Path Optimization for an Earth-Based Demonstration Flight

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The Problem

- The Cassini-Huygens spacecraft mission revealed some incredible observational data on Titan, including the likely existence of high-latitude hydrocarbon oceans and equatorial sand dunes

- NASA intends on going back to Titan, likely using Montgolfier balloons that would be able to operate and gather data on Titan for several years

- While the balloon is on Titan, intervening with the flight-plan on a short time-scale is not possible
    - Require autonomous decision-making

- Demonstration balloon flight is planned for the Mojave Desert in 2009
    - We would like the balloon to autonomously navigate the desert by exploiting winds to minimize control, time of flight, etc.

- Hopefully methods can be translated to solve a similar problem on Titan

Discrete Mechanics and Optimal Control

- Finds optimal trajectory that satisfies a system's equations of motion
- Implemented as fmincon in Matlab's Optimization Toolbox
  - Optimization routine that finds local minimum of a constrained, nonlinear multivariable objective function
- Requires:
  - Objective function
  - Constraint equations
  - Initial guess

DMOC

- We are given the initial position, q_0, final position, q_f, and initial velocity, v_0
  - May or may not care about final velocity
- Discrete trajectory with N time nodes between q_0 and q_f such that $(N+1) \cdot h = t_f$
  - h is constant time step and t_f is time of travel
- Want to find $\{q_i\}$, $\{v_i\}$, and $\{f_i\}$ that satisfy constraints and minimize objective function
  - q_i is generalized position, v_i is generalized impulse, and f_i is control force
- Choose not to fix t_f, so we also want an optimal h

Objective Function

- Depends on the problem
  - Currently concerned with minimizing control required for balloon navigation
  - Could minimize time of travel or other parameters

- We are choosing to use the l^2-norm as the measure of control
  - Want to minimize

$$\sum_i f_i^2$$

- If we were to choose a more sophisticated model of balloon dynamics, with fuel as an explicit parameter, we can also minimize fuel consumption

Constraint Equations

- Trajectory must obey physical laws (i.e. balloon's equations of motion)

- We use the following simple model of balloon dynamics

  - Consider the trajectory $\{q_0, q_1, \ldots, q_N, q_f\}$ - we approximate velocity of balloon as

$$\frac{q_{i+1} - q_i}{h} = v_i + Wind(\frac{q_{i+1} + q_i}{2}, t_{i+\frac{1}{2}})$$

    where v_i are evaluated on a staggered grid at the midpoint between q_i and q_{i+1}

  - Approximate control force as

$$\frac{v_{i+1} - v_i}{h} = f_{i+1}$$

    where f_{i+1} are evaluated on a staggered grid at the midpoint between v_{i+1} and v_i

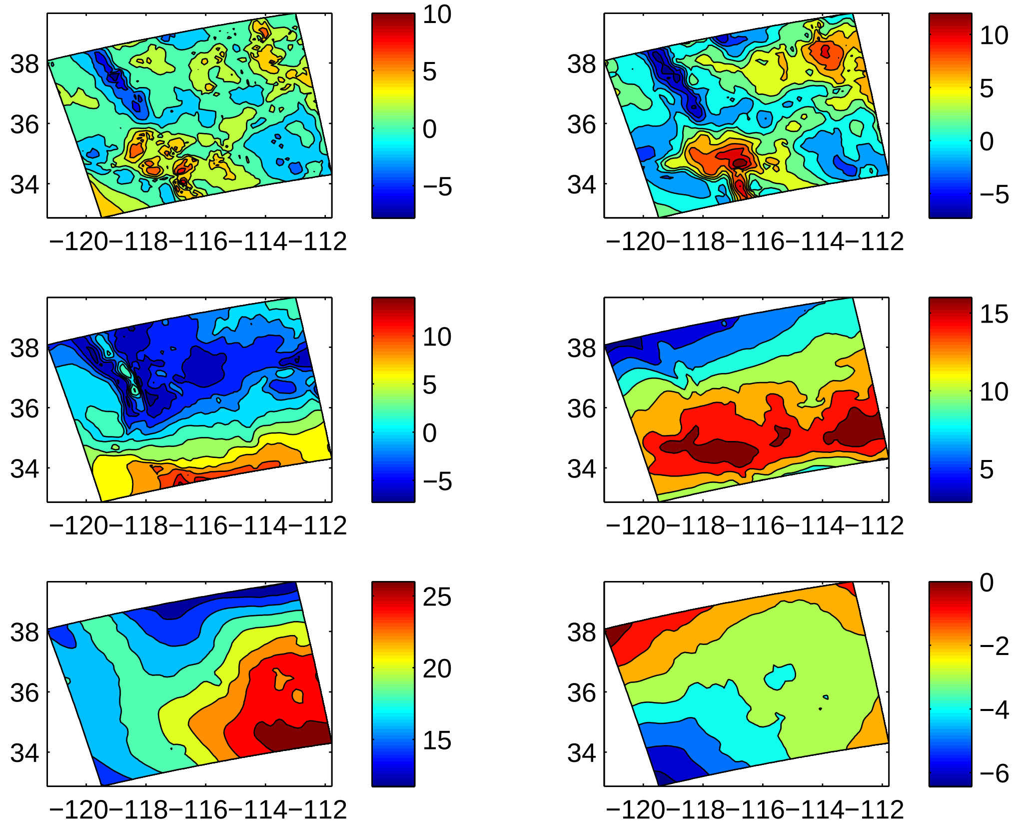
- Wind field is generated using the Weather Research and Forecasting (WRF) model

