OPTIMAL TITAN TRAJECTORY DESIGN

Progress Report

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Overview

- Amendment to Problem Statement
- Invariant Manifolds
  - Background
  - Results
- Gravity Assists
  - Background
  - Energy Kick Function
  - Preliminary Trajectory
- Remaining Tasks

Change in Problem Statement
- Motivation -

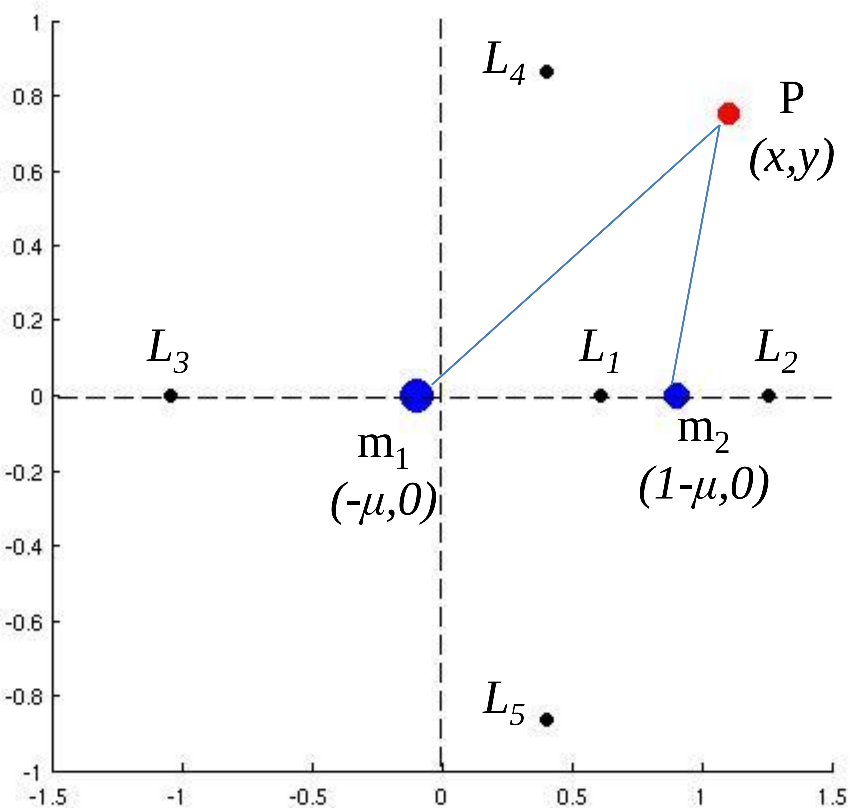
- Previous Problem Statement:
  - Design a trajectory to Titan using Invariant Manifolds and Gravity Assists
    - Compare objective function: ΔV vs. (ΔV and TOF)

- Optimizing for TOF does not produce significant improvements
  - The same initial guess trajectory would traverse same resonances
    - DMOC optimization could only produce a narrow TOF range

Problem Statement

- Design a trajectory to Titan using Invariant Manifolds and Gravity Assists
  - Investigate the effect of the Jacobi Constant on fuel usage
  - Consider the following trajectory:
    - Gravity Assist
    - Invariant Manifold
    - Capture at Titan
  - Expect that varying Jacobi constant will affect the:
    - Target region produced by Poincaré section of Invariant Manifolds at periapsis
    - Resonances traversed by spacecraft
    - ΔV at capture

