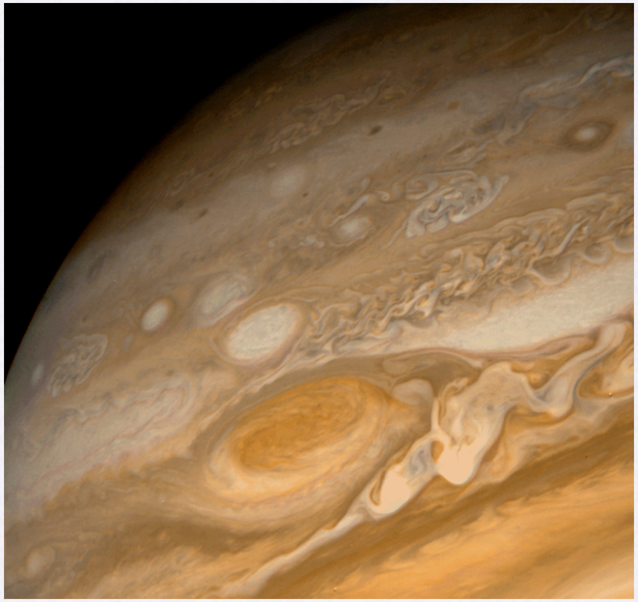

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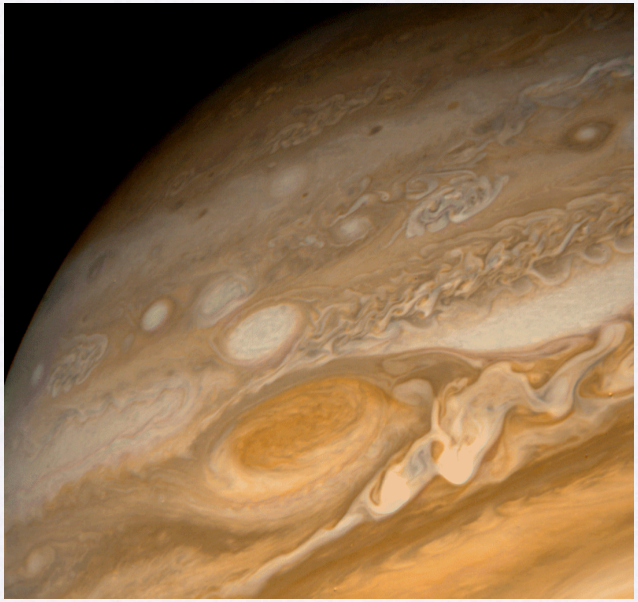

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- George Haller idea—use FTLE (Finite Time Liapunov Exponent Fields) and look for ridges—this was developed in the PhD Theses of Lekien, Shadden.

Invariant Manifolds: Standard View

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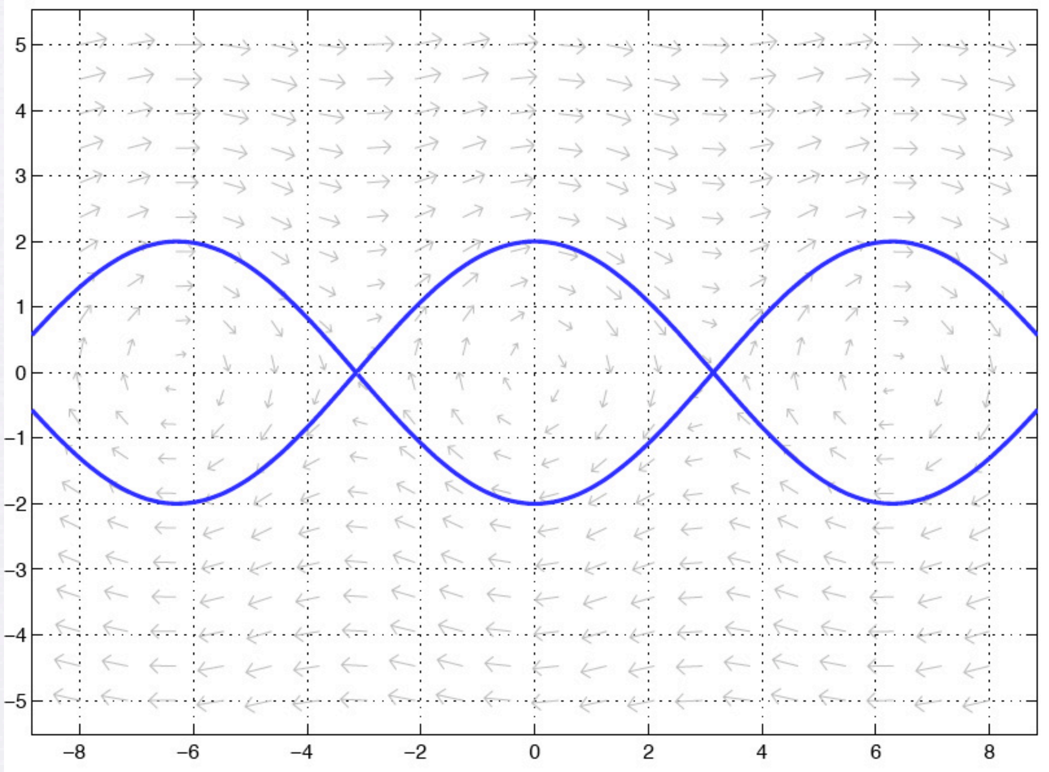

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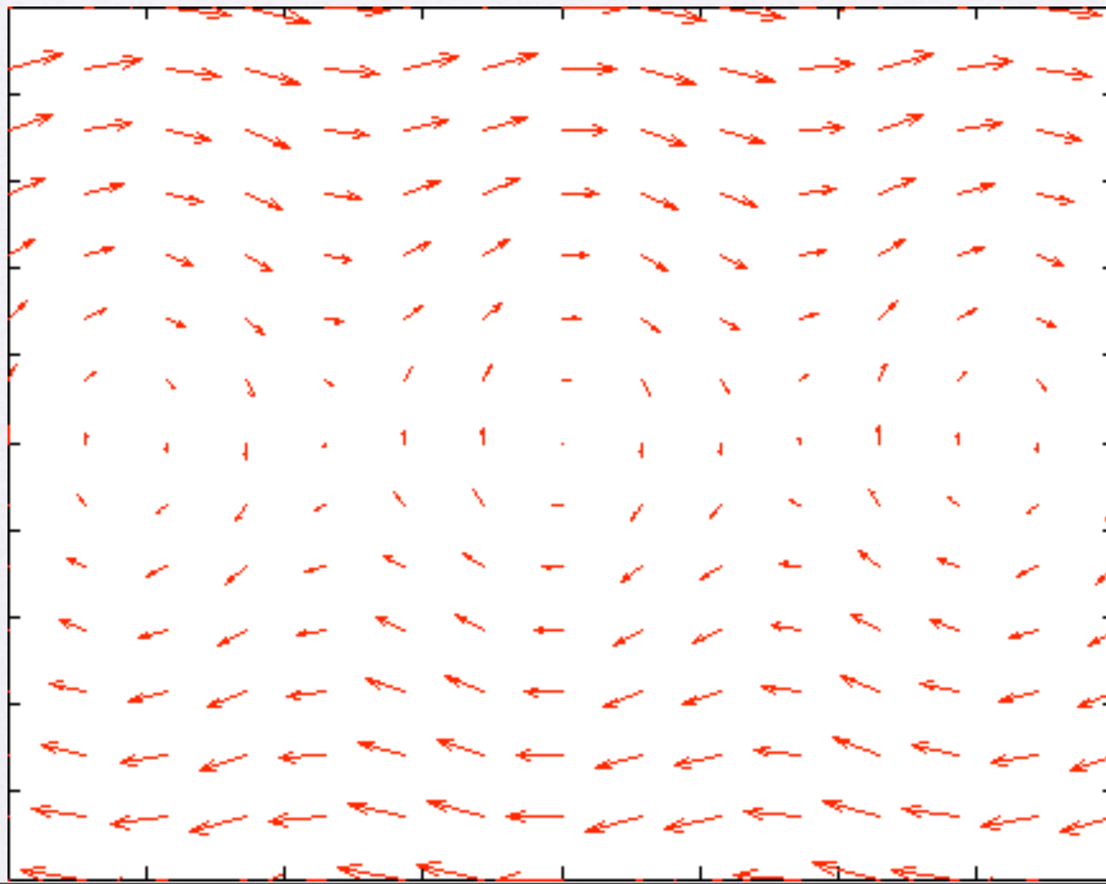

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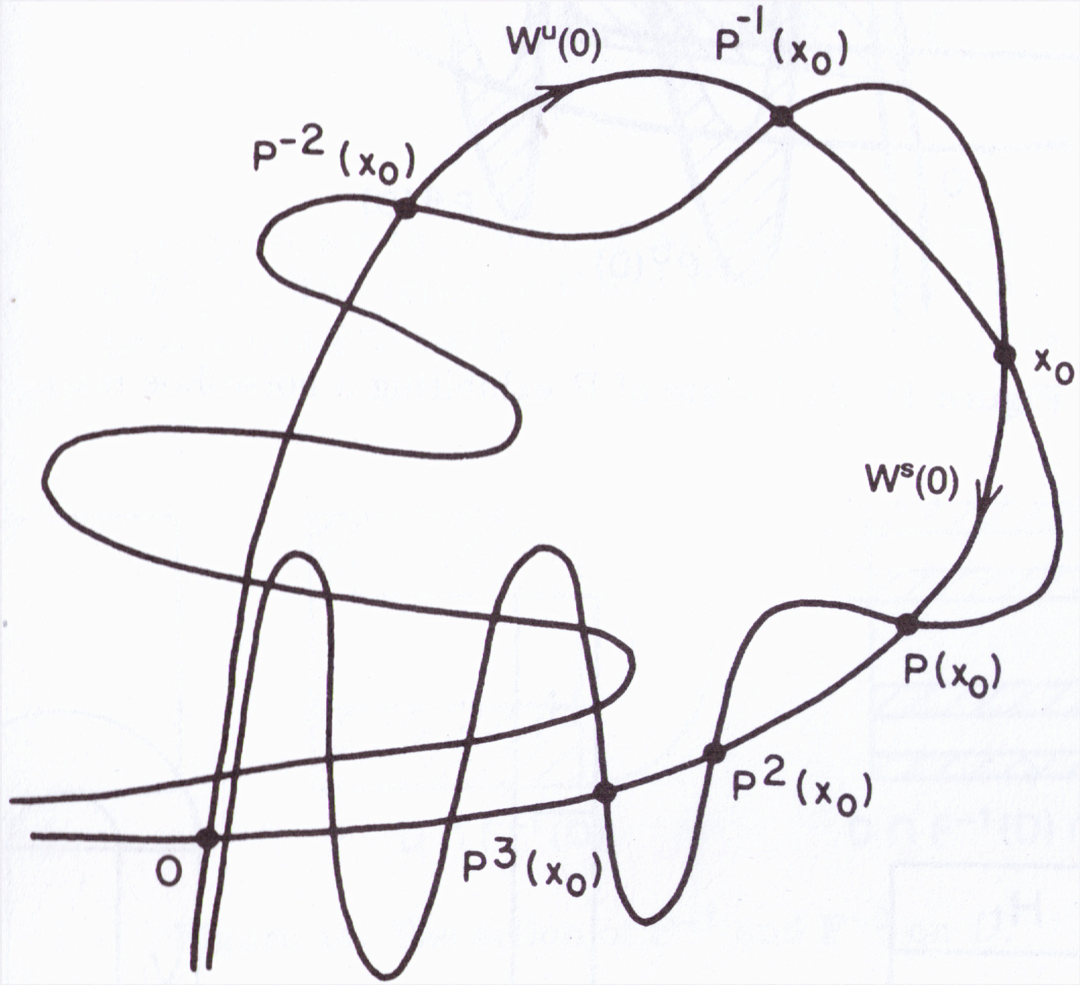

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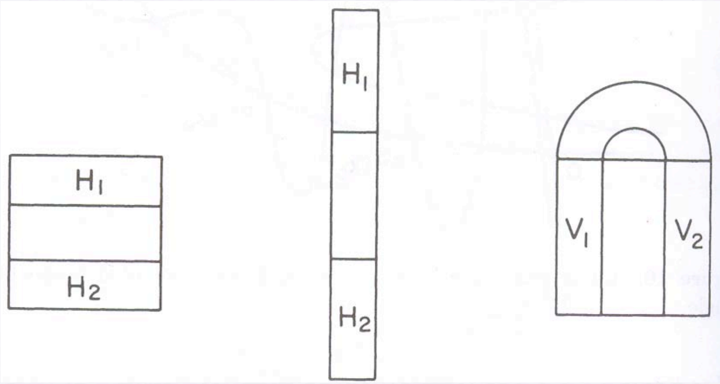

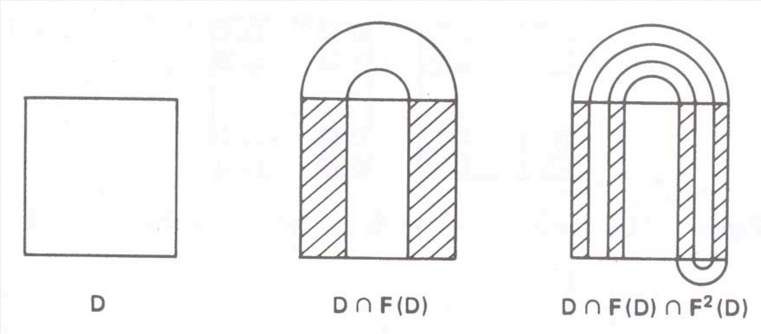

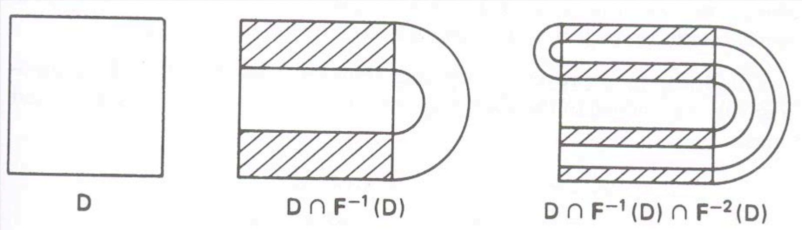

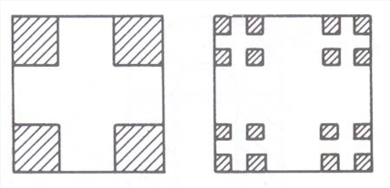

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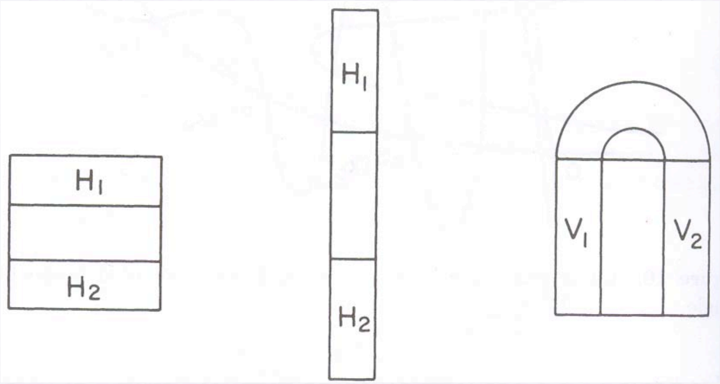

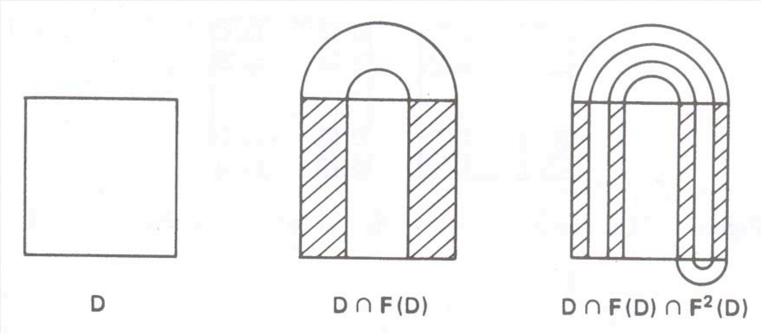

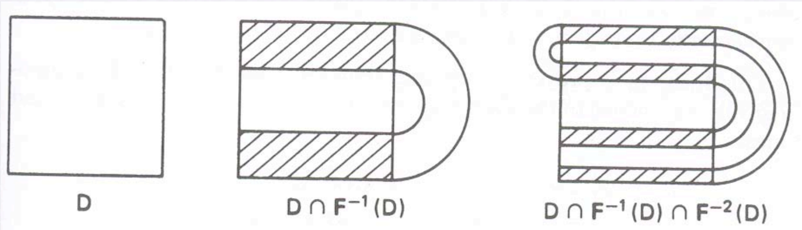

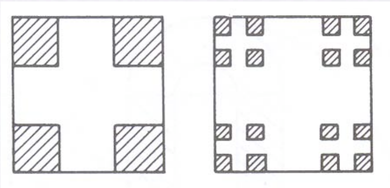

- Proved lots of nice things—eg, an invariant Cantor set.

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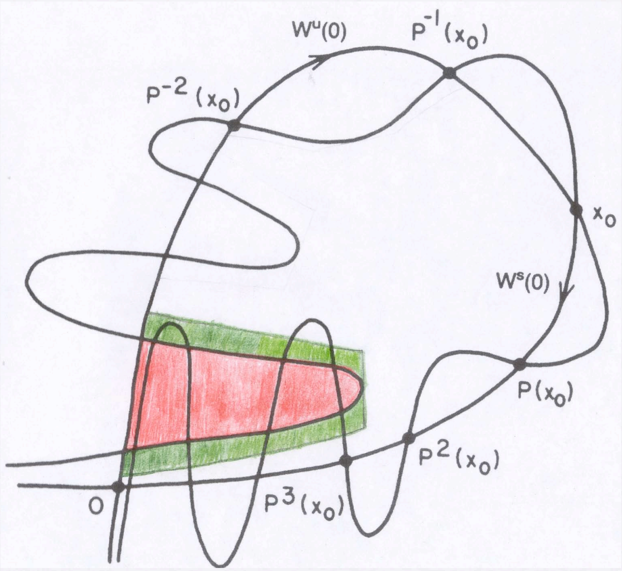

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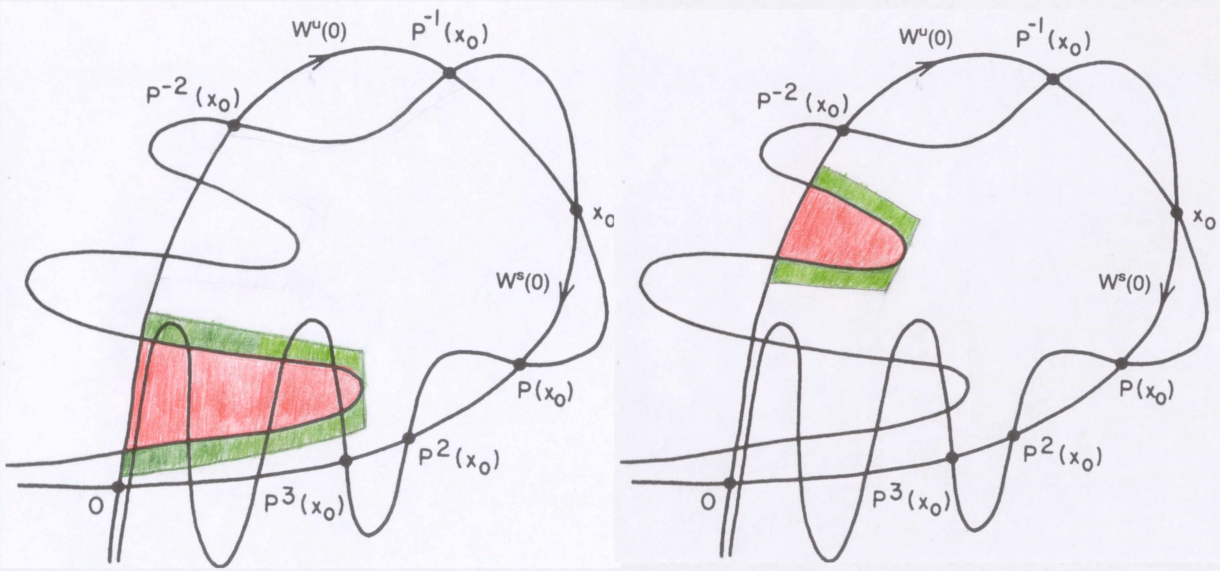

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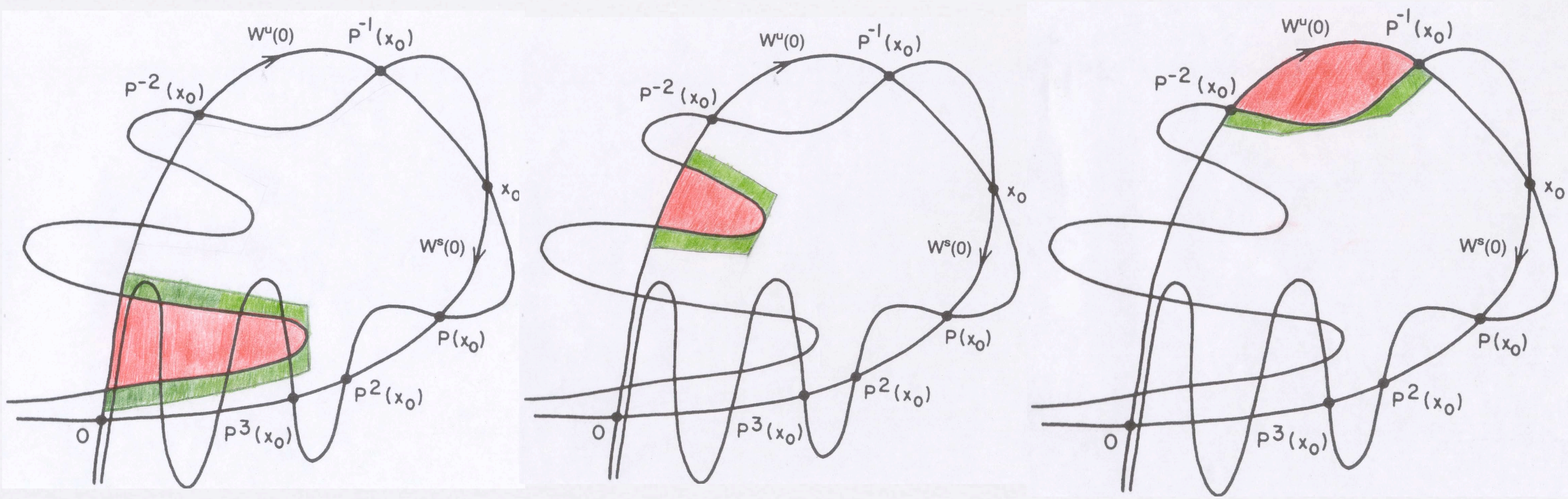

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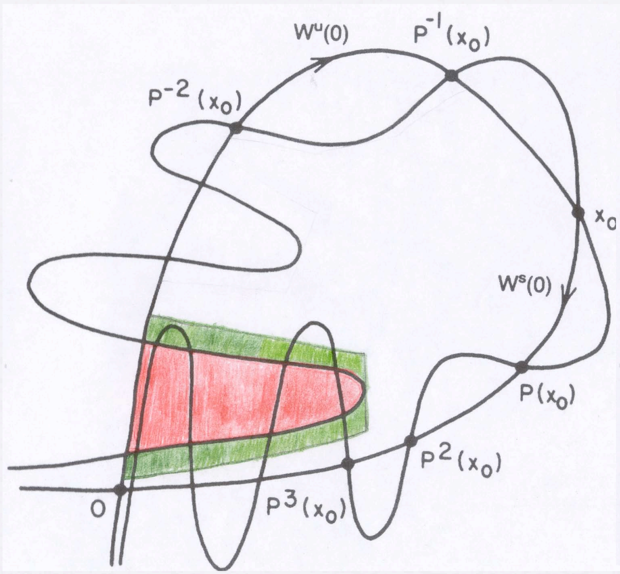

