The Lagrangian-Averaged Navier-Stokes alpha (LANS-α) Turbulence Model in Primitive Equation Ocean Modeling

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Outline

- POP ocean model & climate change assessment
- LANS-α implementation in POP
- Idealized test case: the channel domain
- The real thing: the North Atlantic

D2Hfest LA-UR-05-0887 July 24, 2007

CO_2 ice core record

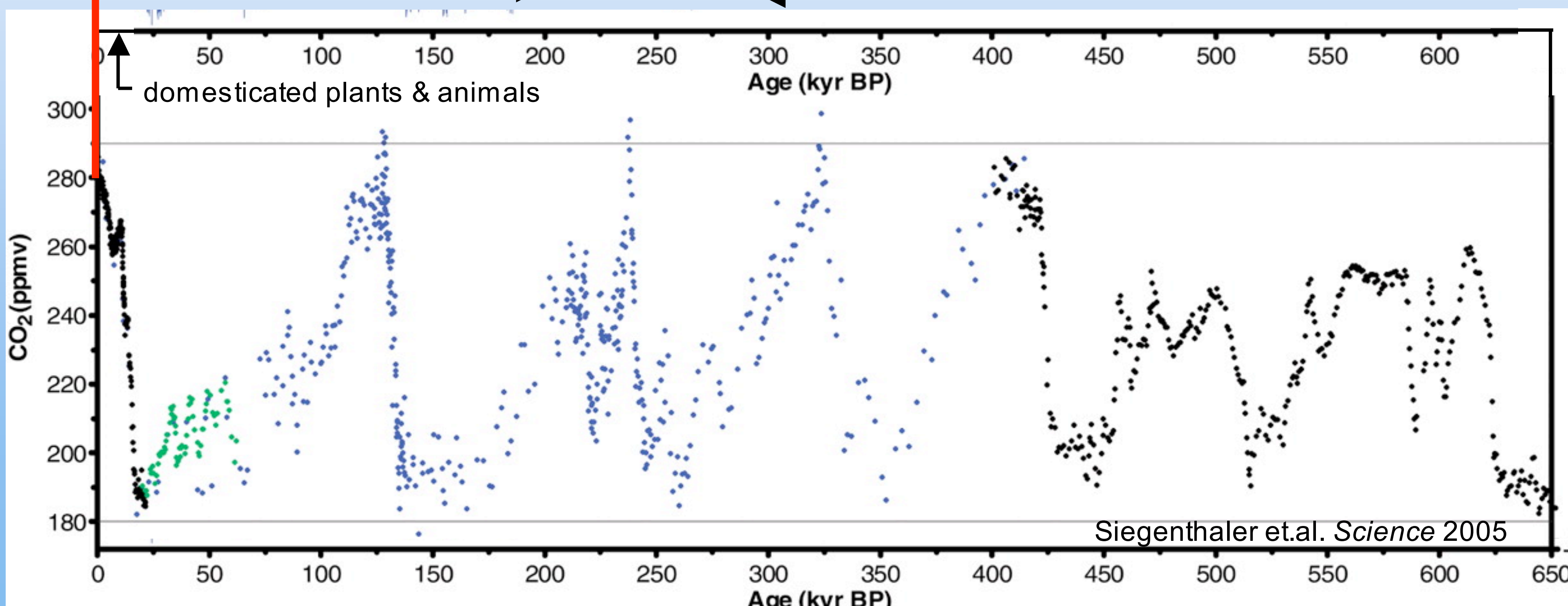
Homo sapiens neanderthalensis

currently:
380 ppm

Homo sapiens archaic

Homo sapiens sapiens

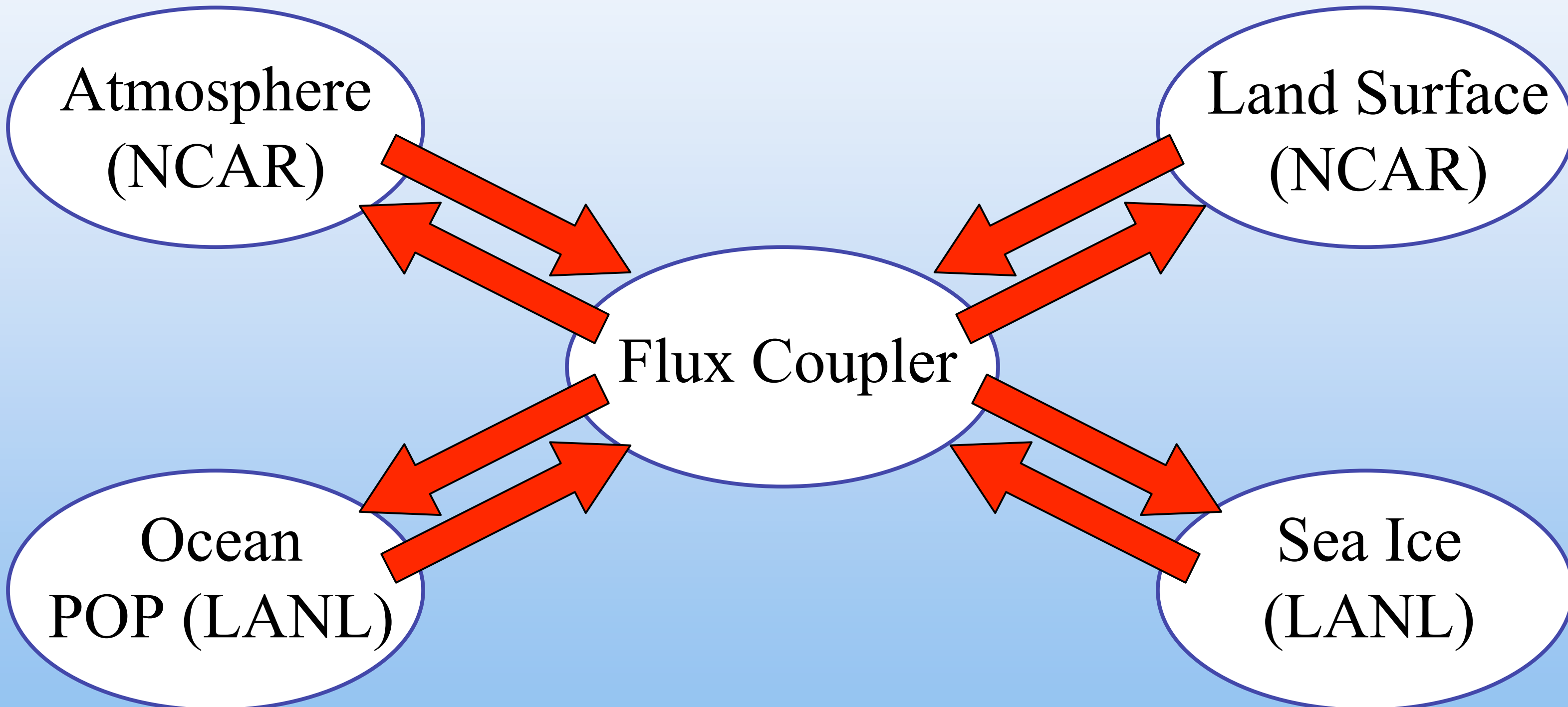
Homo erectus



Community Climate System Model

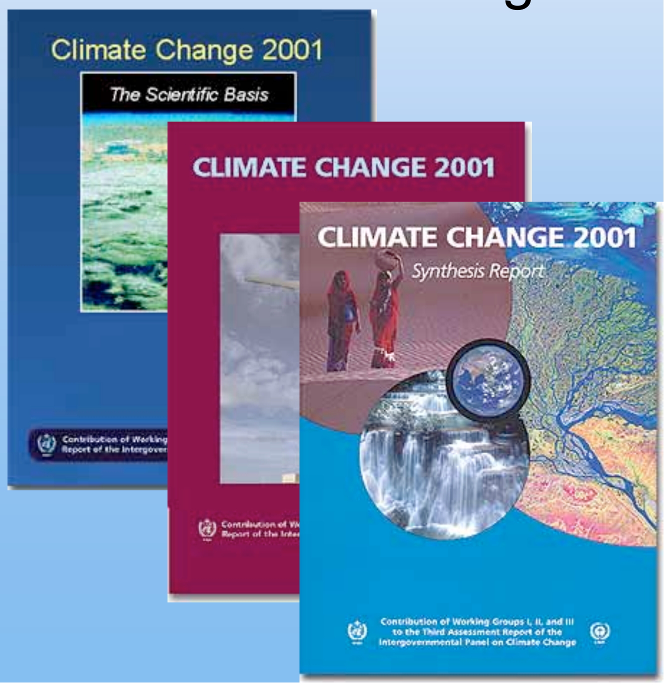
Collaboration of:

