Mathematics 1c: Homework Set 6

Due: Monday, May 17 at 10am.

1. (10 Points) **Section 6.1, Exercise 6** *Let D^* be the parallelogram with vertices*

$$(-1,3), \quad (0,0), \quad (2,-1) \quad \textit{and} \quad (1,2)$$

and D be the rectangle $D = [0,1] \times [0,1]$. Find a transformation T such that D is the image set of D^* under T.

2. (10 Points) **Section 6.2, Exercise 6** *Define $T(u,v) = (u^2 - v^2, 2uv)$. Let D^* be the set of (u,v) with $u^2 + v^2 \leq 1, u \geq 0, v \geq 0$. Find $T(D^*) = D$ and evaluate*

$$\iint_D dx\, dy.$$

3. (10 Points) **Section 6.2, Exercise 8** *Calculate*

$$\iint_R \frac{dx\, dy}{x+y},$$

where R is the region bounded by $x = 0, y = 0, x + y = 1$, and $x + y = 4$ by using the mapping $T(u,v) = (u - uv, uv)$.

4. (10 Points) **Section 6.3, Exercise 4** *Find the center of mass of the region between $y = 0$ and $y = x^2$, where $0 \leq x \leq 1/2$.*

5. (10 Points) **Section 6.4, Exercise 8** *Show that the integral*

$$\int_0^1 \int_0^a \frac{x}{\sqrt{a^2 - y^2}} dy\, dx$$

exists, and compute its value. (You may assume that a is a positive constant).

6. (10 Points) **Review Exercise 4b for Chaper 6** *Perform a change of variables to cylindrical coordinates for*

$$\int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} xyz\, dz\, dx\, dy.$$

7. (10 Points) **Section 7.1, Exercise 4(a)** *Evaluate the path integral of $f(x,y,z) = x \cos z$ along the path $\mathbf{c} : t \mapsto t\mathbf{i} + t^2\mathbf{j}, t \in [0,1]$.*

8. (10 Points) **Section 7.2, Exercise 2** *Evaluate each of the following integrals:*

   (a) $\displaystyle\int_{\mathbf{c}} x\, dy - y\, dx, \quad \mathbf{c}(t) = (\cos t, \sin t), \quad 0 \leq t \leq 2\pi$

   (b) $\displaystyle\int_{\mathbf{c}} x\, dx + y\, dy, \quad \mathbf{c}(t) = (\cos \pi t, \sin \pi t), \quad 0 \leq t \leq 2$

(c) $\int_{\mathbf{c}} yz\,dx + xz\,dy + xy\,dz,$ *where* $\mathbf{c}$ *consists of straight-line segments joining* $(1,0,0)$ *to* $(0,1,0)$ *to* $(0,0,1)$

(d) $\int_{\mathbf{c}} x^2\,dx - xy\,dy + dz$, *where* $\mathbf{c}$ *is the parabola* $z = x^2, y = 0$ *from* $(-1,0,1)$ *to* $(1,0,1)$.