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Perfect multi-contact collisions

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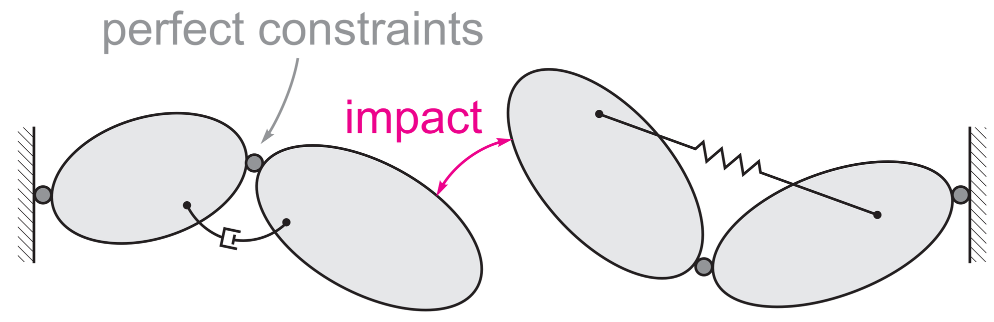
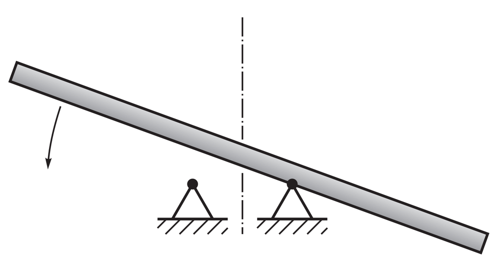
Motivation

Description and parameterization of the set-valued impact law of a multi-contact-colliding finite DOF system

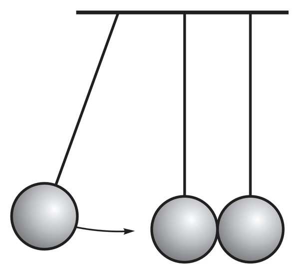


Restrictions: time-independent, perfect constraints and contacts
no excitation, no bilateral contacts

•Examples



rocking rod



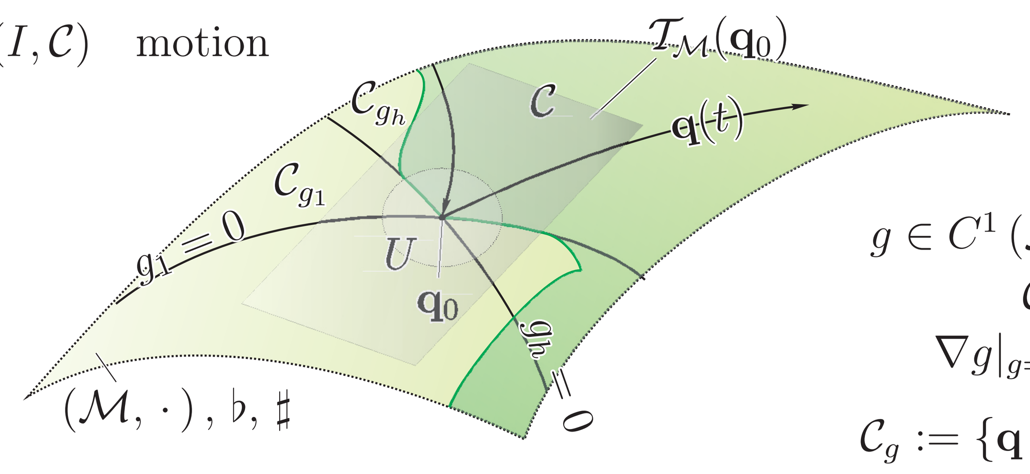
Newton's cradle

The set-valued impact law

•kinematics: displacements and contacts

$\mathcal{C} \subset \mathcal{M}$ admissible displacements $\qquad \mathbf{q}_0 = \mathbf{q}(t_0) \in \mathcal{C}$

