Mathematics 1c: Homework Set 1
Due: Monday, April 5th by 10am.

1. (10 Points) Using the computing site or otherwise, draw the graphs of the following functions:
   
   (a) \( f(x, y) = 3(x^2 + 2y^2)e^{-x^2-y^2} \); **Tip:** On the computing site use \( E[x] \) to take the exponent of \( x \); there is no need to type a * for multiplication; we suggest taking \( x \) and \( y \) between \(-2\) and \(2\).
   
   (b) \( f(x, y) = (x^3 - 3x)/(1 + y^2) \)

   Indicate some key features of these graphs, such as the location of the maxima and minima, important sections, etc.

2. (10 Points) **Section 2.1, parts of Exercises 15, 18.** Sketch the zero level set of the function \( f(x, y, z) = xy + yz \) and the level set for \( c = 1 \) of the function \( f(x, y) = \max(|x|, |y|) \).

3. (15 Points) **Section 2.2, Exercise 12.** Compute the following limits, if they exist
   
   (a) \( \lim_{x \to 0} \frac{\sin 2x - 2x}{x^3} \).
   
   (b) \( \lim_{(x,y) \to (0,0)} \frac{\sin 2x - 2x + y}{x^3 + y} \).
   
   (c) \( \lim_{(x,y,z) \to (0,0,0)} \frac{2x^2y \cos z}{x^2 + y^2} \).

4. (10 Points) **Section 2.3, Exercise 4(d)** Show that the following function is differentiable at each point in its domain. Determine if the function is \( C^1 \).

   \[ f(x, y) = \frac{xy}{\sqrt{x^2 + y^2}}. \]

5. (10 Points) **Section 2.3, Exercise 8(c).** Compute the matrix of partial derivatives of the function \( f(x, y) = (x + y, x - y, xy) \).

6. (10 Points) **Section 2.3 Exercise 10.** Why should the graphs of \( f(x, y) = x^2 + y^2 \), and \( g(x, y) = -x^2 - y^2 + xy^3 \) be called “tangent” at \((0,0)\)?

7. (15 Points) **Section 2.4, Exercise 18.** Suppose that a particle following the path

   \[ c(t) = (e^t, e^{-t}, \cos(t)) \]

flies off on a tangent at \( t_0 = 1 \). Compute the position of the particle at time \( t_1 = 2 \).