Titan Mission Trajectory Design

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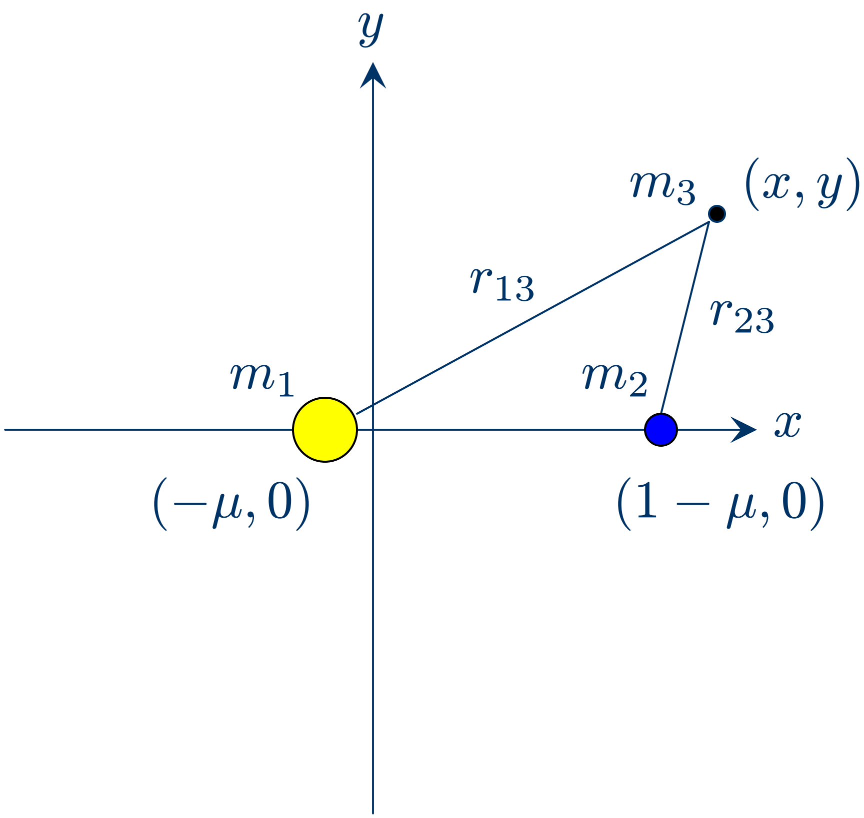
Outline

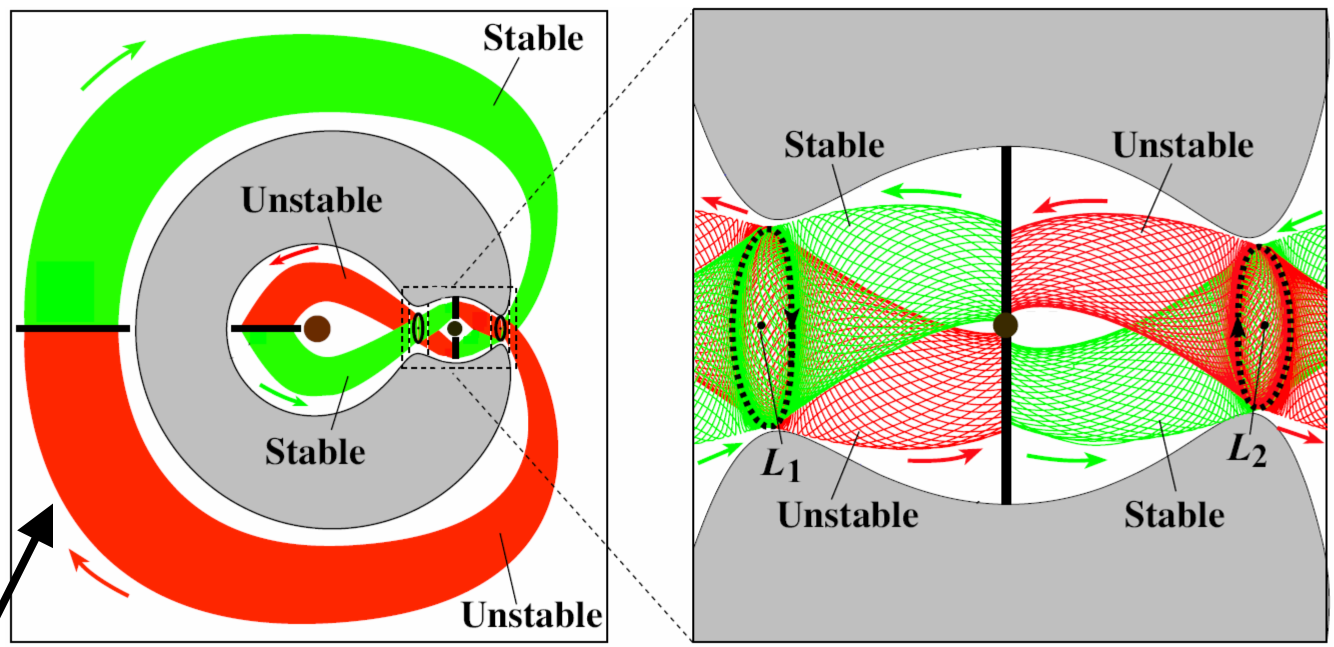
- Shoot the Moon
  - DMOC
  - Invariant Manifolds
- Saturnian moon tour
  - Resonant Gravity Assists
  - Invariant Manifolds