Invariant Manifolds
- PCR3BP -



Equations of Motion for a test particle (spacecraft)

$$\ddot{x} - 2\dot{y} = \frac{\partial \Omega}{\partial x}$$

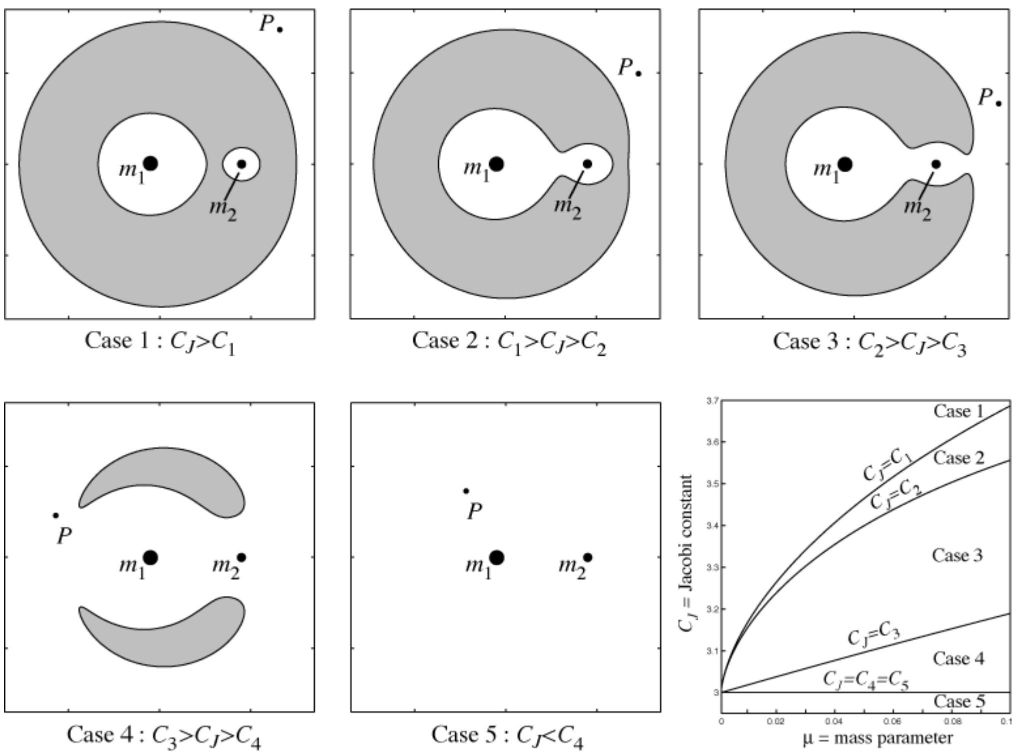
$$\ddot{y} + 2\dot{x} = \frac{\partial \Omega}{\partial y}$$

$$\Omega = \frac{x^2 + y^2}{2} + \frac{1-\mu}{\sqrt{(x+\mu)^2 + y^2}} + \frac{\mu}{\sqrt{(x-1+\mu)^2 + y^2}}$$

Motion exhibits constant, non-negative energy

$$E = \frac{1}{2}(\dot{x}^2 + \dot{y}^2) - \Omega(x, y)$$

Invariant Manifolds
- Allowable Motion -



Ross, S D; Scheeres, D J. Multiple Gravity Assists, Capture and Escape in the Restricted Three-Body Problem. 2007. pg 4

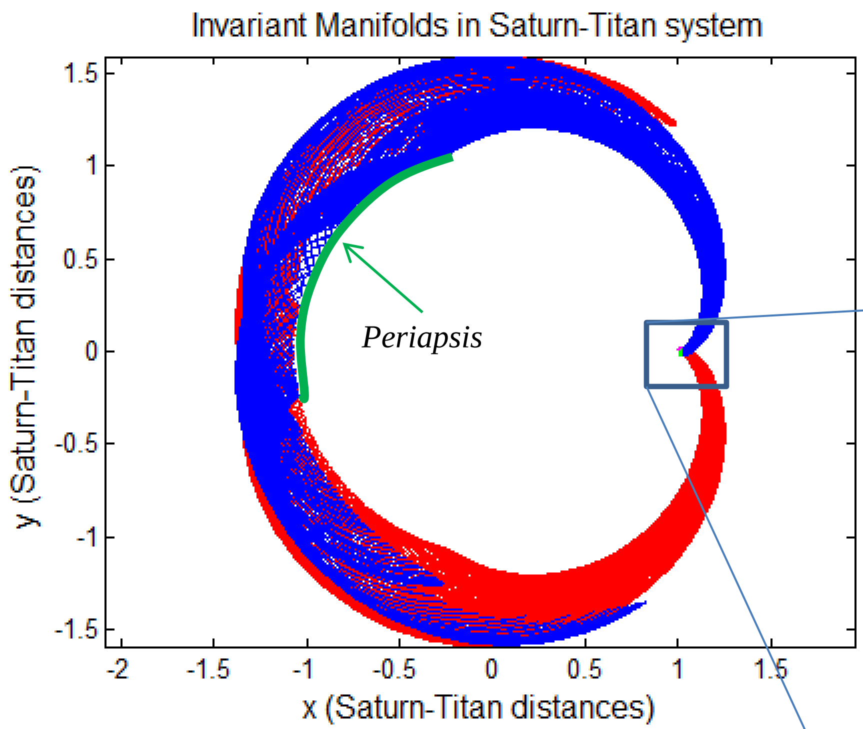
- Access to Titan from exterior region for

