Direct simulation and regularization modelling of rotating turbulence

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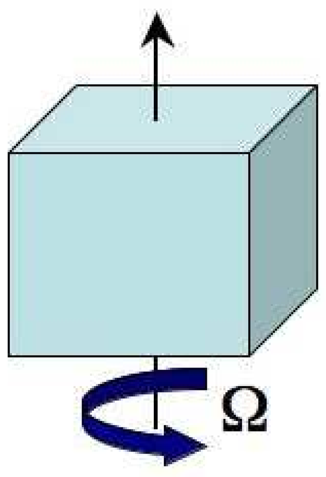
Rotating decaying turbulence

Importance

- Engineering flows: jet engines, turbines
- Large-scale flows: Earth's atmosphere, ocean circulation

Turbulence in a rotating frame of reference



Rotating cubic box with angular velocity Ω

Approach

Incompressible Navier-Stokes equations with Coriolis force:

$$\begin{cases} \partial_t \mathbf{v} - \frac{1}{Re}\Delta \mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} + \nabla p = \frac{\mathbf{v}\times\mathbf{\Omega}}{Ro} \\ \nabla\cdot\mathbf{v} = 0 \end{cases}$$

Characteristic numbers:

$$Re = \frac{U_r L_r}{\nu} \quad ; \quad Ro = \frac{U_r}{2\Omega L_r}$$

Regularization models

Regularized velocity: $\bar{\mathbf{v}} = (1-\alpha^2\Delta)^{-1}\mathbf{v}$, α - lagrangian distance of fluctuations from the mean trajectory

Leray:

$$\begin{cases} \partial_t \mathbf{v} - \frac{1}{Re}\Delta \mathbf{v} + (\bar{\mathbf{v}}\cdot\nabla)\mathbf{v} + \nabla p = \frac{\bar{\mathbf{v}}\times\mathbf{\Omega}}{Ro} \\ \nabla\cdot\mathbf{v} = 0 \end{cases}$$

LANS-α:

$$\begin{cases} \partial_t \mathbf{v} - \frac{1}{Re}\Delta \mathbf{v} + (\bar{\mathbf{v}}\cdot\nabla)\mathbf{v} + \sum_{j=1}^{3} v_j \nabla \bar{v}_j + \nabla P = \frac{\bar{\mathbf{v}}\times\mathbf{\Omega}}{Ro} \\ \nabla\cdot\mathbf{v} = 0 \end{cases}$$

where modified pressure $P = p - \frac{1}{2}(|\bar{\mathbf{v}}|^2 + \alpha^2|\nabla\bar{\mathbf{v}}|^2)$

Computational method

- spin-up of decaying turbulence
- pseudo-spectral fully dealiased simulations
- resolution from 128^3 (LES) up to 512^3 (DNS)
- spectral helical-wave decomposition – larger time-steps
- initial Taylor-Reynolds number $R_\lambda = 92$ and 200
- number of rotation rates $\Omega = 0, 1, 2, 5, 10, 20, 50, 100$