The Circular Restricted Three-Body Problem (CR3BP)



Invariant Manifolds

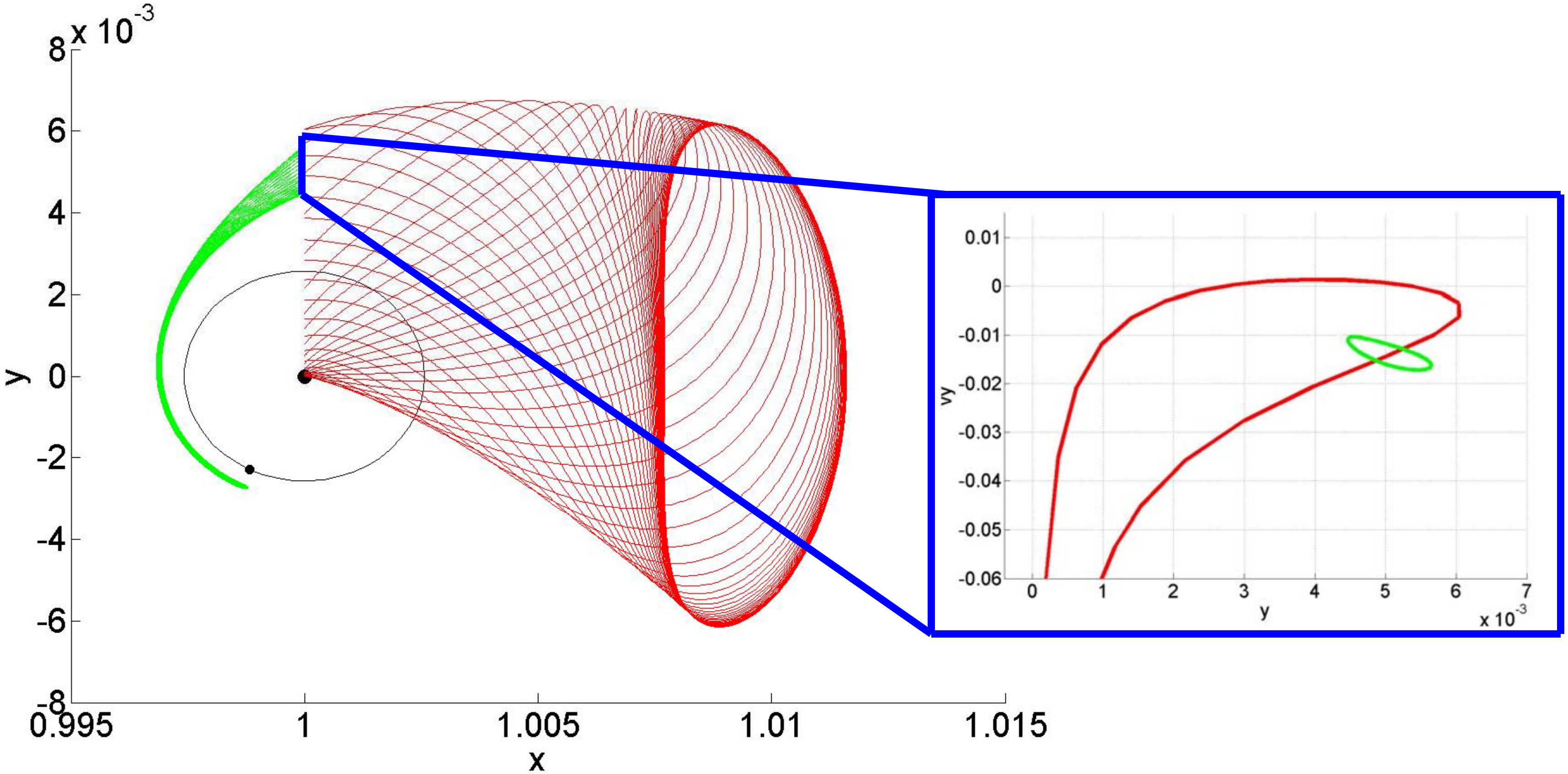
Stable and unstable manifolds of the L_1 and L_2 Lyapunov orbits