- National Center for Atmospheric Research (NCAR) in Boulder, CO
- Los Alamos National Laboratory (LANL)



IPCC - Intergovernmental Panel on Climate Change

- Created in 1988 by World Meteorological Organization (WMO) and United Nations Environment Programme (UNEP)
- Role of IPCC: assess on a comprehensive, objective, open and transparent basis the scientific, technical and socio-economic information relevant to understanding:
  - the **scientific basis** of risk of human-induced climate change
  - its potential **impacts** and
  - options for **adaptation and mitigation**.
- Main activity: Assessment reports
  - Third Assessment Report: 2001
  - Fourth Assessment Report: 2007
  - Fifth: planned for 2013

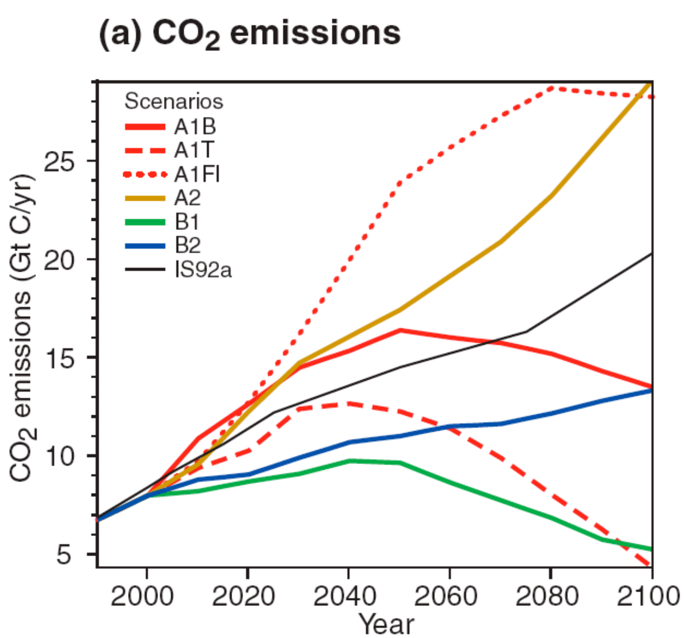


IPCC scenarios of future emissions

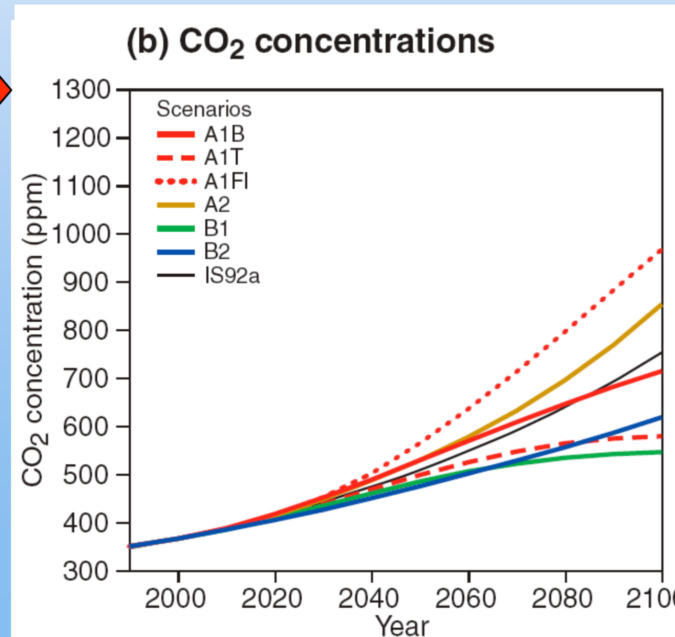
	A: *slower* conversion to clean & efficient technologies	**B**: *faster* conversion to clean & efficient technologies
1: global population levels off, declines after 2050	**A1FI**: fossil intensive **A1T**: non-fossil intensive **A1B**: balance of F&T	**B1**
2: continuously increasing population	**A2**	**B2**

IS92a: business as usual (extrapolation from current rates of increase)

economic models



carbon cycle models

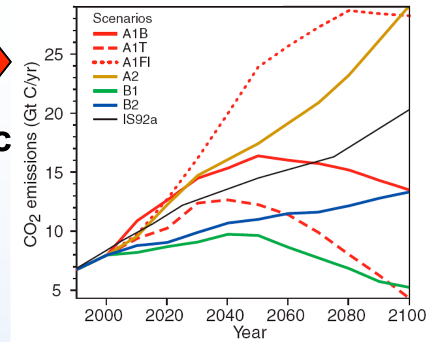


scenarios

	A: *slower* conversion to clean & efficient technologies	**B**: *faster* conversion to clean & efficient technologies
1: global population levels off, declines after 2050	**A1FI**: fossil intensive **A1T**: non-fossil intensive **A1B**: balance of F&T	**B1**
2: continuously increasing population	**A2**	**B2**