Decay exponent n and spectral exponent p

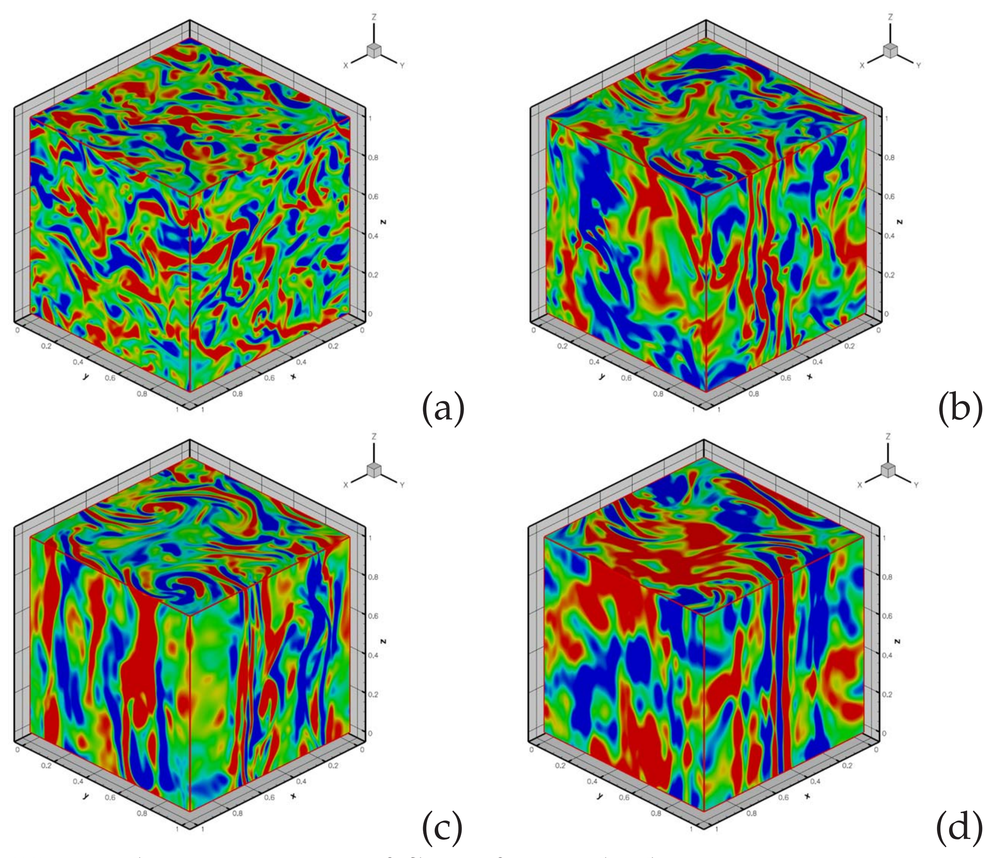
	p	Nonconfined	Confined
$\Omega = 0$:	5/3	$n = 6/5$	$n = 2$
$\Omega \neq 0$ (I):	2	$n = 3/5$	$n = 1$
$\Omega \neq 0$ (II):	3	$n = 0$	$n = 0$

Rotating (I) - energy transfer time-scale Ω^{-1}
Rotating (II) - totally inhibited energy transfer (Kraichnan)

Testing hypothesis with DNS

Direct Simulations

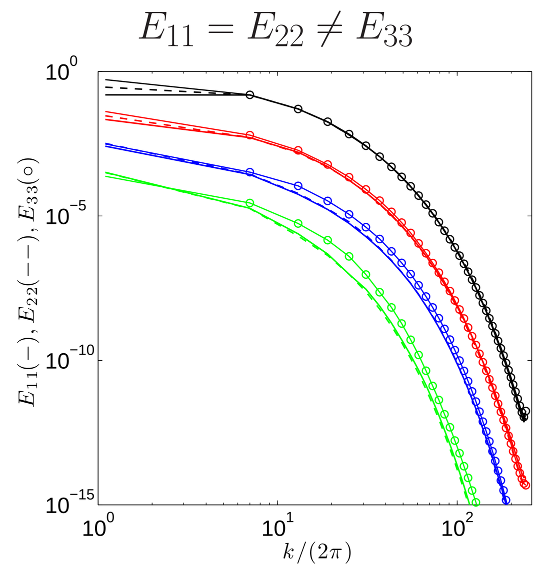
Competition: 3D turbulence - 2D structuring



Vertical structuring of flow for turbulence at $R_\lambda = 92$: vorticity ω_y contours at $t = 2.5$ for $Ro = \infty, 0.2, 0.1, 0.02$ (a,b,c,d)

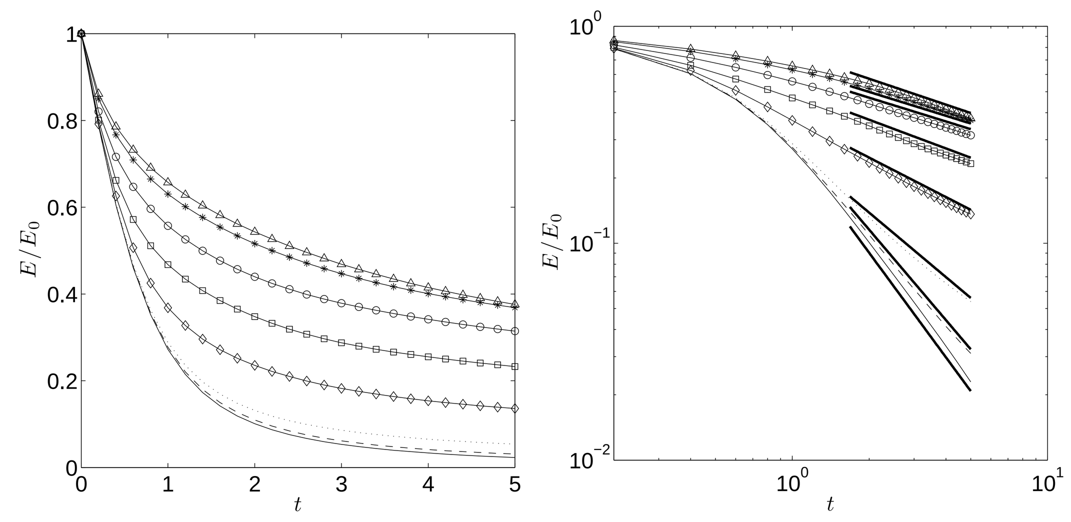
Anisotropic alteration of energy spectrum