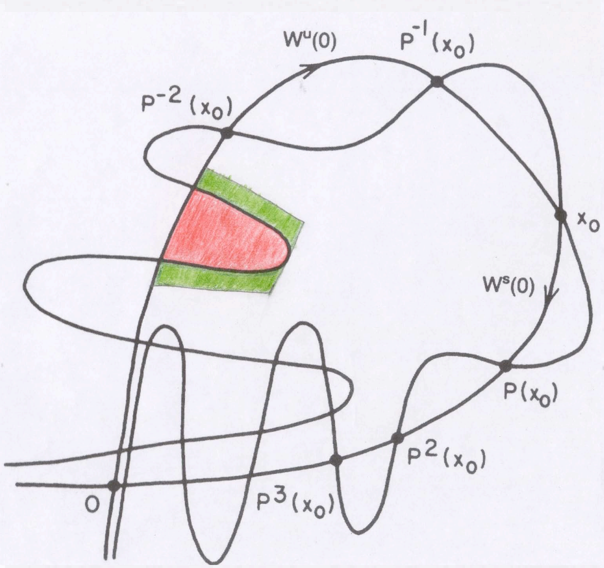

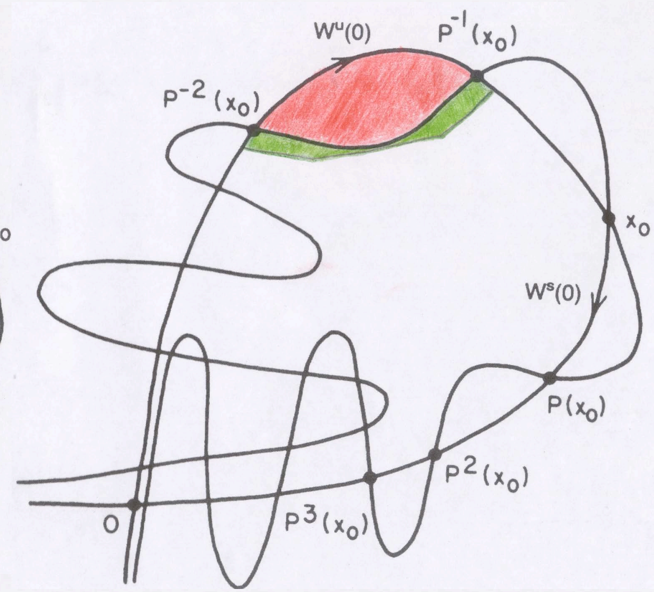

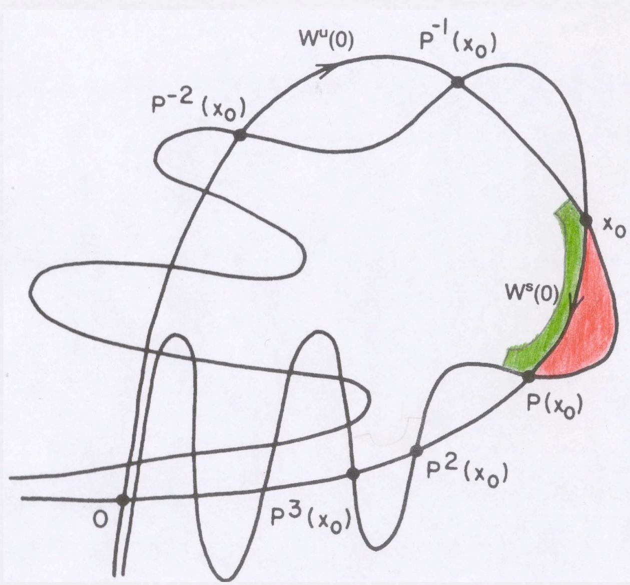

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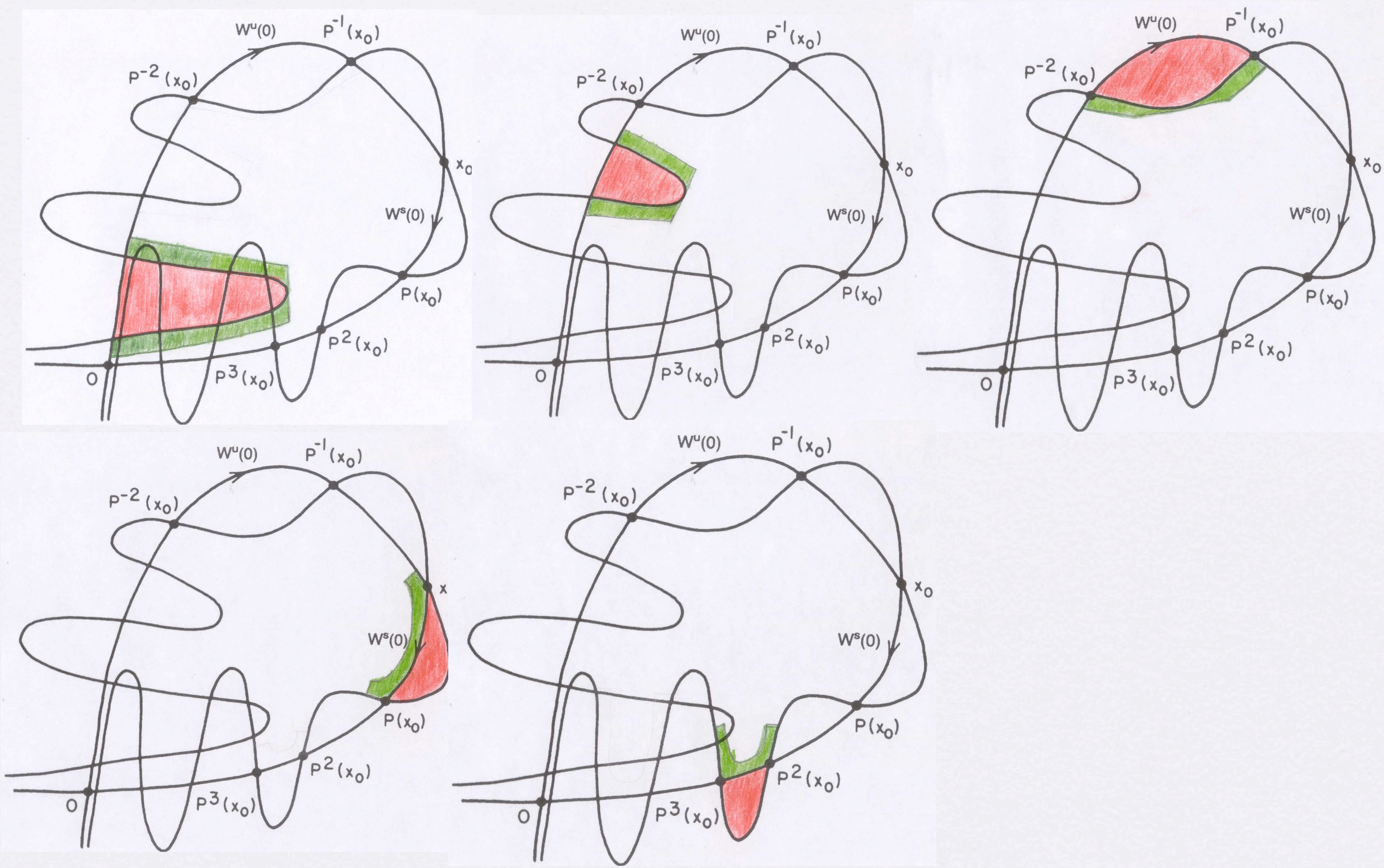

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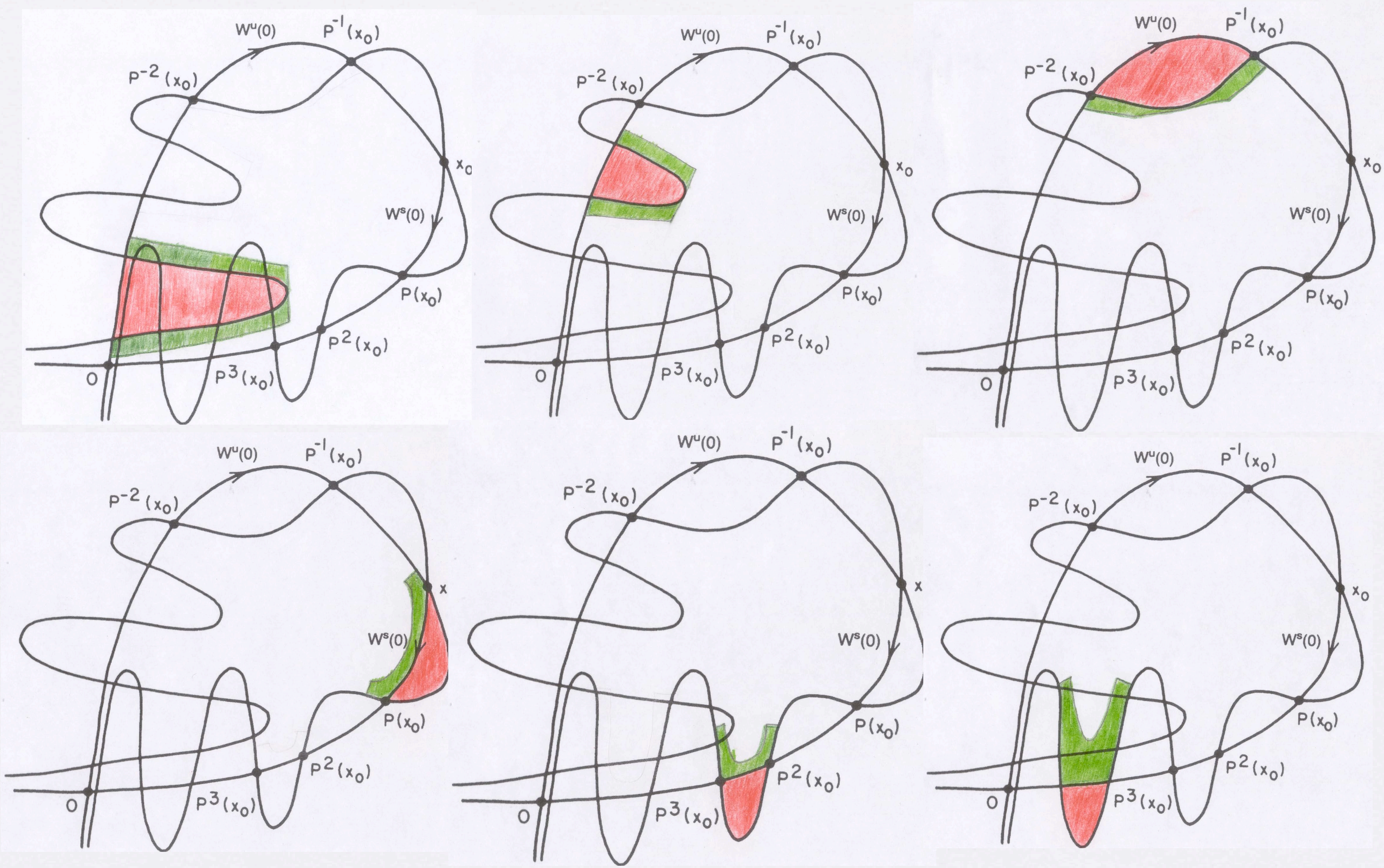

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Poincaré, one of the creators of modern dynamical systems, 1890

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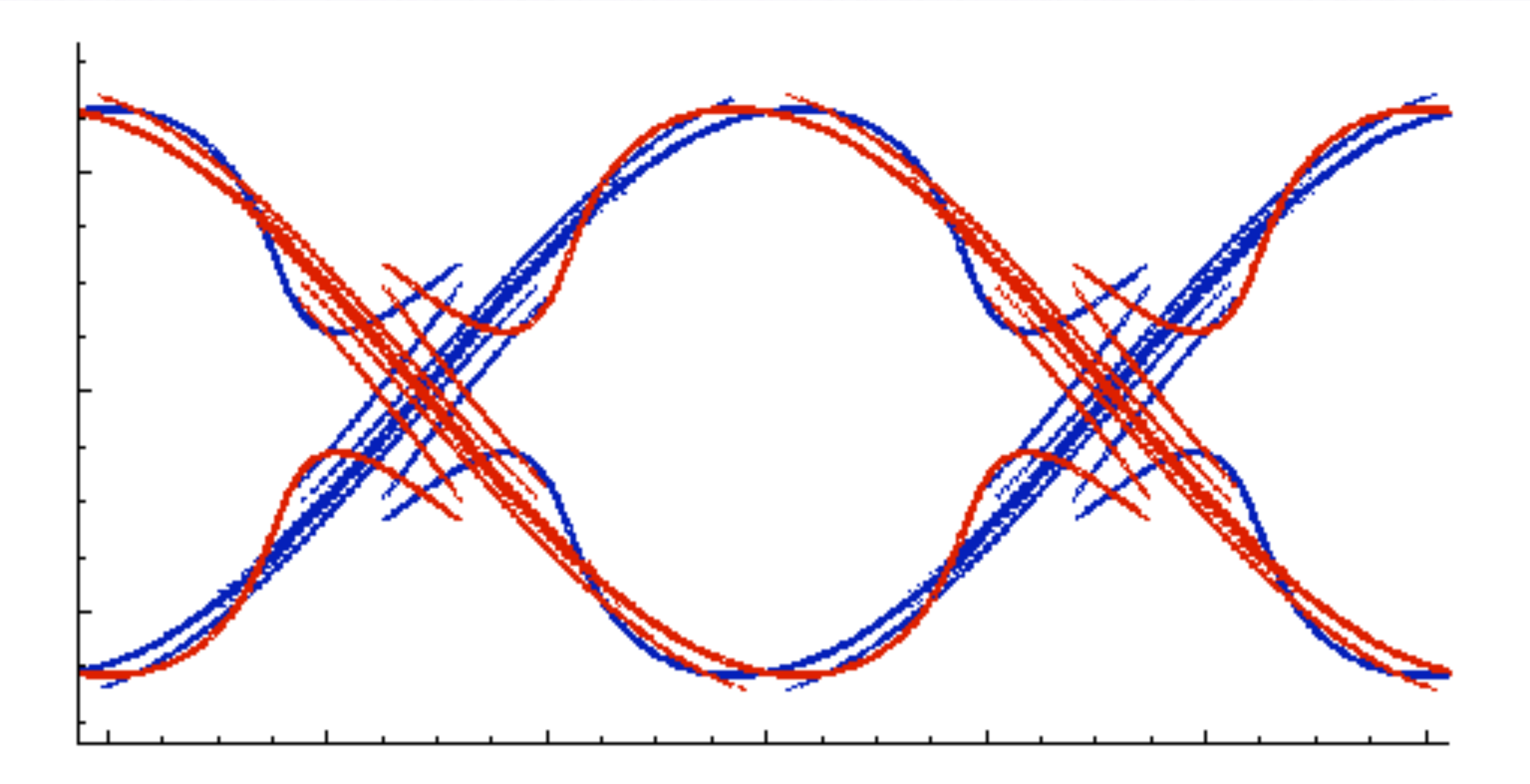
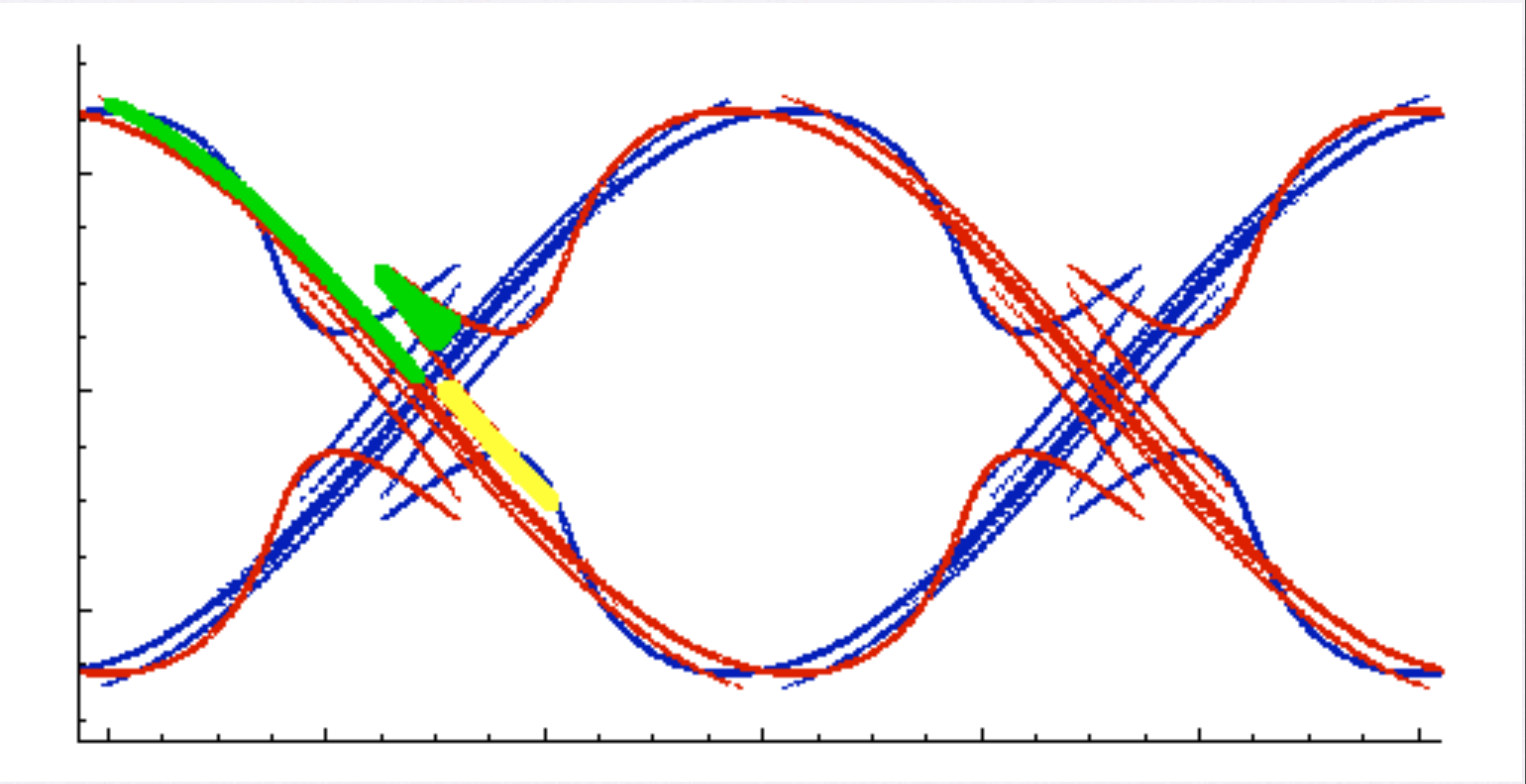
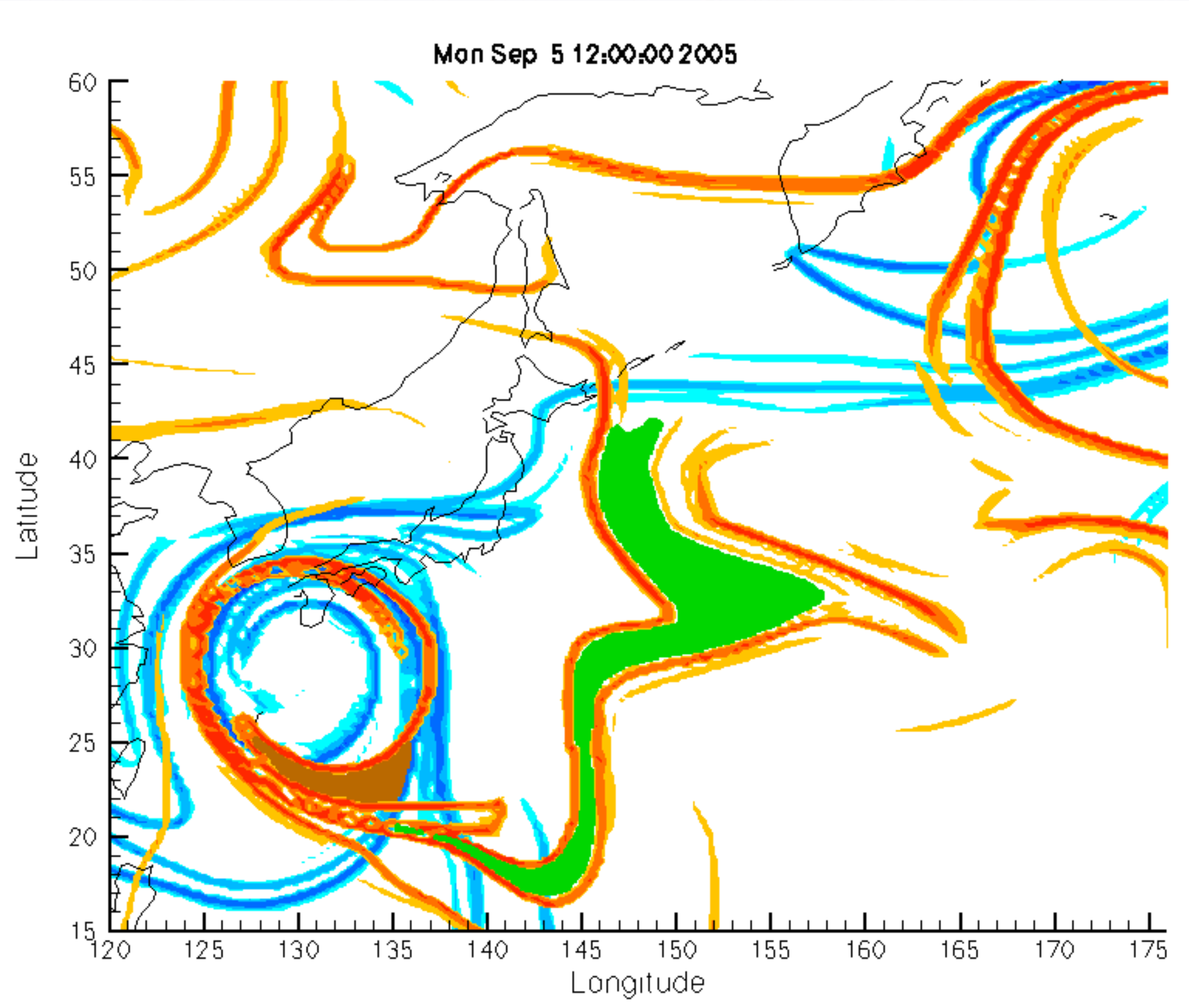


Look at lobes, mixing, dynamically



Back to Nabi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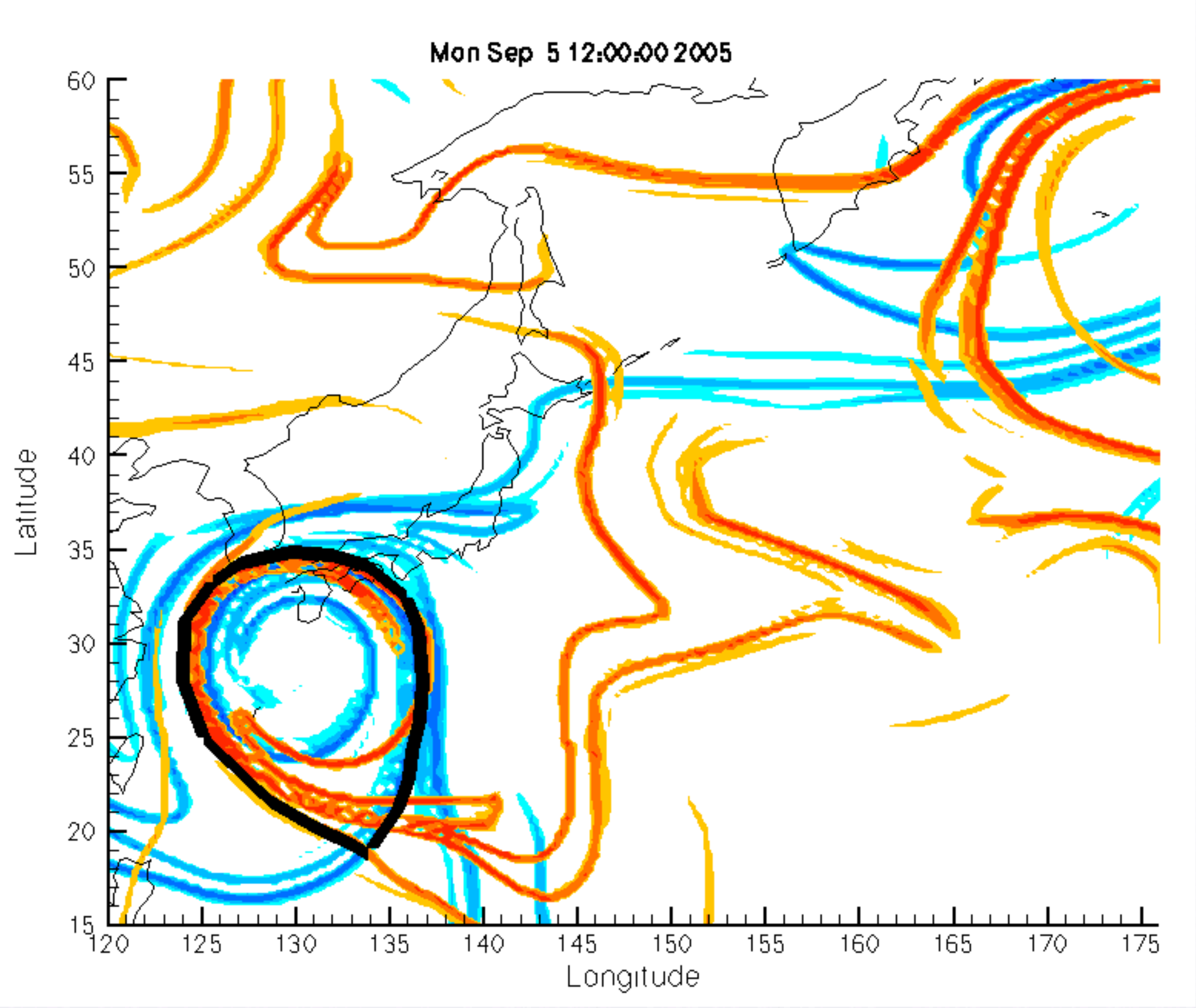
Nabi has horseshoes in it