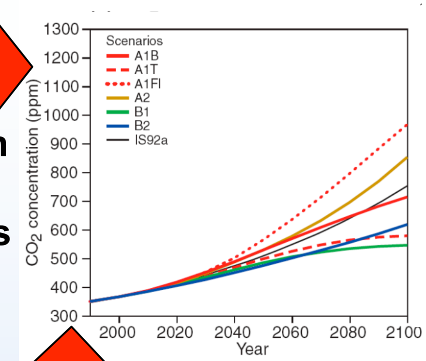
economic models

CO_2 emissions



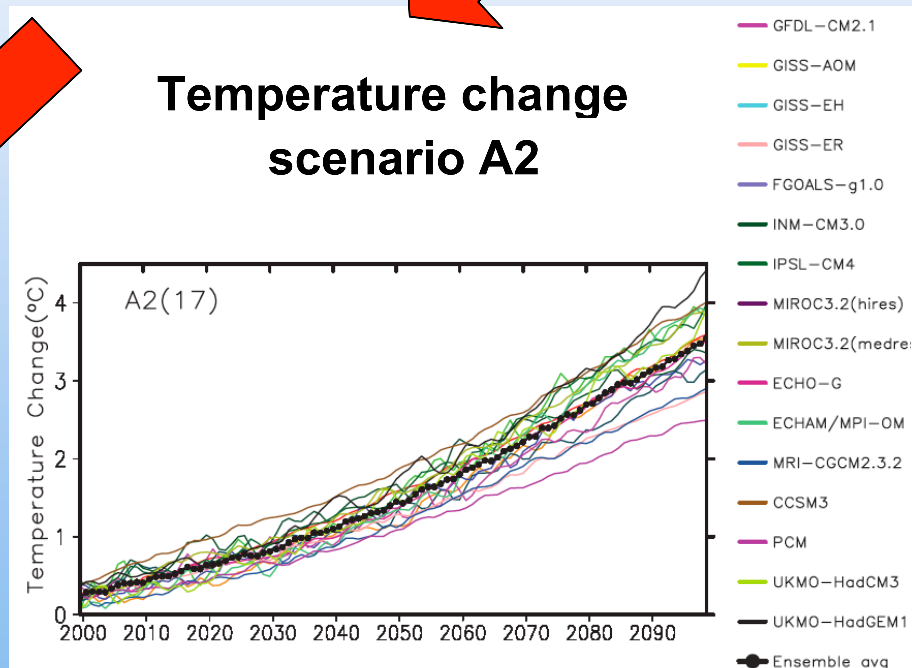
carbon cycle models

CO_2 concentration



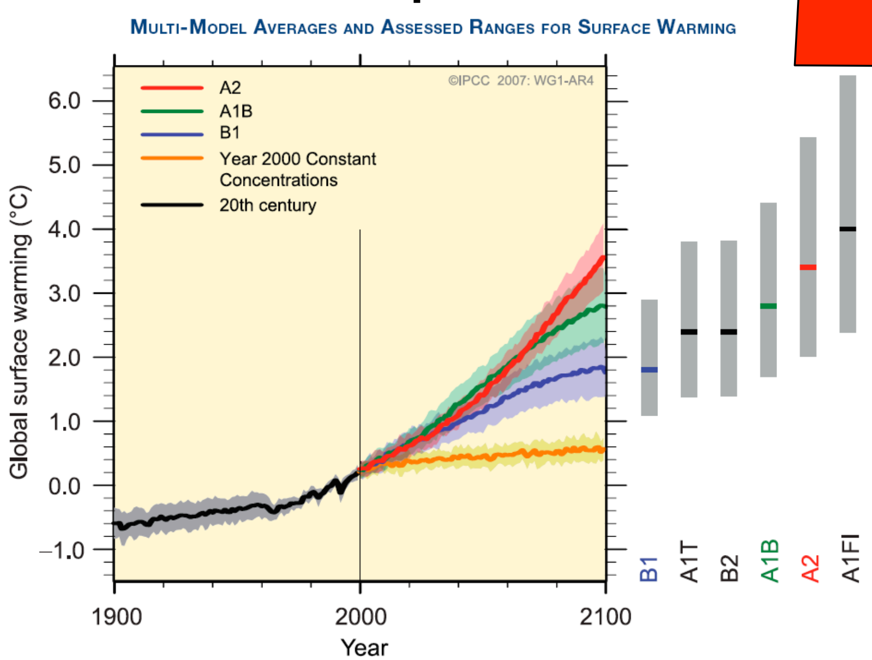
ensemble of climate models

Temperature change scenario A2

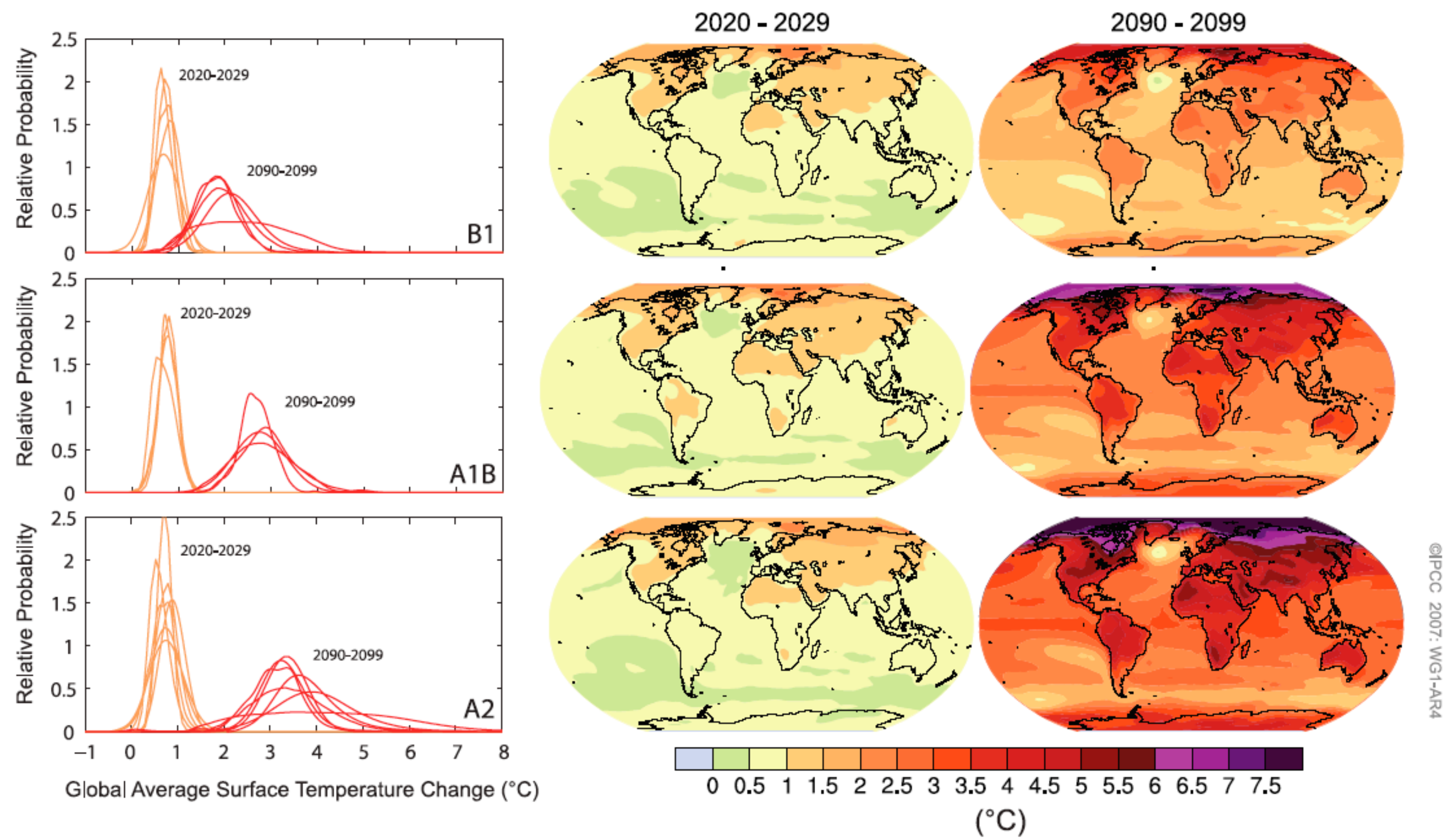


average of ensemble

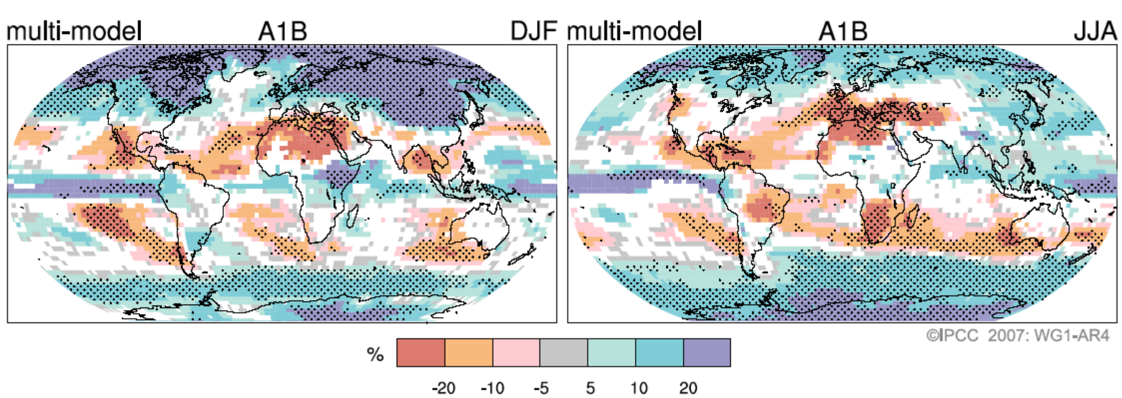
Final product



Projections of Surface Temperatures



Projected Patterns of Precipitation Changes



IPCC: Estimates of confidence

Table 4: Estimates of confidence in observed and projected changes in extreme weather and climate events. The table depicts an assessment of confidence in observed changes in extremes of weather and climate during the latter half of the 20th century (left column) and in projected changes during the 21st century (right column)[a]. This assessment relies on observational and modelling studies, as well as physical plausibility of future projections across all commonly used scenarios and is based on expert judgement (see Footnote 4). [Based upon Table 9.6]

Confidence in observed changes (latter half of the 20th century)	Changes in Phenomenon	Confidence in projected changes (during the 21st century)
Likely	**Higher maximum temperatures and more hot days over nearly all land areas**	Very likely
Very likely	**Higher minimum temperatures, fewer cold days and frost days over nearly all land areas**	Very likely
Very likely	**Reduced diurnal temperature range over most land areas**	Very likely
Likely, over many areas	**Increase of heat index[8] over land areas**	Very likely, over most areas
Likely, over many Northern Hemisphere mid- to high latitude land areas	**More intense precipitation events[b]**	Very likely, over many areas
Likely, in a few areas	**Increased summer continental drying and associated risk of drought**	Likely, over most mid-latitude continental interiors (Lack of consistent projections in other areas)
Not observed in the few analyses available	**Increase in tropical cyclone peak wind intensities[c]**	Likely, over some areas
Insufficient data for assessment	**Increase in tropical cyclone mean and peak precipitation intensities[c]**	Likely, over some areas

[a] For more details see Chapter 2 (observations) and Chapters 9, 10 (projections).