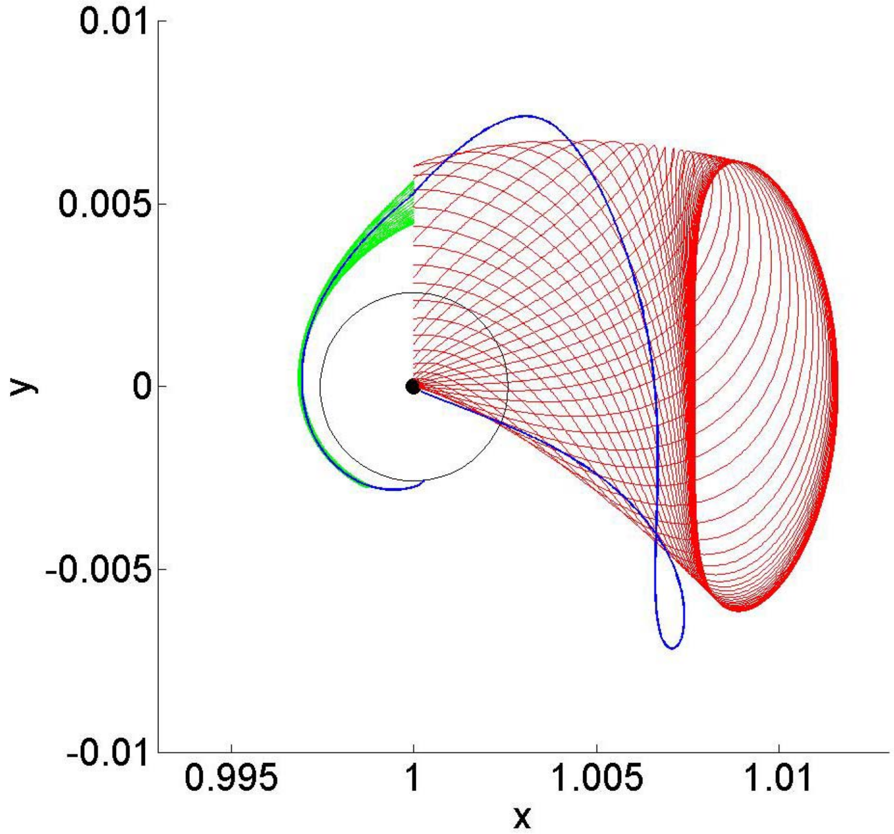
Projection of cylindrical
tubes onto position space

Ross, S.D., "Cylindrical Manifolds and Tube Dynamics in the Restricted Three-Body Problem" (PhD thesis, California Institute of Technology, 2004), 109.

