Mathematics 1c: Homework Set 6
Due: Monday, May 17 at 10am.

1. (10 Points) Section 6.1, Exercise 6 Let $D^*$ be the parallelogram with vertices 

$$(-1,3), \ (0,0), \ (2,-1) \ \text{and} \ (1,2)$$

and $D$ be the rectangle $D = [0,1] \times [0,1]$. Find a transformation $T$ such that $D$ is the image set of $D^*$ under $T$.

2. (10 Points) Section 6.2, Exercise 6 Define $T(u,v) = (u^2 - v^2, 2uv)$. Let $D^*$ be the set of $(u,v)$ with $u^2 + v^2 \leq 1$, $u \geq 0$, $v \geq 0$. Find $T(D^*) = D$ and evaluate

$$\iint_D dx \, dy.$$

3. (10 Points) Section 6.2, Exercise 8 Calculate

$$\iint_R \frac{dx \, dy}{x + y},$$

where $R$ is the region bounded by $x = 0$, $y = 0$, $x + y = 1$, and $x + y = 4$ by using the mapping $T(u,v) = (u - uv, uv)$.

4. (10 Points) Section 6.3, Exercise 4 Find the center of mass of the region between $y = 0$ and $y = x^2$, where $0 \leq x \leq 1/2$.

5. (10 Points) Section 6.4, Exercise 8 Show that the integral

$$\int_0^1 \int_0^a \frac{x}{\sqrt{a^2 - y^2}} dy \, dx$$

exists, and compute its value. (You may assume that $a$ is a positive constant).

6. (10 Points) Review Exercise 4b for Chapter 6 Perform a change of variables to cylindrical coordinates for

$$\int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} xyz \, dz \, dx \, dy.$$

7. (10 Points) Section 7.1, Exercise 4(a) Evaluate the path integral of $f(x,y,z) = x \cos z$ along the path $c : t \mapsto t \mathbf{i} + t^2 \mathbf{j}$, $t \in [0,1]$.

8. (10 Points) Section 7.2, Exercise 2 Evaluate each of the following integrals:

(a) $\int_c x \, dy - y \, dx$, \quad $c(t) = (\cos t, \sin t)$, \quad $0 \leq t \leq 2\pi$

(b) $\int_c x \, dx + y \, dy$, \quad $c(t) = (\cos \pi t, \sin \pi t)$, \quad $0 \leq t \leq 2$
(c) $\int_c yz \, dx + xz \, dy + xy \, dz$, where $c$ consists of straight-line segments joining $(1,0,0)$ to $(0,1,0)$ to $(0,0,1)$

(d) $\int_c x^2 \, dx - xy \, dy + dz$, where $c$ is the parabola $z = x^2, y = 0$ from $(-1,0,1)$ to $(1,0,1)$. 