[b] For other areas there are either insufficient data of conflicting analyses.

[c] Past and future changes in tropical cyclone location and frequency are uncertain.

[8] Heat index: A combination of temperature and humidity that measures effects on human comfort

TAR
p.82

Parallel Ocean Program (POP)

- Bryan-Cox type model, z-level vertical grid, finite difference model

conservation of momentum

$$\partial_t \mathbf{u} + \underbrace{\mathbf{u}\cdot\nabla\mathbf{u}}_{\text{advection}} - \underbrace{f\times\mathbf{u}}_{\text{Coriolis}} = \underbrace{-\rho_0^{-1}\nabla p}_{\text{pressure gradient}} + \underbrace{A_M\nabla_h^2\mathbf{u} + \partial_z\mu\partial_z\mathbf{u}}_{\text{diffusion}}$$

conservation of mass for incompressible fluid

$$\nabla_h\cdot\mathbf{u} + \partial_z w = 0$$

conservation of tracers (temperature, salinity)

$$\partial_t\varphi + \underbrace{\mathbf{u}\cdot\nabla\varphi}_{\text{advection}} = \underbrace{A_H\nabla_h^2\varphi + \partial_z\kappa\partial_z\varphi}_{\text{diffusion}} + \underbrace{Q}_{\text{source/sink}}$$

hydrostatic in the vertical

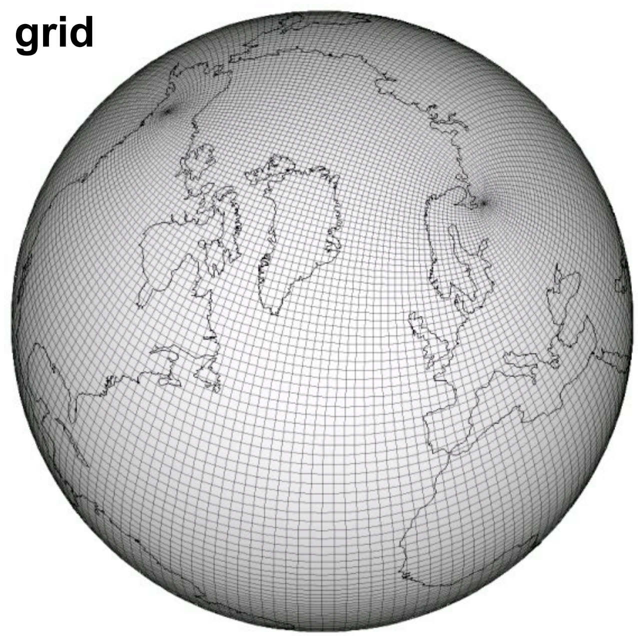
$$\frac{\partial p}{\partial z} = -\rho g$$

equation of state

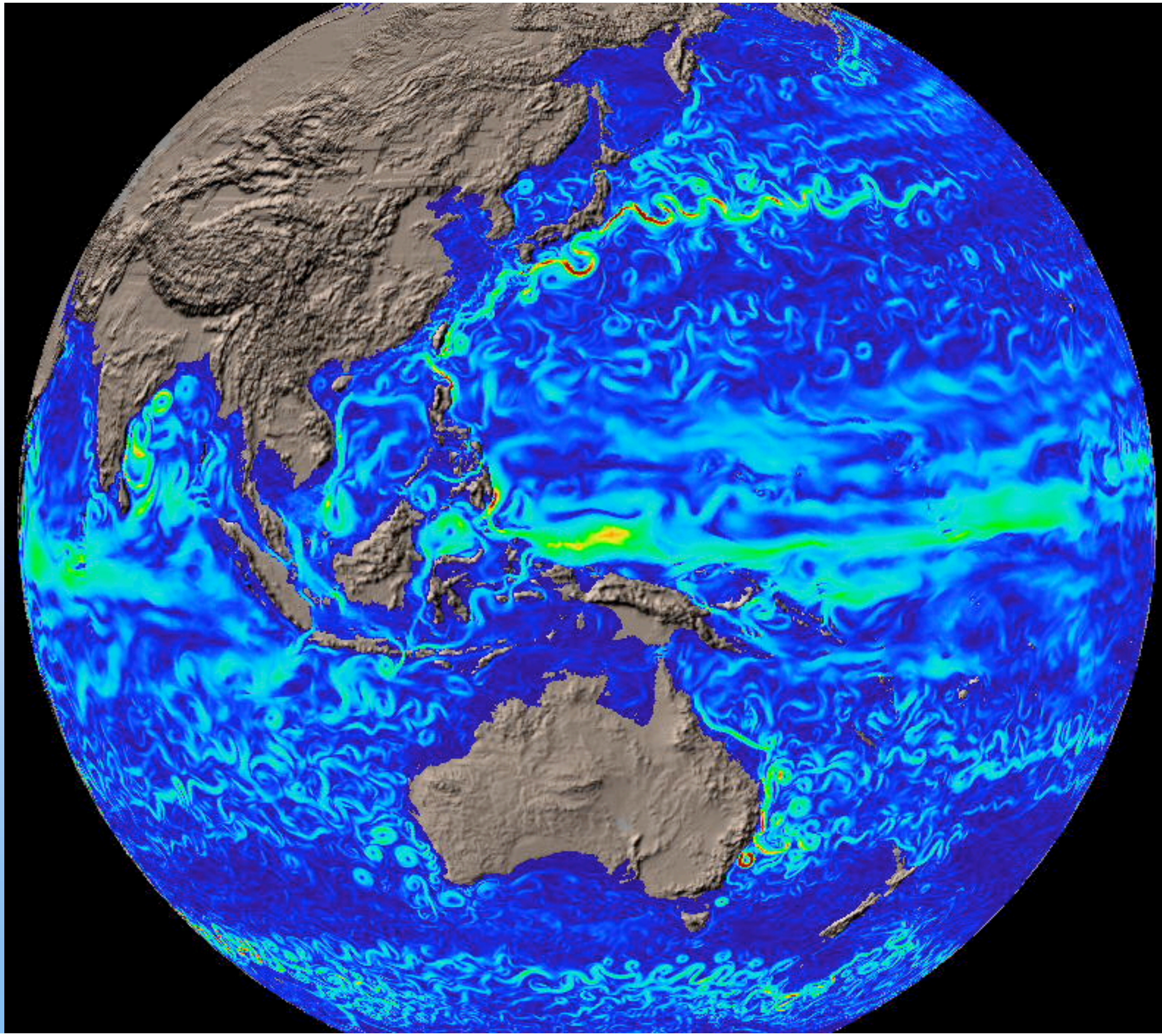
$$\rho = \rho(T,S,p)$$

$\mathbf{u}$	hor. velocity
w	vertical velocity
φ	tracer
t	time
p	pressure
ρ_0	density
T	temperature
S	salinity

grid



POP: 0.1º resolution, speed

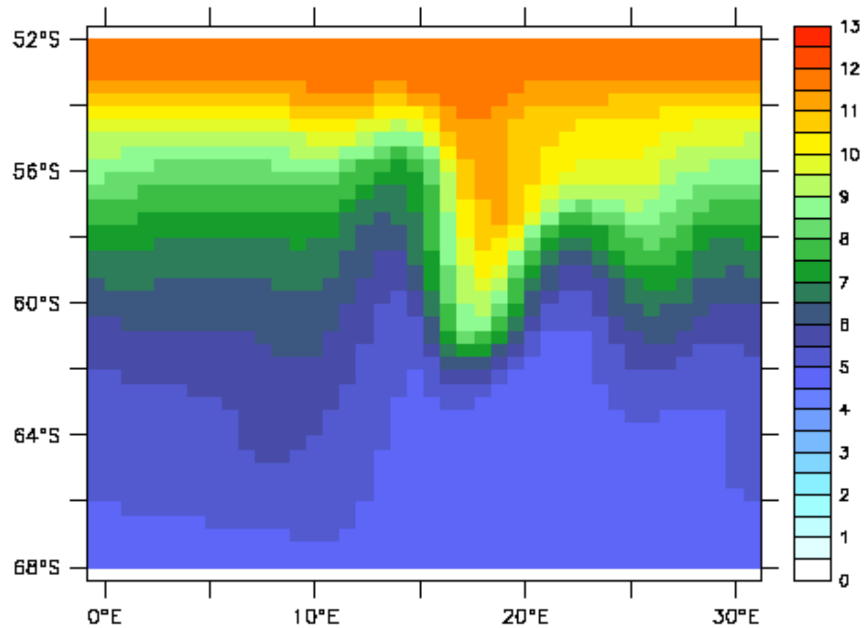


Parallel Ocean Program (POP)