  $C_2 > C_J > C_3$

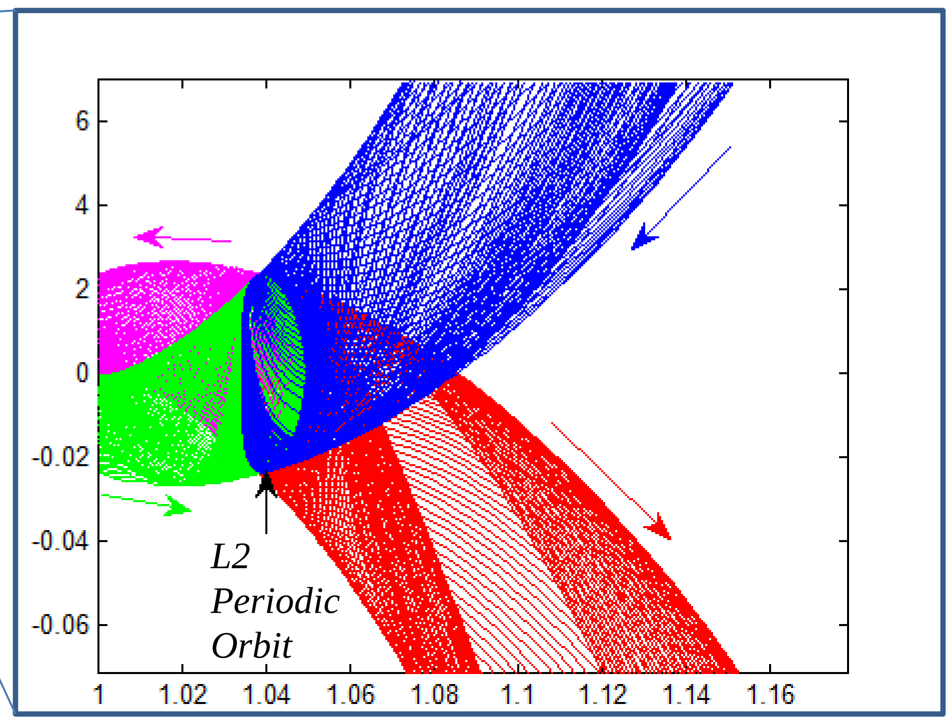
- Where Jacobi Constant, $C_J = -2E$ and $C_2 = C_J(L_2)$
- For Saturn-Titan Problem