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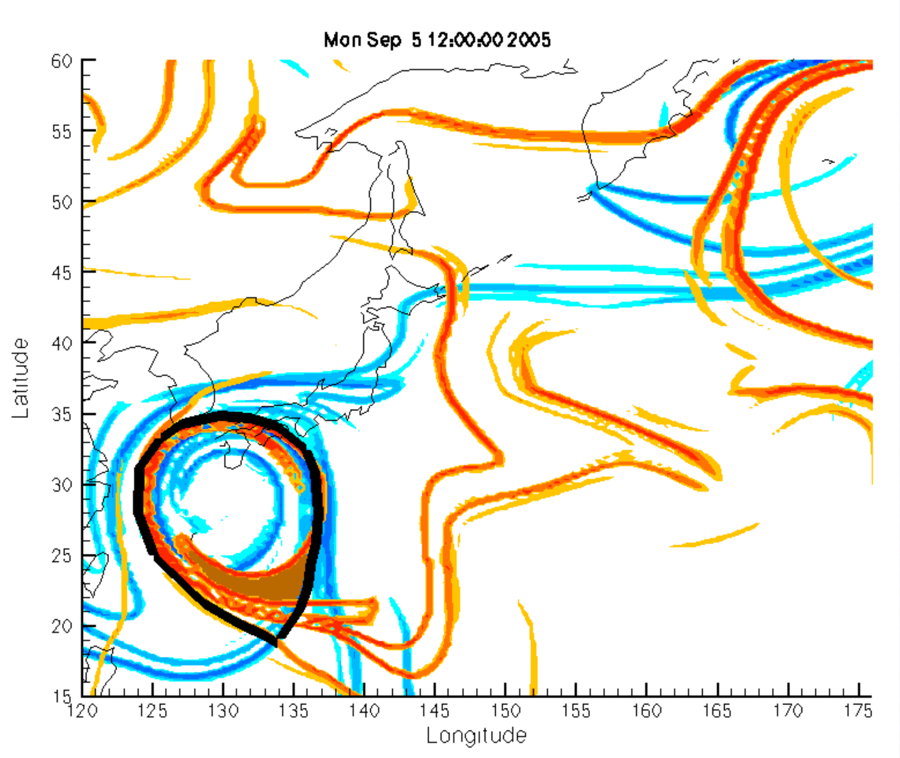

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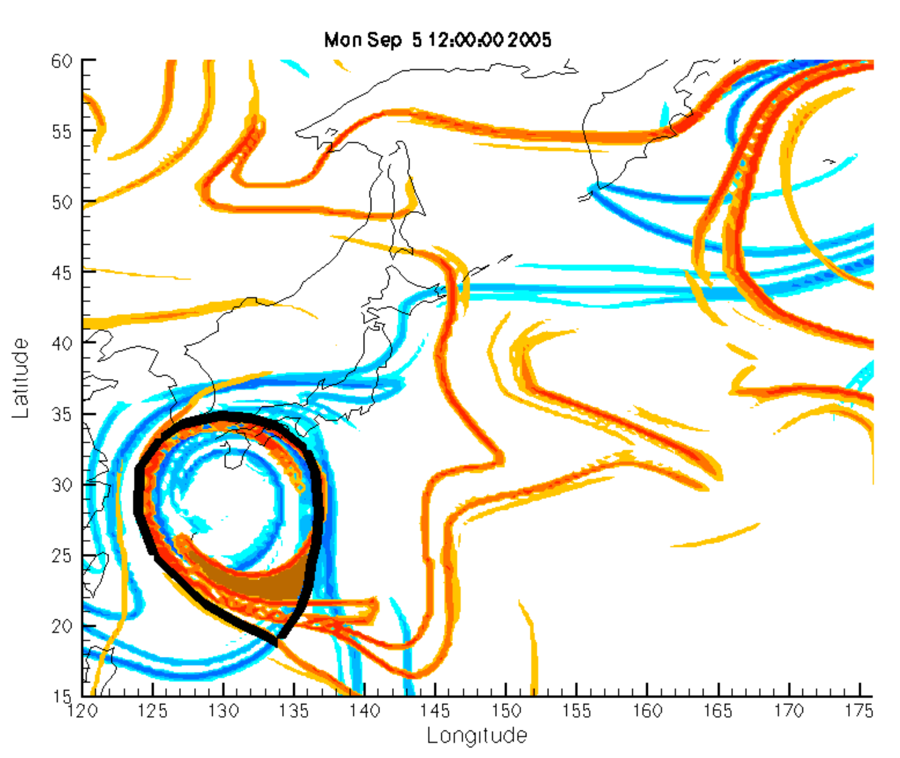

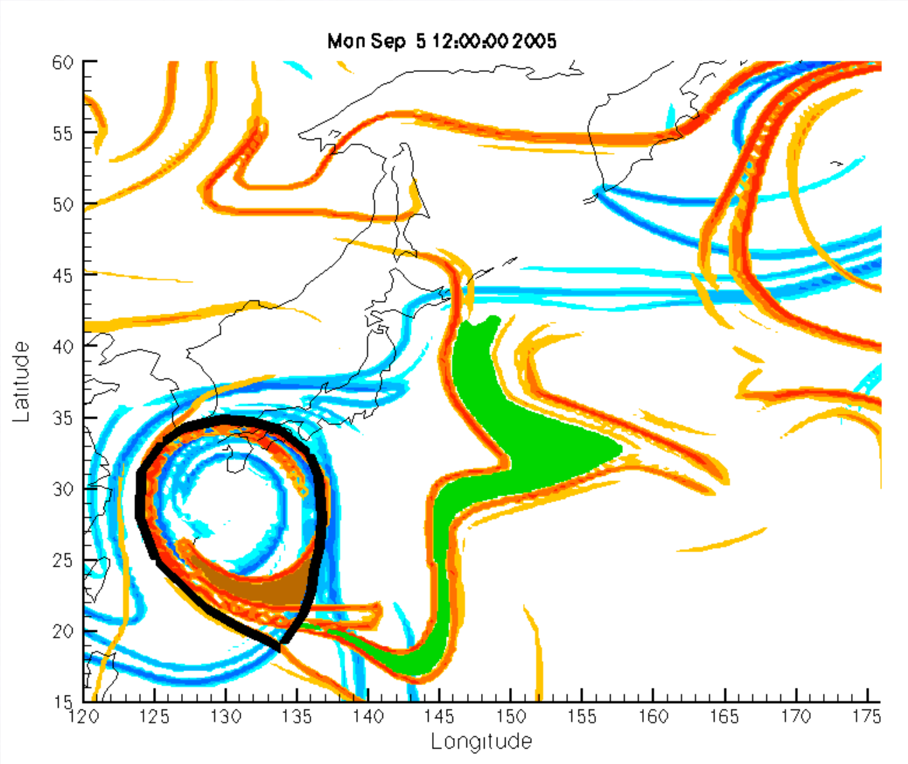

Even Shades of a Cantor Set

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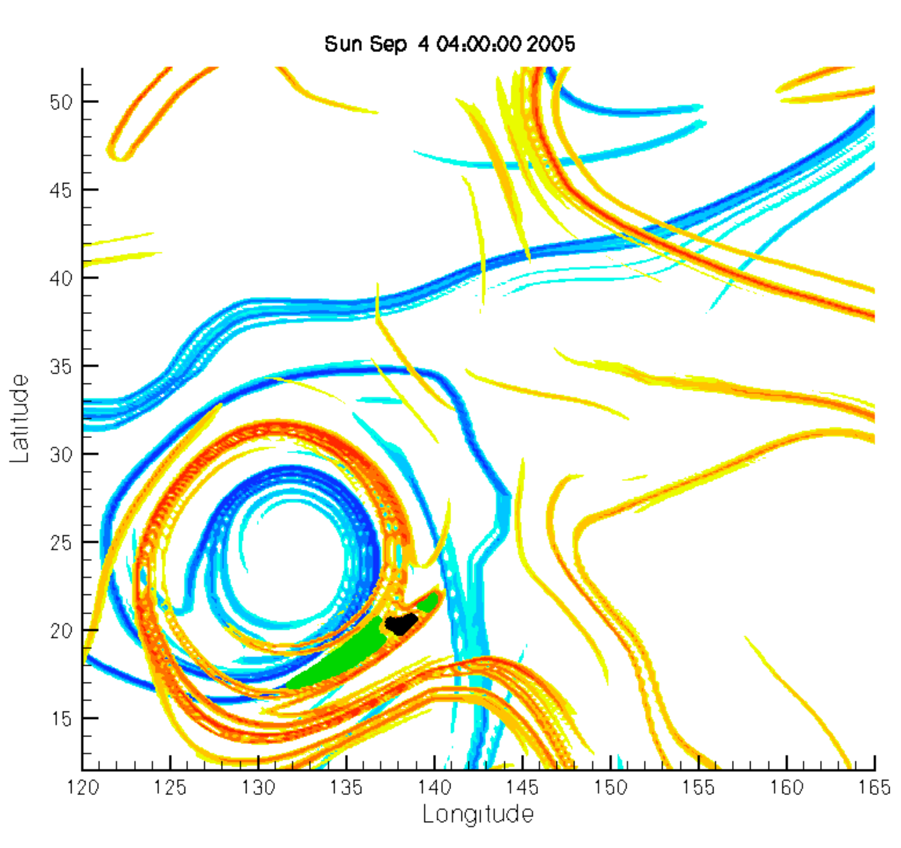
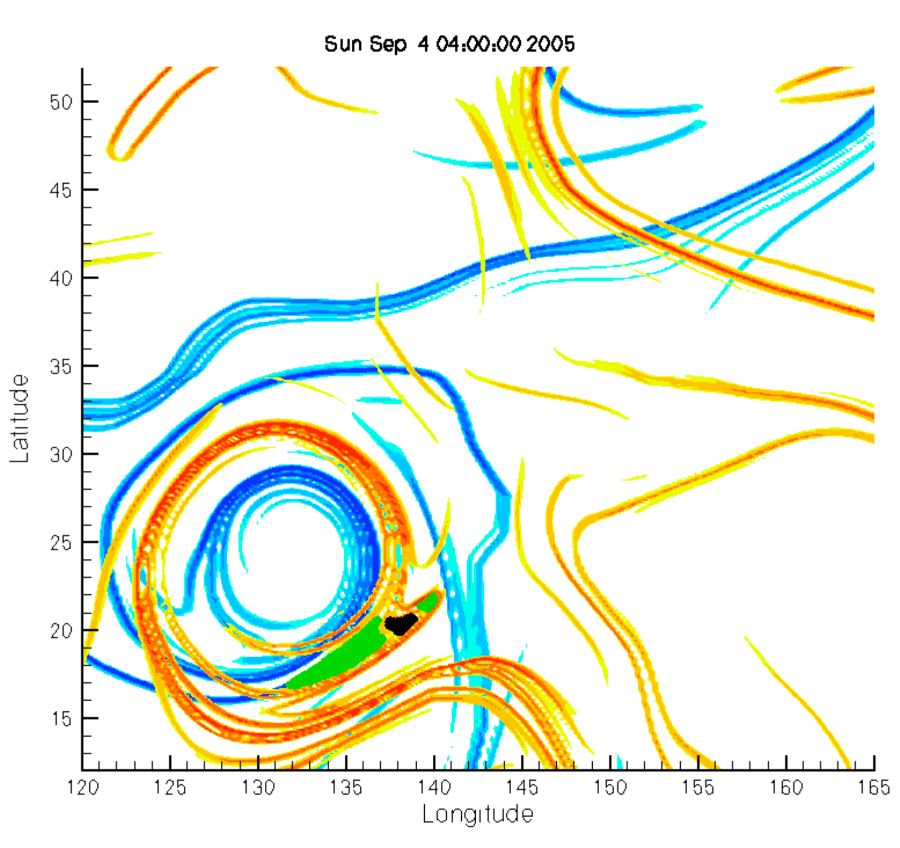
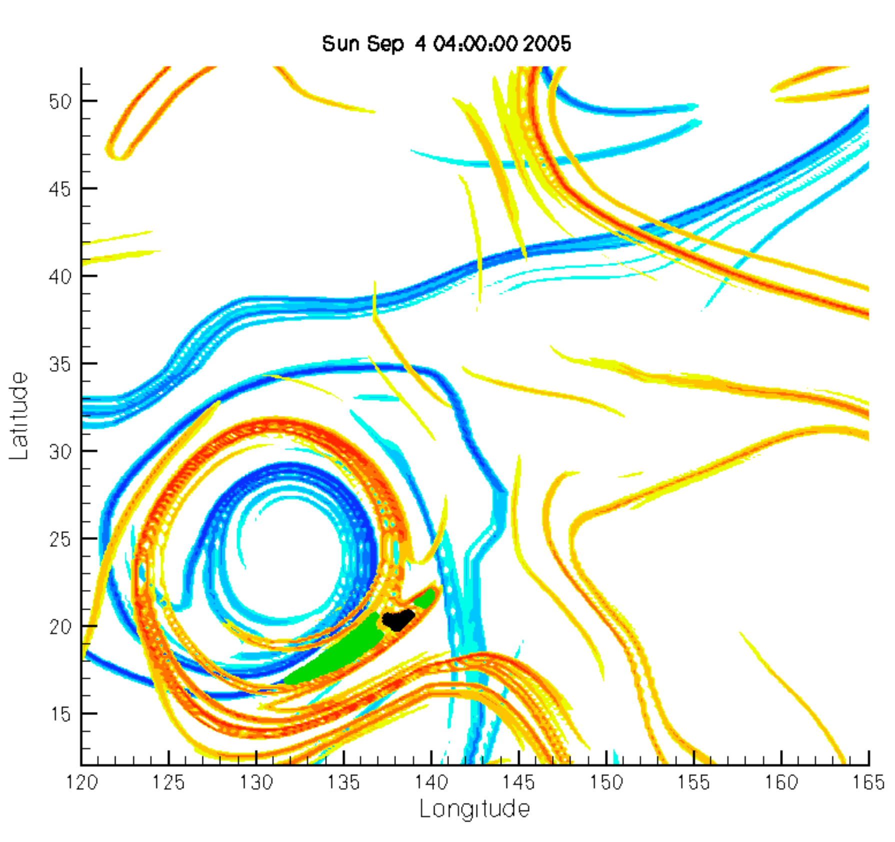


Even Shades of a Cantor Set





3D Structures

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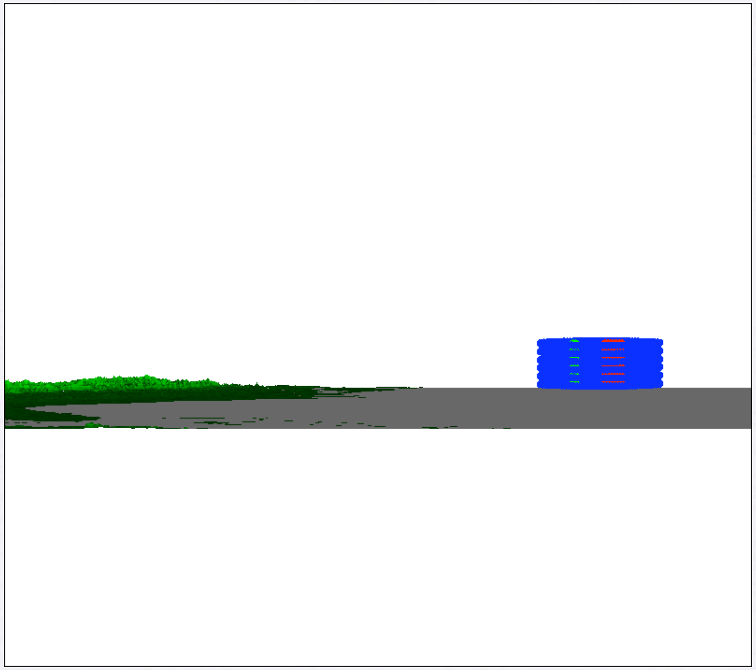

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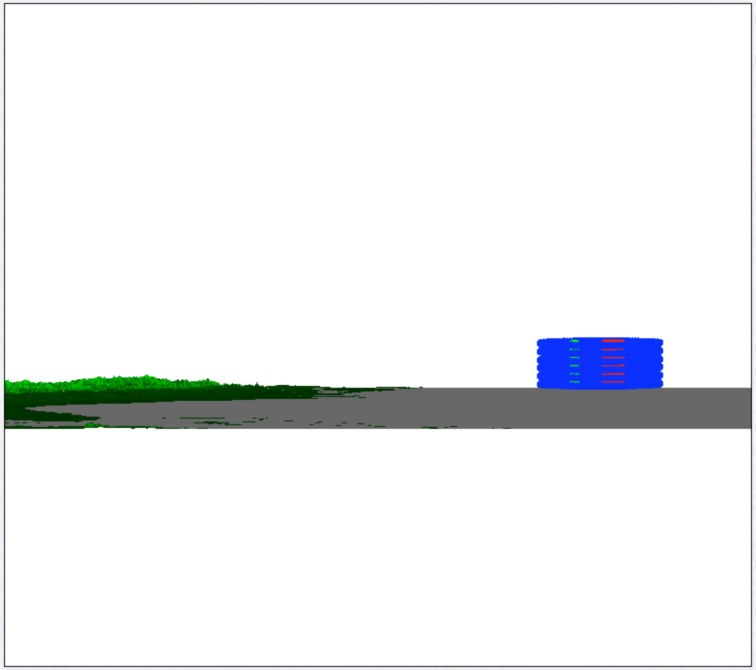

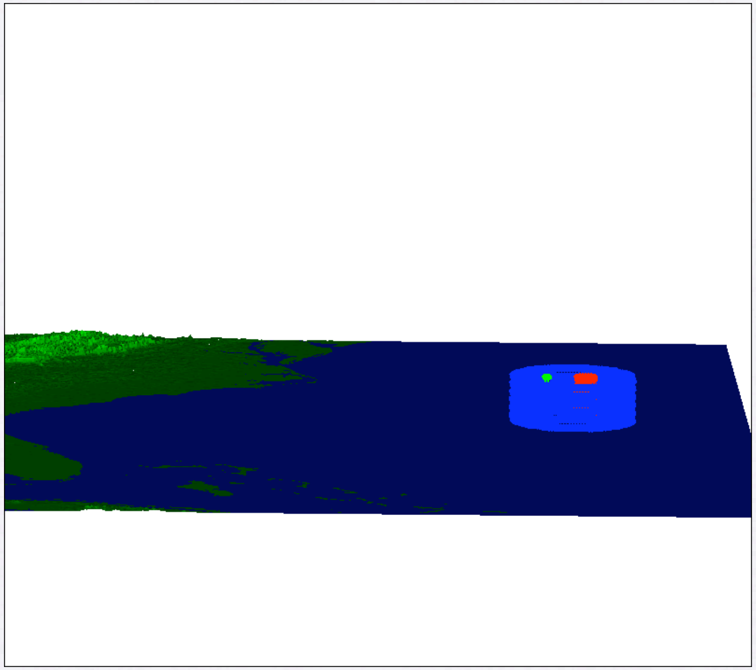

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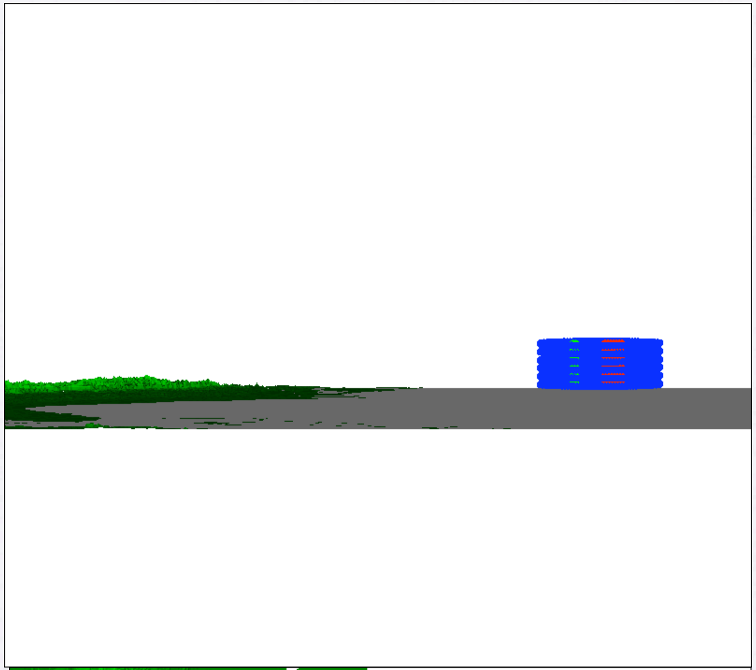