Resolution is costly, but critical to the physics

Climate simulations

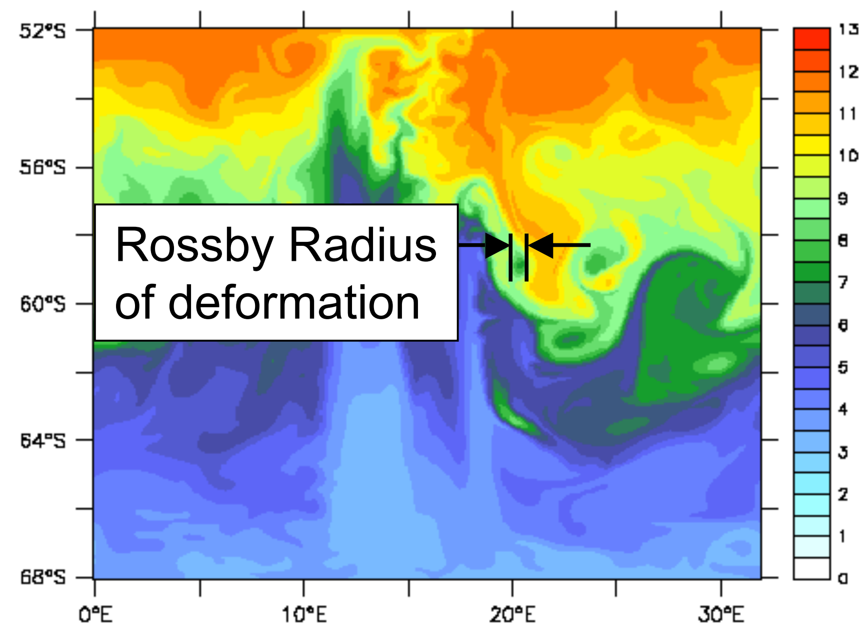
- low resolution: 1 deg (100 km)
- long duration: 100s of years
- fully coupled to atmosphere, etc.



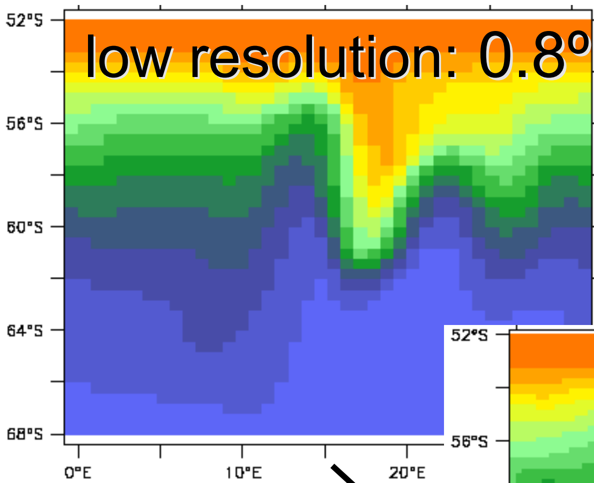
Potential temperature
0.8° x 0.8° grid

Eddy-resolving sim.

- high resolution: 0.1 deg (10 km)
- short duration: 50-100 years
- ocean only



Potential temperature
0.1° x 0.1° grid



What do you get with higher resolution?

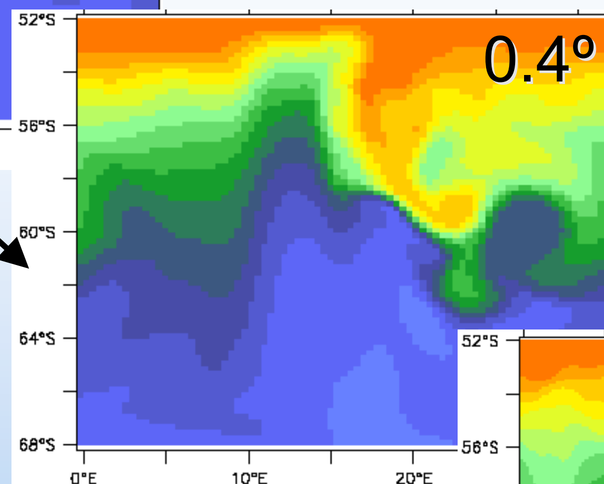
Small-scale turbulence and eddies transport energy and heat.

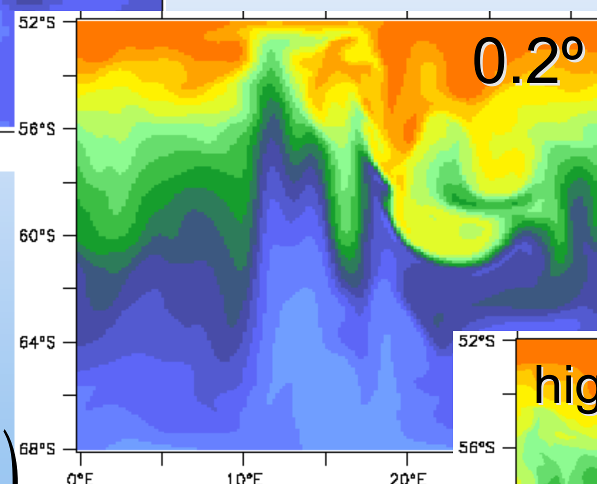
Reynolds decomposition:

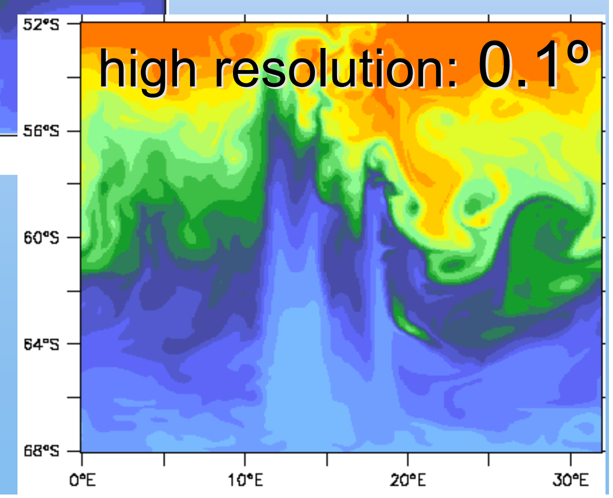
$$u = \overline{u} + u'$$

total — time average — perturbation

cost of doubling horizontal grid is factor of 10







These become more realistic with higher resolution:

- eddy heat transport: $\overline{u_2' T'}$
- eddy kinetic energy: $\frac{1}{2}\left(\overline{u_1'^2} + \overline{u_2'^2}\right)$
- feedback of small-scale features on the large-scale mean flow - important for oceanic jets
- vertical temperature profile

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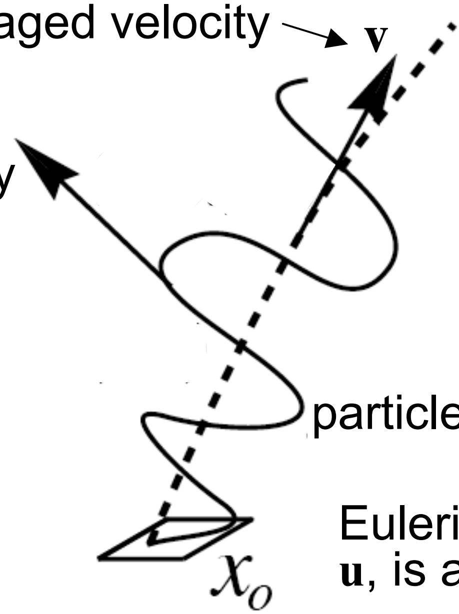
Lagrangian-Averaged Navier-Stokes Equation (LANS-α)