Weather Research and Forecasting Model

- Numerical weather prediction system for both research and forecasting applications
- Influence of wind is a significant component of constraint equations
- Balloon demonstration flight planned for the Mojave Desert in 2009
- For our purposes, we ran the model for the Mojave Desert starting July 5, 2005 (around the Death Valley region)

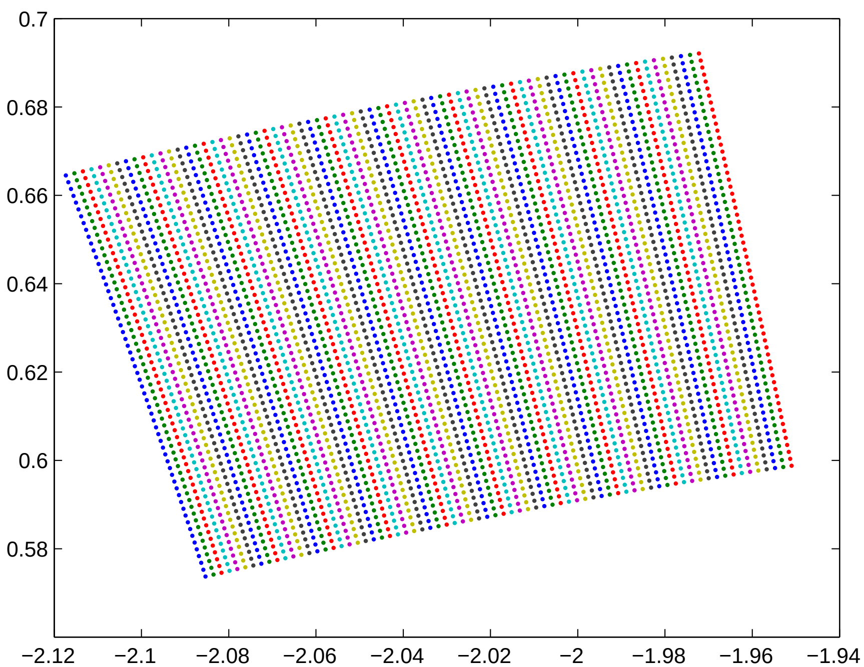
WRF Output

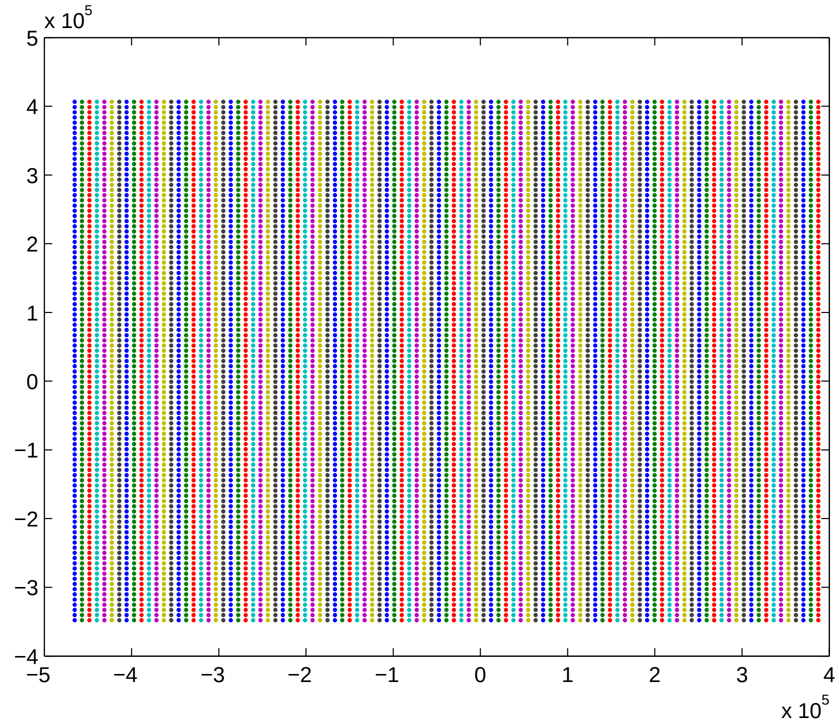
- Plot of the x-direction wind velocity at different sigma levels
  - Sigma is the ratio of the pressure at a point in the atmosphere to the pressure of the surface of the Earth beneath it



Modifying WRF Output

- Wind speeds are on a curvilinear grid but we would like them to be on a uniform, rectangular Cartesian grid





Modifying WRF Output

