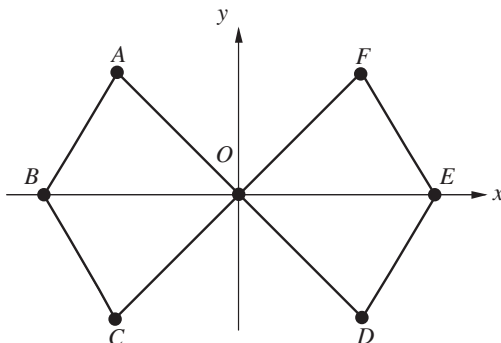


Example 1. Let \mathbf{F} be a conservative vector field in the plane. In the Figure, suppose that in the integral of \mathbf{F} along AOF is 3, along OF is 2, and along AB is -5 . Compute the integral of \mathbf{F} along the path BOEF.



Solution. Since \mathbf{F} is conservative, the line integral depends only on the endpoints. Thus, we can take the path BAOF instead. The integral along BAOF = (integral along BA) + (integral along AOF) = $-(-5) + 3 = 8$. \diamond

Example 2. Suppose that the kinetic energy of a particle that moves in a circular path under the influence a force field \mathbf{F} according to Newton's second law (that is, $\mathbf{F} = m\mathbf{a}$) increases after the particle makes one circuit. Can the force field governing the particle's motion be conservative?

Solution. No. If the force field $\mathbf{F}(\mathbf{r})$ is conservative, then

$$\int_C \mathbf{F} \cdot d\mathbf{s} = (\text{Energy at start}) - (\text{Energy at finish}) = 0,$$

which is a contradiction.