$\mathbf{q}(t) \in \mathrm{AC}\,(I, \mathcal{C})$ motion



$$\left.\begin{array}{r} g \in C^1(\mathcal{M}, \mathbb{R}) \\ \mathcal{C}_g \neq \emptyset \\ \nabla g|_{g=0} \neq 0 \end{array}\right\} \text{ contact}$$

$$\mathcal{C}_g := \{\mathbf{q} \in \mathcal{M} \mid g(\mathbf{q}) \geq 0\}$$

$$\nabla = \sharp \mathbf{d}$$

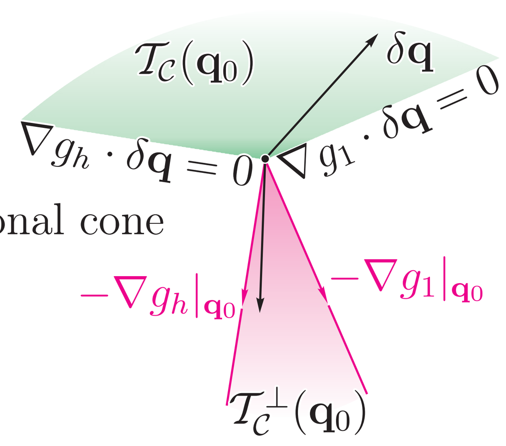
$$\left.\begin{array}{l} \mathcal{H} = \{g_1, \cdots, g_h\}, \quad 0 < h < \infty, \ \mathbf{q}_0 \in U \subset \mathcal{M} \text{ open} \\ \mathcal{C} \cap U = \mathcal{C}_{g_1} \cap \cdots \cap \mathcal{C}_{g_h} \cap U \text{ and } g(\mathbf{q}_0) = 0 \ \forall g \in \mathcal{H} \end{array}\right\} \text{ multi-contact collision}$$

•kinematics: velocities, tangent space and cones

$\mathcal{T}_{\mathcal{C}}(\mathbf{q}_0) = \{\delta\mathbf{q} \in \mathcal{T}_{\mathcal{M}}(\mathbf{q}_0) \mid 0 \leq \delta g = \nabla g|_{\mathbf{q}_0} \cdot \delta\mathbf{q} \quad \forall g \in \mathcal{H}\}$ Tangent cone

$\dot{\mathbf{q}}^+ \in \mathcal{T}_{\mathcal{C}}(\mathbf{q}_0)$ kinematically admissible right-velocities

$\mathcal{T}_{\mathcal{C}}^{\perp}(\mathbf{q}_0) = \left\{-\sum_{g \in \mathcal{H}} \nabla g(\mathbf{q}_0)\,\Lambda_g \mid \Lambda_g \geq 0 \ \forall g \in \mathcal{H}\right\}$ Orthogonal cone



•dynamics $\quad T(\mathbf{u}) := \frac{1}{2}|\mathbf{u}|^2 := \frac{1}{2}\mathbf{u} \cdot \mathbf{u}$ kinetic energy

$\flat\,(\dot{\mathbf{q}}^+ - \dot{\mathbf{q}}^-) = \mathbf{dR}_{,\delta_t} + \mathbf{dP}_{,\delta_t}$ impact equation