- First, interpolate all necessary data (i.e. wind speed and geopotential height) onto Cartesian grid

- Change of coordinate from longitude and latitude to Cartesian distance

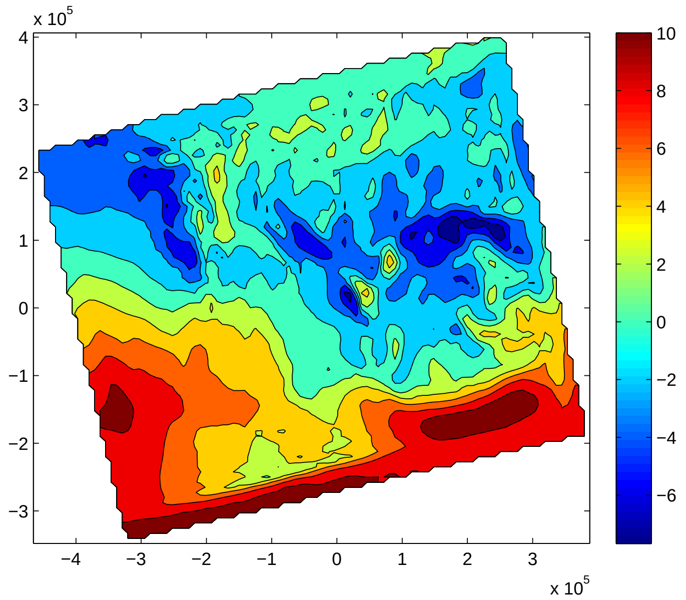
$$x = \frac{\pi}{180^\circ} \cdot r \cdot \phi \cdot \cos(\lambda - \lambda_0)$$

$$y = \frac{\pi}{180^\circ} \cdot r \cdot (\phi - \phi_0)$$

  - ϕ is latitude, λ is longitude, and $r = 6,378 \cdot 10^3 m$ is the radius of the earth
  - (λ_0, ϕ_0) is origin, which in this case is $(-116^\circ E, 36^\circ N)$

Modifying WRF Output

- Sigma coordinates complicate calculations
  - Must make a change of variable to altitude coordinates
  - Use geopotential height divided by Earth's gravity to obtain altitude
- We use interpolation to find wind velocity at altitude of our choice
- Example of x-direction wind velocity on July 6, 2:00am GMT, at an altitude of $4500m$