$E_{11} = E_{22} \neq E_{33}$



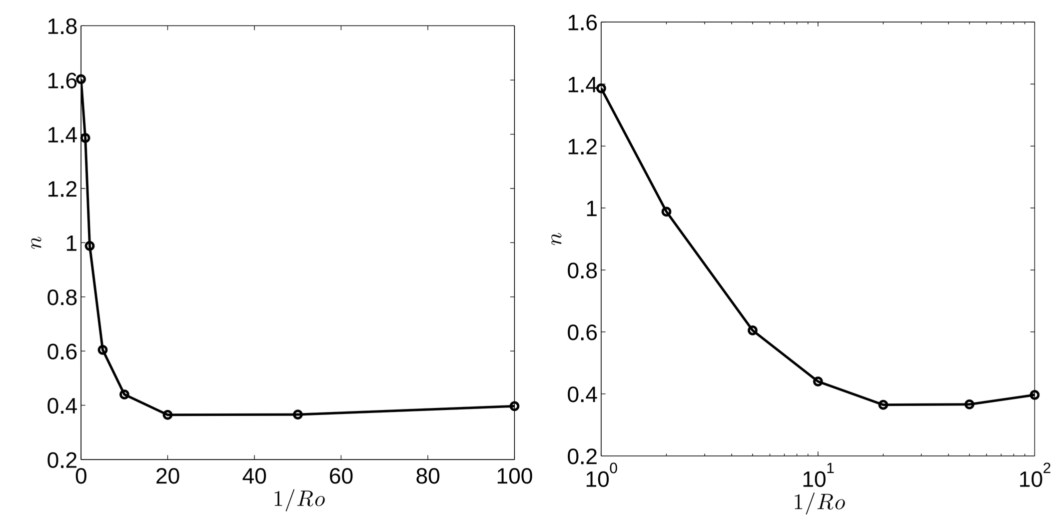
Energy spectra components at $t = 2.5$ for $Ro = \infty$, 0.2, 0.1, 0.02

Decay of kinetic energy



Slower kinetic energy decay at stronger rotation: $Ro = \infty, 1, 0.5, 0.2, 0.1, 0.05, 0.02, 0.01$