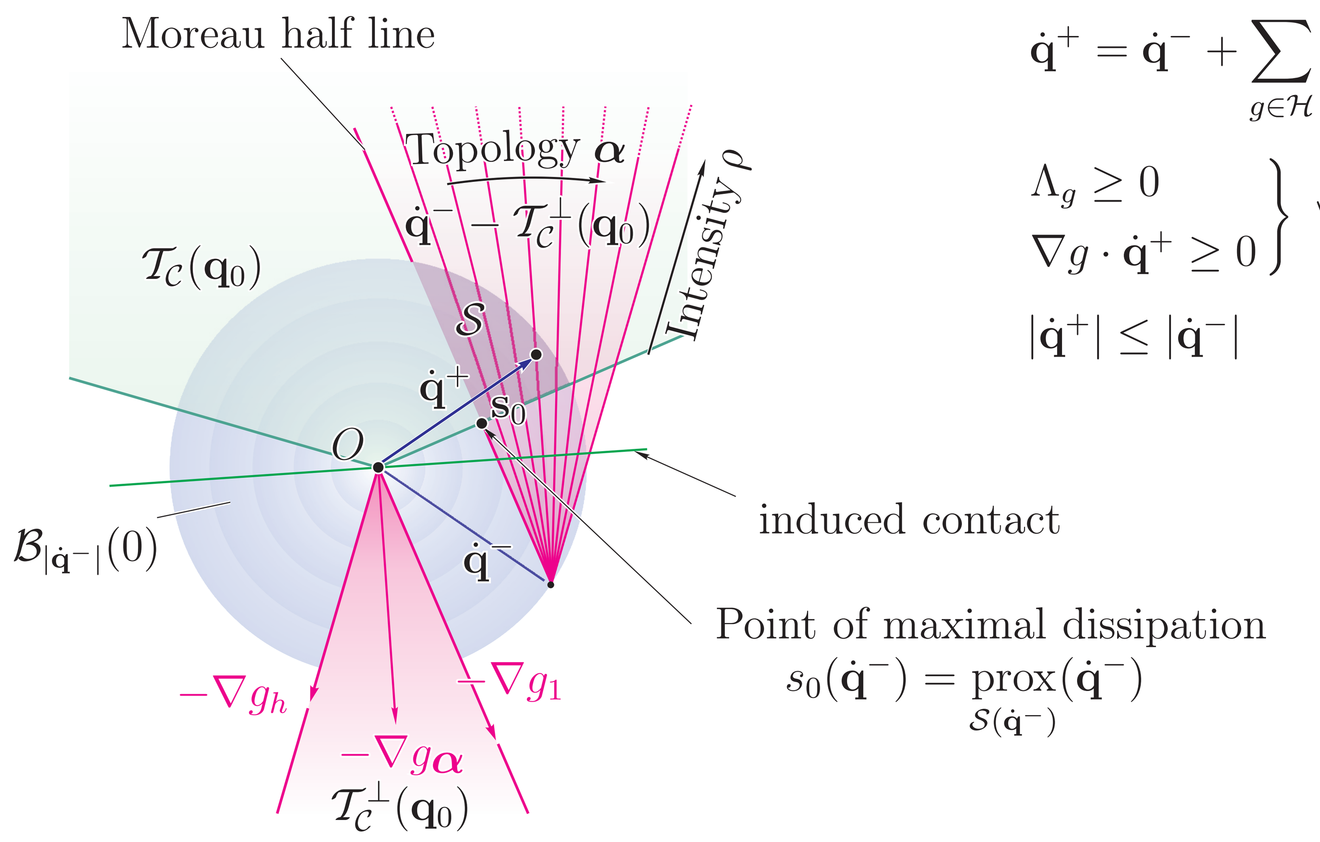
$\mathbf{dP}_{,\delta_t} = 0$ and $T^+ \leq T^-$ no excitation

$-\mathbf{dR}_{,\tau} \cdot \mathcal{T}_{\mathcal{C}}(\mathbf{q}_0) \leq 0$ perfect constraints $\qquad \tau$: positive measure

$\dot{\mathbf{q}}^+ - \dot{\mathbf{q}}^- \in -\mathcal{T}_{\mathcal{C}}^{\perp}(\mathbf{q}_0)$ kinetically admissible right-velocities

$|\dot{\mathbf{q}}^+| \leq |\dot{\mathbf{q}}^-|$ energetically admissible right-velocities

•set of possible right-velocities $\quad \mathcal{S}(\dot{\mathbf{q}}^-) = \mathcal{T}_{\mathcal{C}}(\mathbf{q}_0) \cap (\dot{\mathbf{q}}^- - \mathcal{T}_{\mathcal{C}}^{\perp}(\mathbf{q}_0)) \cap \mathcal{B}_{|\dot{\mathbf{q}}^-|}(0)$



$$\dot{\mathbf{q}}^+ = \dot{\mathbf{q}}^- + \sum_{g \in \mathcal{H}} \Lambda_g \nabla g$$

$$\left.\begin{array}{l} \Lambda_g \geq 0 \\ \nabla g \cdot \dot{\mathbf{q}}^+ \geq 0 \end{array}\right\} \forall g \in \mathcal{H}$$

$$|\dot{\mathbf{q}}^+| \leq |\dot{\mathbf{q}}^-|$$

$$\mathcal{B}_r(\mathbf{v}) := \{\mathbf{u} \in \mathcal{T}_{\mathcal{M}}(\mathbf{q}_0) \mid |\mathbf{u} - \mathbf{v}| \leq r\}$$