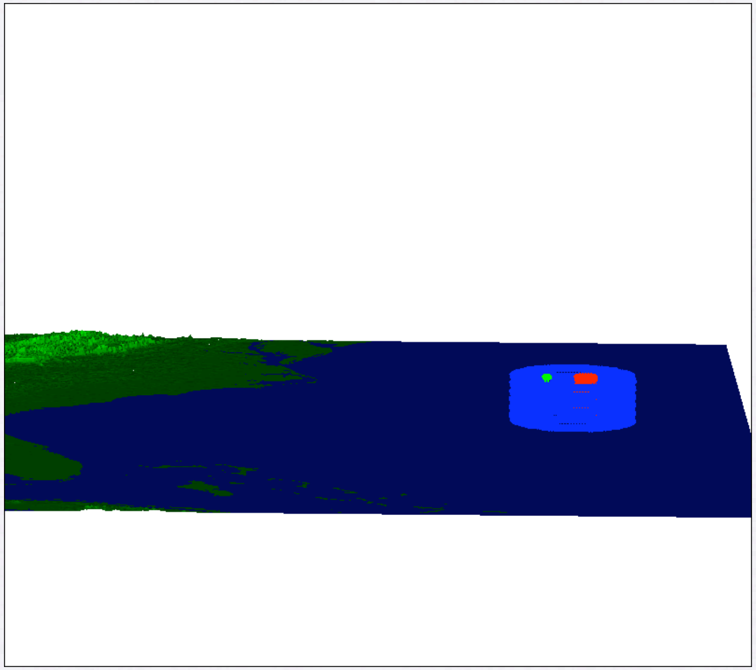

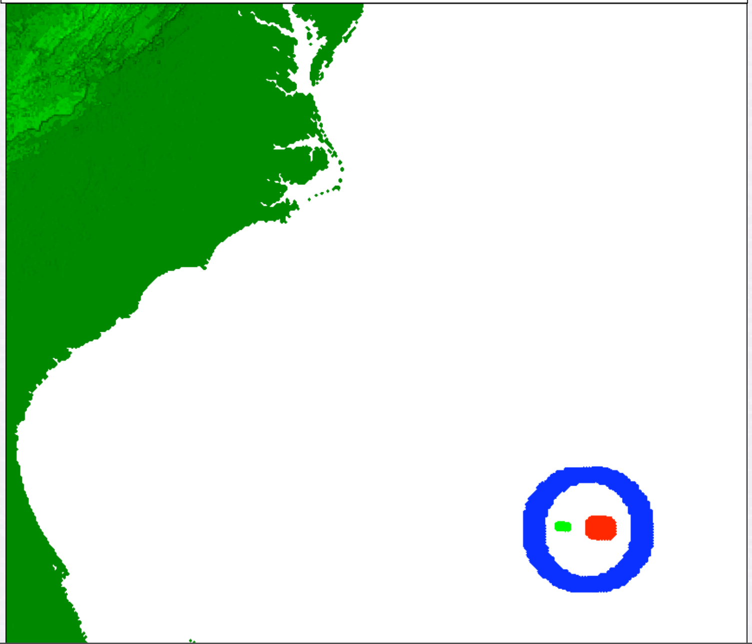

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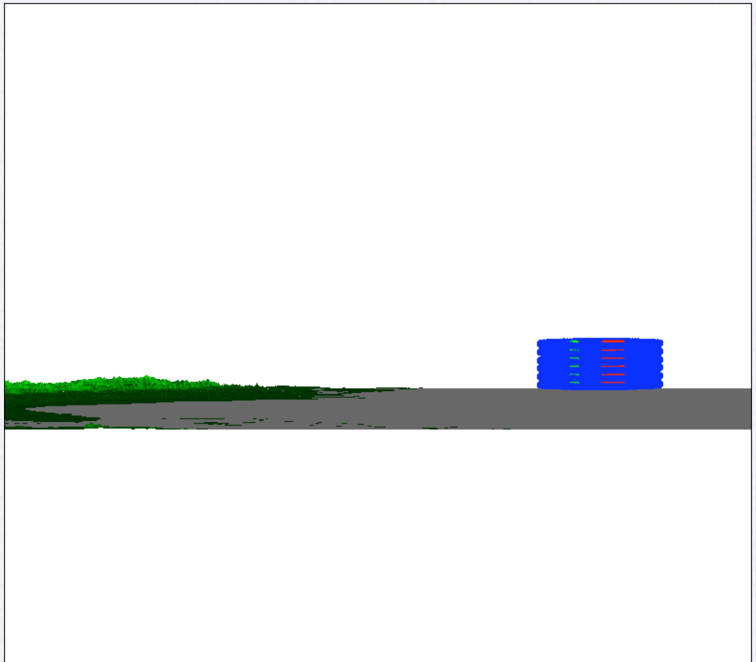

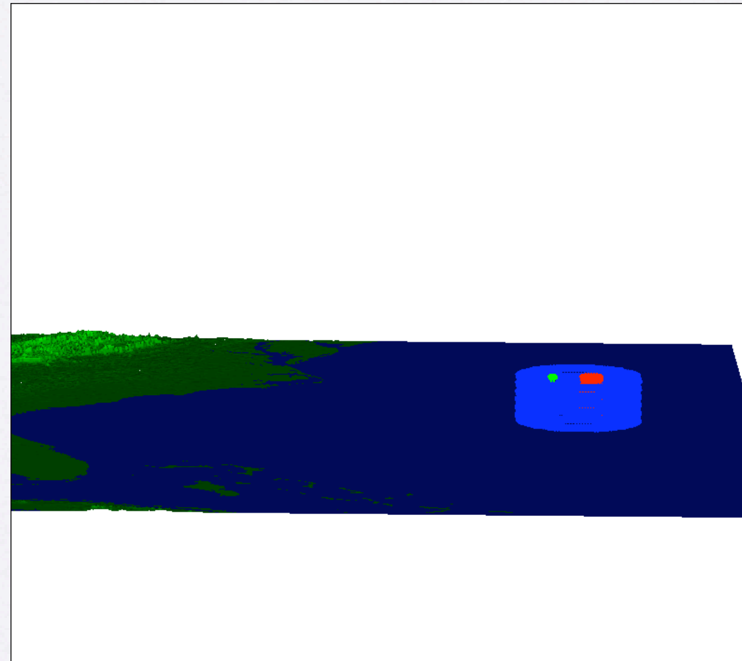

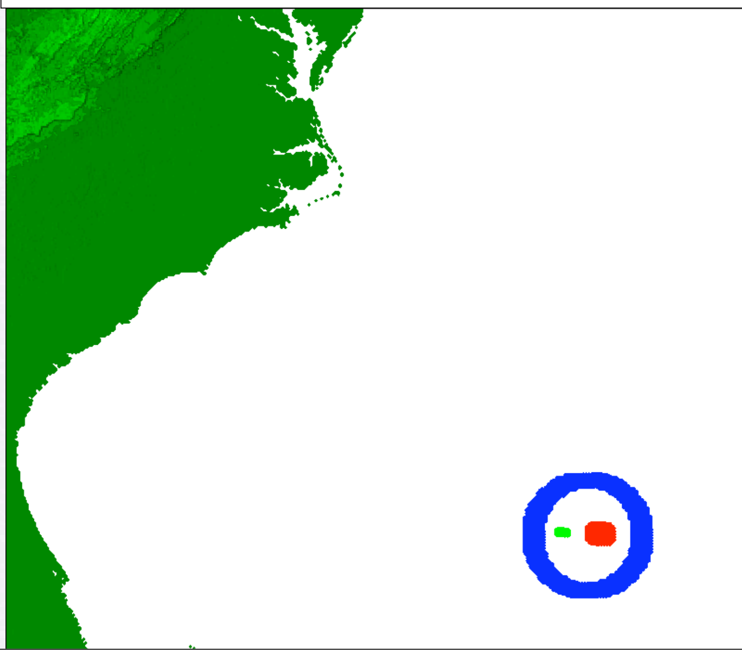

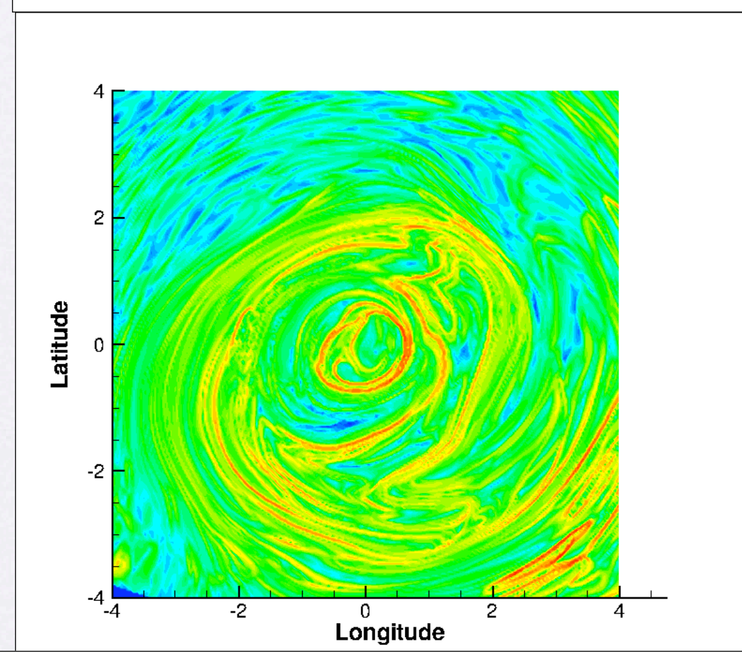

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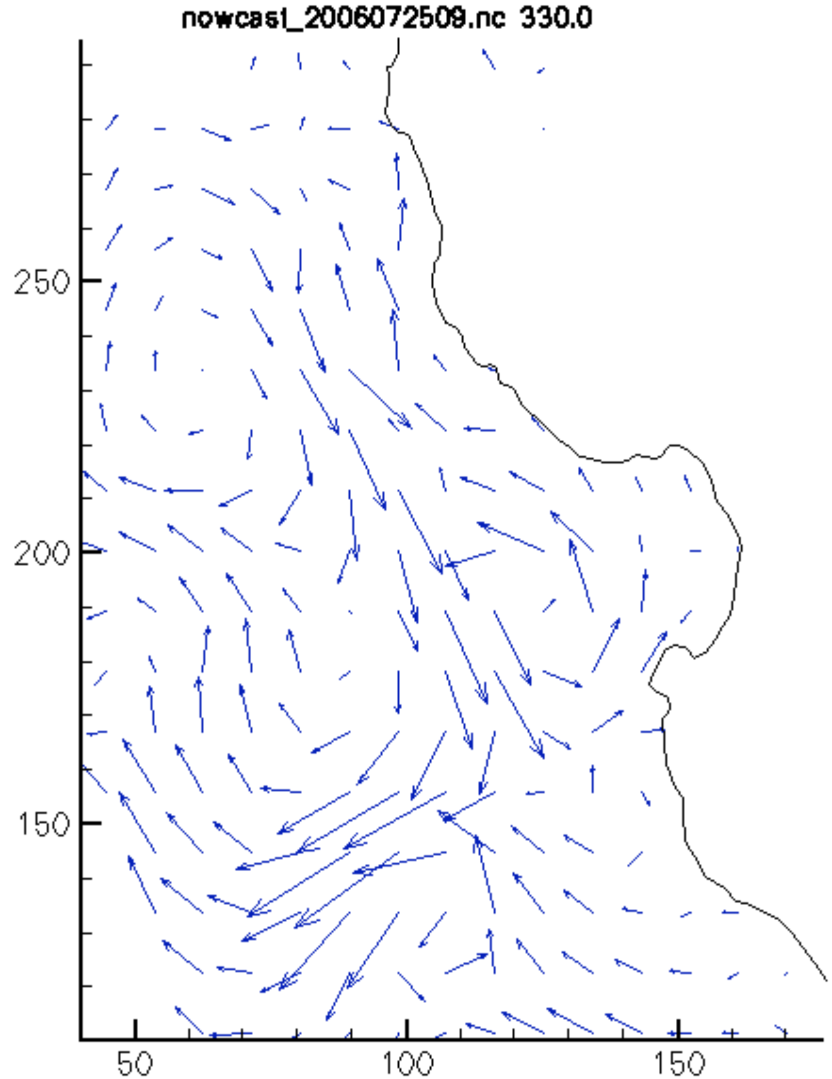
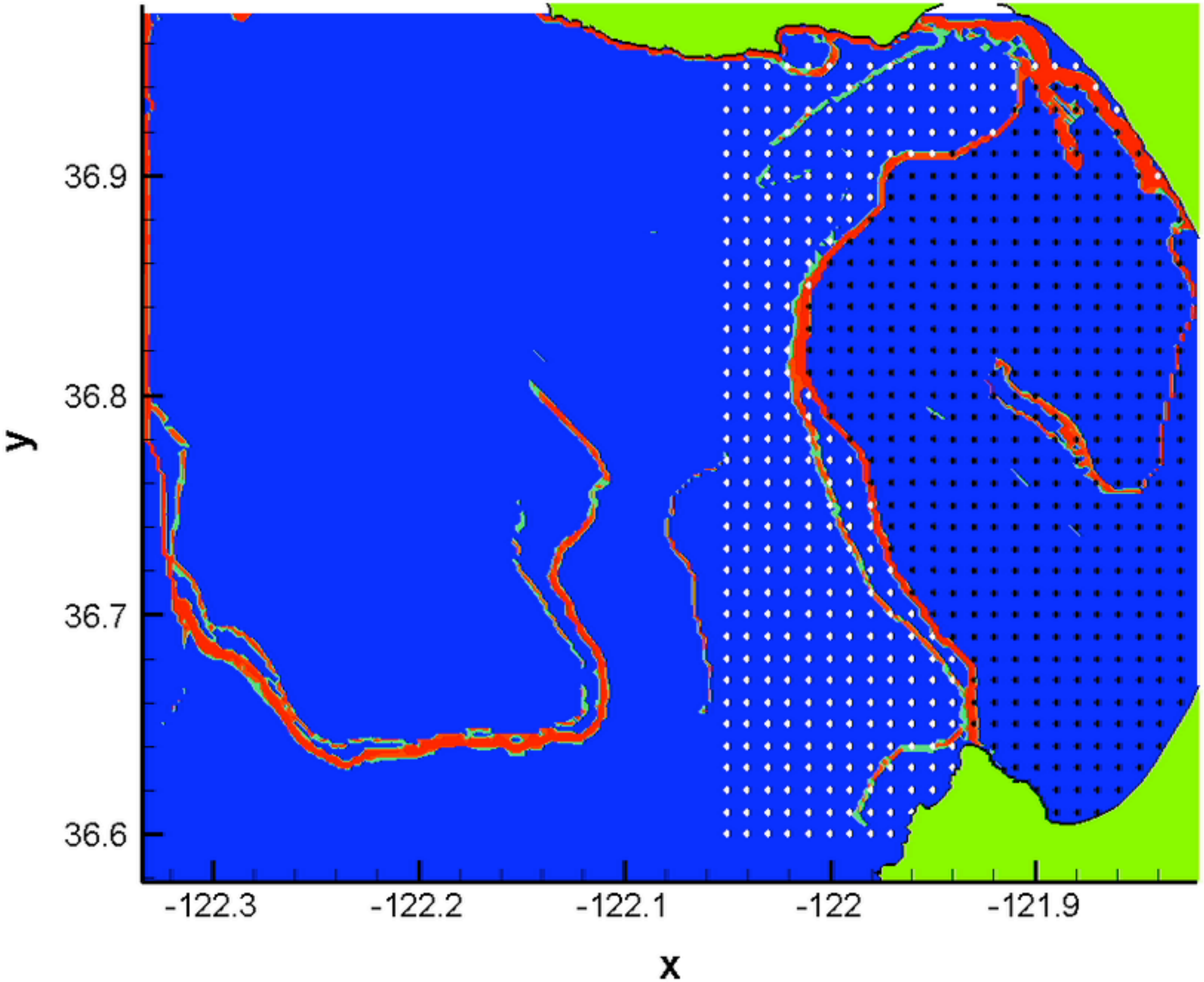
- LCS divide particles with different dynamical fates
- Clear example in ocean dynamics

Lagrangian Coherent Structures in Monterey bay



LCS in the Ocean



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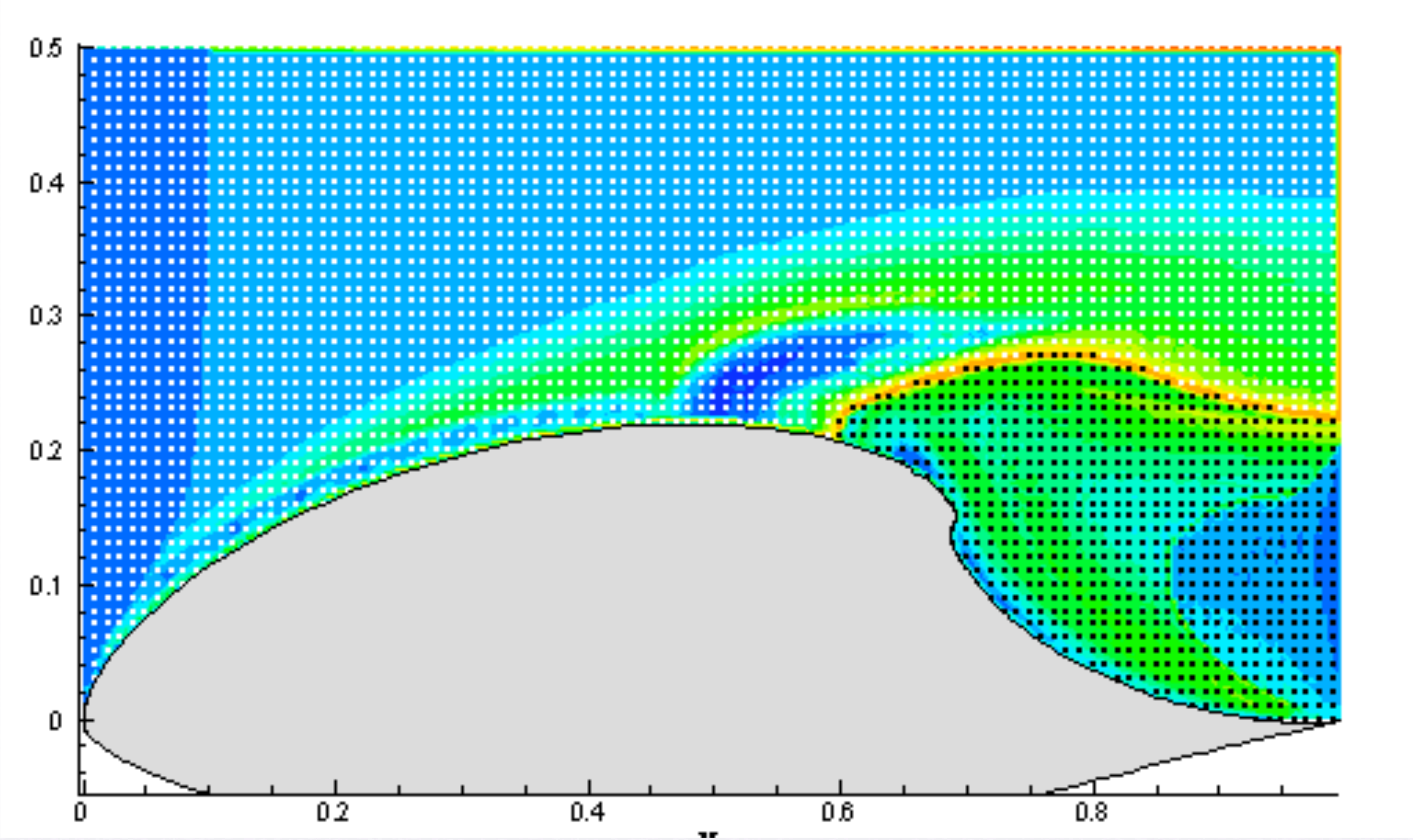
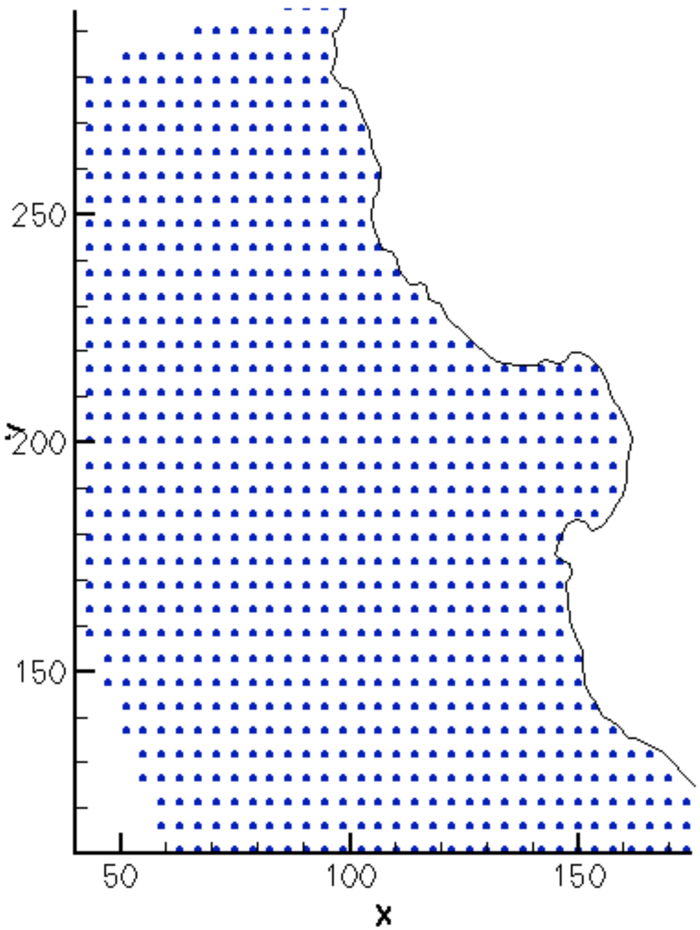
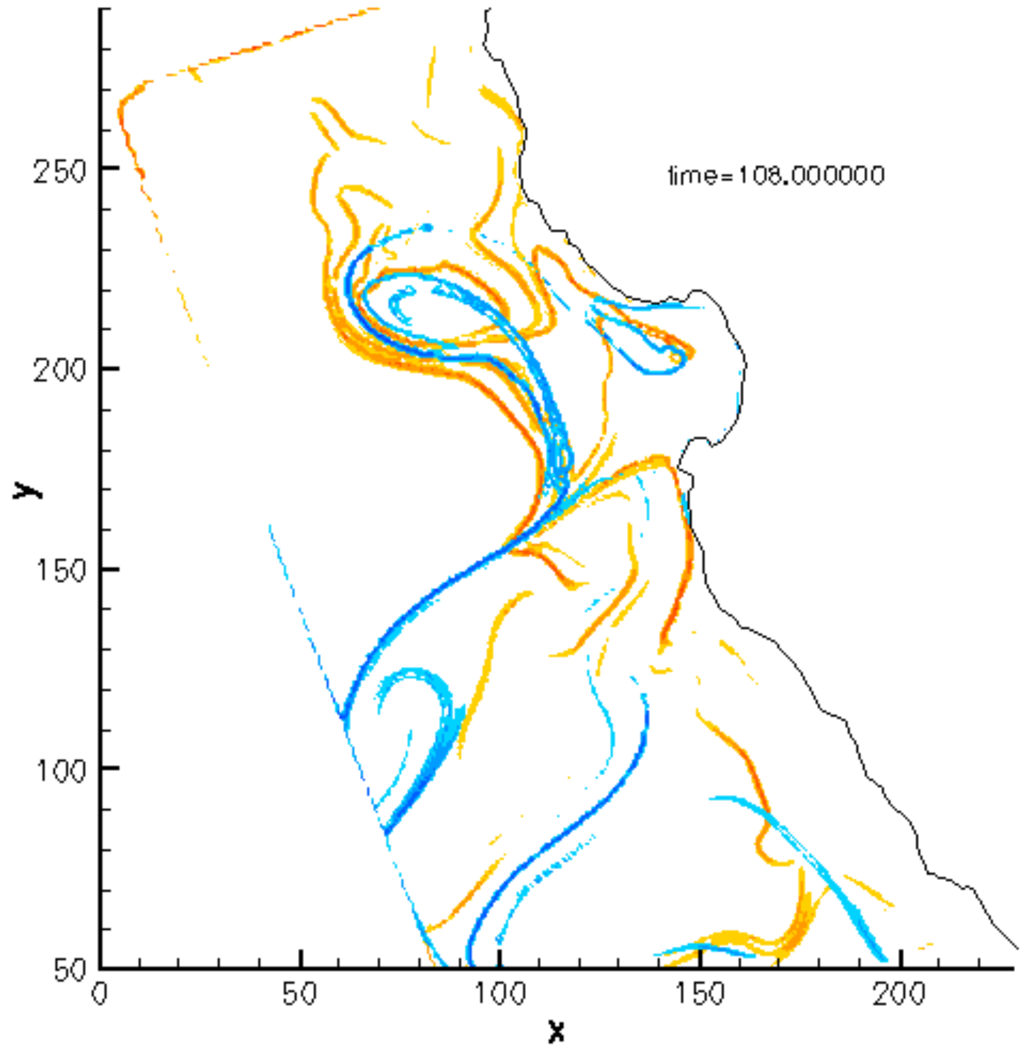
LCS for flow over an Airfoil