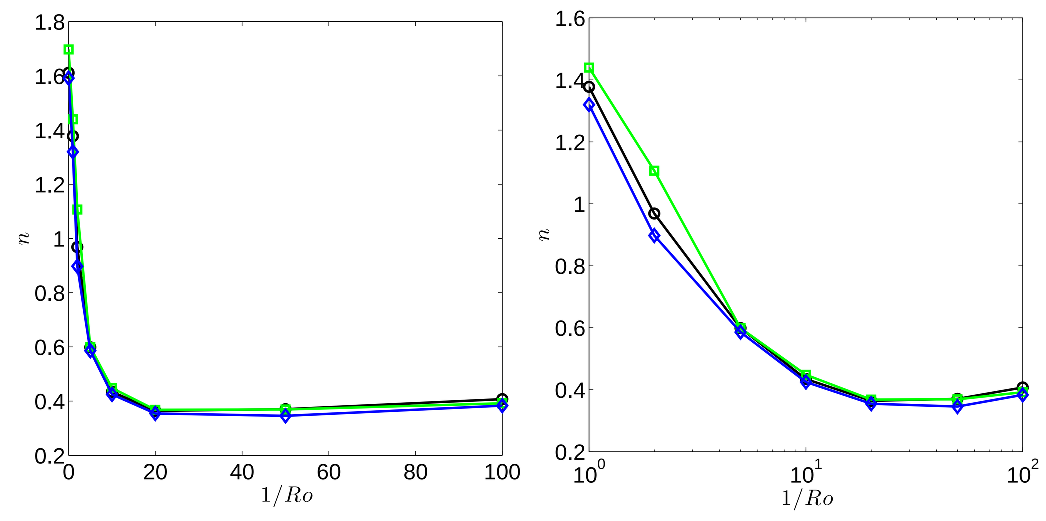
$$E \sim (t_2 - t_1)^{-n}$$



Exponent of the energy decay computed over various time-intervals $t_2 - t_1$

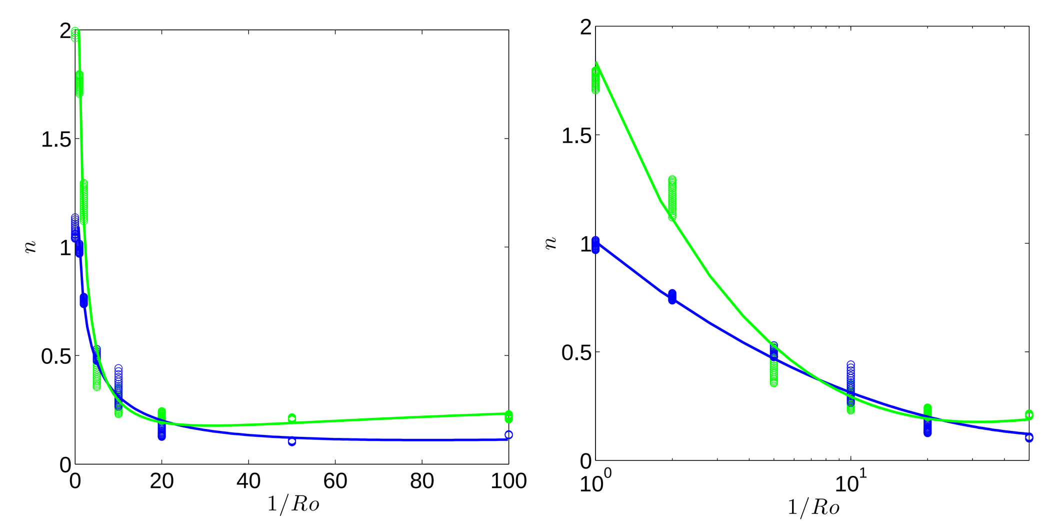
Regularization models

Energy decay exponent at $R_\lambda = 92$



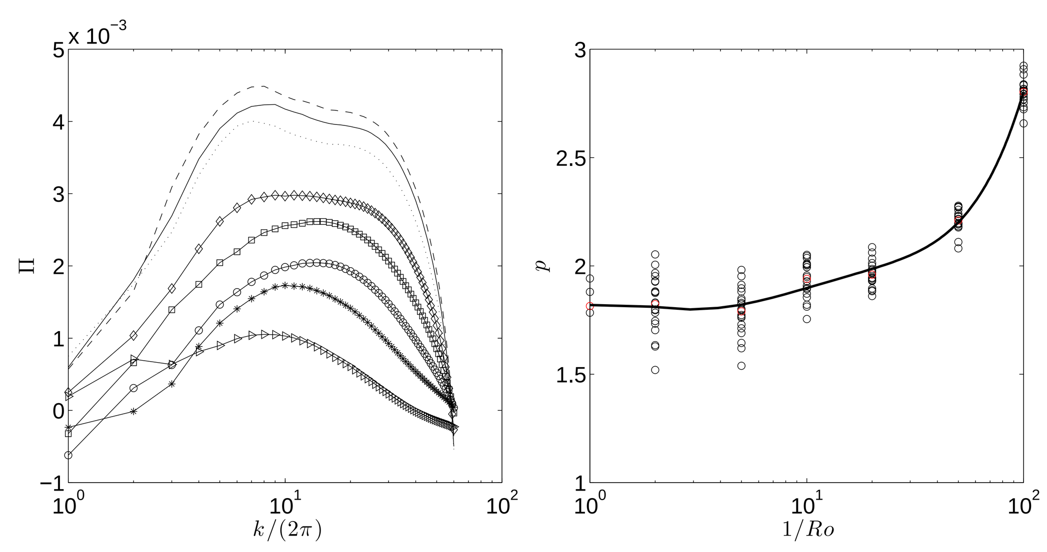
DNS (black), LES simulations: Leray, LANS-α

Energy decay exponent at $R_\lambda = 200$



LES simulations: Leray, LANS-α

Leray model - spectral exponent at $R_\lambda = 200$



Average transport power spectra Π (evidence of inertial range) and energy spectrum exponent p computed in various spectral regions

Conclusions

- cross-over from 3D to 2D dynamics as Ro decreases (seen in flow structuring and energy spectra)
- accurate predictions of Leray and LANS-α models for lower Reynolds number
- energy exponent transition from $-5/3$ ($Ro \to \infty$) to -3 ($Ro \to 0$) for the Leray model at higher Reynolds number

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