Mathematics 1c: Homework Set 5

Due: Monday, May 10th at 10am.

1. (10 Points) **Section 5.1, Exercise 4** *Using Cavalieri's principle, compute the volume of the structure shown in Figure* 5.1.11 *of the textbook; each section is a rectangle of length* 5 *and width* 3.

2. (20 Points) **Section 5.2, Exercise 8** *Let f be continuous on $R = [a, b] \times [c, d]$. For $a < x < b$, and $c < y < d$, define*

$$F(x, y) = \int_a^x \int_c^y f(u, v) dv\, du.$$

Show that

$$\frac{\partial^2 F}{\partial x \partial y} = \frac{\partial^2 F}{\partial y \partial x} = f(x, y).$$

Use this example to discuss the relationship between Fubini's Theorem and the equality of mixed partial derivatives.

3. (10 Points)**Section 5.3, Exercise 2(a).** *Evaluate and sketch the region of integration*

$$\int_{-3}^{2} \int_0^{y^2} (x^2 + y) dx\, dy.$$

4. (10 Points)**Section 5.4, Exercise 2(a).** *Find*

$$\int_{-1}^{1} \int_{|y|}^{1} (x + y)^2 dx\, dy.$$

5. (10 Points)**Section 5.4, Exercise 8.** *Compute the double integral*

$$\iint_D f(x, y) dA$$

where

$$f(x, y) = y^2 \sqrt{x}$$

and D is the set of (x, y) where $x > 0, y > x^2$, and $y < 10 - x^2$.

6. (10 Points)**Section 5.5, Exercise 15.** *Evaluate*

$$\iiint_W (x^2 + y^2 + z^2) dx\, dy\, dz,$$

where W is the region bounded by $x + y + z = a$ (where $a > 0$ is a given constant), $x = 0, y = 0$, and $z = 0$.

7. (10 Points)**Section 5.5, Exercise 16.** *Evaluate*

$$\iiint_W z\, dx\, dy\, dz$$

where W is the region bounded by the planes $x = 0, y = 0, z = 0, z = 1$, and the cylinder $x^2 + y^2 = 1$, with $x \geq 0, y \geq 0$.