

Mathematics 1c: Homework Set 1

Due: Monday, April 5th by 10am.

1. (10 Points) Using the computing site or otherwise, draw the graphs of the following functions:

(a) $f(x, y) = 3(x^2 + 2y^2)e^{-x^2 - y^2}$; **Tip:** On the computing site use $E[x]$ to take the exponent of x ; there is no need to type a $*$ for multiplication; we suggest taking x and y between -2 and 2 .

(b) $f(x, y) = (x^3 - 3x)/(1 + y^2)$

Indicate some key features of these graphs, such as the location of the maxima and minima, important sections, etc

2. (10 Points) **Section 2.1, parts of Exercises 15, 18.** Sketch the zero level set of the function $f(x, y, z) = xy + yz$ and the level set for $c = 1$ of the function $f(x, y) = \max(|x|, |y|)$.

3. (15 Points) **Section 2.2, Exercise 12.** Compute the following limits, if they exist

(a) $\lim_{x \rightarrow 0} \frac{\sin 2x - 2x}{x^3}$.

(b) $\lim_{(x,y) \rightarrow (0,0)} \frac{\sin 2x - 2x + y}{x^3 + y}$.

(c) $\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{2x^2y \cos z}{x^2 + y^2}$.

4. (10 Points) **Section 2.3, Exercise 4(d)** Show that the following function is differentiable at each point in its domain. Determine if the function is C^1 .

$$f(x, y) = \frac{xy}{\sqrt{x^2 + y^2}}.$$

5. (10 Points) **Section 2.3, Exercise 8(c).** Compute the matrix of partial derivatives of the function $f(x, y) = (x + y, x - y, xy)$.

6. (10 Points) **Section 2.3 Exercise 10.** Why should the graphs of $f(x, y) = x^2 + y^2$, and $g(x, y) = -x^2 - y^2 + xy^3$ be called "tangent" at $(0, 0)$?

7. (15 Points) **Section 2.4, Exercise 18.** Suppose that a particle following the path

$$\mathbf{c}(t) = (e^t, e^{-t}, \cos(t))$$

flies off on a tangent at $t_0 = 1$. Compute the position of the particle at time $t_1 = 2$.