Mathematics 1c: Homework Set 2

Due: Monday, April 12th by 10am.

1. (10 Points) **Section 2.5, Exercise 8** *Suppose that a function is given in terms of rectangular coordinates by* $u = f(x, y, z)$. *If* $x = \rho\cos\theta\sin\phi, y = \rho\sin\theta\sin\phi, z = \rho\cos\phi$, *express the partial derivatives* $\partial u/\partial\rho, \partial u/\partial\theta$, *and* $\partial u/\partial\phi$ *in terms of* $\partial u/\partial x, \partial u/\partial y$, *and* $\partial u/\partial z$.

2. (10 Points) **Section 2.5, Exercise 12** *Suppose that the temperature at the point* (x, y, z) *in space is* $T(x, y, z) = x^2 + y^2 + z^2$. *Let a particle follow the right circular helix* $\sigma(t) = (\cos t, \sin t, t)$ *and let* $T(t)$ *be its temperature at time* t.

   (a) *What is* $T'(t)$*?*

   (b) *Find an approximate value for the temperature at* $t = (\pi/2) + 0.01$.

3. (10 Points) **Section 2.6, Exercise 3(c)** *Compute the directional derivative of the function*

$$f(x, y, z) = xyz$$

   at the point $(x_0, y_0, z_0) = (1, 0, 1)$ *in the direction of the unit vector parallel to the vector* $\mathbf{d} = (1, 0, -1)$.

4. (20 Points) **Section 2.6, Exercise 16**. *Captain Ralph is in trouble near the sunny side of Mercury and notices that the hull of his ship is beginning to melt. The temperature in his vicinity is given by*

$$T = e^{-x^2} + e^{-2y^2} + e^{-3z^2}.$$

   (a) *If he is at the point* $(1, 1, 1)$, *in what direction should he proceed in order to cool fastest?*

   (b) *If the ship travels at* e^8 *meters per second, how fast will the temperature decrease if he proceeds in that direction?*

   (c) *Unfortunately, the metal of the hull will crack if cooled at a rate greater than* $\sqrt{14}\mathrm{e}^2$ *degrees per second. Describe the set of possible directions in which he may proceed to bring the temperature down at no more than that rate.*

5. (10 Points) **Review Exercise 20 for Chapter 2**. *Let* $(x(t), y(t))$ *be a path in the plane,* $0 \le t \le 1$, *and let* $f(x, y)$ *be a* C^1 *function of two variables and let* f_x *and* f_y *denote the two partial derivatives. Assume that*

$$\left(\frac{dx}{dt}\right) f_x + \left(\frac{dy}{dt}\right) f_y \le 0.$$

   Show that $f(x(1), y(1)) \le f(x(0), y(0))$.

6. (10 Points) **Review Exercise 22 for Chapter 2**. *Find the direction in which the function*
$$w = x^2 + xy$$
increases most rapidly at the point $(-1, 1)$. *What is the magnitude of* ∇w *at this point? Interpret this magnitude geometrically.*

7. (10 Points) **Review Exercise 42 for Chapter 2**. *Use the chain rule to find a formula for*
$$\frac{d}{dt}(f(t)^{g(t)}).$$