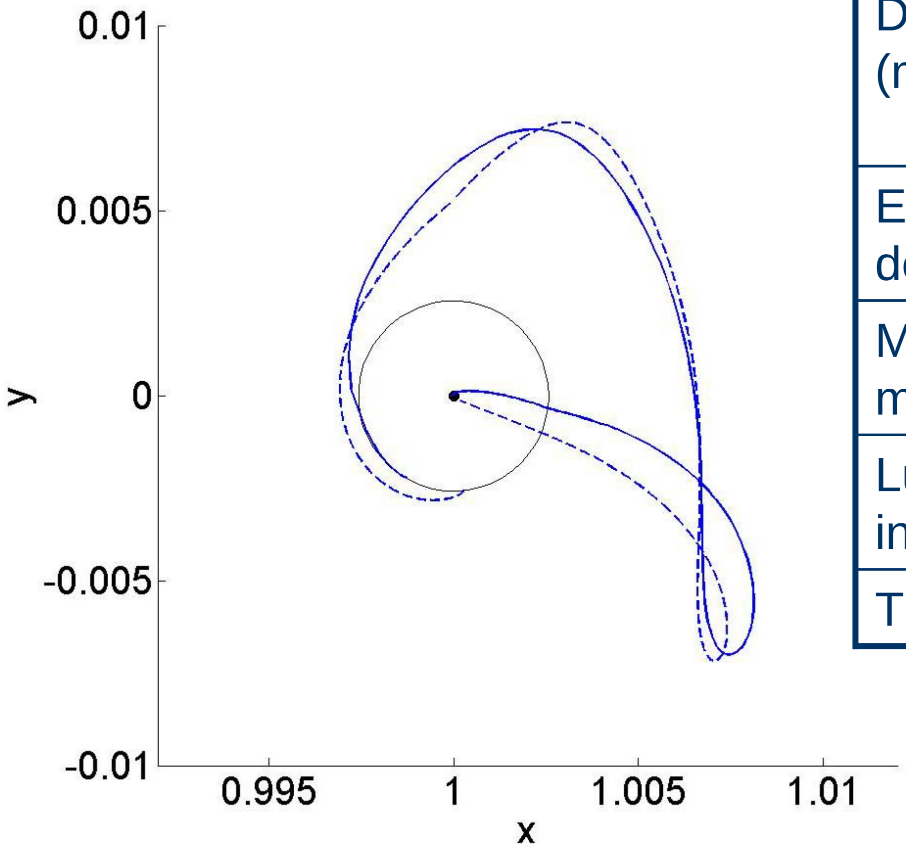
Shoot the Moon



Shoot the Moon



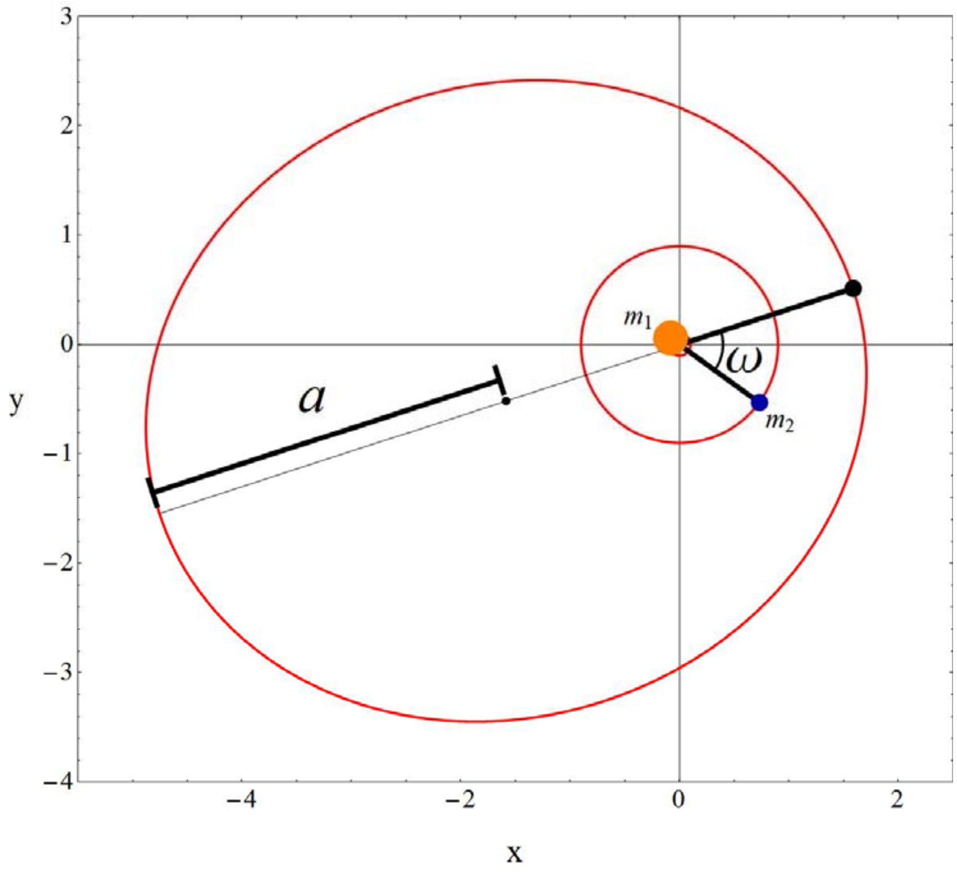
Shoot the Moon



Delta V (m/s)	Classic Hohmann Transfer	Multibody Approach
Earth orbit departure	3150	3210
Mid-course maneuvers	0	~0
Lunar orbit insertion	810	640
Total	3960	3850