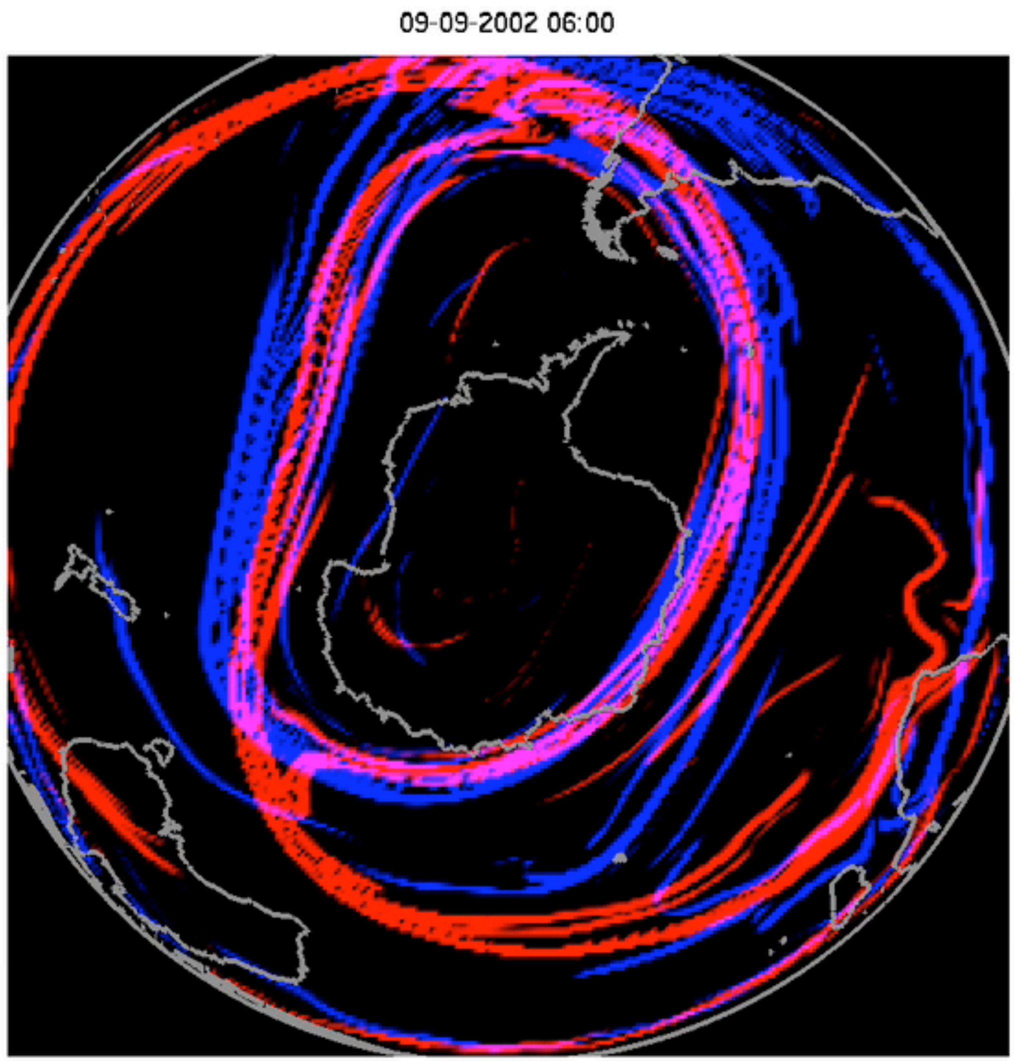


Two Types of LCS: Attracting and Repelling



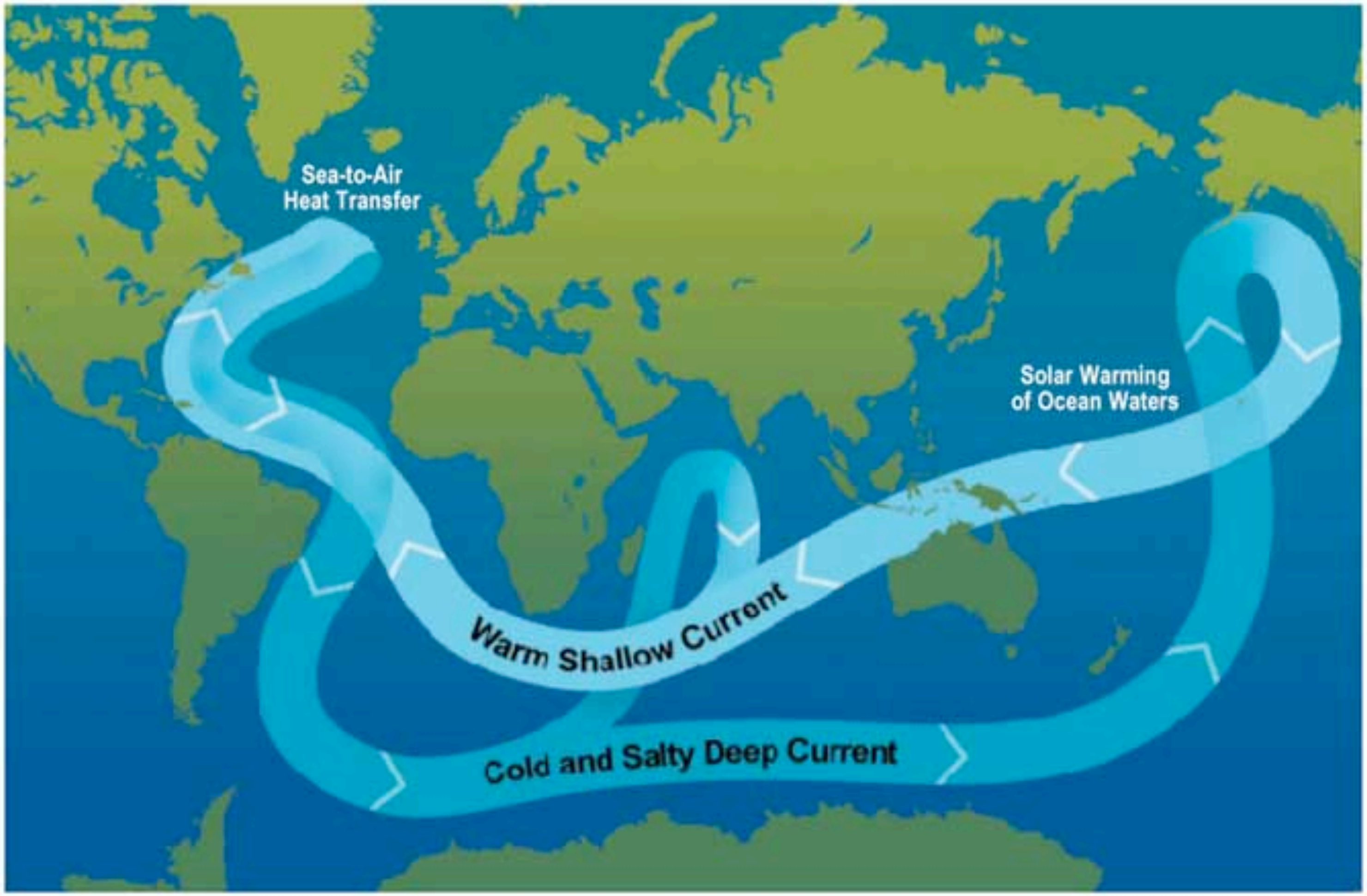


LCS for Ozone Hole Breakup



Thermohaline Circulation

Ocean Circulation Conveyor Belt



The ocean plays a major role in the distribution of the planet's heat through deep sea circulation. This simplified illustration shows this "conveyor belt" circulation which is driven by differences in heat and salinity. Records of past climate suggest that there is some chance that this circulation could be altered by the changes projected in many climate models, with impacts to climate throughout lands bordering the North Atlantic.

Vortex Rings

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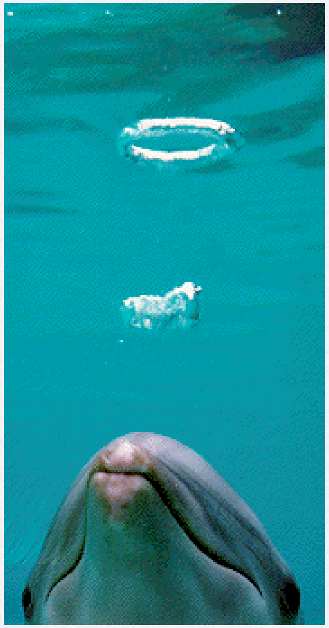

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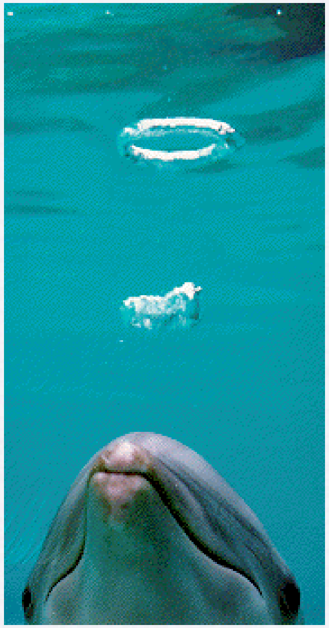

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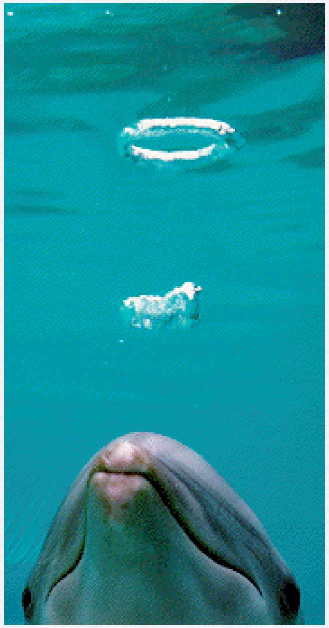

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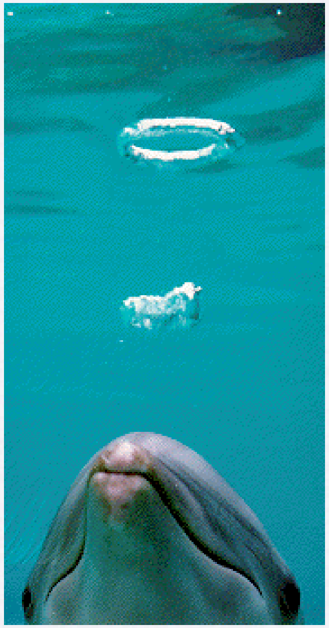
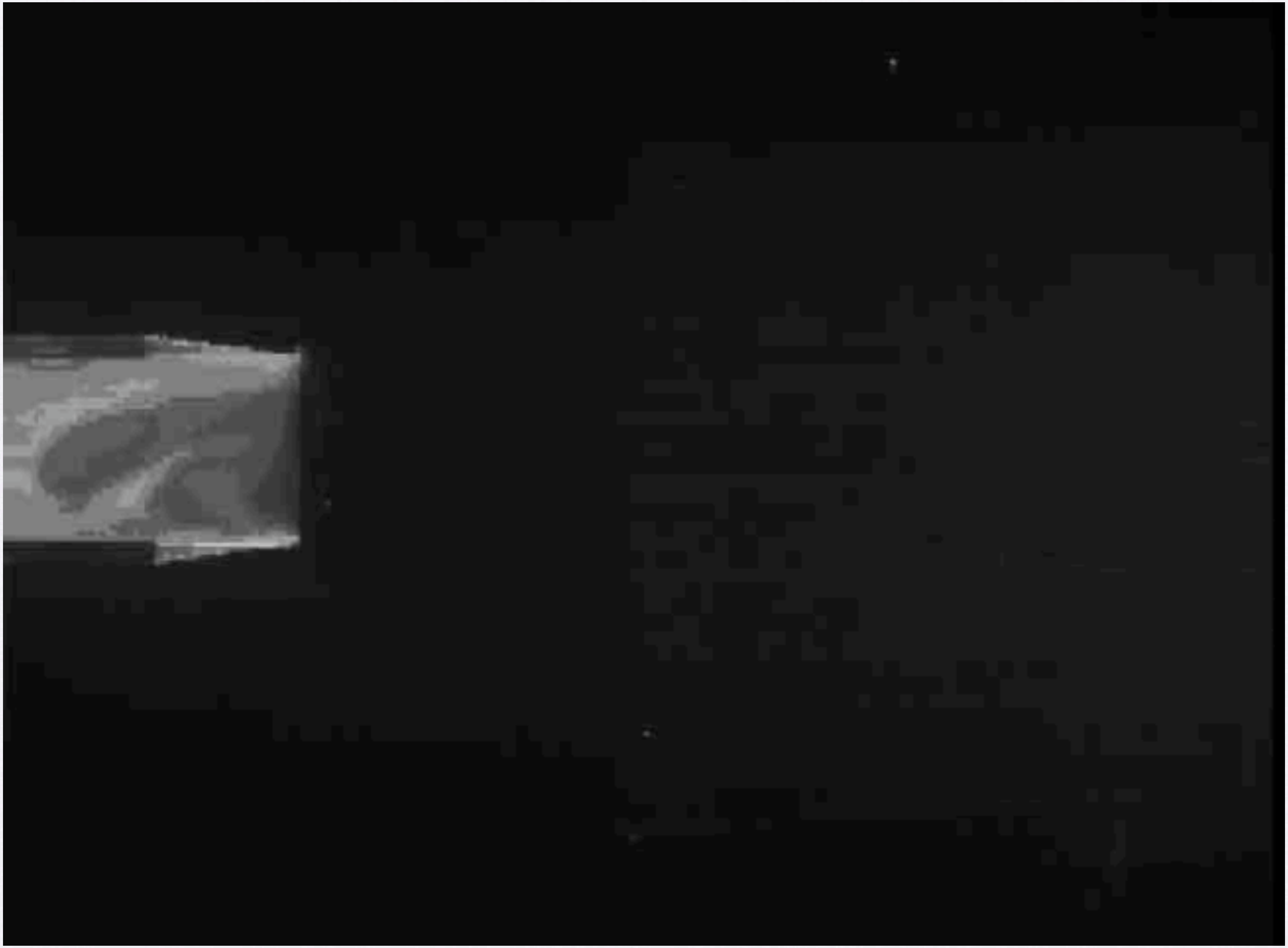
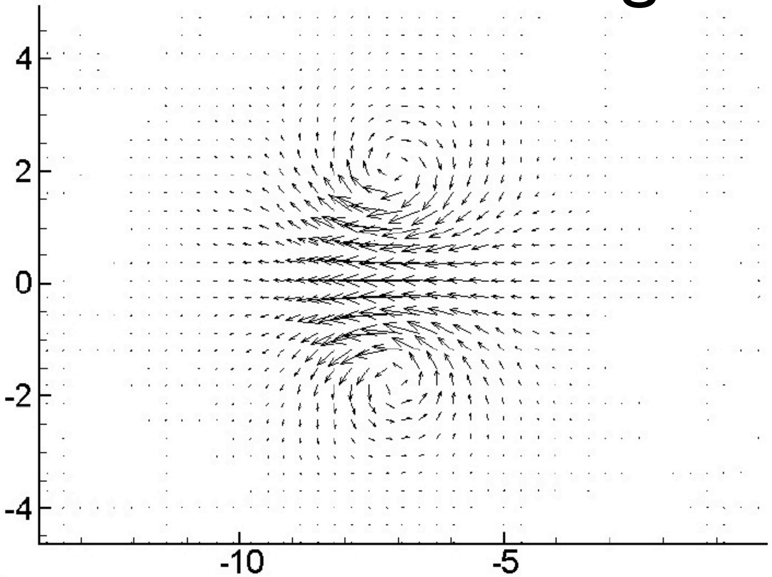


Laboratory Vortex Rings



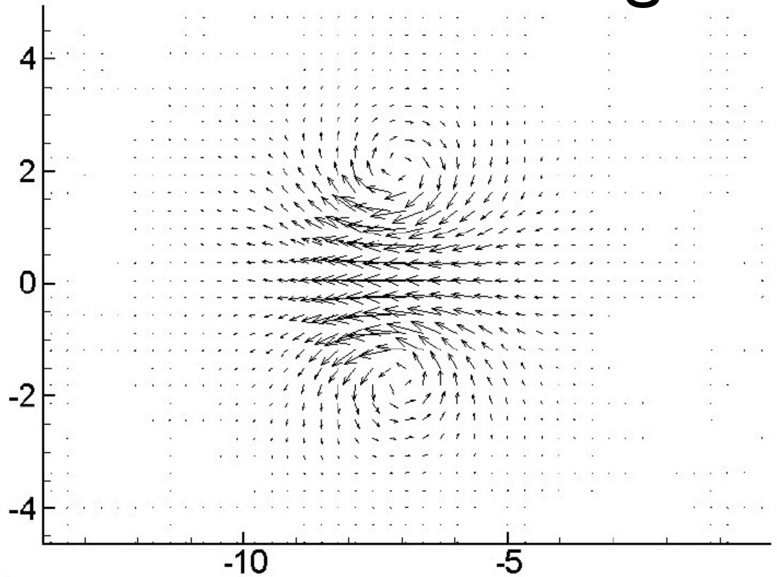
LCS for the vortex ring



Vortex velocity field

Shawn Shadden, Stanford

LCS for the vortex ring



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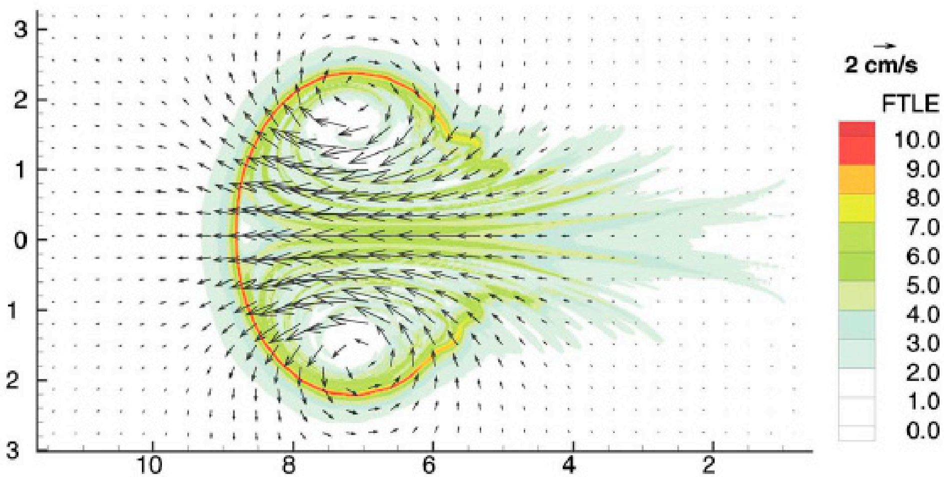

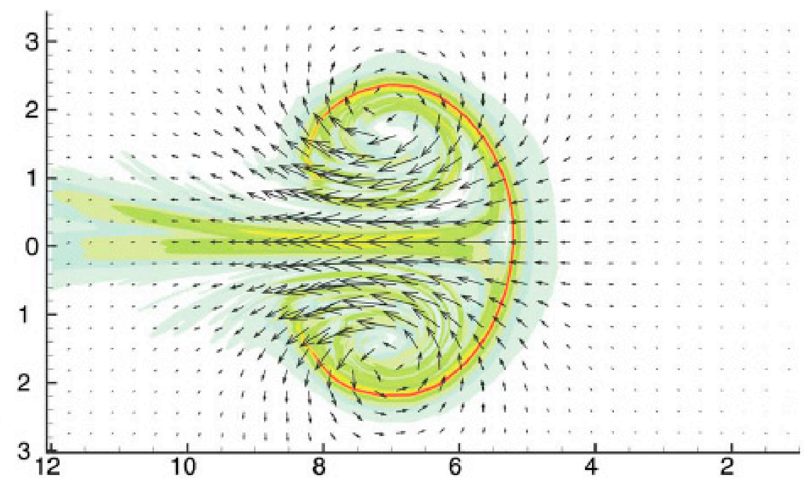

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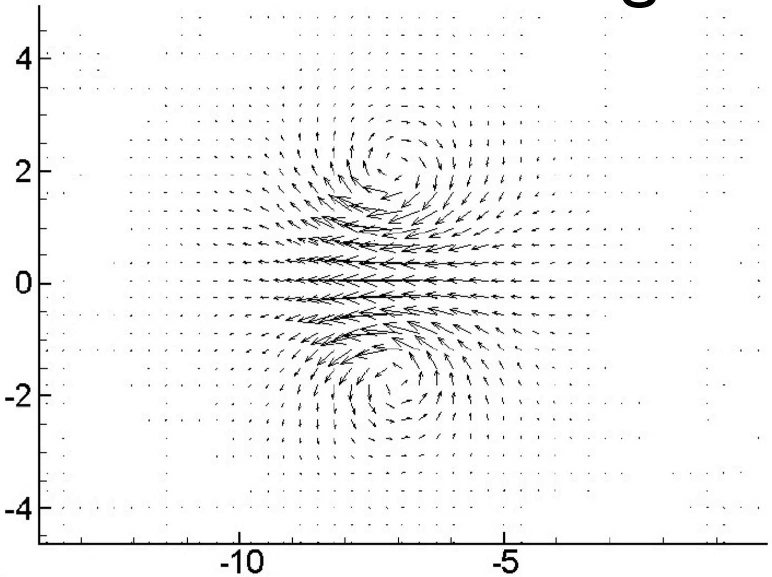

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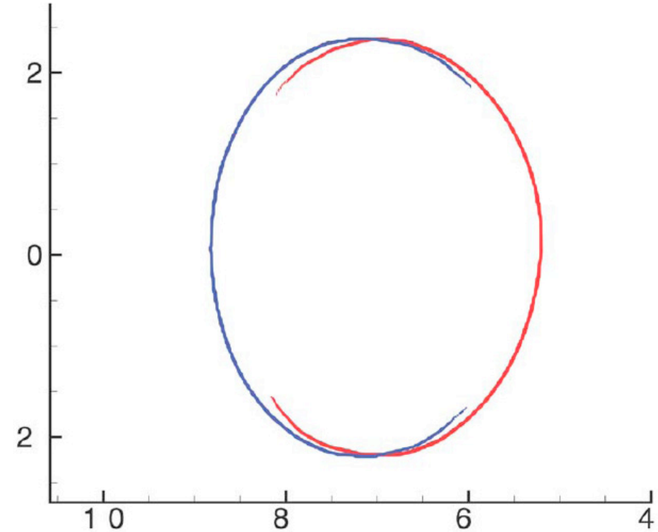

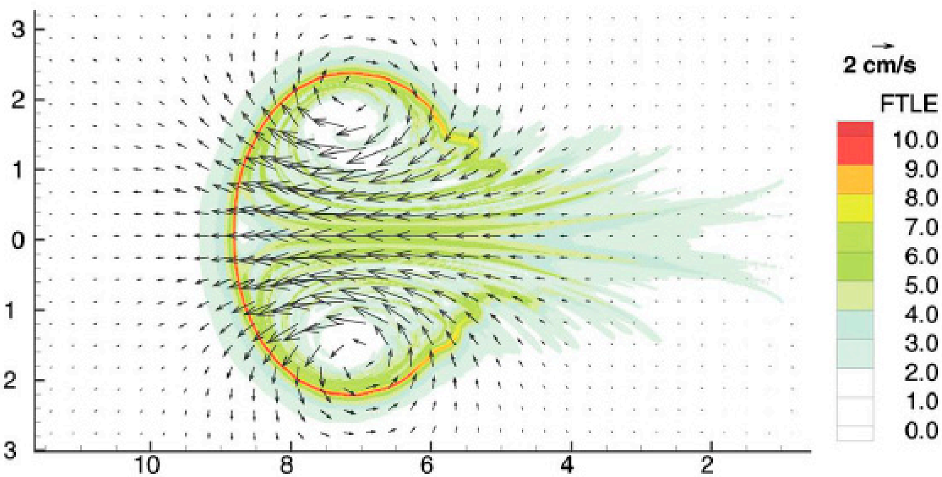

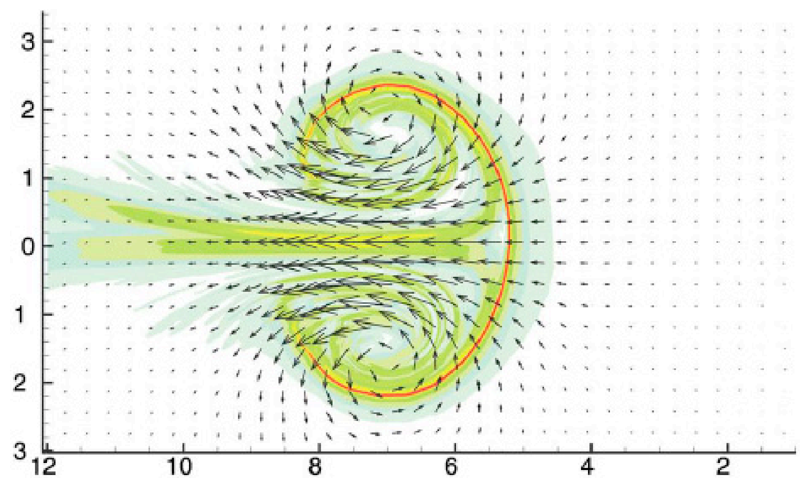

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LCS and Vortex Ring Boundaries

