Overview

Amendment to Problem Statement

Invariant Manifolds
  Background
  Results

Gravity Assists
  Background
  Energy Kick Function
  Preliminary Trajectory

Remaining Tasks
Change in Problem Statement

- Motivation -

- Previous Problem Statement:
  - Design a trajectory to Titan using Invariant Manifolds and Gravity Assists
    - Compare objective function: $\Delta V$ vs. $(\Delta V$ and $TOF$)

- Optimizing for TOF does not produce significant improvements
  - The same initial guess trajectory would traverse same resonances
    - DMOC optimization could only produce a narrow TOF range
Problem Statement

• Design a trajectory to Titan using Invariant Manifolds and Gravity Assists
  • Investigate the effect of the Jacobi Constant on fuel usage
  • Consider the following trajectory:
    – Gravity Assist
    – Invariant Manifold
    – Capture at Titan
  • Expect that varying Jacobi constant will affect the:
    – Target region produced by Poincaré section of Invariant Manifolds at periapsis
    – Resonances traversed by spacecraft
    – $\Delta V$ at capture
Invariant Manifolds
- PCR3BP -

Equations of Motion for a test particle (spacecraft)

\[ \dot{x} - 2\dot{y} = \frac{\partial \Omega}{\partial x} \]
\[ \dot{y} + 2\dot{x} = \frac{\partial \Omega}{\partial y} \]

\[ \Omega = \frac{x^2 + y^2}{2} + \frac{1 - \mu}{\sqrt{(x + \mu)^2 + y^2}} + \frac{\mu}{\sqrt{(x - 1 + \mu)^2 + y^2}} \]

Motion exhibits constant, non-negative energy

\[ E = \frac{1}{2}(\dot{x}^2 + \dot{y}^2) - \Omega(x, y) \]
Invariant Manifolds
- Allowable Motion -

- Access to Titan from exterior region for $C_2 > C_J > C_3$
- Where Jacobi Constant, $C_J = -2E$
  and $C_2 = C_J(L_2)$
- For Saturn-Titan Problem
  $3.0157 > C_J > 3.0005$

Ross, S D; Scheeres, D J. Multiple Gravity Assists, Capture and Escape in the Restricted Three-Body Problem. 2007. pg 4
Invariant manifolds shown in Saturn-Titan reference frame emanating from L$_2$ Lyapunov orbit
Using $C_J = 3.012$
Resonant Gravity Assists - Geometry -

Keplerian Energy \( K = \frac{-1}{2a} \)

Ross, S D; Scheeres, D J. Multiple Gravity Assists, Capture and Escape in the Restricted Three-Body Problem. 2007. pg 4
Resonant Gravity Assists
- Energy Kick Function -

Energy kick function depends on average semimajor axis

Changes $\omega_{\text{target}}$ periapsis angle to be targeted for maximum energy kick

Increase accuracy in model by interpolating $\omega_{\text{target}}$ for current semimajor axis
Resonant Gravity Assists
- Keplerian Map -

\[
\begin{bmatrix}
\omega_n \\
K_n
\end{bmatrix} = \begin{bmatrix}
\omega_n - 2\pi(-2K_{n+1})^{\frac{3}{2}} \pmod{2\pi} \\
K_n + \mu f(\omega_n)
\end{bmatrix}
\]

- Poincaré Section taken at periapsis for \( C_j = 3.012 \)
  - Red = target region, invariant manifolds
Resonant Gravity Assists
- Preliminary Trajectory -

Total $\Delta V = 7.75\text{m/s}$

However – still need slightly more fuel to land within the invariant manifolds
Summer Schedule

• This week:
  – Optimize targeting of invariant manifolds
  – Combine gravity assists and invariant manifolds with capture at Titan to create overall trajectory

• Beyond:
  – Optimize initial guess trajectory with DMOC
  – Create multiple trajectories using different Jacobi Constants