Impact laws

$\mathbf{s}\colon\ \dot{\mathbf{q}}^- \mapsto \dot{\mathbf{q}}^+ \in \mathcal{S}(\dot{\mathbf{q}}^-)$

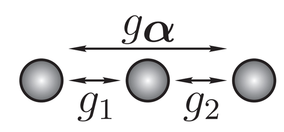
•characteristic entities and effects

Intensity $\rho = |\dot{\mathbf{q}}^+ - \dot{\mathbf{q}}^-|$

Topology (Topologies): $\quad \boldsymbol{\alpha} = \dfrac{\boldsymbol{\Lambda}}{|\boldsymbol{\Lambda}|_1} \qquad \boldsymbol{\Lambda} = (\Lambda_{g_1}, \cdots, \Lambda_{g_h})^{\top}, \quad |\boldsymbol{\Lambda}|_1 = \Lambda_{g_1} + \cdots + \Lambda_{g_2}$

•Induced contacts and related one-point collisions

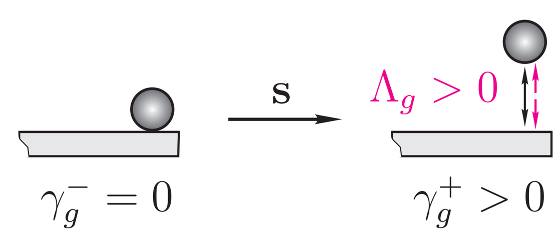
$$\left.\begin{array}{ll} g_{\boldsymbol{\alpha}} = \sum_{g \in \mathcal{H}} \alpha_g\, g & \text{induced contact} \\ & \boldsymbol{\alpha}\text{: topology} \\ \Lambda_{\boldsymbol{\alpha}} = |\boldsymbol{\Lambda}|_1 & \text{induced intensity} \end{array}\right\} \begin{array}{c} \dot{\mathbf{q}}^+ = \dot{\mathbf{q}}^- + \Lambda_{\boldsymbol{\alpha}} \nabla g_{\boldsymbol{\alpha}} \\ \text{one-point collision} \end{array}$$



$\boldsymbol{\alpha} = \left(\frac{1}{2}, \frac{1}{2}\right)^{\top}$

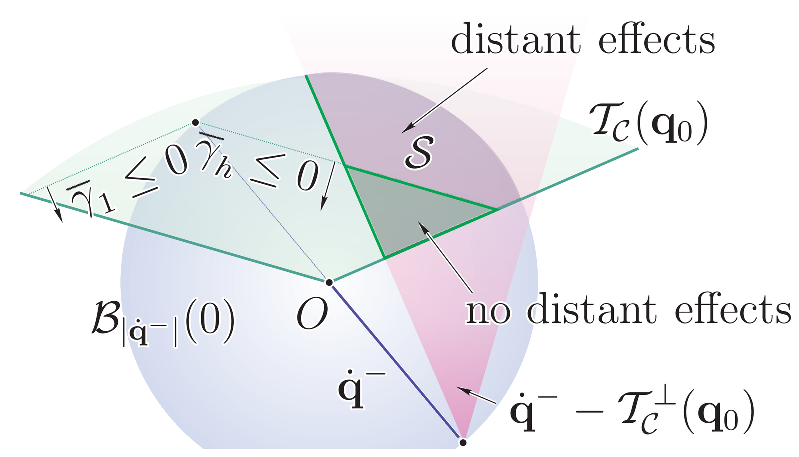
•Distant effect

$\exists g \in \mathcal{H} \ \exists \boldsymbol{\Lambda}\colon \quad \Lambda_g \overline{\gamma}_g > 0$ antidissipative contact



