Mathematics 1c: Homework Set 4

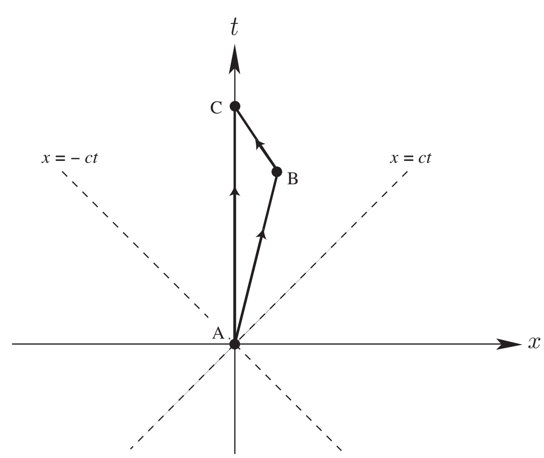
Due: Monday, April 26 at 10am.

1. (10 Points) **Section 4.1, Exercise 14** *Show that, at a local maximum or minimum of the quantity* $\|\mathbf{r}(t)\|$, $\mathbf{r}'(t)$ *is perpendicular to* $\mathbf{r}(t)$.

2. (10 Points)**Section 4.1, Exercise 18.** *Let* $\mathbf{c}$ *be a path in* $\mathbb{R}^3$ *with zero acceleration. Prove that* $\mathbf{c}$ *is a straight line or a point.*

3. (10 Points) **Section 4.2, Exercise 4.** *Compute the arc length of the curve described by*
$$\left(t+1, \frac{2\sqrt{2}}{3}t^{3/2}+7, \frac{1}{2}t^2\right)$$
on the interval $1 \leq t \leq 2$.

4. (20 points) **Exercise 18.** *In special relativity, the* ***proper time*** *of a path* $\mathbf{c} : [a,b] \to \mathbb{R}^4$ *with* $\mathbf{c}(\lambda) = (x(\lambda), y(\lambda), z(\lambda), t(\lambda))$ *is defined to be the quantity*
$$\frac{1}{c}\int_a^b \sqrt{-[x'(\lambda)]^2 - [y'(\lambda)]^2 - [z'(\lambda)]^2 + c^2[t'(\lambda)]^2} d\lambda,$$
where c is the velocity of light, a constant. Referring to the Figure, show that, using self-explanatory notation,
$$\text{proper time (AB)} + \text{proper time (BC)} < \text{proper time (AC)}.$$
(*This inequality is a special case of what is known as the* ***twin paradox***.)



5. (10 points) **Section 4.3, Exercise 14.** *Show that the curve*
$$\mathbf{c}(t) = (t^2, 2t-1, \sqrt{t}), t > 0$$
is a flow line of the velocity vector field
$$\mathbf{F}(x,y,z) = (y+1, 2, 1/2z).$$

6. (10 points) **Section 4.4, Exercise 10.** *Find the divergence of the vector field* $\mathbf{v}(x, y, z) = y\mathbf{i} - x\mathbf{j}$.

7. (10 Points) **Exercise 16.** *Find the curl of the vector field*

$$\mathbf{F}(x, y, z) = \frac{yz\mathbf{i} - xz\mathbf{j} + xy\mathbf{k}}{x^2 + y^2 + z^2}.$$