Net Savings: 110 m/s

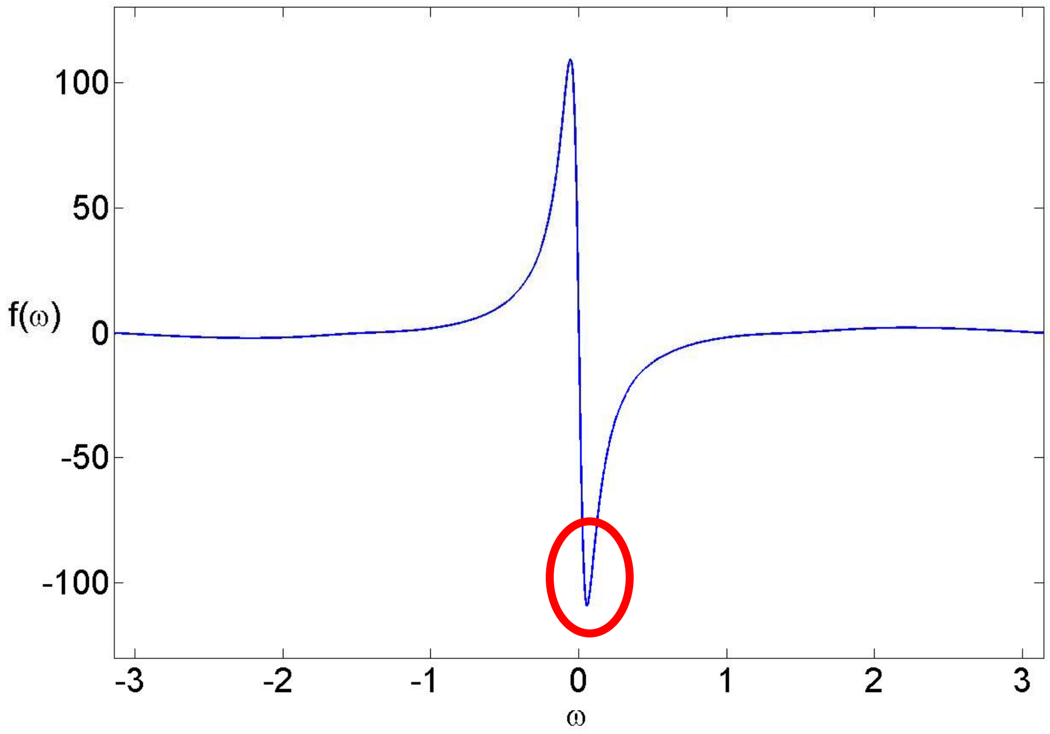
Saturnian Moon Tour



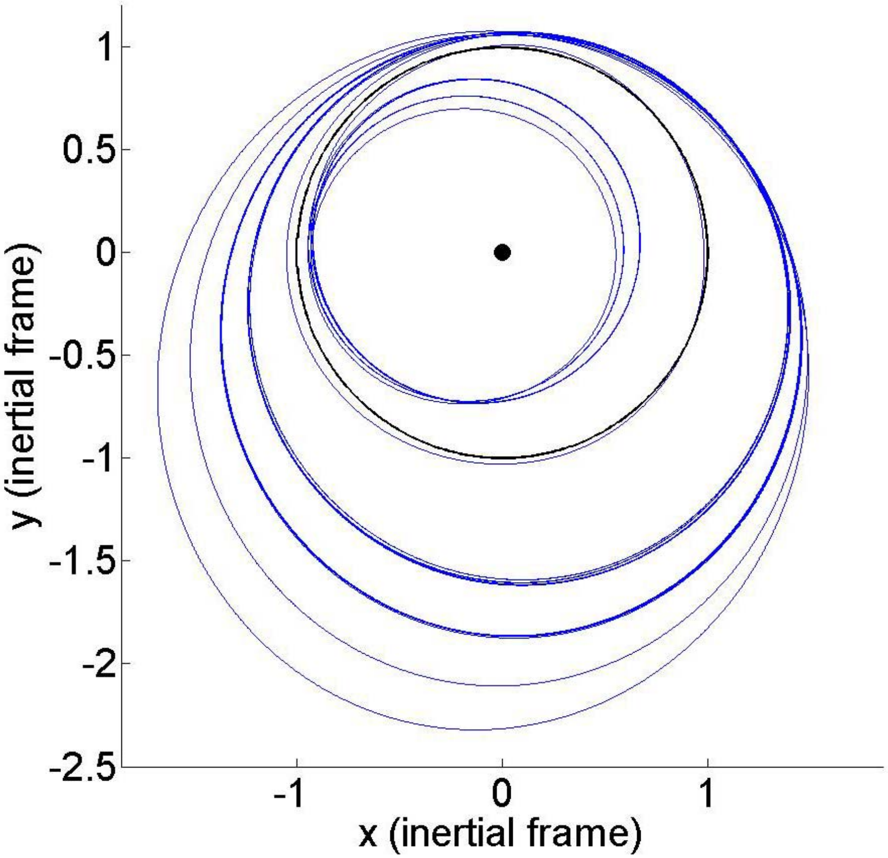
$$K = -\frac{1}{2a}$$

Saturnian Moon Tour

$$\begin{pmatrix} \omega_{n+1} \\ K_{n+1} \end{pmatrix} = \begin{pmatrix} \omega_n - 2\pi(-2K_{n+1})^{-3/2} \mod 2\pi \\ K_n + \mu f(\omega_n) \end{pmatrix}$$

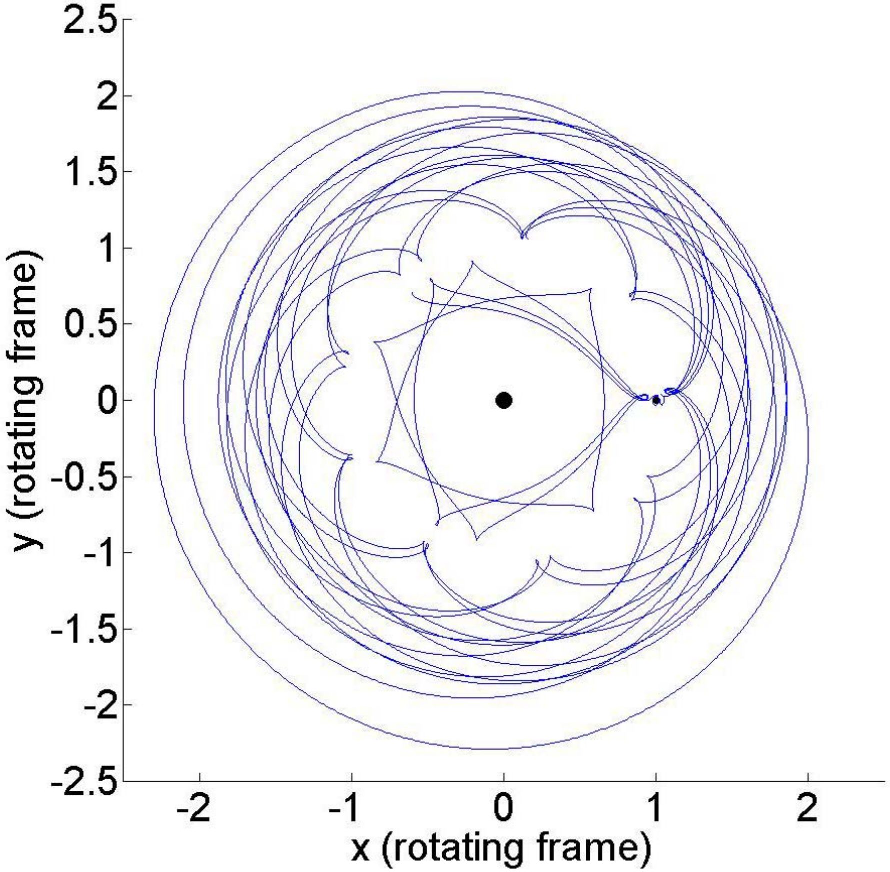


Saturnian Moon Tour



$$\Delta V = 8 \text{ m/s}$$

Saturnian Moon Tour



$$\Delta V = 8 \text{ m/s}$$

Further Study

- Continuation to inner moons of Saturn
- Comparison with standard trajectory design techniques
- Analysis of trade-off between Delta-V vs. time-of-flight

Further Reading

- W. S. Koon, M. W. Lo, J. E. Marsden, and S. D. Ross, *Dynamical Systems, the Three-Body Problem and Space Mission Design*. Springer, 2006.