LCS gives much sharper boundaries than vorticity

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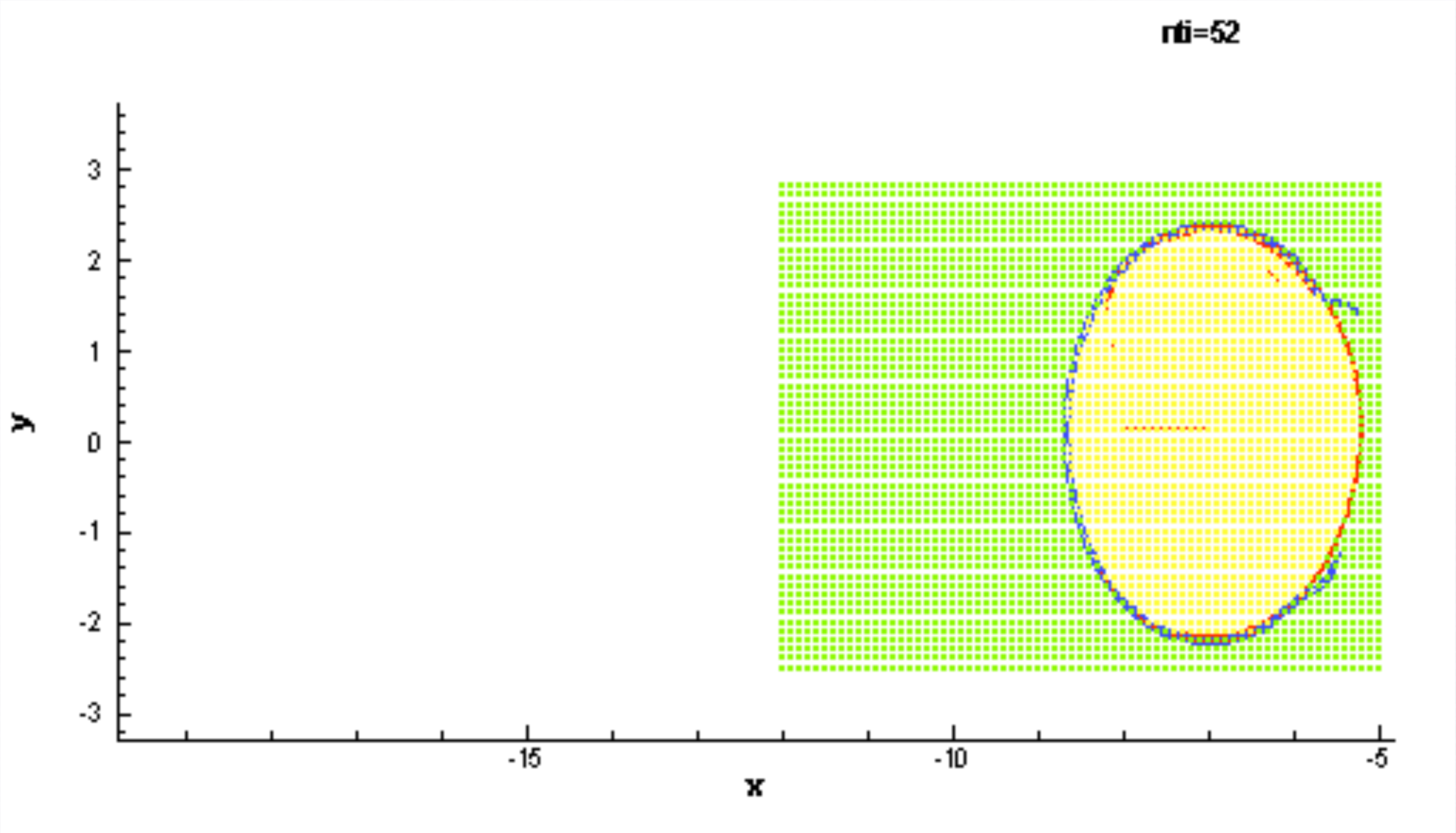

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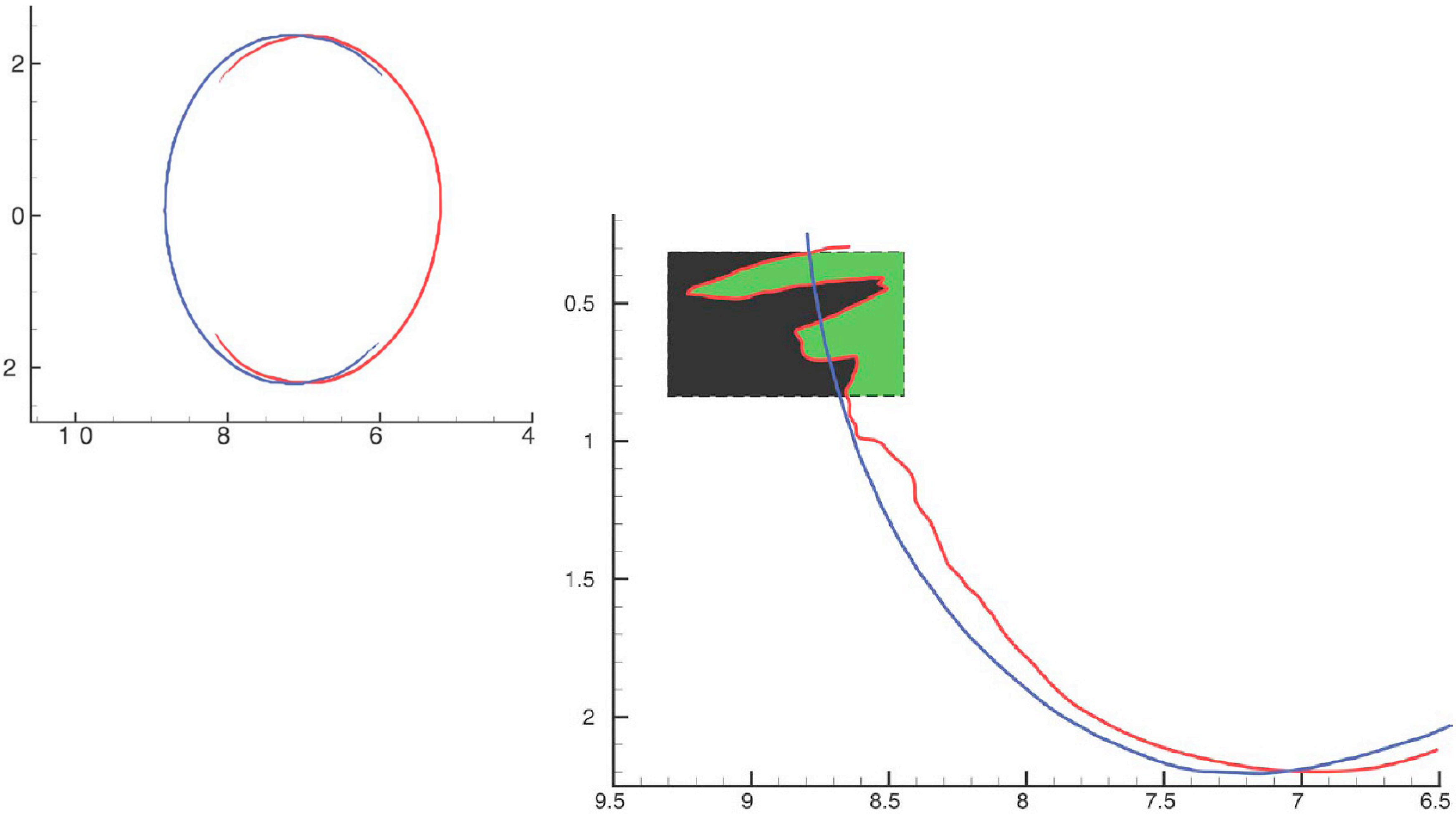
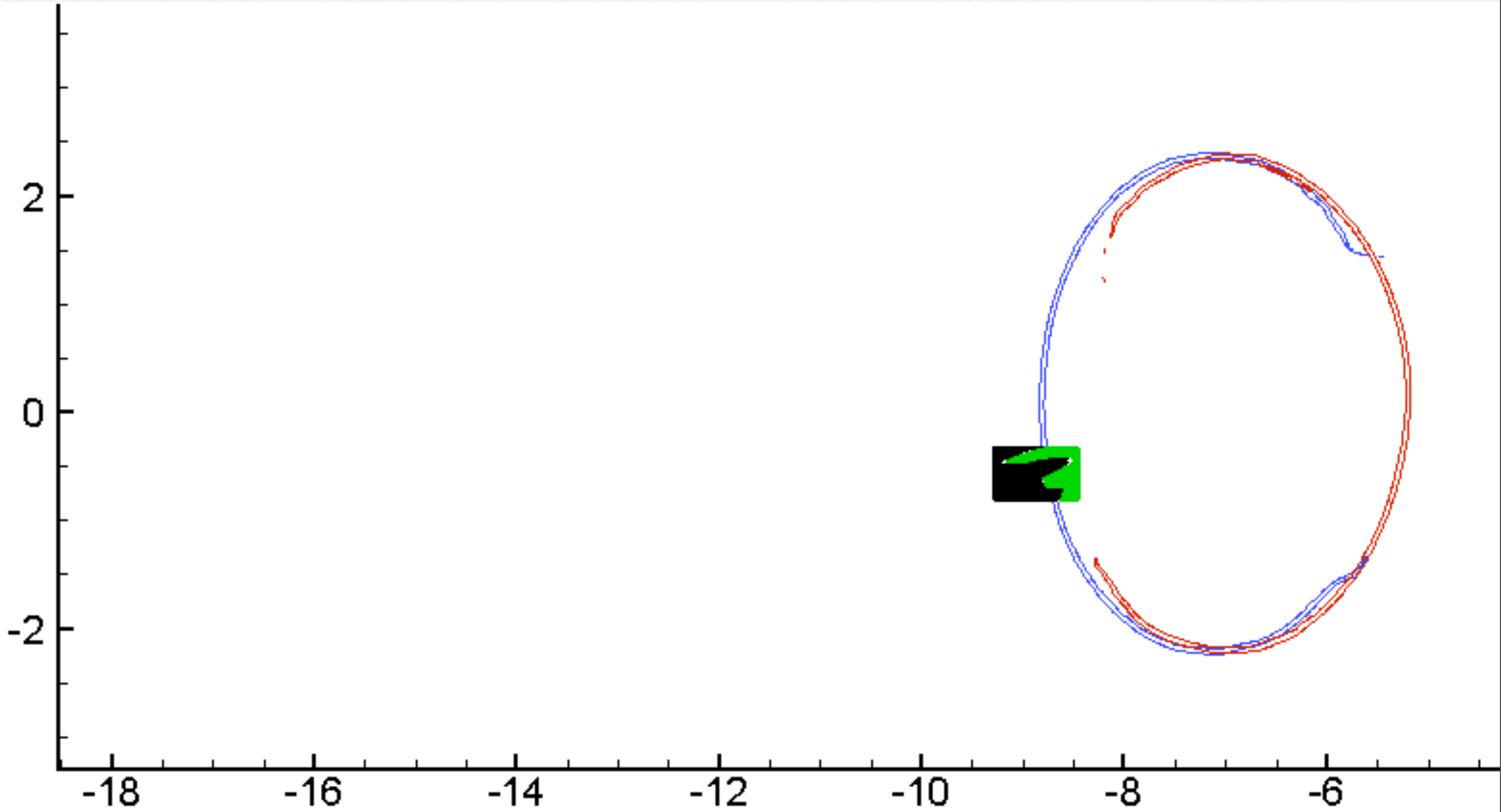
Lobes in the vortex ring



Lobes, Mixing, Transport



Jellyfish

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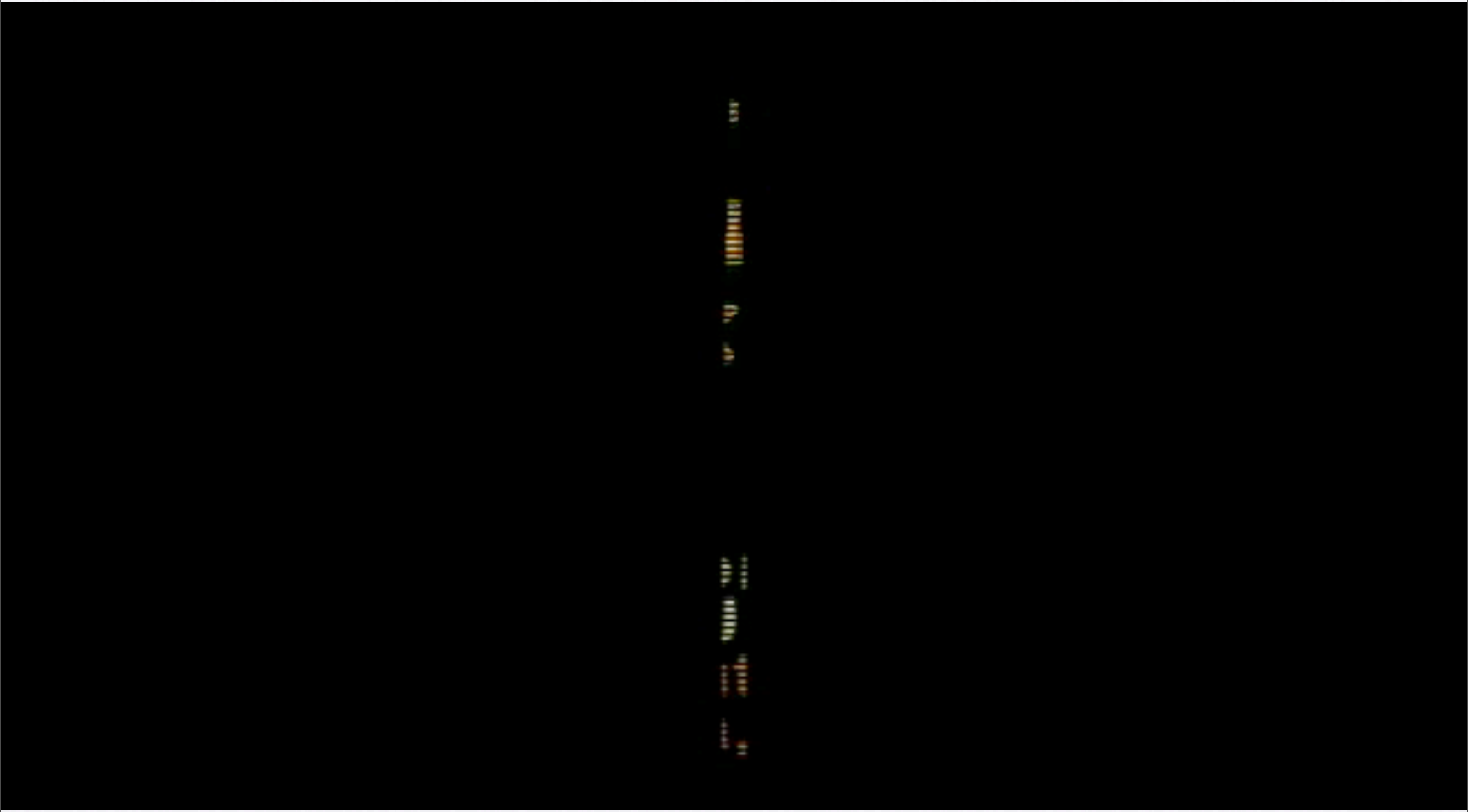
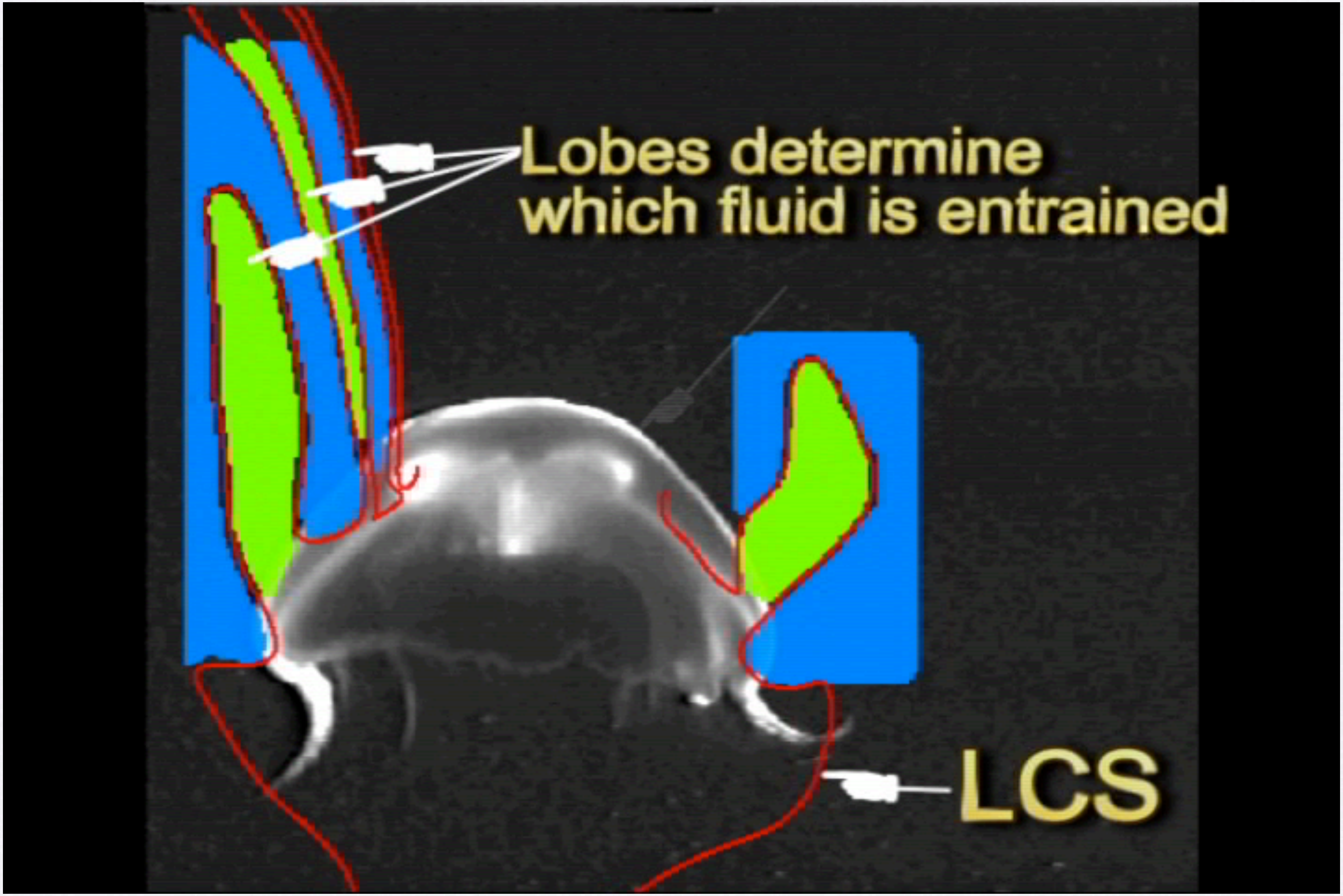
Vortex Rings and Jellyfish



Jellyfish one more time



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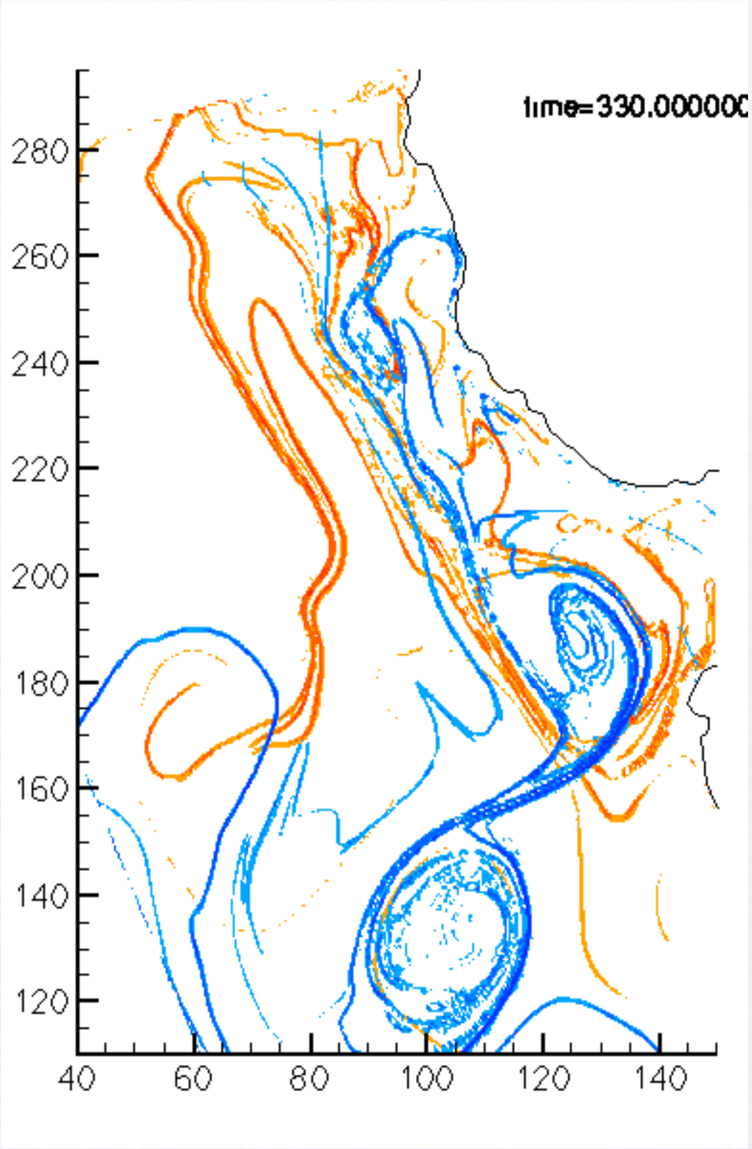

3D LCS-Meddies

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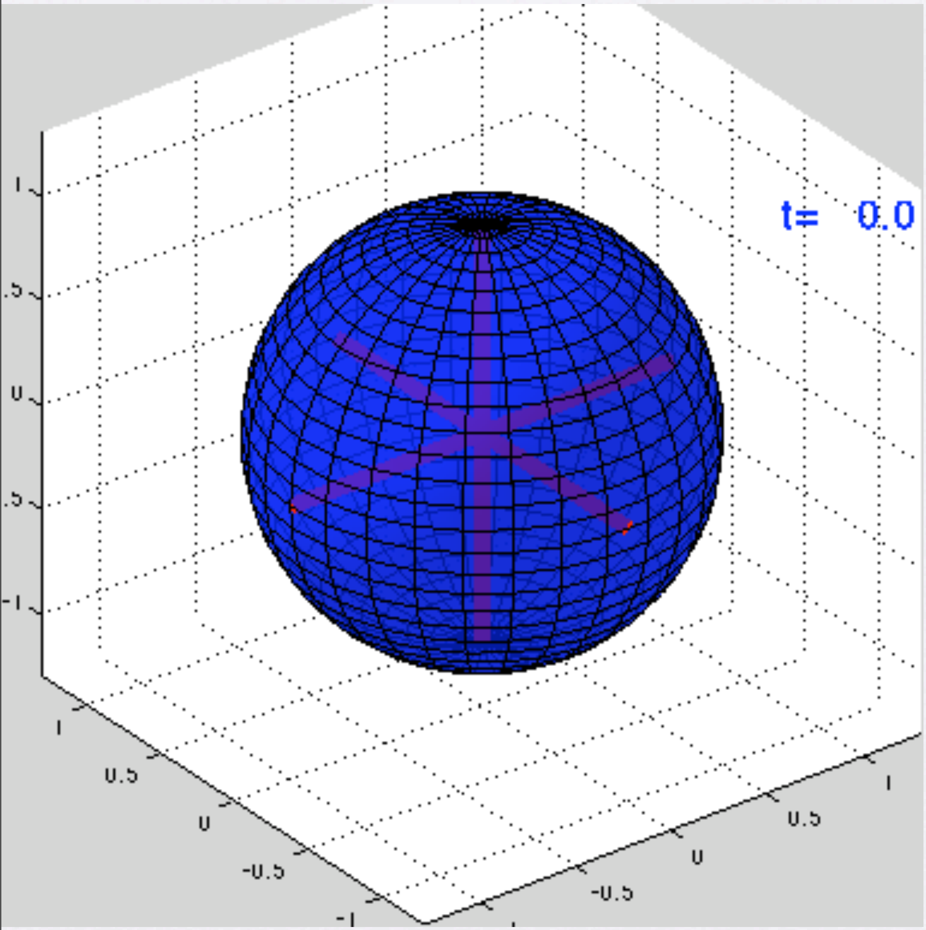