Two ways to take averages: Lagrangian and Eulerian



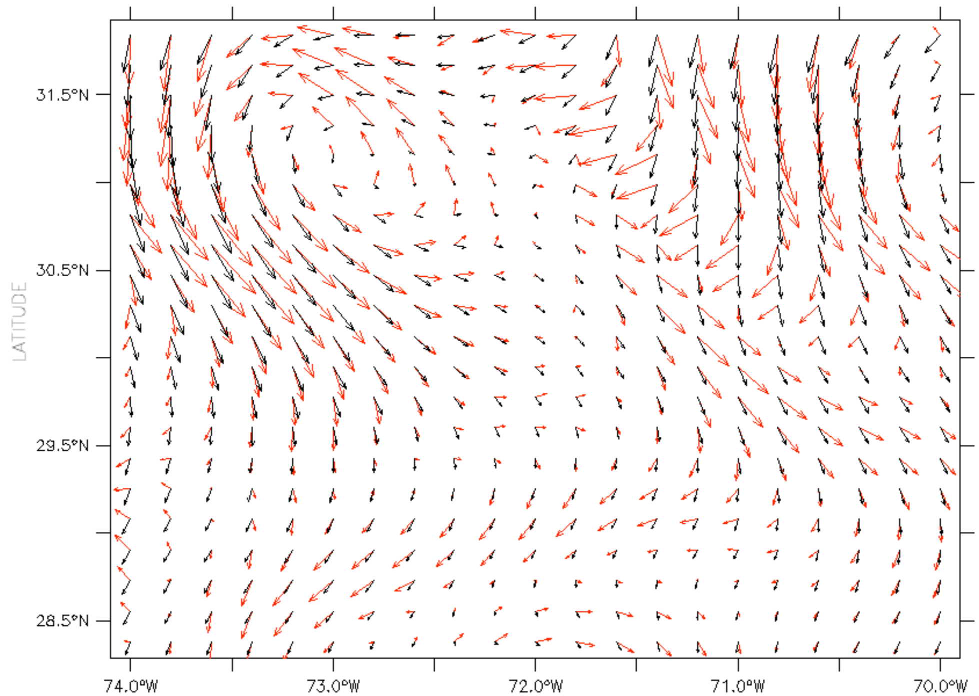
v Lagrangian averaged velocity

u Eulerian averaged velocity

$$\mathbf{u} = \left(1 - \alpha^2 \nabla^2\right)^{-1} \mathbf{v}$$

smooth ($\mathbf{u}$) — Helmholtz operator ($\left(1 - \alpha^2 \nabla^2\right)^{-1}$) — rough ($\mathbf{v}$)

Lagrangian-Averaged Navier-Stokes Equation (LANS-α)



v rough

u smooth

larger α smooths more

v Lagrangian averaged velocity

u Eulerian averaged velocity

$$\mathbf{u} = \underbrace{\left(1 - \alpha^2 \nabla^2\right)^{-1}}_{\text{Helmholtz operator}} \mathbf{v}$$

smooth ($\mathbf{u}$), rough ($\mathbf{v}$)

$$\frac{\partial \mathbf{v}}{dt} + \underbrace{\mathbf{u} \cdot \nabla \mathbf{v}}_{\text{advection}} + \underbrace{v_j \nabla u_j}_{\text{extra nonlinear term}} - \underbrace{\mathbf{f} \times \mathbf{u}}_{\text{Coriolis}} = -\underbrace{\tfrac{1}{\rho_0} \nabla p}_{\text{pressure gradient}} + \underbrace{\nu \nabla^2 \mathbf{v}}_{\text{diffusion}} + \mathbf{F}$$

Lagrangian-Averaged Navier-Stokes Equation (LANS-α)

$$\underbrace{\frac{\partial \mathbf{v}}{dt} + \mathbf{u}\cdot\nabla\mathbf{v}}_{\text{advection}} + \underbrace{v_j \nabla u_j}_{\text{extra nonlinear term}} - \underbrace{\mathbf{f}\times\mathbf{u}}_{\text{Coriolis}} = \underbrace{-\tfrac{1}{\rho_0}\nabla p}_{\text{pressure gradient}} + \underbrace{\nu\nabla^2\mathbf{v}}_{\text{diffusion}} + \mathbf{F}$$

$$v_j \nabla u_j = \begin{pmatrix} v_1\partial_x u_1 + v_2\partial_x u_2 + v_3\partial_x u_3 \\ v_1\partial_y u_1 + v_2\partial_y u_2 + v_3\partial_y u_3 \\ v_1\partial_z u_1 + v_2\partial_z u_2 + v_3\partial_z u_3 \end{pmatrix}$$

Extra nonlinear term is in alpha model, but not in Leray model. This term is required for conservation of PV.

Standard POP

tracer equation

$$\frac{\partial \varphi}{dt} + \underbrace{\mathbf{u}\cdot\nabla\varphi}_{\text{advection}} = \underbrace{D_H(\varphi) + D_V(\varphi)}_{\text{diffusion}}$$

momentum equation

$$\frac{\partial \mathbf{u}}{dt} + \underbrace{\mathbf{u}\cdot\nabla\mathbf{u}}_{\text{advection}} - \underbrace{\mathbf{f}\times\mathbf{u}}_{\text{Coriolis}} + \underbrace{\mathrm{metric}(\mathbf{u})}_{\text{e.g. centrifugal}} = \underbrace{-\rho_0^{-1}\nabla p}_{\text{pressure gradient}} + \underbrace{F_H(\mathbf{u}) + F_V(\mathbf{u})}_{\text{diffusion}}$$

POP-alpha

rough velocity, $\mathbf{v}$

smooth velocity, $\mathbf{u}$

tracer equation

$$\frac{\partial \varphi}{dt} + \underbrace{\mathbf{u}\cdot\nabla\varphi}_{\text{advection}} = \underbrace{D_H(\varphi) + D_V(\varphi)}_{\text{diffusion}}$$

momentum equation

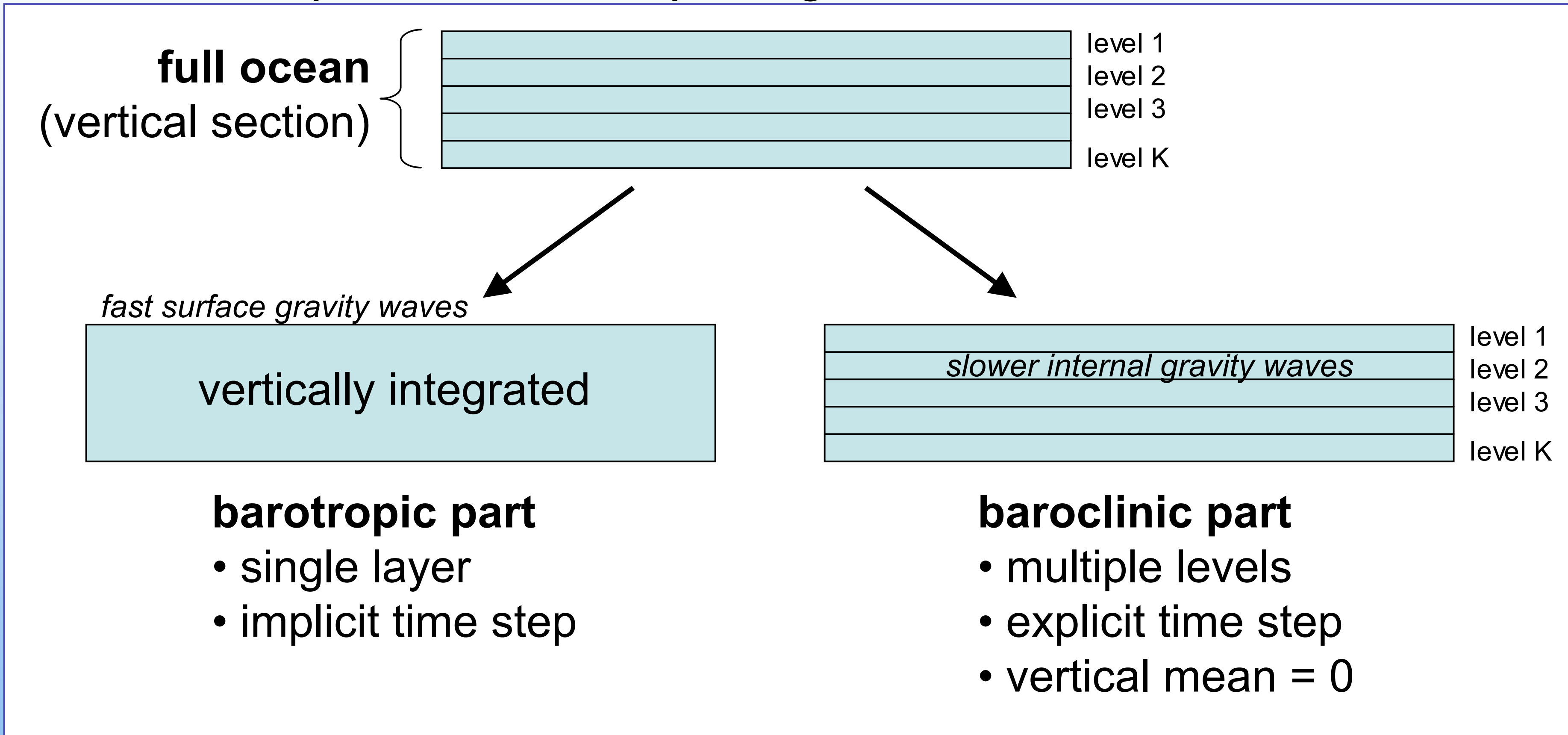
$$\frac{\partial \mathbf{v}}{dt} + \underbrace{\mathbf{u}\cdot\nabla\mathbf{v}}_{\text{advection}} + \underbrace{u_j\nabla v_j}_{\text{extra nonlinear term}} - \underbrace{\mathbf{f}\times\mathbf{u}}_{\text{Coriolis}} + \underbrace{\mathrm{metric}(\mathbf{u})}_{\text{e.g. centrifugal}} = \underbrace{-\rho_0^{-1}\nabla p}_{\text{pressure gradient}} + \underbrace{F_H(\mathbf{v}) + F_V(\mathbf{v})}_{\text{diffusion}}$$

