Direct simulation and regularization modelling of rotating turbulence

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Rotating decaying turbulence

Importance
• Engineering flows: jet engines, turbines
• Large-scale flows: Earth’s atmosphere, ocean circulation

Turbulence in a rotating frame of reference

Rotating cubic box with angular velocity $\Omega$.

Approach
Incompressible Navier-Stokes equations with Coriolis force:
$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{v} + \frac{\Omega \times \mathbf{v}}{Re}.$$

Characteristic numbers:
$$Re = \frac{U L}{v} ; \quad Ro = \frac{\Omega L}{v}.$$

Regularity models
Regularized velocity: $\mathbf{\bar{v}} = (1 - \alpha^2) \mathbf{v} - \frac{\alpha}{Re} \mathbf{\bar{v}}$.

Leray model:
$$\frac{\partial \mathbf{\bar{v}}}{\partial t} + (\mathbf{\bar{v}} \cdot \nabla) \mathbf{\bar{v}} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{\bar{v}} + \frac{\Omega \times \mathbf{\bar{v}}}{Re}.$$

LANS-$\alpha$
$$\frac{\partial \mathbf{\bar{v}}}{\partial t} + (\mathbf{\bar{v}} \cdot \nabla) \mathbf{\bar{v}} + \sum_{l=1}^{3} \sigma_l \nabla v_l + \nabla p = \frac{\Omega \times \mathbf{\bar{v}}}{Re},$$

where modified pressure $P = p - \frac{1}{3} |\mathbf{\bar{v}}|^2$.

Computational method
• Spin-up of decaying turbulence
• Pseudo-spectral fully dealiased simulations
• Resolution from $128^3$ (LES) up to $512^3$ (DNS)
• Spectral helical wave decomposition – larger time-steps
• Initial Taylor-Reynolds number $Re_L = 92$ and 200
• Number of rotation rates $\Omega = 0.1, 2.5, 10, 20, 50, 100$

Decay exponent $\alpha$ and spectral exponent $\rho$

<table>
<thead>
<tr>
<th>$\Omega$</th>
<th>$\rho$</th>
<th>Nonconfined</th>
<th>Confined</th>
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<tbody>
<tr>
<td>$\Omega = 0$</td>
<td>$\rho = \frac{5}{3}$</td>
<td>$\alpha = 6/5$</td>
<td>$\alpha = 2$</td>
</tr>
<tr>
<td>$\Omega \neq 0$</td>
<td>$\rho = 2$</td>
<td>$\alpha = 3/5$</td>
<td>$\alpha = 1$</td>
</tr>
<tr>
<td>$\Omega \neq 0$</td>
<td>$\rho = 3$</td>
<td>$\alpha = 0$</td>
<td>$\alpha = 0$</td>
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Rotating (I) - energy transfer time-scale $T^{-1}$
Rotating (II) - totally inhibited energy transfer (Kraichnan)

Testing hypothesis with DNS

Direct Simulations

Competition: 3D turbulence - 2D structuring

Vertical structuring of flow for turbulence at $Ro = 92$.

Anisotropic alteration of energy spectrum

Energy spectra components at $t = 2.5$ for $Ro = \infty$, 0.2, 0.1, 0.02.

Decay of kinetic energy

Slower kinematic energy decay at stronger rotation:

$Ro = \infty$, 1.0, 0.5, 0.2, 0.1, 0.05, 0.02, 0.01

$E \sim (t_2 - t_1)^{\lambda_1}$

Exponent of the energy decay computed over various time-intervals $t_2 - t_1$.

Conclusions

• Cross-over from 3D to 2D dynamics as $Ro$ decreases (seen in flow structuring and energy spectra)
• Accurate predictions of Leray and LANS-$\alpha$ models for lower Reynolds number
• Energy exponent transition from $-5/3$ ($Ro \rightarrow \infty$) to $-3$ ($Ro = 0$) for the Leray model at higher Reynolds number