Path Optimization for an Earth-Based Demonstration Flight

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Summer Undergraduate Research Fellowship
August 8, 2008
The Problem

- The Cassini-Huygens spacecraft mission revealed some incredible observational data on Titan, including the likely existence of high-latitude hydrocarbon oceans and equatorial sand dunes.

- NASA intends on going back to Titan, likely using Montgolfier balloons that would be able to operate and gather data on Titan for several years.

- While the balloon is on Titan, intervening with the flight-plan on a short time-scale is not possible.
  - Require autonomous decision-making.

- Demonstration balloon flight is planned for the Mojave Desert in 2009.
  - We would like the balloon to autonomously navigate the desert by exploiting winds to minimize control, time of flight, etc.

- Hopefully methods can be translated to solve a similar problem on Titan.
Discrete Mechanics and Optimal Control

• Finds optimal trajectory that satisfies a system’s equations of motion

• Implemented as fmincon in Matlab’s Optimization Toolbox
  ○ Optimization routine that finds local minimum of a constrained, nonlinear multivariable objective function

• Requires:
  ○ Objective function
  ○ Constraint equations
  ○ Initial guess
DMOC

- We are given the initial position, $q_0$, final position, $q_f$, and initial velocity, $v_0$
  - May or may not care about final velocity
- Discrete trajectory with $N$ time nodes between $q_0$ and $q_f$ such that $(N + 1) \cdot h = t_f$
  - $h$ is constant time step and $t_f$ is time of travel
- Want to find $\{q_i\}$, $\{v_i\}$, and $\{f_i\}$ that satisfy constraints and minimize objective function
  - $q_i$ is generalized position, $v_i$ is generalized impulse, and $f_i$ is control force
- Choose not to fix $t_f$, so we also want an optimal $h$
Objective Function

- Depends on the problem
  - Currently concerned with minimizing control required for balloon navigation
  - Could minimize time of travel or other parameters

- We are choosing to use the $l^2$-norm as the measure of control
  - Want to minimize
    \[ \sum_{i} f_i^2 \]

- If we were to choose a more sophisticated model of balloon dynamics, with fuel as an explicit parameter, we can also minimize fuel consumption
Constraint Equations

- Trajectory must obey physical laws (i.e. balloon’s equations of motion)

- We use the following simple model of balloon dynamics
  - Consider the trajectory \(\{q_0, q_1, \ldots, q_N, q_f\}\) - we approximate velocity of balloon as
    \[
    \frac{q_{i+1} - q_i}{h} = v_i + Wind\left(\frac{q_{i+1} + q_i}{2}, t_{i+\frac{1}{2}}\right)
    \]
    where \(v_i\) are evaluated on a staggered grid at the midpoint between \(q_i\) and \(q_{i+1}\)
  - Approximate control force as
    \[
    \frac{v_{i+1} - v_i}{h} = f_{i+1}
    \]
    where \(f_{i+1}\) are evaluated on a staggered grid at the midpoint between \(v_{i+1}\) and \(v_i\)

- Wind field is generated using the Weather Research and Forecasting (WRF) model
Weather Research and Forecasting Model

- Numerical weather prediction system for both research and forecasting applications
- Influence of wind is a significant component of constraint equations
- Balloon demonstration flight planned for the Mojave Desert in 2009
- For our purposes, we ran the model for the Mojave Desert starting July 5, 2005 (around the Death Valley region)
WRF Output

- Plot of the $x$-direction wind velocity at different sigma levels
  - Sigma is the ratio of the pressure at a point in the atmosphere to the pressure of the surface of the Earth beneath it
• Wind speeds are on a curvilinear grid but we would like them to be on a uniform, rectangular Cartesian grid
Modifying WRF Output

- First, interpolate all necessary data (i.e. wind speed and geopotential height) onto Cartesian grid

- Change of coordinate from longitude and latitude to Cartesian distance

\[
x = \frac{\pi}{180^\circ} \cdot r \cdot \phi \cdot \cos(\lambda - \lambda_0)
\]

\[
y = \frac{\pi}{180^\circ} \cdot r \cdot (\phi - \phi_0)
\]

○ \(\phi\) is latitude, \(\lambda\) is longitude, and \(r = 6,378 \cdot 10^3 m\) is the radius of the earth

○ \((\lambda_0, \phi_0)\) is origin, which in this case is \((-116^\circ E, 36^\circ N)\)
Modifying WRF Output

- Sigma coordinates complicate calculations
  - Must make a change of variable to altitude coordinates
  - Use geopotential height divided by Earth’s gravity to obtain altitude

- We use interpolation to find wind velocity at altitude of our choice

- Example of $x$-direction wind velocity on July 6, 2:00am GMT, at an altitude of 4500m
Problems

- One major issue with our current model is the lack of drag due to wind
  - We would like to implement a more sophisticated model of balloon dynamics, but...

- fmincon is a local optimization routine so initial guess is critical

- For first implementation of DMOC, we chose a linear initial guess
  - Finite differences was used to compute initial velocity and control force
  - Does not necessarily satisfy constraints of the problem
First Example

- $N = 50$, initial position is $(4, 0, 2)$, final position is $(9, 0, 3)$, no initial velocity, wind is $(0.2z, 0, 0)$

- Linear initial guess, initial guess for $h = 0.16$

- Optimal $h = 0.207$
First Example

• Same as before except now initial guess for \( h = 1.4 \)

• Optimal \( h = 8.75 \)
Second Example

- \( N = 50 \), initial position is \((7, 0, 2)\), final position is \((10, 0, 3)\), no initial velocity, wind is \((1, 0, 0)\)

- Linear initial guess

- If wind is \((x, 0, 0)\), the optimization routine is unable to find a trajectory that satisfies the constraints although one should exist
Second Example

- Changed the wind less drastically using the same linear initial guess

- We chose winds of \((1 + \alpha x, 0, 0)\) where \(\alpha \in [0.001, 1]\)
  - Optimization succeeded for \(\alpha \leq 0.6\)

- Used the optimal output from \(\alpha = 0.5\) as the initial guess for \(\alpha = 1\)
  - Optimization succeeded

- Conclusion: need to generate a better initial guess!
Future Goals

• Currently working on implementing DMOC with actual wind fields from WRF
  ◦ Will likely use “receding-horizon” to build in time dependence (i.e. assume wind fields are constant over hourly intervals)

• Use Lagrangian Coherent Structures to initiate DMOC
  ◦ Demonstrated that LCS “provide a good correspondence with optimal trajectories for autonomous underwater gliders in the ocean”
  ◦ Asked question: “Can computations of optimal trajectories be sped up by using information of LCS to initialize the optimization code?”

• Use DMOC primitives for real-time computation
  ◦ Curse of dimensionality?

• Implement a more sophisticated model of balloon dynamics