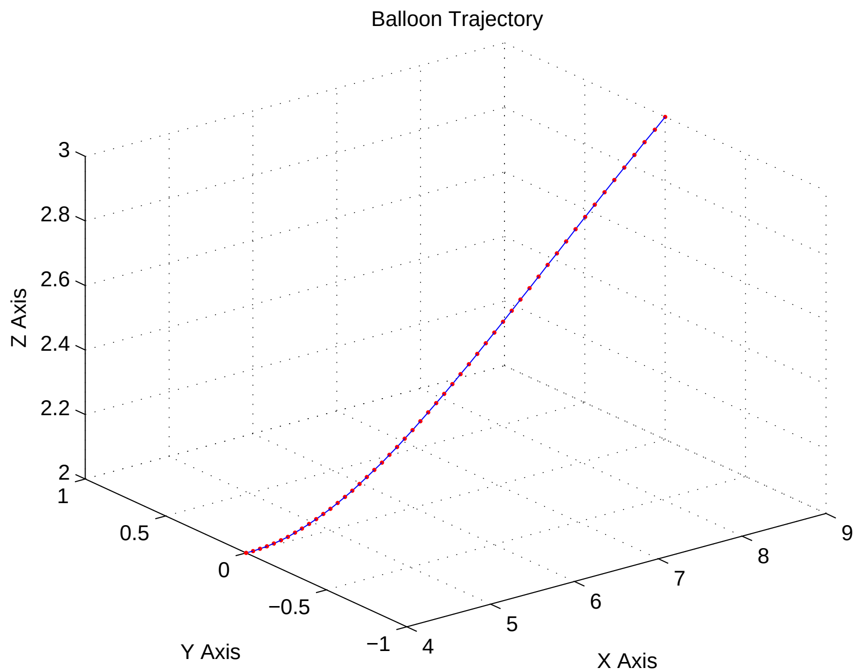


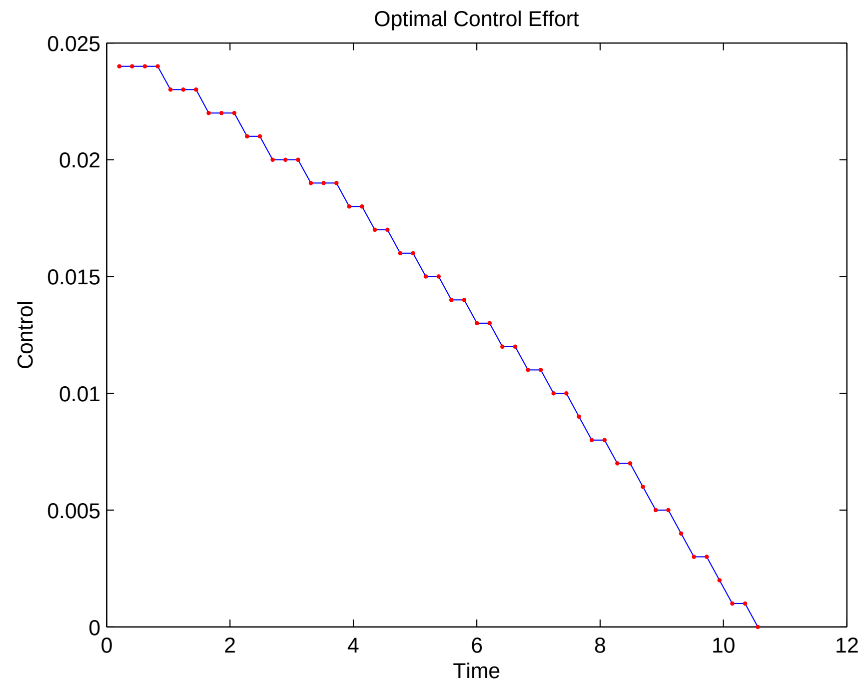
Problems

- One major issue with our current model is the lack of drag due to wind
  - We would like implement a more sophisticated model of balloon dynamics, but...
- fmincon is a local optimization routine so initial guess is critical
- For first implementation of DMOC, we chose a linear initial guess
  - Finite differences was used to compute initial velocity and control force
  - Does not necessarily satisfy constraints of the problem

First Example

- $N = 50$, initial position is $(4, 0, 2)$, final position is $(9, 0, 3)$, no initial velocity, wind is $(0.2z, 0, 0)$
- Linear initial guess, initial guess for $h = 0.16$

