Mathematics 1c: Solutions, Homework Set 3

Due: Monday, April 19th by 10am.

1. (10 Points) **Section 3.1, Exercise 16** *Let* $w = f(x, y)$ *be a function of two variables, and let*
$$x = u + v, \quad y = u - v.$$
Show that
$$\frac{\partial^2 w}{\partial u \partial v} = \frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial y^2}.$$

2. (10 Points) **Section 3.1, Exercise 22**
   (a) *Show that the function*
   $$g(x, t) = 2 + e^{-t} \sin x$$
   satisfies the heat equation: $g_t = g_{xx}$. [*Here* $g(x, t)$ *represents the temperature in a metal rod at position* x *and time* t.]
   (b) *Sketch the graph of* g *for* $t \geq 0$. (*Hint: Look at sections by the planes* $t = 0, t = 1,$ *and* $t = 2$.)
   (c) *What happens to* $g(x, t)$ *as* $t \to \infty$*? Interpret this limit in terms of the behavior of heat in the rod.*

3. (10 Points) **Section 3.2, Exercise 6** *Determine the second-order Taylor formula for the function*
$$f(x, y) = e^{(x-1)^2} \cos y$$
expanded about the point $x_0 = 1, y_0 = 0$.

4. (10 Points) **Section 3.3, Exercise 7** *Find the critical points for the function*
$$f(x, y) = 3x^2 + 2xy + 2x + y^2 + y + 4$$
and determine if they are maxima, minima or saddle points.

5. (10 Points) **Section 3.3, Exercise 25** *Write the number* 120 *as a sum of three positive numbers so that the sum of the products taken two at a time is a maximum.*

6. (10 Points) **Section 3.4, Exercise 2** *Find the extrema of* $f(x, y) = x - y$ *subject to the constraint* $x^2 - y^2 = 2$.

7. (10 Points) **Section 3.4, Exercise 20** *A light ray travels from point* A *to point* B *crossing a boundary between two media* (*see Figure* 3.4.7 *of the text*). *In the first medium its speed is* v_1 *and in the second* v_2. *Show that the trip is made in minimum time when Snell's law holds:*
$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2}.$$

8. (10 Points) **Section 3.4, Exercise 22** *Let* P *be a point on a surface* S *in* $\mathbb{R}^3$ *defined by the equation* $f(x, y, z) = 1$*, where* f *is of class* C^1*. Suppose that* P *is a point where the distance from the origin to* S *is maximized. Show that the vector emanating from the origin and ending at* P *is perpendicular to* S*.*