- O. Junge, J. E. Marsden, and S. Ober-Blöbaum, "Discrete mechanics and optimal control," Proceedings of the 16th IFAC Conference on Decision and Control, Prague, 2005, pp. 1–6.

- S. D. Ross and D. J. Scheeres, "Multiple gravity assists, capture, and escape in the restricted three-body problem," SIAM Journal on Applied Dynamical Systems, Vol. 6, 2007, pp. 576–596.

Acknowledgments

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- The Aerospace Corporation

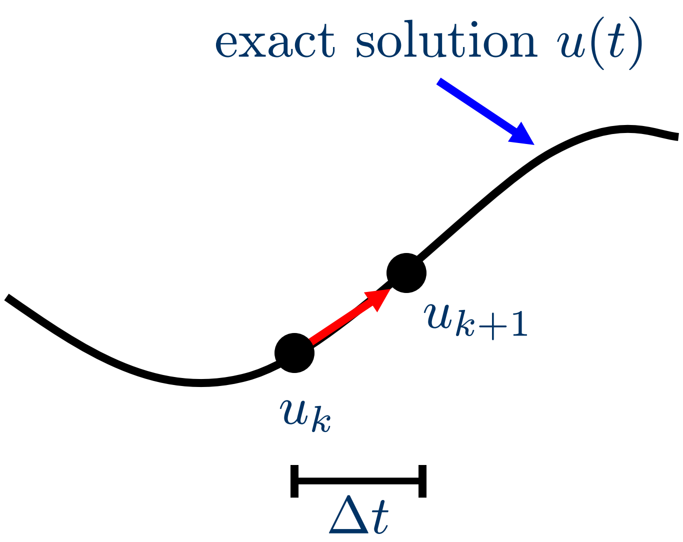
Numerical Methods

- Standard numerical integration algorithms:

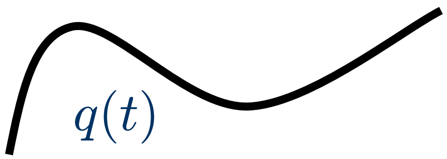
$$M\ddot{q} = F(q)$$

$$\begin{aligned}\dot{u} &= v \\ \dot{v} &= M^{-1}F(u)\end{aligned}$$

$$\begin{aligned}u_{k+1} &= u_k + \Delta t\, v_k \\ v_{k+1} &= v_k + \Delta t\, M^{-1}F(u_k)\end{aligned}$$



Variational Integrators

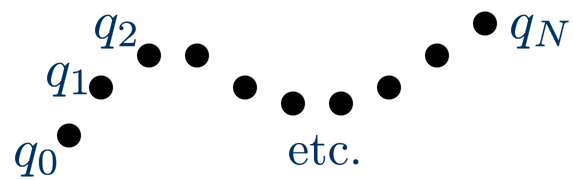


Continuous:
Extremize the integral

$$\int_0^T L(q, \dot{q})\,\mathrm{d}t$$

Arrive at the
Euler-Lagrange equations

$$\frac{\partial L}{\partial q} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}} = 0$$



Discrete:
Extremize the sum

$$\sum_{k=0}^{N-1} L_d(q_k, q_{k+1}, \Delta t)$$

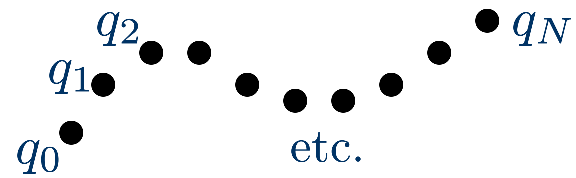
Arrive at the discrete
Euler-Lagrange equations

$$D_2 L_d(q_{k-1}, q_k, \Delta t) + D_1 L_d(q_k, q_{k+1}, \Delta t) = 0$$

DMOC

Minimize:

$$\Delta V = \sum_{k=0}^{N-1} ||f_k|| \, \Delta t$$



Subject to:

$$D_2 L_d(q_{k-1}, q_k, \Delta t) + D_1 L_d(q_k, q_{k+1}, \Delta t) + \underbrace{f_{k-1}^{+} + f_k^{-}}_{\text{Thrust}} = 0$$