  $3.0157 > C_J > 3.0005$

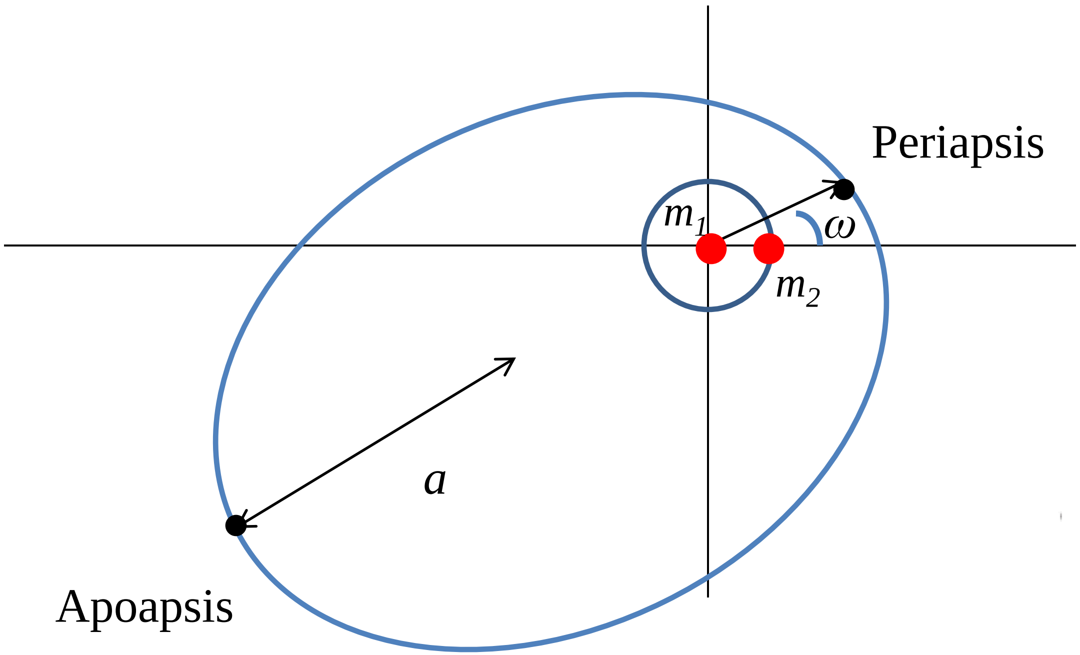
Invariant Manifolds
- Saturn-Titan System -

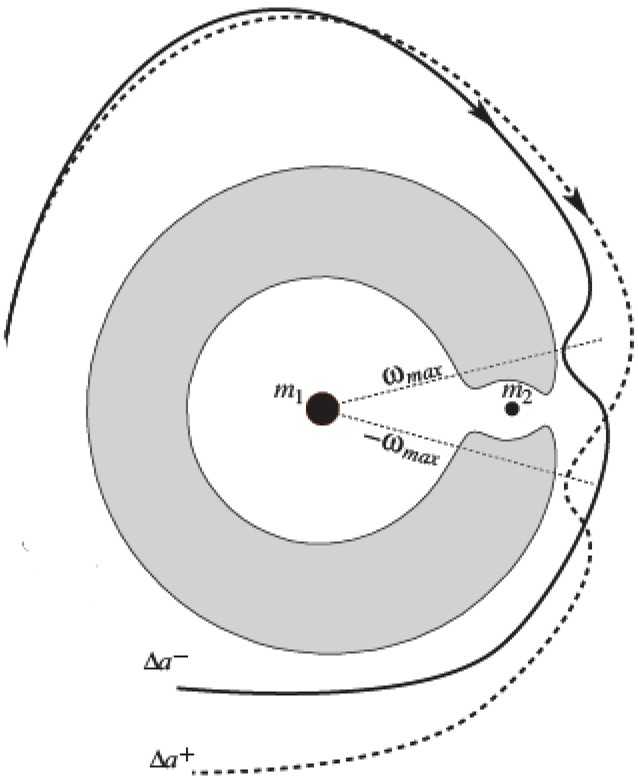


Invariant manifolds shown in Saturn-Titan reference frame emanating from L_2 Lyapunov orbit
Using $C_J = 3.012$



