Outline

- Shoot the Moon
  - DMOC
  - Invariant Manifolds
- Saturnian moon tour
  - Resonant Gravity Assists
  - Invariant Manifolds
The Circular Restricted Three-Body Problem (CR3BP)
Invariant Manifolds

Stable and unstable manifolds of the $L_1$ and $L_2$ Lyapunov orbits

Projection of cylindrical tubes onto position space

Shoot the Moon
Shoot the Moon
Shoot the Moon

<table>
<thead>
<tr>
<th>Delta V (m/s)</th>
<th>Classic Hohmann Transfer</th>
<th>Multibody Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth orbit departure</td>
<td>3150</td>
<td>3210</td>
</tr>
<tr>
<td>Mid-course maneuvers</td>
<td>0</td>
<td>~0</td>
</tr>
<tr>
<td>Lunar orbit insertion</td>
<td>810</td>
<td>640</td>
</tr>
<tr>
<td>Total</td>
<td>3960</td>
<td>3850</td>
</tr>
</tbody>
</table>

Net Savings: 110 m/s
Saturnian Moon Tour

\[ K = -\frac{1}{2a} \]
Saturnian Moon Tour

\[
\begin{pmatrix}
\omega_{n+1} \\
K_{n+1}
\end{pmatrix} = \begin{pmatrix}
\omega_n - 2\pi (-2K_{n+1})^{-3/2} \\
K_n + \mu f(\omega_n) \pmod{2\pi}
\end{pmatrix}
\]
Saturnian Moon Tour

\[ \Delta V = 8 \, \text{m/s} \]
Saturnian Moon Tour

\[ \Delta V = 8 \text{ m/s} \]
Further Study

- Continuation to inner moons of Saturn
- Comparison with standard trajectory design techniques
- Analysis of trade-off between Delta-V vs. time-of-flight
Further Reading


Acknowledgments

- Dr. Jerrold Marsden
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- Sigrid Leyendecker
- Caltech SURF Program
- The Aerospace Corporation
**Numerical Methods**

- **Standard numerical integration algorithms:**

  \[ M\ddot{q} = F(q) \]
  \[ \dot{u} = v \]
  \[ \dot{v} = M^{-1}F(u) \]

  \[ u_{k+1} = u_k + \Delta t v_k \]
  \[ v_{k+1} = v_k + \Delta t M^{-1}F(u_k) \]

\[ \text{exact solution } u(t) \]
Variational Integrators

Continuous:
Extremize the integral

$$\int_0^T L(q, \dot{q}) \, dt$$

Arrive at the Euler-Lagrange equations

$$\frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} = 0$$

Discrete:
Extremize the sum

$$\sum_{k=0}^{N-1} L_d(q_k, q_{k+1}, \Delta t)$$

Arrive at the discrete Euler-Lagrange equations

$$D_2 L_d(q_{k-1}, q_k, \Delta t) + D_1 L_d(q_k, q_{k+1}, \Delta t) = 0$$
Minimize:
\[ \Delta V = \sum_{k=0}^{N-1} \|f_k\| \Delta t \]

Subject to:
\[ D_2 L_d(q_{k-1}, q_k, \Delta t) + D_1 L_d(q_k, q_{k+1}, \Delta t) + f_{k-1}^+ + f_k^- = 0 \]
Thrusted