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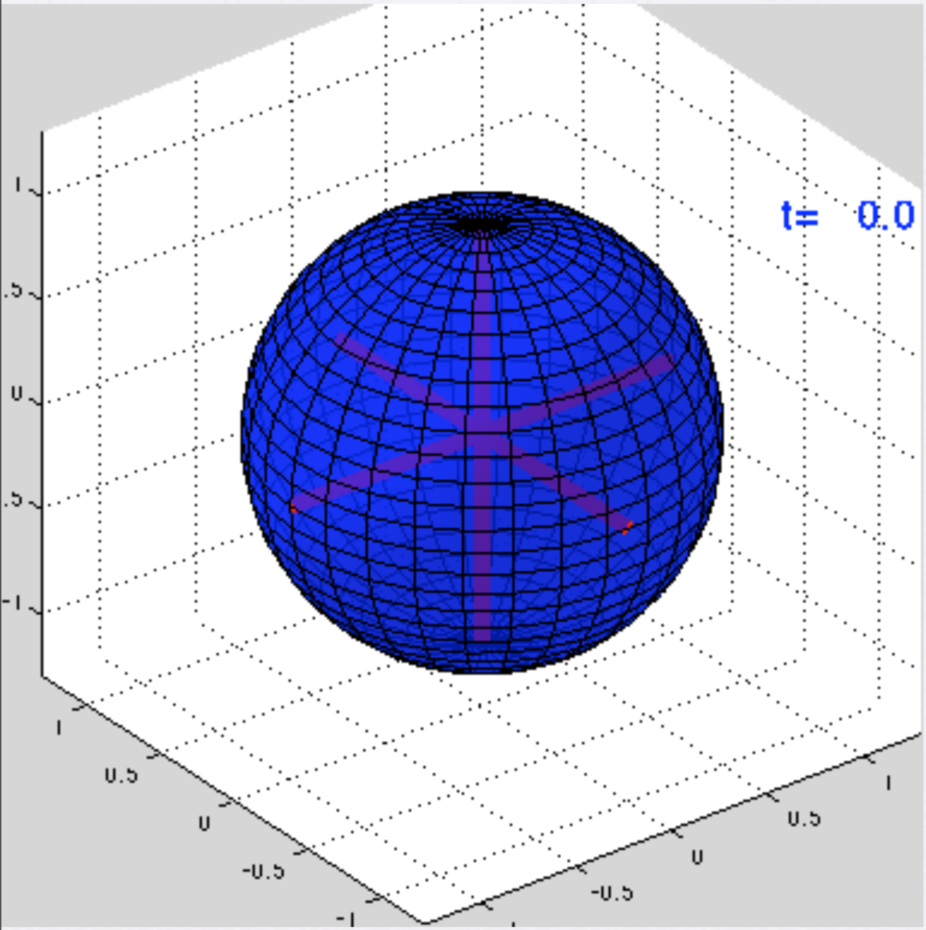

Ellipsoid of vorticity

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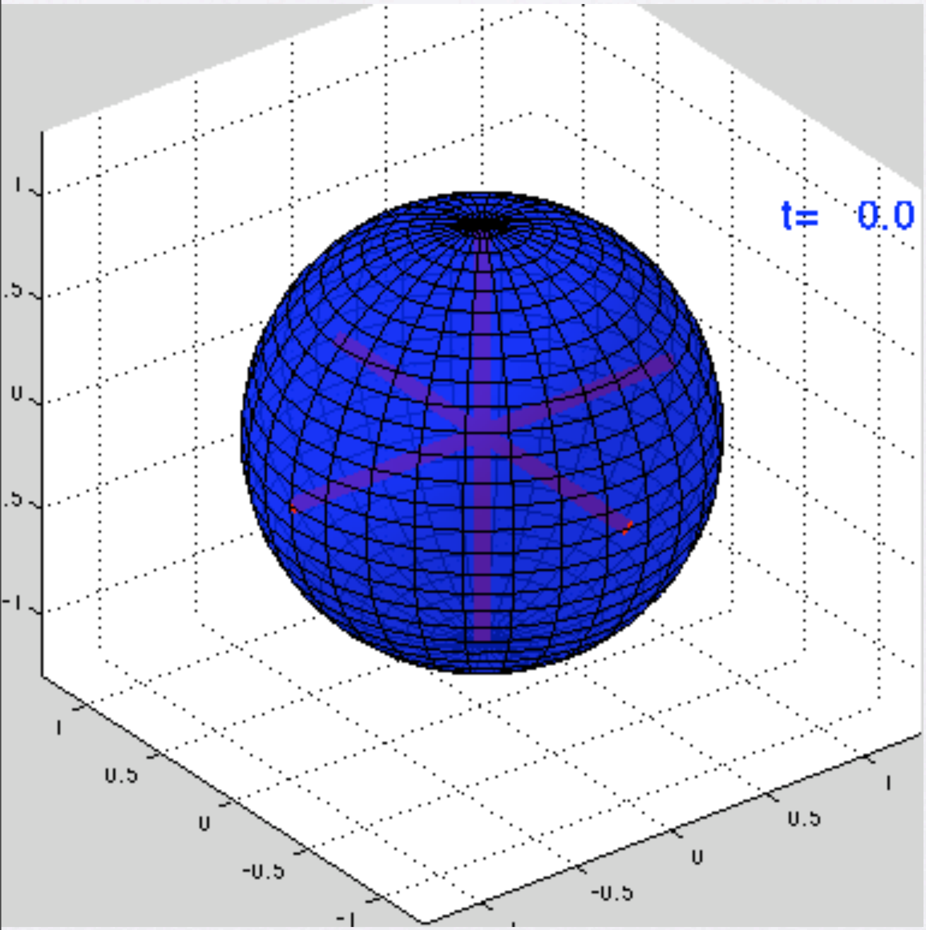

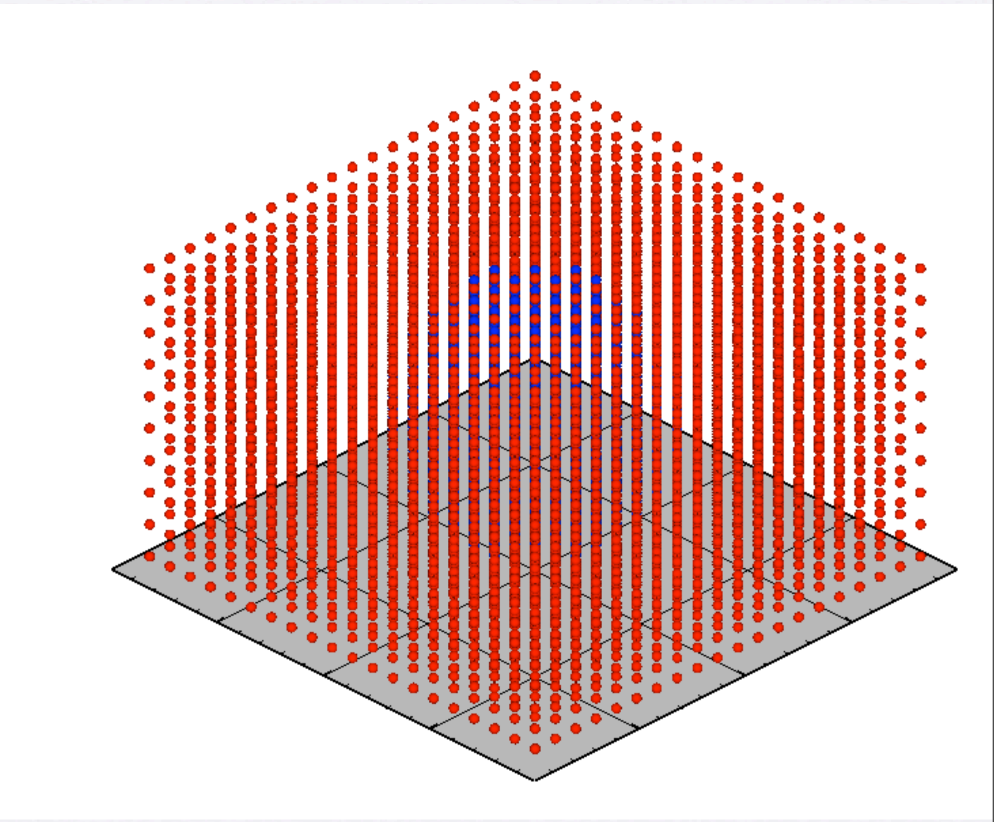

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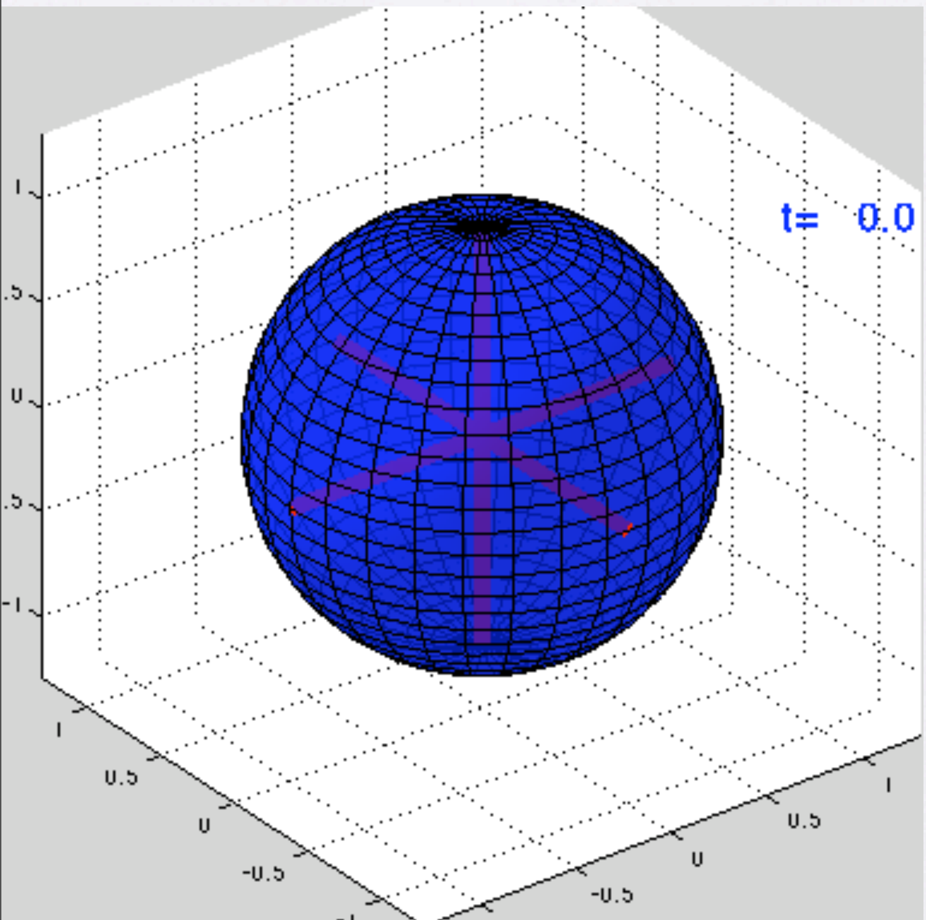

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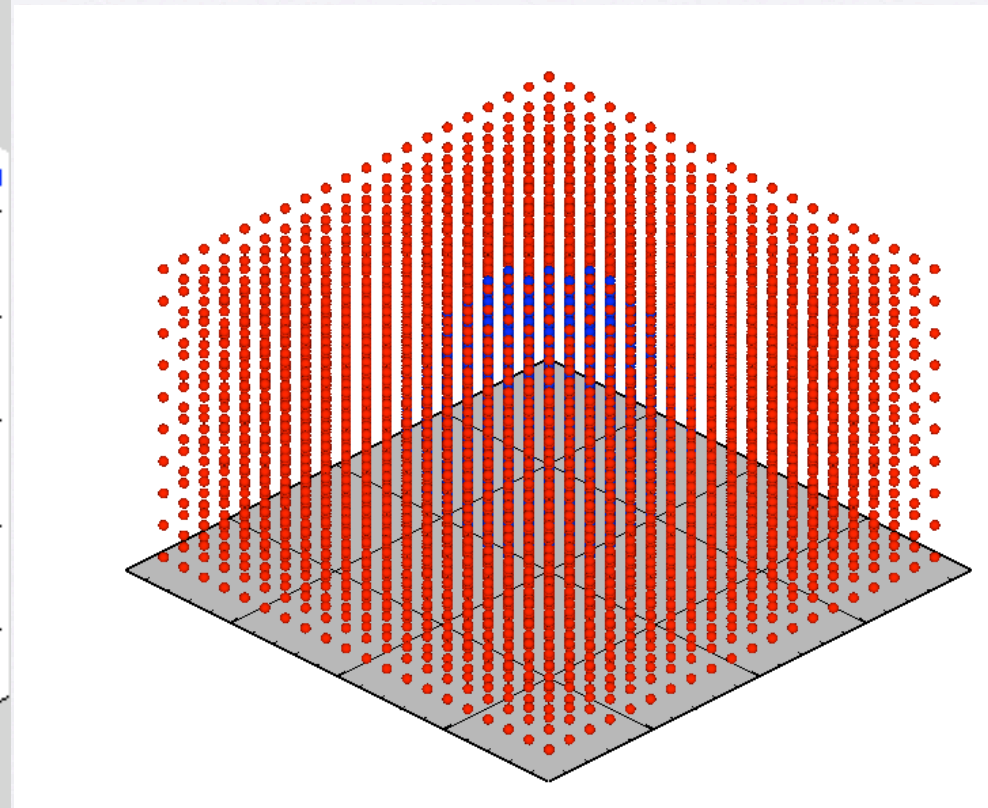

Particle trajectories

3D LCS

- Mediterranean Salt Lenses in the Atlantic

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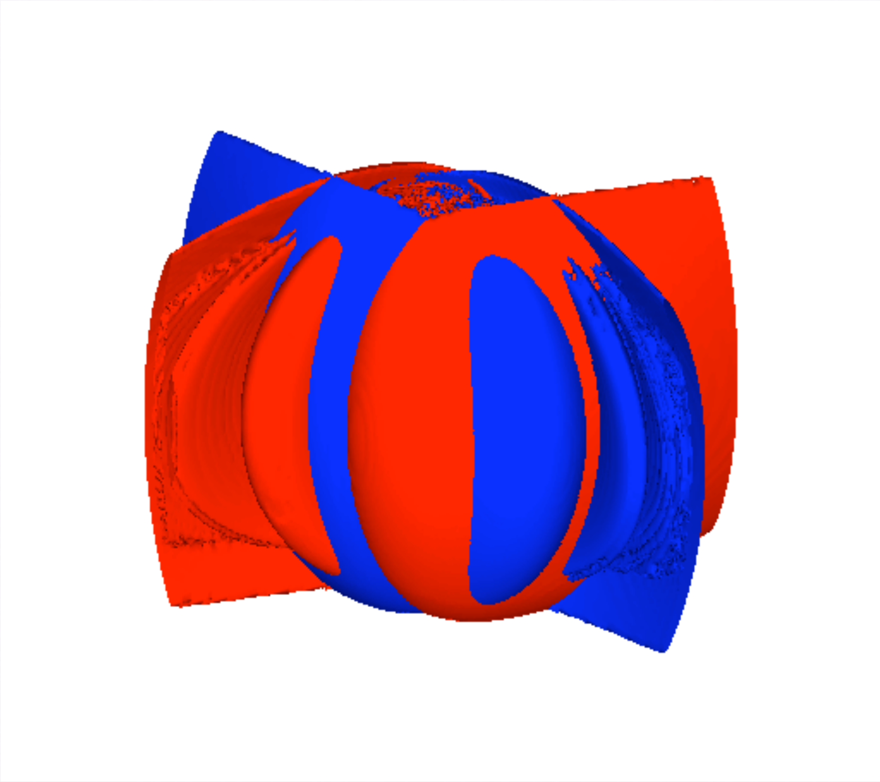

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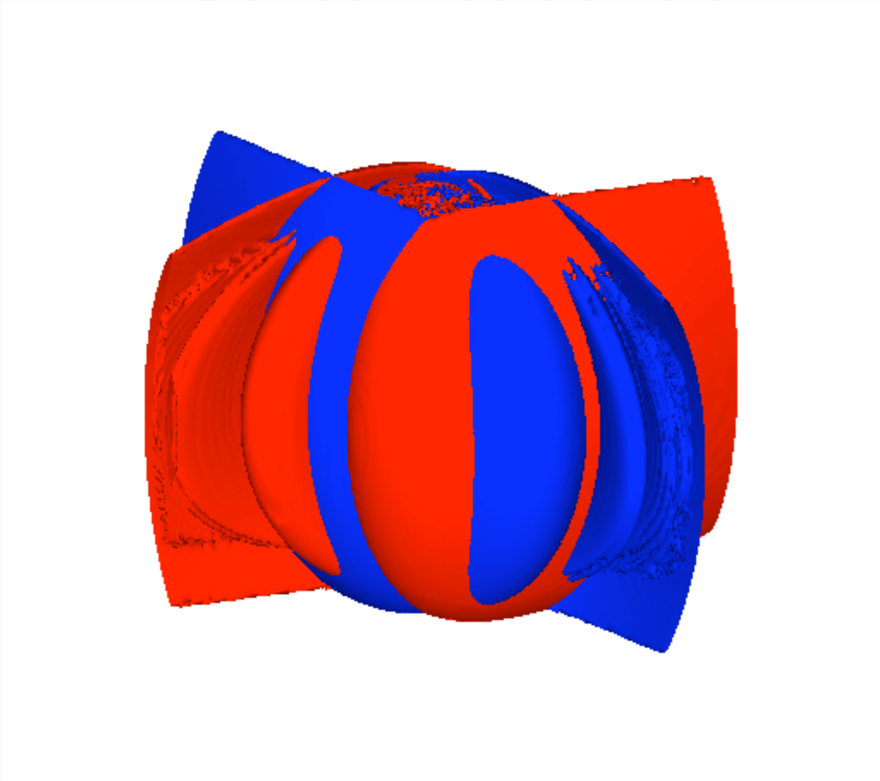

Side view

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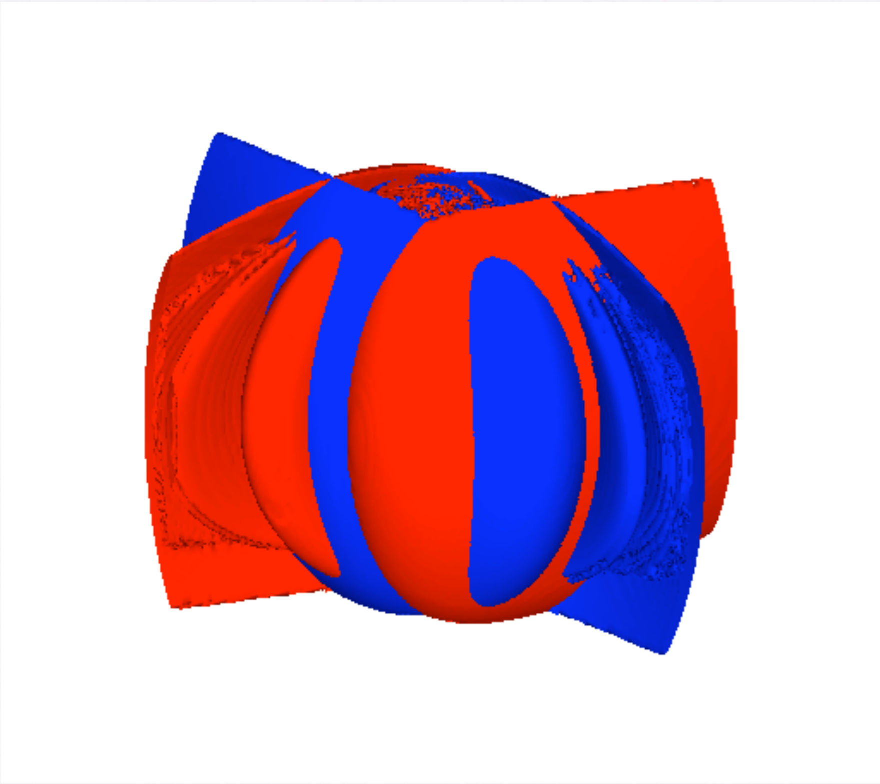

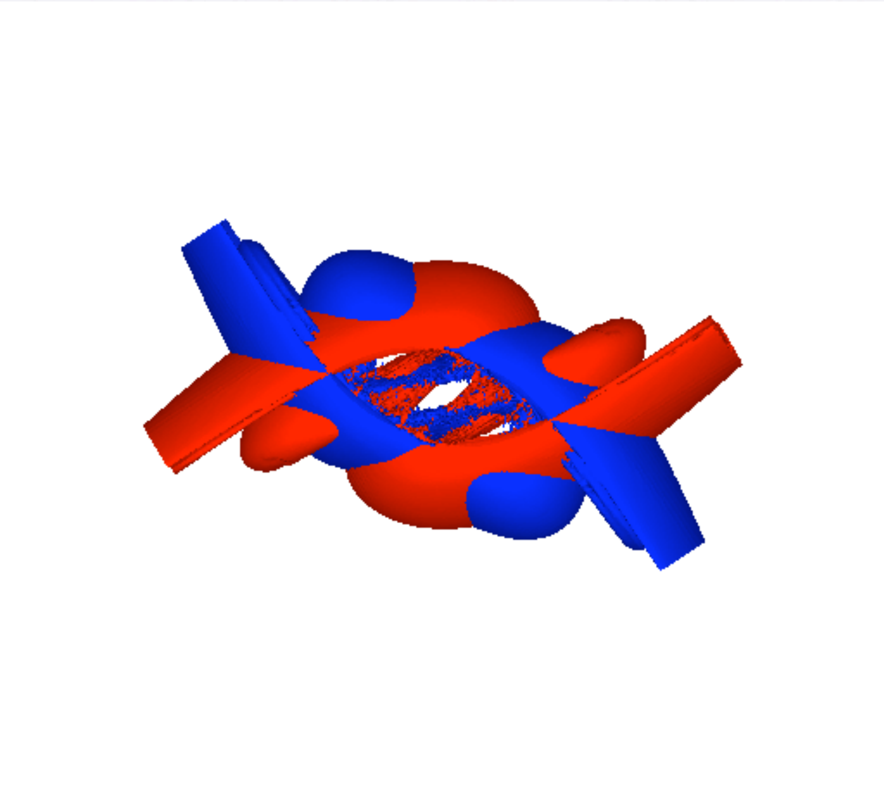

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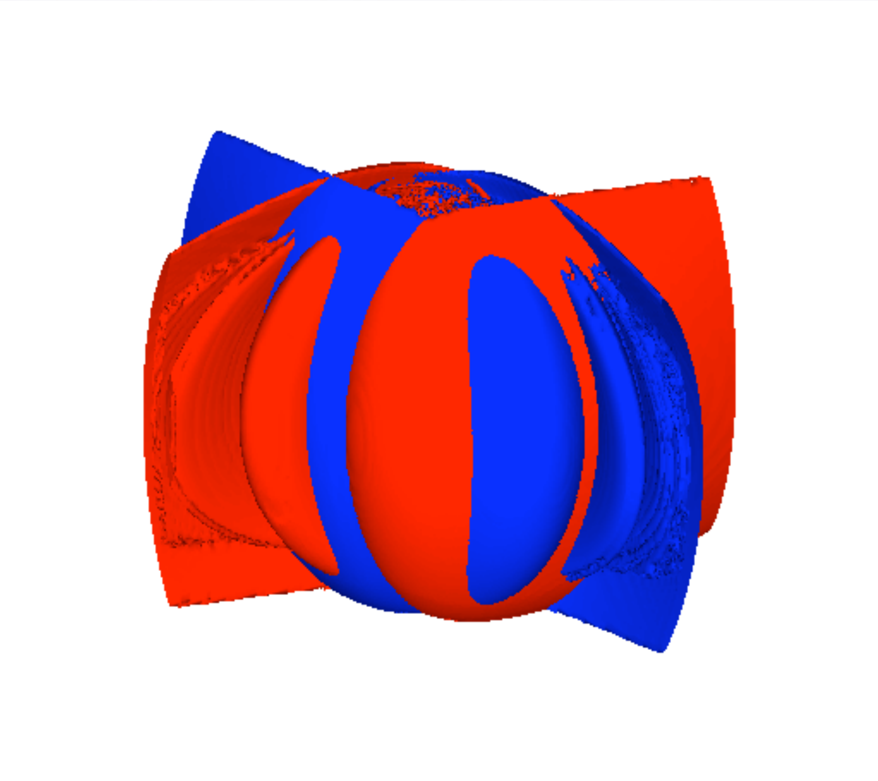