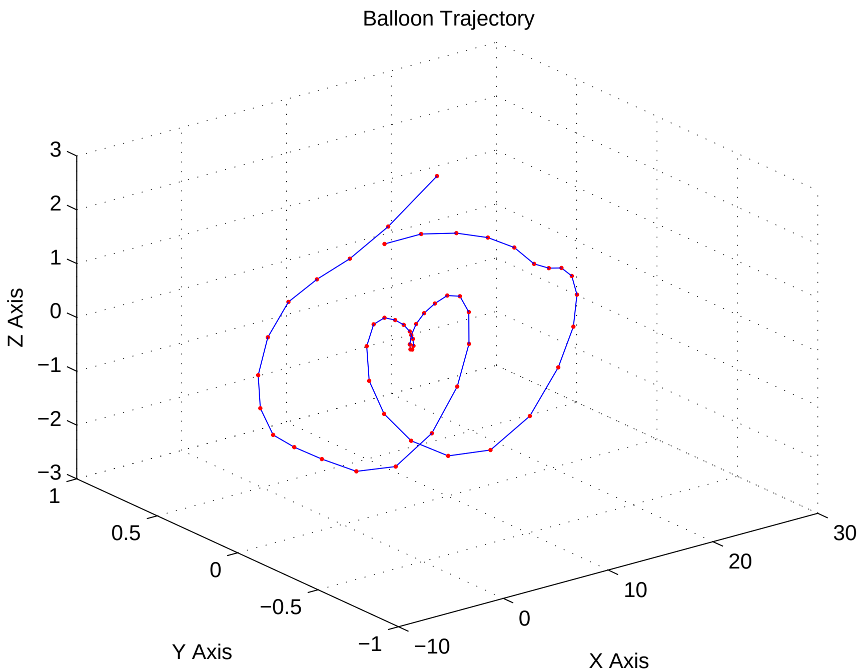


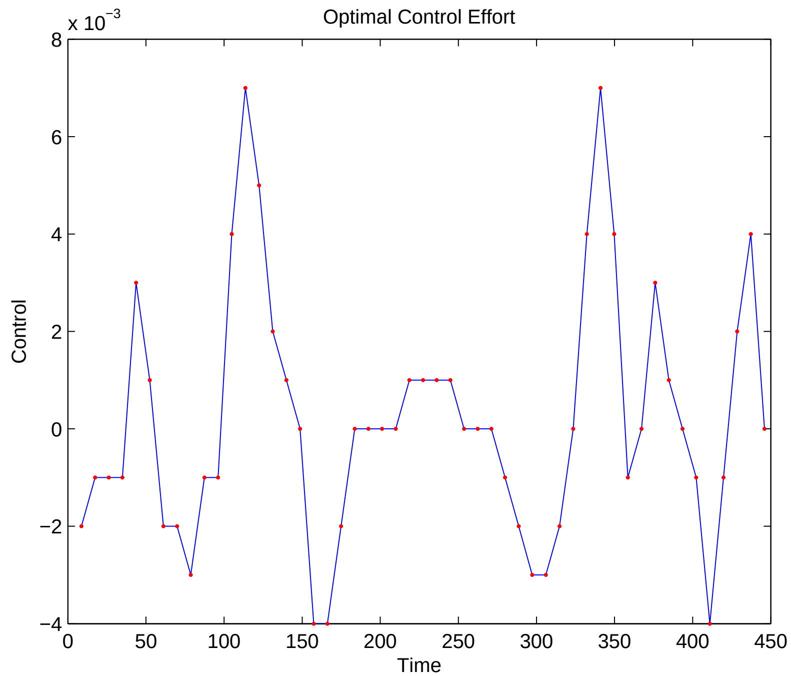


- Optimal $h = 0.207$

First Example

- Same as before except now initial guess for $h = 1.4$



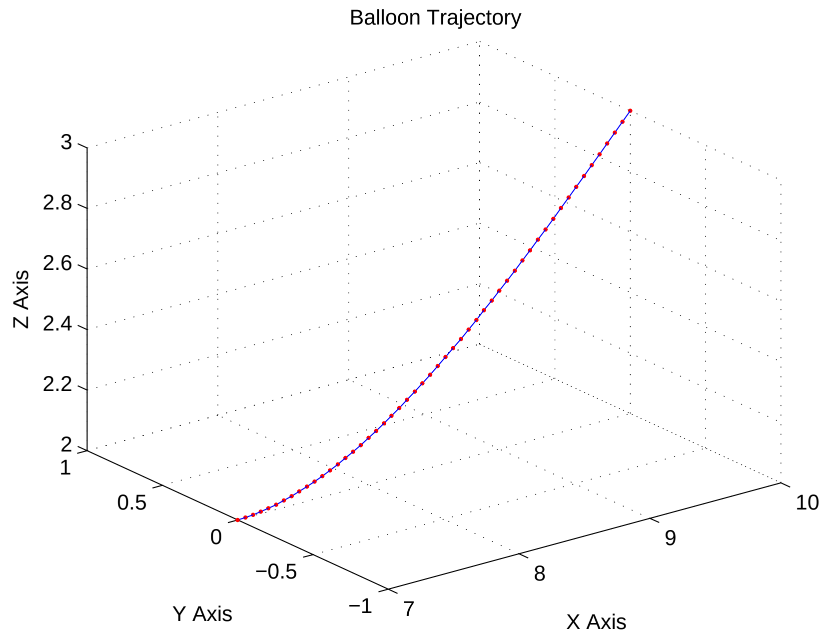


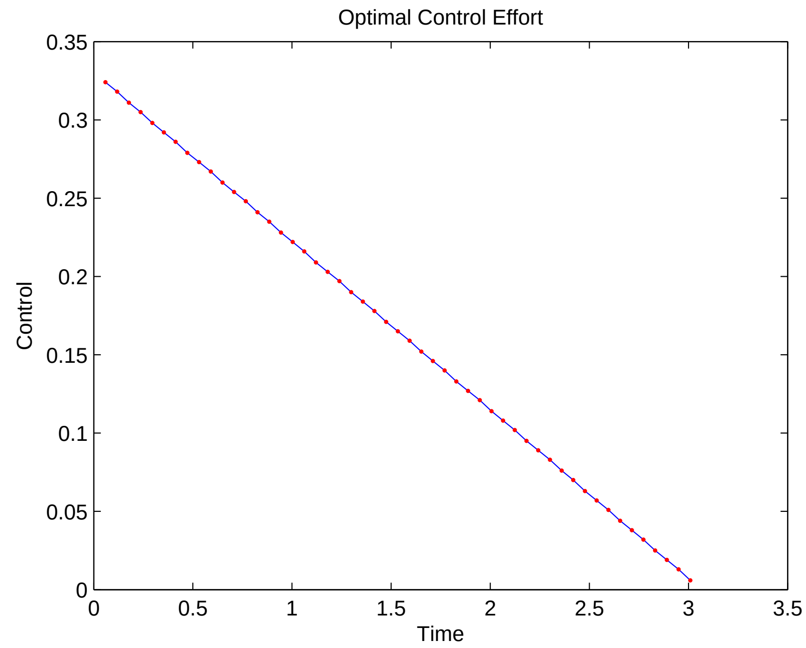
- Optimal $h = 8.75$

Second Example

- $N = 50$, initial position is $(7, 0, 2)$, final position is $(10, 0, 3)$, no initial velocity, wind is $(1, 0, 0)$

- Linear initial guess





- If wind is $(x, 0, 0)$, the optimization routine is unable to find a trajectory that satisfies the constraints although one should exist

Second Example

- Changed the wind less drastically using the same linear initial guess
- We chose winds of $(1 + \alpha x, 0, 0)$ where $\alpha \in [0.001, 1]$
  - Optimization succeeded for $\alpha \leq 0.6$
- Used the optimal output from $\alpha = 0.5$ as the initial guess for $\alpha = 1$
  - Optimization succeeded
- Conclusion: need to generate a better initial guess!

Future Goals

- Currently working on implementing DMOC with actual wind fields from WRF
  - Will likely use "receding-horizon" to build in time dependence (i.e. assume wind fields are constant over hourly intervals)

- Use Lagrangian Coherent Structures to initiate DMOC
  - Inanc, T., S.C. Shadden and J.E. Marsden Optimal trajectory generation in ocean flows, Proc. of 2005 American Control Conference, (2005), 674-679.
    - Demonstrated that LCS "provide a good correspondence with optimal trajectories for autonomous underwater gliders in the ocean"
    - Asked question: "Can computations of optimal trajectories be sped up by using information of LCS to initialize the optimization code?"

- Use DMOC primitives for real-time computation
  - Curse of dimensionality?

- Implement a more sophisticated model of balloon dynamics