Mathematics 1c: Homework Set 7

Due: Monday, May 24 at 10am.

1. (10 Points) **Section 7.3, Exercise 6**. *Find an expression for a unit vector normal to the surface*

$$x = 3\cos\theta\sin\phi, \quad y = 2\sin\theta\sin\phi, \quad z = \cos\phi$$

for θ in $[0, 2\pi]$ and ϕ in $[0, \pi]$.

2. (15 Points) **Section 7.3, Exercise 15**

   (a) *Find a parameterization for the hyperboloid $x^2 + y^2 - z^2 = 25$.*

   (b) *Find an expression for a unit normal to this surface.*

   (c) *Find an equation for the plane tangent to the surface at $(x_0, y_0, 0)$, where $x_0^2 + y_0^2 = 25$.*

   (d) *Show that the pair of lines $(x_0, y_0, 0) + t(-y_0, x_0, 5)$ and $(x_0, y_0, 0) + t(y_0, -x_0, 5)$ lie in the surface and as well as in the tangent plane found in part* (c)*.*

3. (10 Points) **Section 7.4, Exercise 6**. *Find the area of the portion of the unit sphere that is cut out by the cone*

$$z \geq \sqrt{x^2 + y^2}.$$

4. (10 Points) **Section 7.5, Exercise 2**. Evaluate

$$\iint_S xyz\, dS$$

where S is the triangle with vertices $(1, 0, 0), (0, 2, 0)$ and $(0, 1, 1)$.

5. (10 Points) **Section 7.6, Exercise 7**. *Calculate the integral* $\iint_S \mathbf{F} \cdot d\mathbf{S}$*, where S is the surface of the half-ball $x^2 + y^2 + z^2 \leq 1, z \geq 0$, and where*

$$\mathbf{F} = (x + 3y^5)\mathbf{i} + (y + 10xz)\mathbf{j} + (z - xy)\mathbf{k}.$$

6. (10 Points) **Section 7.6, Exercise 15.** *Let the velocity field of a fluid be given by $\mathbf{v} = \mathbf{i} + x\mathbf{j} + z\mathbf{k}$ in meters/second. How many cubic meters of fluid per second are crossing the surface $x^2 + y^2 + z^2 = 1, z \geq 0$? (Distances are in meters.)*

7. (15 Points) **Section 7.6, Exercise 18**. *If S is the upper hemisphere*

$$\{(x, y, z) \mid x^2 + y^2 + z^2 = 1, z \geq 0\}$$

oriented by the normal pointing out of the sphere, compute

$$\iint_S \mathbf{F} \cdot d\mathbf{S}$$

for parts (a) *and* (b)*.*

(a) $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j}$

(b) $\mathbf{F}(x, y, z) = y\mathbf{i} + x\mathbf{j}$

(c) *for each of the vector fields above, compute*

$$\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} \quad \textit{and} \quad \int_C \mathbf{F} \cdot d\mathbf{S},$$

where C is the unit circle in the xy plane traversed in the counterclockwise direction (as viewed from the positive z axis). (Notice that C is the boundary of S. The phenomenon illustrated here will be studied more thoroughly in the next chapter, using Stokes' theorem.)