Helmholtz inversion $\mathbf{u} = \left(1 - \alpha^2\nabla^2\right)^{-1}\mathbf{v}$

The POP-alpha model

Issues:

1. How do we implement the alpha model within the barotropic/baroclinic splitting of POP?



Outline of algorithm - Standard POP

Start: variables known at step *n*+1

- temperature, salinity, density, pressure at *n*
- baroclinic velocities $\mathbf{u}_k^n$
- barotropic velocity $\mathbf{U}^n$

Baroclinic computations levels $k = 1 \ldots km$

leap frog step:

$$\hat{\mathbf{u}}_k^{n+1} = \mathbf{u}_k^{n-1} + 2\Delta t RHS_k^n$$

where RHS contains:

- advection
- metric (centrifugal)
- Coriolis
- diffusion

subtract depth average:

$$\tilde{\mathbf{u}}_k^{n+1} = \hat{\mathbf{u}}_k^{n+1} - \frac{1}{H}\sum_{k=1}^{km} \hat{\mathbf{u}}_k^{n+1} dz_k$$

Barotropic computations

implicit solve for the free surface height η^{n+1}

new barotropic velocity:

$$\mathbf{U}^{n+1} = \mathbf{U}^{n-1} + 2\Delta t RHS$$

where RHS contains:

- vertically integrated forcing
- $\nabla \eta^{n-1}, \nabla \eta^n, \nabla \eta^{n+1}$

add barotropic velocity back to baroclinic:

$$\mathbf{u}_k^{n+1} = \tilde{\mathbf{u}}_k^{n+1} + \mathbf{U}^{n+1}$$

step *n*+1 complete

Outline of algorithm - POP-alpha

Start: variables known at step *n*+1

- temperature, salinity, density, pressure at *n*
- baroclinic *smooth* and *rough* velocities $\mathbf{u}_k^n, \mathbf{v}_k^n$
- barotropic *smooth* and *rough* velocities $\mathbf{U}^n, \mathbf{V}^n$

Baroclinic computations levels $k = 1 \ldots km$

leap frog step:

$$\hat{\mathbf{v}}_k^{n+1} = \mathbf{v}_k^{n-1} + 2\Delta t RHS_k^n$$

where RHS contains:

- advection
- metric (centrifugal)
- Coriolis
- diffusion
- extra nonlinear term, $\nabla \mathbf{u}^{\mathrm{T}} \cdot \mathbf{v}$

subtract depth average:

$$\tilde{\mathbf{v}}_k^{n+1} = \hat{\mathbf{v}}_k^{n+1} - \frac{1}{H}\sum_{k=1}^{km} \hat{\mathbf{v}}_k^{n+1} dz_k$$

$$\tilde{\mathbf{u}}_k^{n+1} = smooth(\hat{\mathbf{v}}_k^{n+1})$$

$$\tilde{\mathbf{u}}_k^{n+1} = \hat{\mathbf{u}}_k^{n+1} - \frac{1}{H}\sum_{k=1}^{km} \hat{\mathbf{u}}_k^{n+1} dz_k$$

Barotropic computations

implicit solve for the free surface height η^{n+1}

new barotropic velocity:

$$\mathbf{V}^{n+1} = \mathbf{V}^{n-1} + 2\Delta t RHS$$

where RHS contains:

- vertically integrated forcing
- $\nabla \eta^{n-1}, \nabla \eta^n, \nabla \eta^{n+1}$

$$\mathbf{U}^{n+1} = smooth(\mathbf{V}^{n+1})$$

add barotropic velocity back to baroclinic:

$$\mathbf{u}_k^{n+1} = \tilde{\mathbf{u}}_k^{n+1} + \mathbf{U}^{n+1}$$

$$\mathbf{v}_k^{n+1} = \tilde{\mathbf{v}}_k^{n+1} + \mathbf{V}^{n+1}$$

step *n*+1 complete

Barotropic Algorithm - Pop-alpha

Simultaneously solve for: free surface height η^{n+1} and both velocities, $\mathbf{U}^{n+1}, \mathbf{V}^{n+1}$

Invert using iterative CG routine

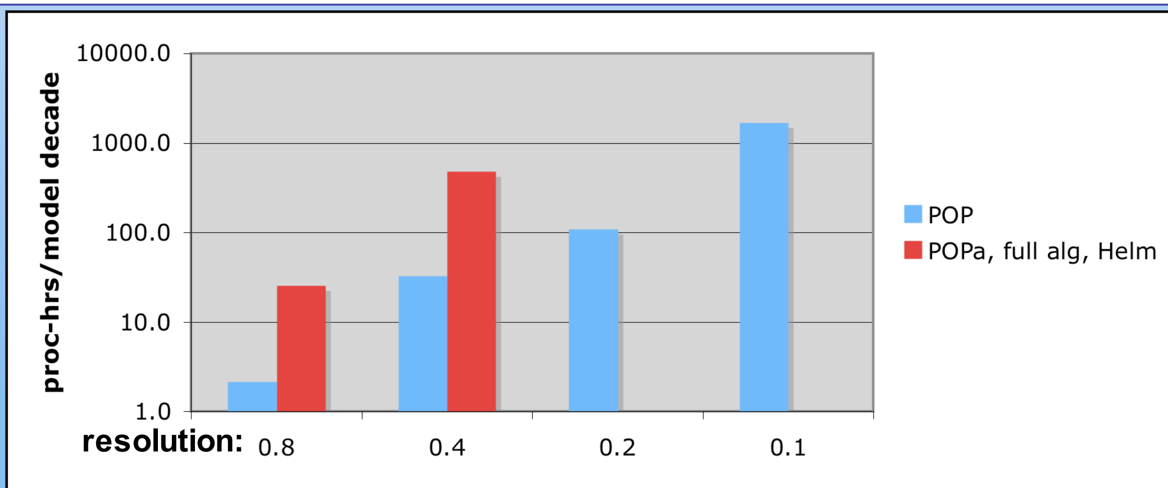
smoothing *within* each iteration is too costly!

$$\left(\nabla \cdot H(1-\alpha^2\nabla^2)^{-1}\nabla - \frac{2}{\gamma g \tau^2}\right)\eta^{n+1} =$$

$$\frac{1}{\tau\gamma g}\nabla \cdot H\mathbf{U}^{n-1} - \frac{2}{\gamma g \tau^2}\eta^n + \frac{1}{\gamma g}\nabla \cdot H(1-\alpha^2\nabla^2)^{-1}\Big[\underbrace{\mathbf{G}^n - \mathbf{B}\mathbf{U}^n - \gamma g \nabla\left(\eta^{n-1} + \eta^n\right)}_{\text{momentum forcing terms}}\Big]$$

$$\mathbf{V}^{n+1} = \mathbf{V}^{n-1} + 2\Delta t\Big[\underbrace{\mathbf{G}^n - \mathbf{B}\mathbf{U}^n - \gamma g \nabla\left(\eta^{n-1} + \eta^n + \eta^{n+1}\right)}_{\text{momentum forcing terms}}\Big]$$

$$\mathbf{U}^{n+1} = (1-\alpha^2\nabla^2)^{-1}\mathbf{V}^{n+1}$$



Barotropic Algorithm - Pop-alpha

Simultaneously solve for: free surface height η^{n+1} and both velocities, $\mathbf{U}^{n+1}, \mathbf{V}^{n+1}$

Invert using iterative CG routine

smoothing *within* each iteration is too costly!