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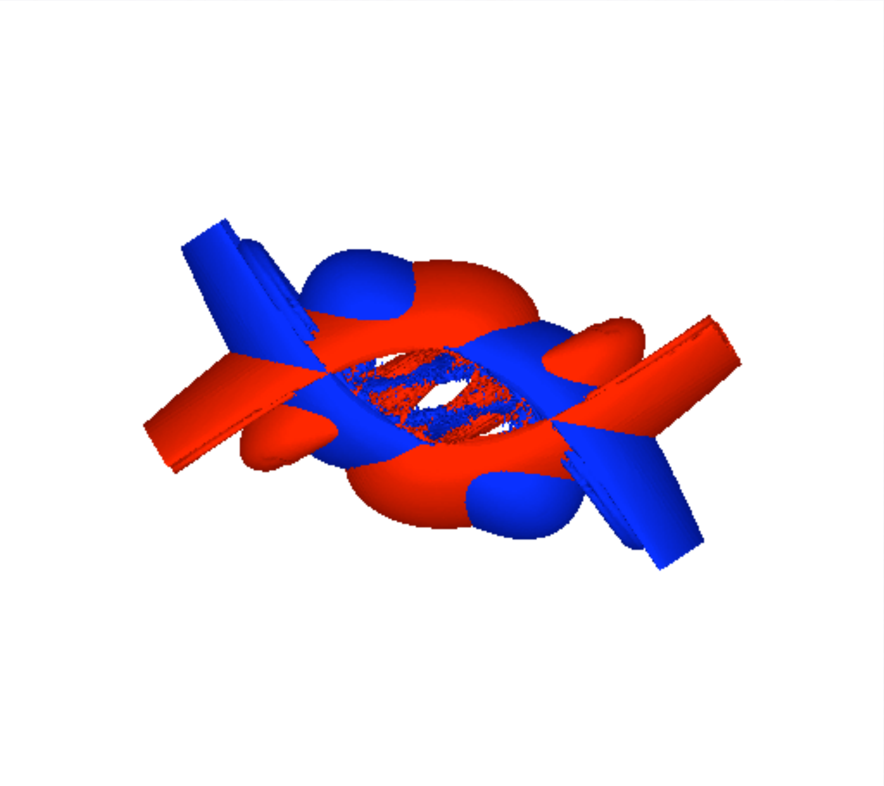
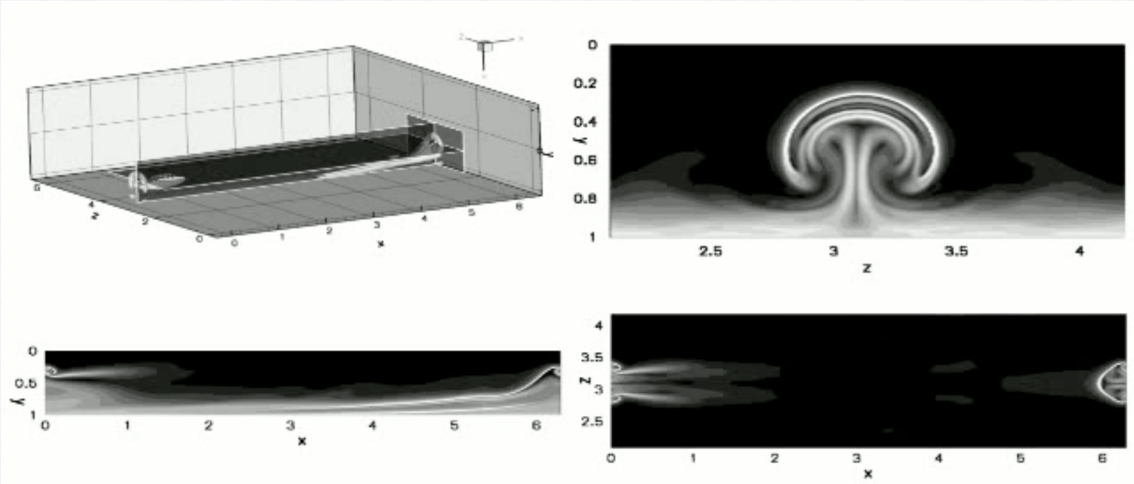


Top view

Hairpin vortices in near wall turbulent flow

- Melissa Green,
  George Haller,
  Clancy Rowley

Hairpin vortices in near wall turbulent flow



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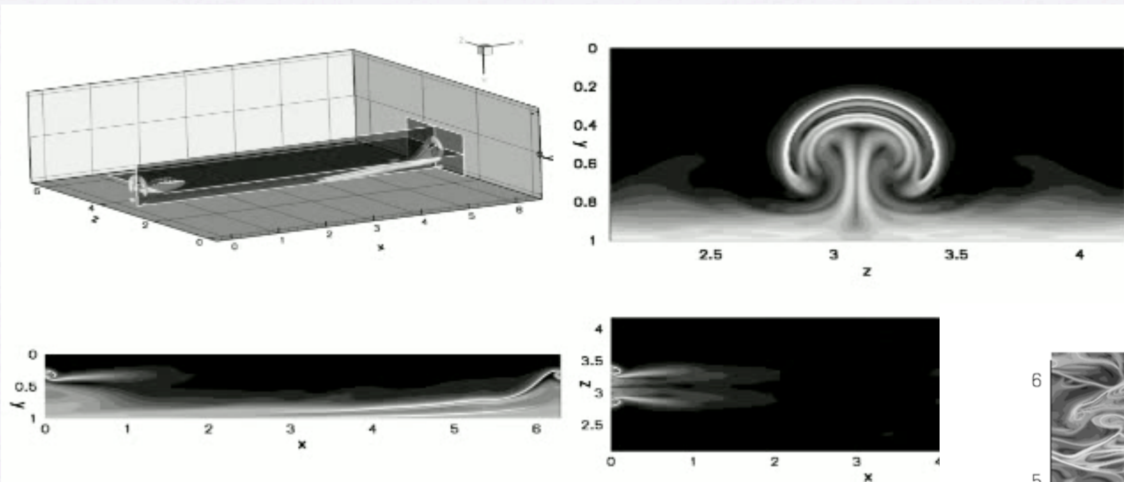

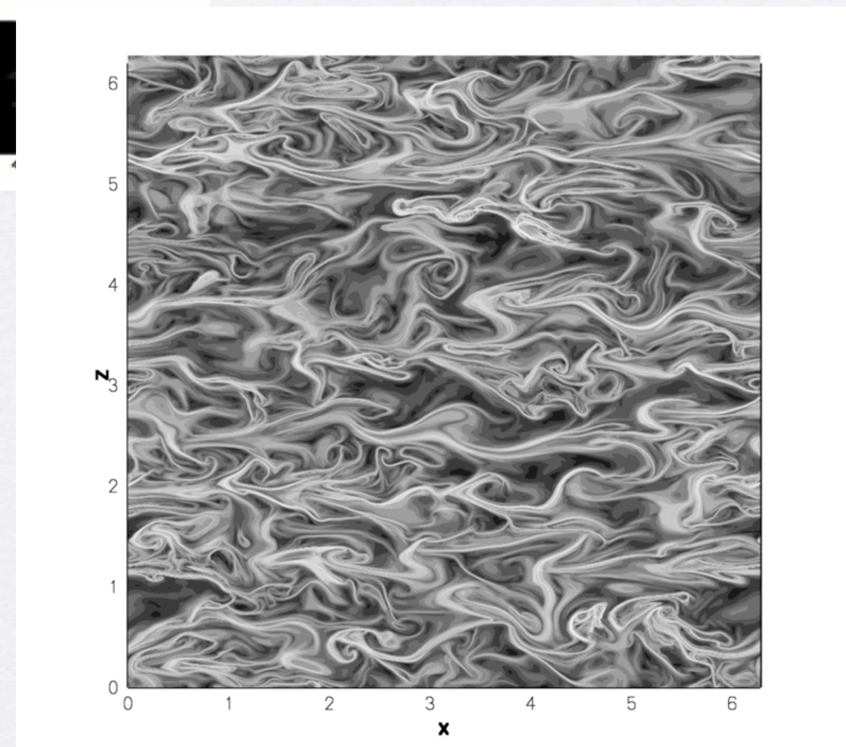

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A Word on Computing LCS

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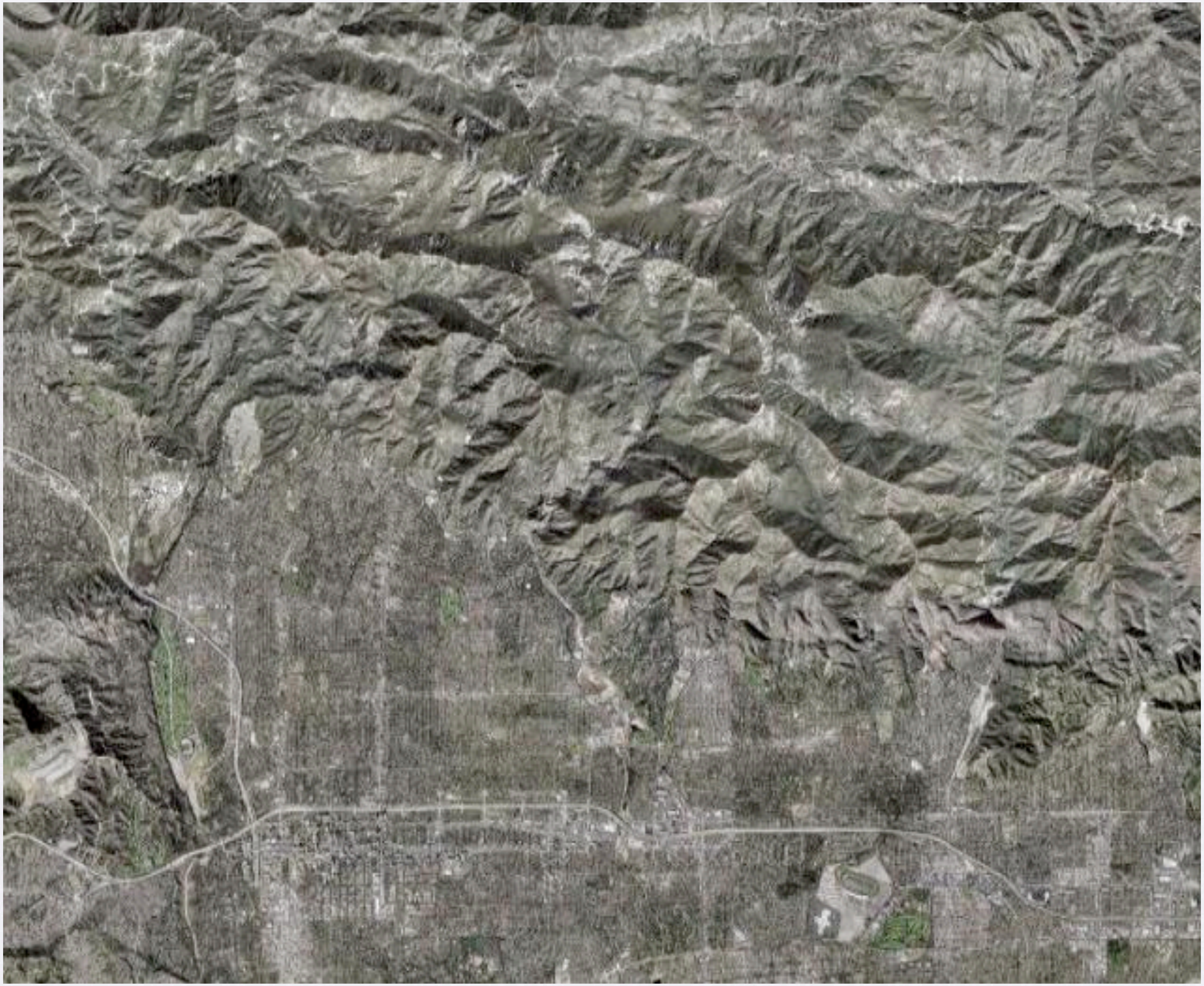
- Run time backwards for the *attracting LCS*
- Computations in 2d can be done on a laptop, but in 3d it requires a hefty computer.

Ridges can be complicated



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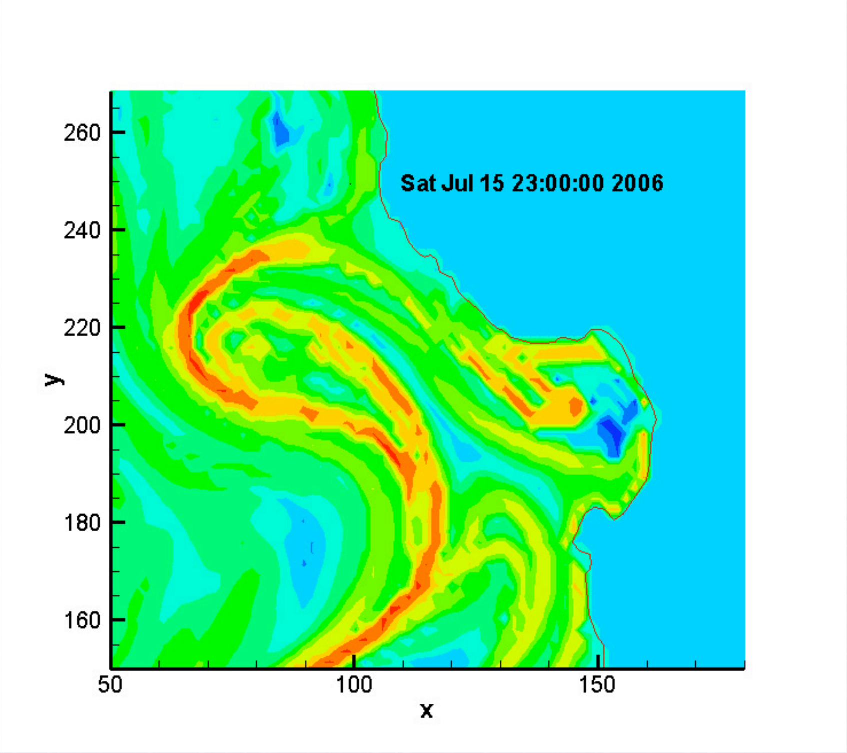

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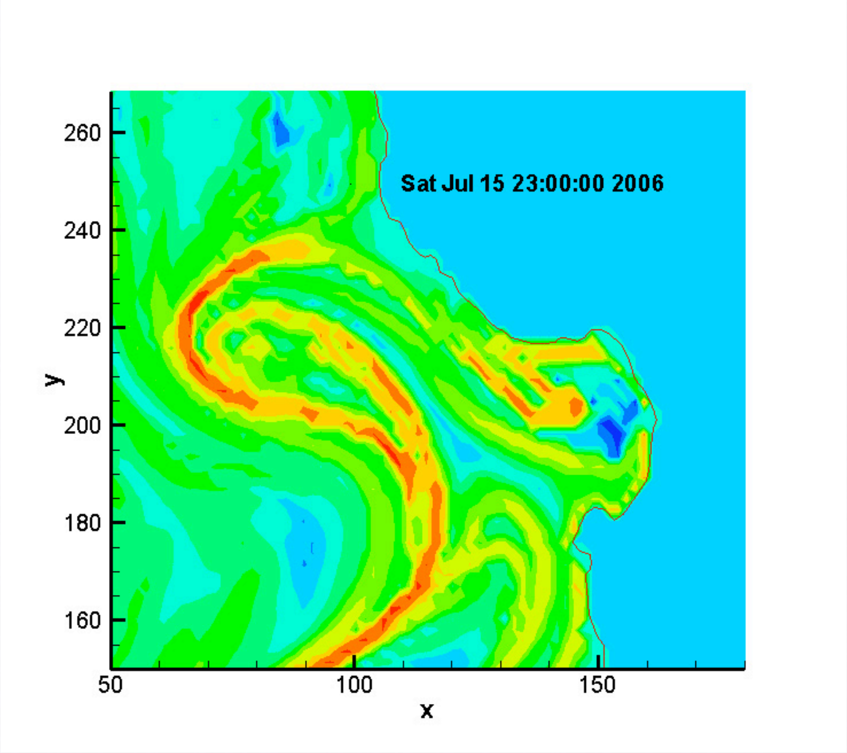

Attracting LCS low res

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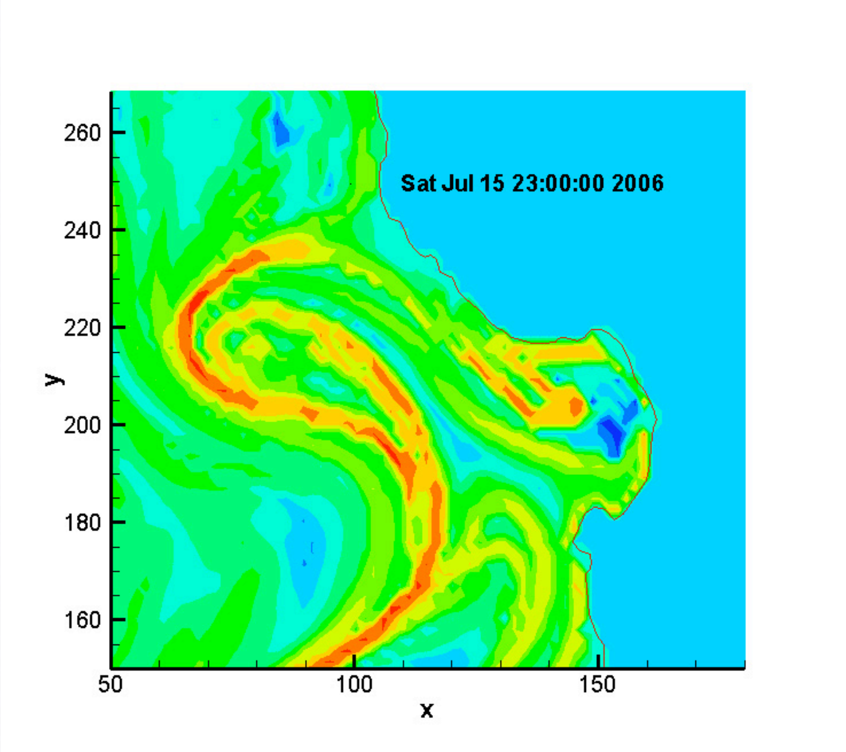

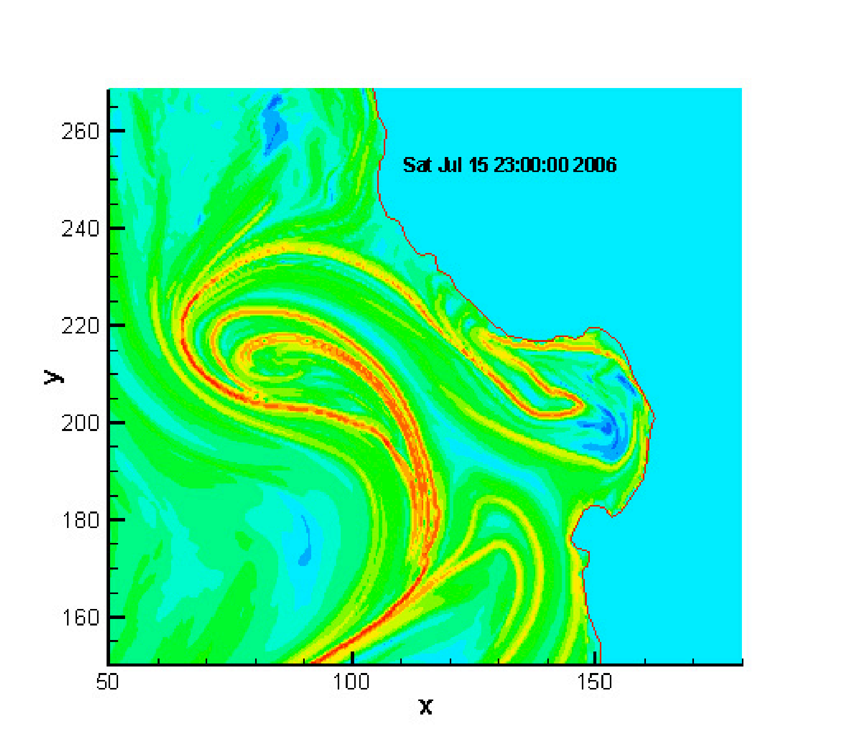

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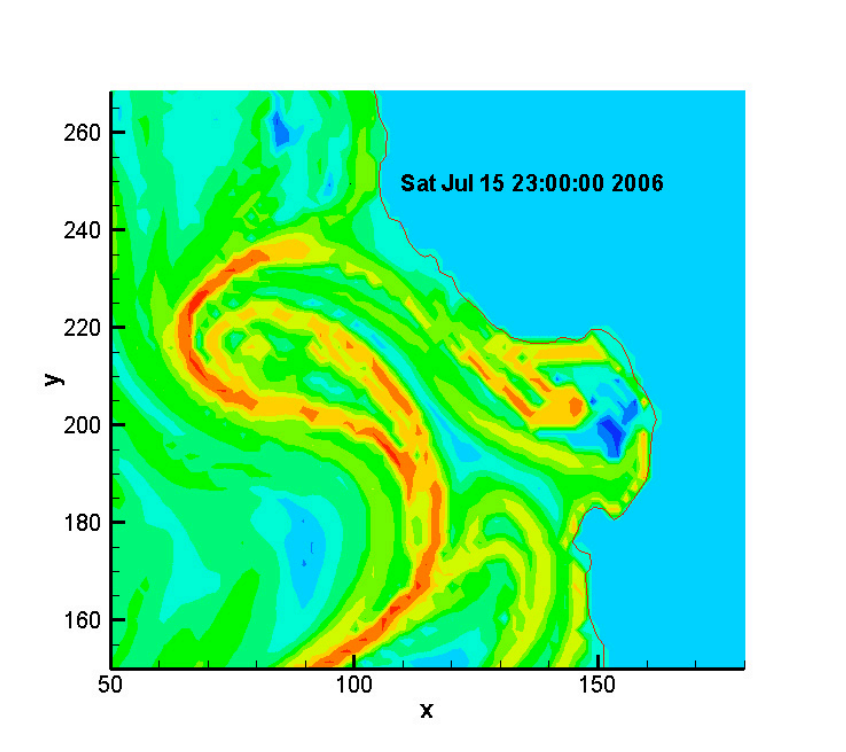
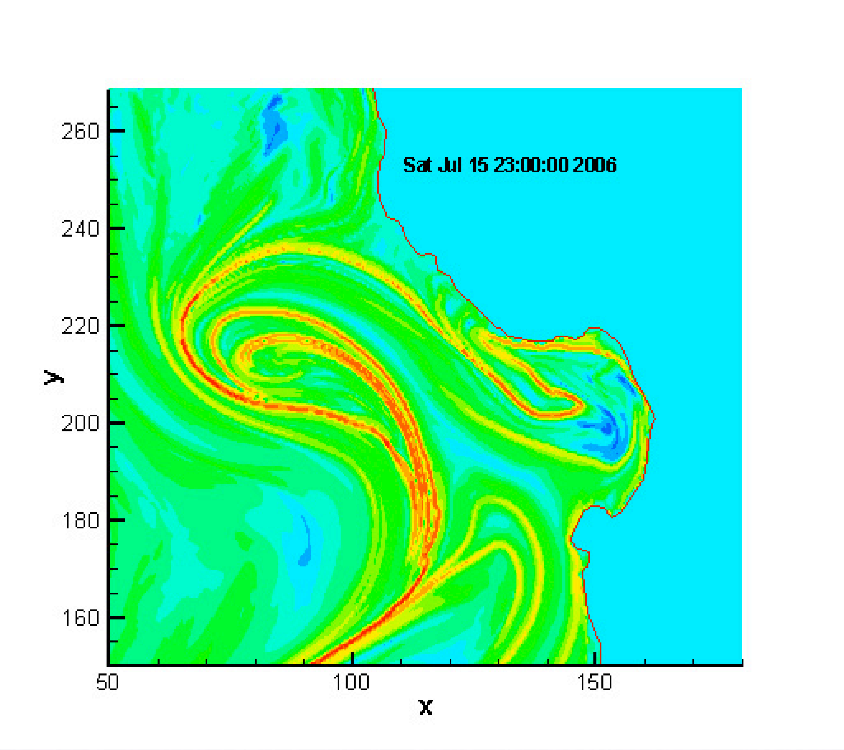


Attracting LCS low res



Attracting LCS high res

Cardiovascular Applications

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- Emerging field of virtual surgery-Charlie Taylor, Stanford

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