Resonant Gravity Assists
- Geometry -





$$\textit{Keplerian Energy} \quad K = \frac{-1}{2a}$$

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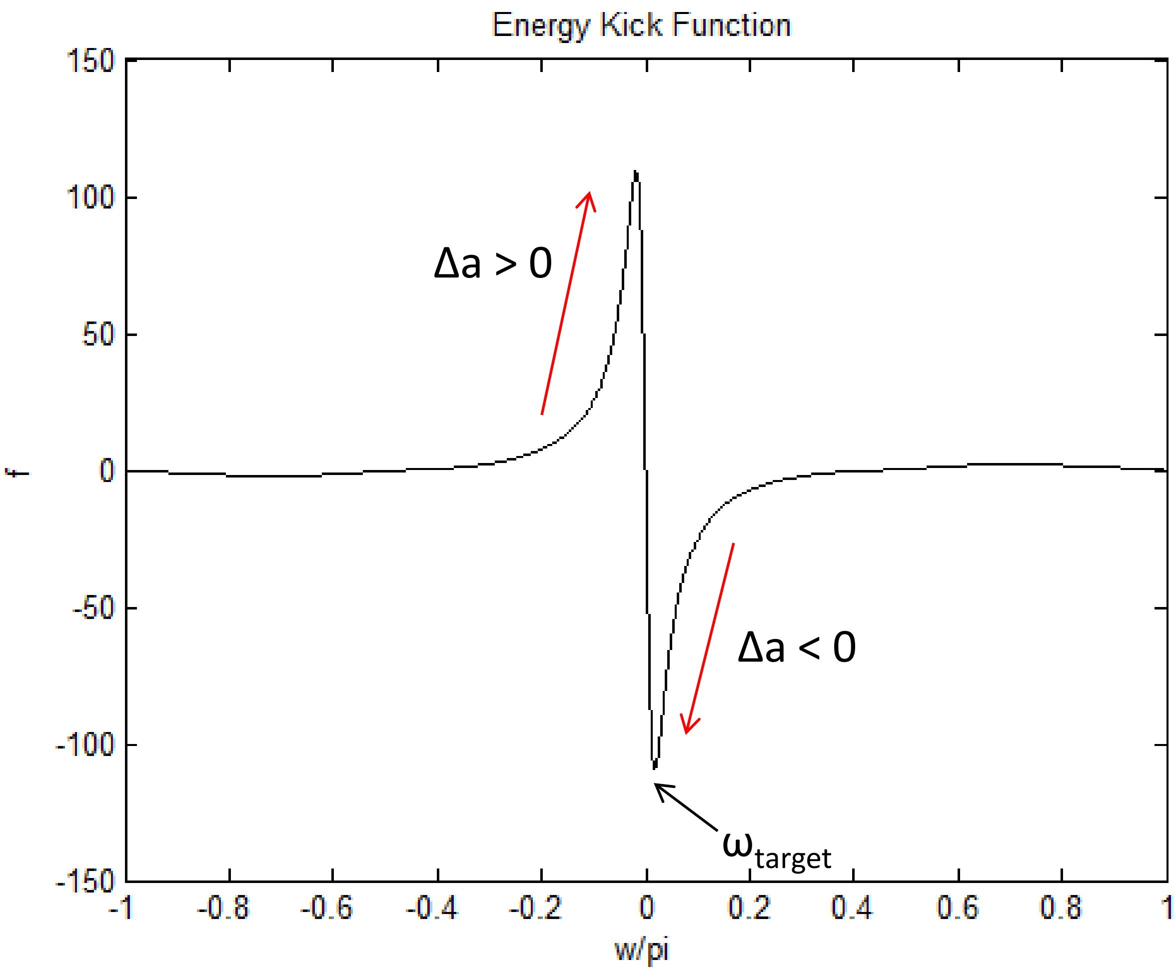
Resonant Gravity Assists
- Energy Kick Function -