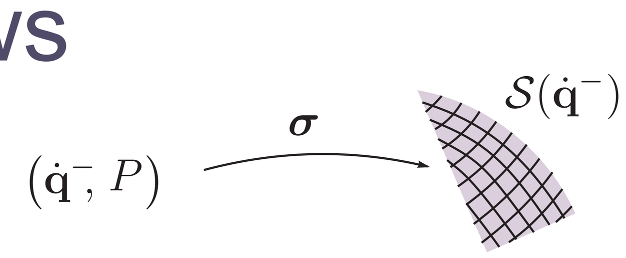
$\overline{\gamma}_g = \frac{1}{2}(\gamma_g^- + \gamma_g^+)$

$\gamma_g^{\pm} = \nabla g \cdot \dot{\mathbf{q}}^{\pm}$



Parametrized impact laws

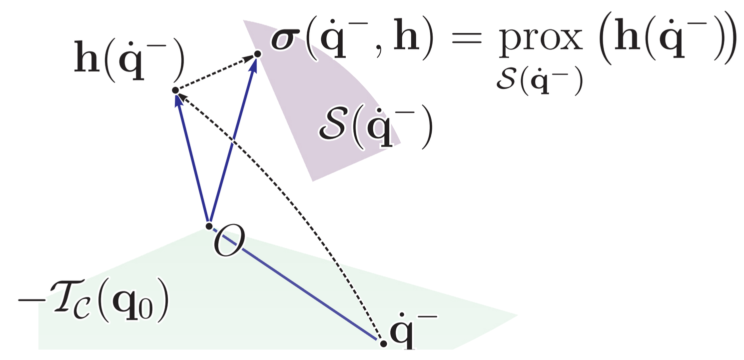
$$\begin{array}{l} \boldsymbol{\sigma} : -\mathcal{T}_{\mathcal{C}}(\mathbf{q}_0) \times P \to \mathcal{T}_{\mathcal{M}}(\mathbf{q}_0) \\ \qquad (\dot{\mathbf{q}}^-, \mathbf{p}) \mapsto \dot{\mathbf{q}}^+ \in \mathcal{S}(\dot{\mathbf{q}}^-) \end{array}$$



•Canonical paramized impact law

$P = \{\mathbf{h} : -\mathcal{T}_{\mathcal{C}}(\mathbf{q}_0) \to \mathcal{T}_{\mathcal{M}}(\mathbf{q}_0)\}$

$\boldsymbol{\sigma}(\dot{\mathbf{q}}^-, \mathbf{h}) = \mathrm{prox}_{\mathcal{S}(\dot{\mathbf{q}}^-)}(\mathbf{h}(\dot{\mathbf{q}}^-))$

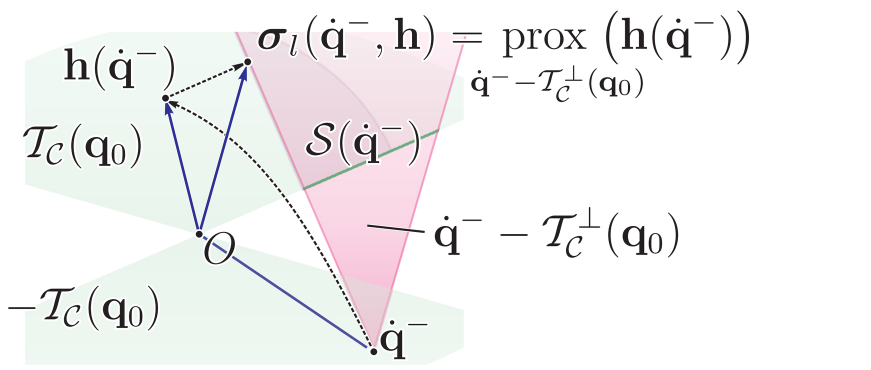


•Linear parametrized impact law

$P_l = \{\mathbf{h} : -\mathcal{T}_{\mathcal{C}}(\mathbf{q}_0) \to \mathcal{T}_{\mathcal{C}}(\mathbf{q}_0) \text{ linear}\}$