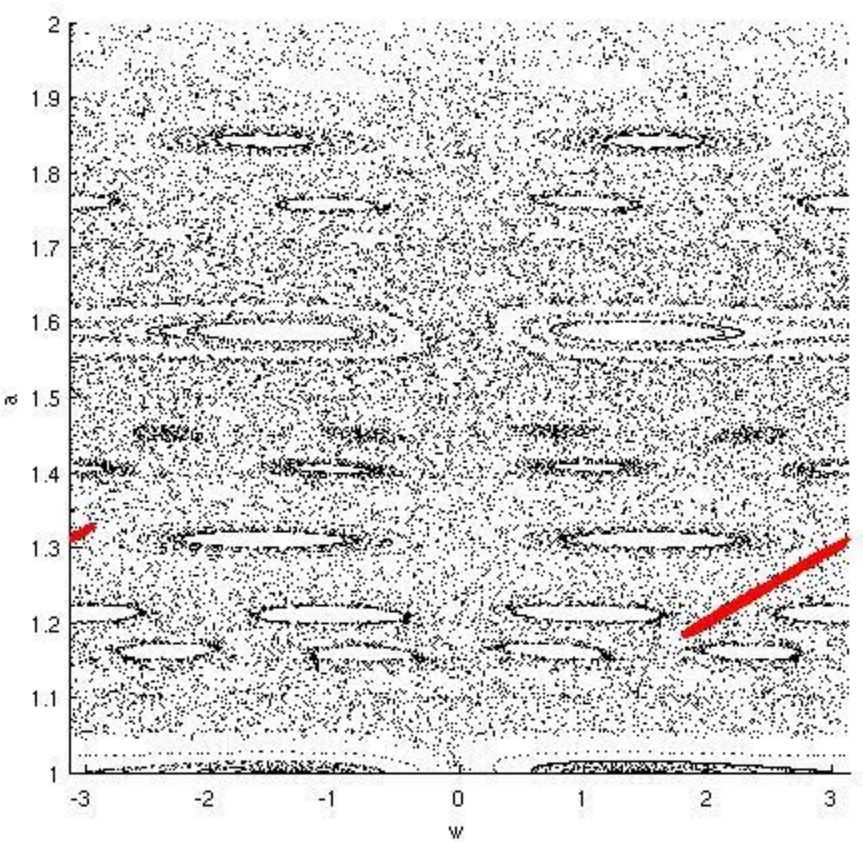


Energy kick function depends on average semimajor axis

Changes ω_{target} periapsis angle to be targeted for maximum energy kick

Increase accuracy in model by interpolating ω_{target} for current semimajor axis

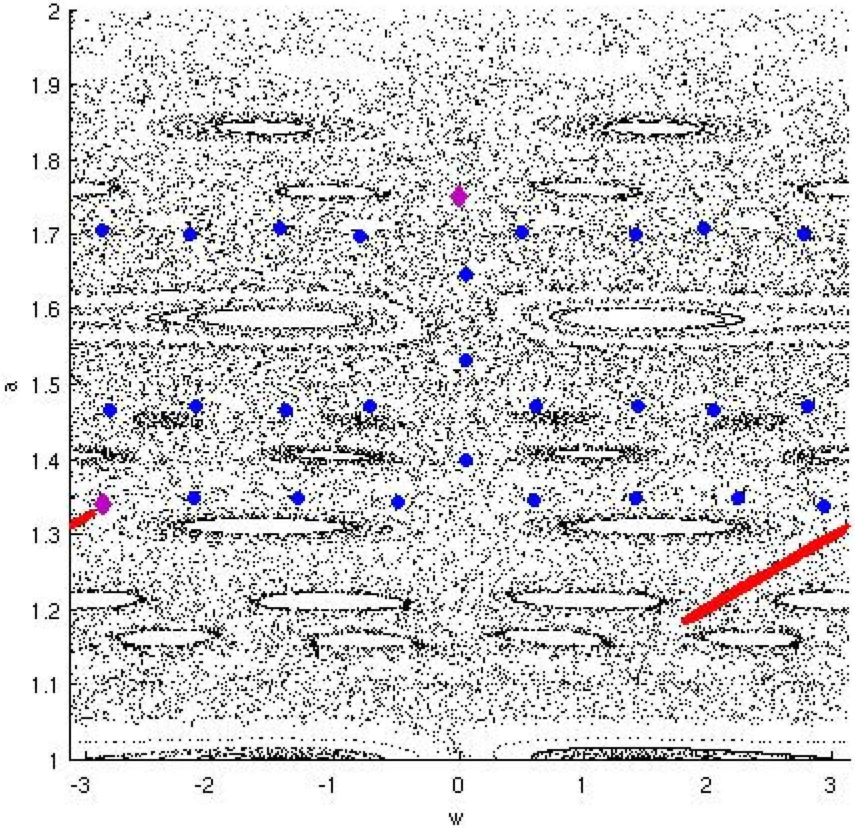
Resonant Gravity Assists
- Keplerian Map -