$\boldsymbol{\sigma}_l(\dot{\mathbf{q}}^-, \mathbf{h}) = \mathrm{prox}_{\dot{\mathbf{q}}^- - \mathcal{T}_{\mathcal{C}}^{\perp}(\mathbf{q}_0)}(\mathbf{h}(\dot{\mathbf{q}}^-))$

$\Updownarrow$



$$\left.\begin{array}{ll} 0 \leq \Lambda_g \perp (\gamma_g^+ - \gamma_g^h) \geq 0 & \forall g \in \mathcal{H} \\ \dot{\mathbf{q}}^+ = \dot{\mathbf{q}}^- + \sum_{g \in \mathcal{H}} \Lambda_g \nabla g & \begin{array}{l} \gamma_g^{\pm} = \nabla g \cdot \dot{\mathbf{q}}^{\pm} \\ \gamma_g^h = -\sum_{\tilde{g} \in \mathcal{H}} \varepsilon_{g\tilde{g}}\, \gamma_{\tilde{g}}^- \end{array} \end{array}\right\}$$

$\boldsymbol{\epsilon} = (\varepsilon_{g\tilde{g}}) \in M_{h,h}(\mathbb{R})$ Frémond Matrix

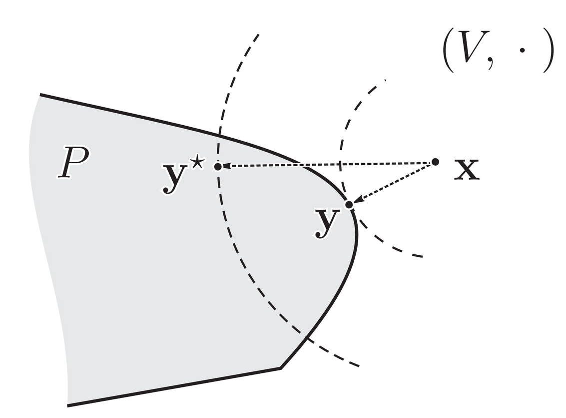
Special cases

$\boldsymbol{\epsilon} = 0$	Point of maximal dissipation
$\boldsymbol{\epsilon} = \varepsilon \mathbb{1}, \ \varepsilon \in [0,1]$	Moreau half line
$\boldsymbol{\epsilon} = \mathrm{diag}\,(\varepsilon_1, \cdots, \varepsilon_h), \ \varepsilon_i \in [0,1]$	Extended Newton Impact law

Outlook

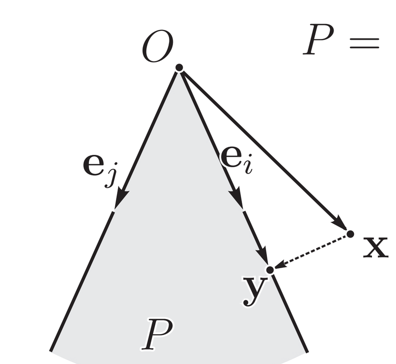
- Implementation of measured/simulated impact laws
- time-dependent constraints
- excitation, C^0-constraints
- friction, non-perfect collisions

proximal point in a closed convex set



$\mathrm{prox}_P(\mathbf{x}) = \mathbf{y} \in P\colon$

$|\mathbf{x} - \mathbf{y}| = \min |\mathbf{x} - P|$



$P = \left\{\sum_i \alpha_i \mathbf{e}_i \mid \alpha_i \geq 0 \, \forall i\right\}$

$\mathbf{y} = \mathrm{prox}_P(\mathbf{x})$

$\Updownarrow$

$$\left.\begin{array}{l} \underbrace{0 \leq (y_i - x_i), \ (y_i - x_i)\,\alpha_i = 0, \ \alpha_i \geq 0}_{0 \leq (y_i - x_i) \perp \alpha_i \geq 0} \\ \mathbf{y} = \sum_j \alpha_j \mathbf{e}_j, \quad x_i = \mathbf{e}_i \cdot \mathbf{x}, \ y_i = \mathbf{e}_i \cdot \mathbf{y} \end{array}\right\} \forall i$$