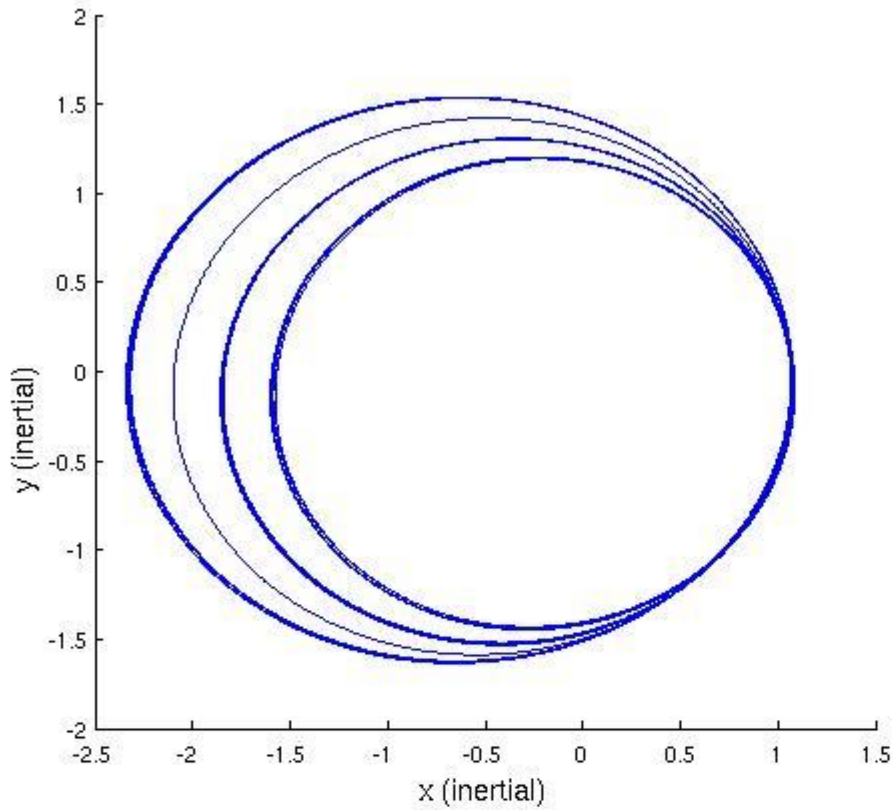


$$\begin{pmatrix} \omega_n \\ K_n \end{pmatrix} = \begin{pmatrix} \omega_n - 2\pi(-2K_{n+1})^{\frac{3}{2}} (\bmod 2\pi) \\ K_n + \mu f(\omega_n) \end{pmatrix}$$

- Poincaré Section taken at periapsis for C_J = 3.012
  - Red = target region, invariant manifolds

Resonant Gravity Assists
- Preliminary Trajectory -





Total ΔV = 7.75m/s

However – still need slightly more fuel to land within the invariant manifolds

Summer Schedule

- This week:
  - Optimize targeting of invariant manifolds
  - Combine gravity assists and invariant manifolds with capture at Titan to create overall trajectory
- Beyond:
  - Optimize initial guess trajectory with DMOC
  - Create multiple trajectories using different Jacobi Constants