What if we eliminate just this one smoothing step?

$$\left(\nabla \cdot H(1-\alpha^2\nabla^2)^{-1}\nabla - \frac{2}{\gamma g \tau^2}\right)\eta^{n+1} =$$

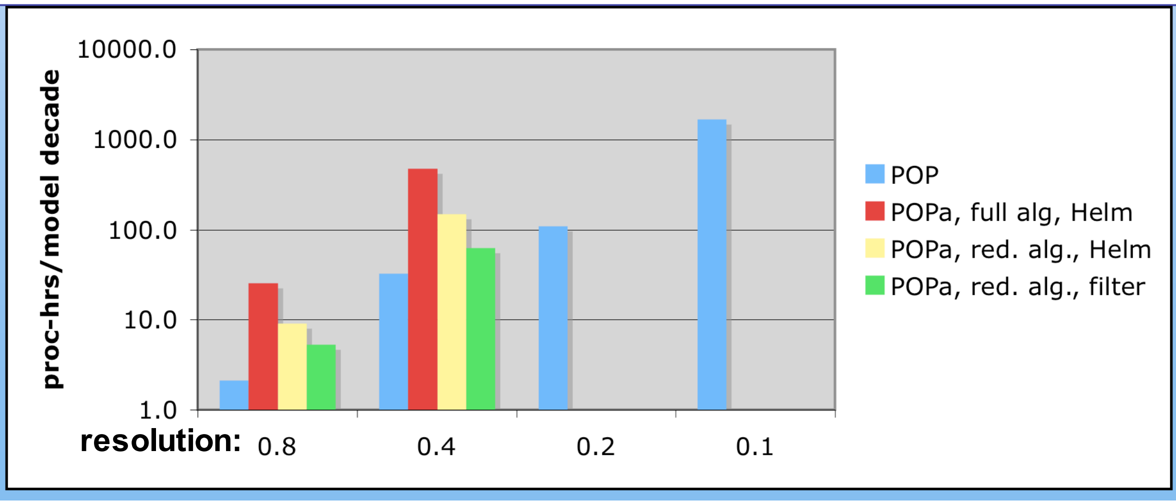
$$\frac{1}{\tau\gamma g}\nabla \cdot H\mathbf{U}^{n-1} - \frac{2}{\gamma g \tau^2}\eta^n + \frac{1}{\gamma g}\nabla \cdot H(1-\alpha^2\nabla^2)^{-1}\left[\mathbf{G}^n - \mathbf{B}\mathbf{U}^n - \gamma g \nabla\left(\eta^{n-1} + \eta^n\right)\right]$$

momentum forcing terms

$$\mathbf{V}^{n+1} = \mathbf{V}^{n-1} + 2\Delta t\left[\mathbf{G}^n - \mathbf{B}\mathbf{U}^n - \gamma g \nabla\left(\eta^{n-1} + \eta^n + \eta^{n+1}\right)\right]$$

momentum forcing terms

$$\mathbf{U}^{n+1} = (1-\alpha^2\nabla^2)^{-1}\mathbf{V}^{n+1}$$



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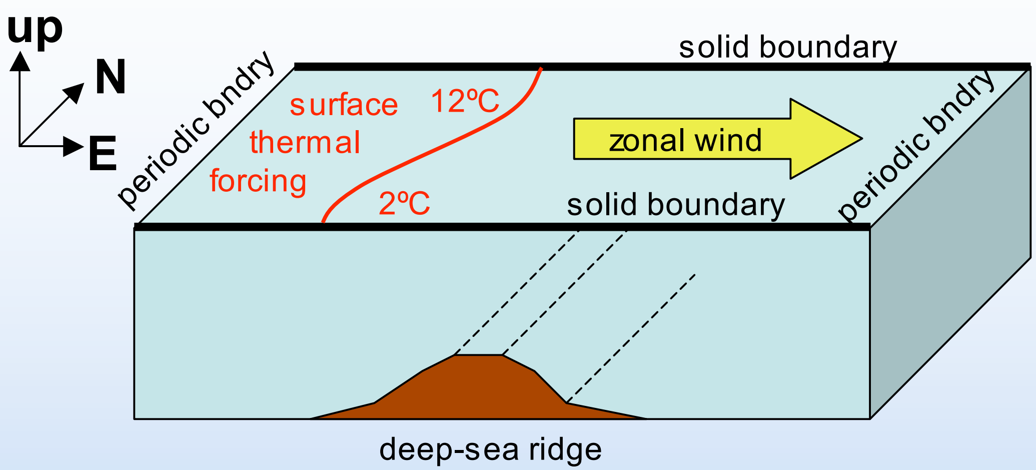
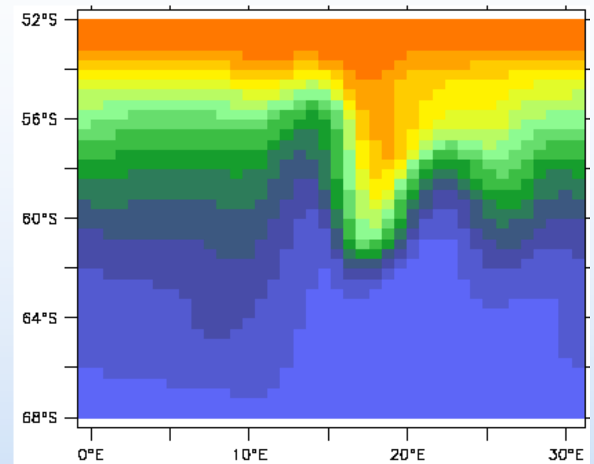
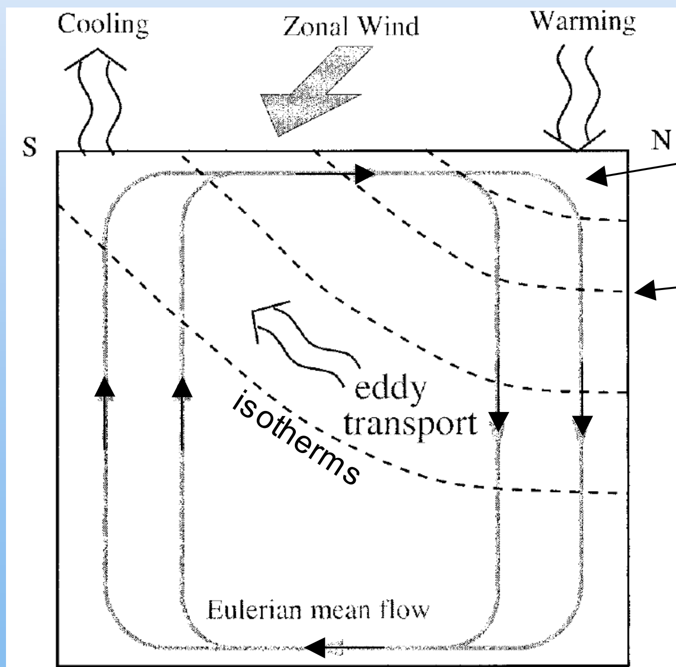
The test problem:
Idealization of Antarctic Circumpolar Current



Surface temperature





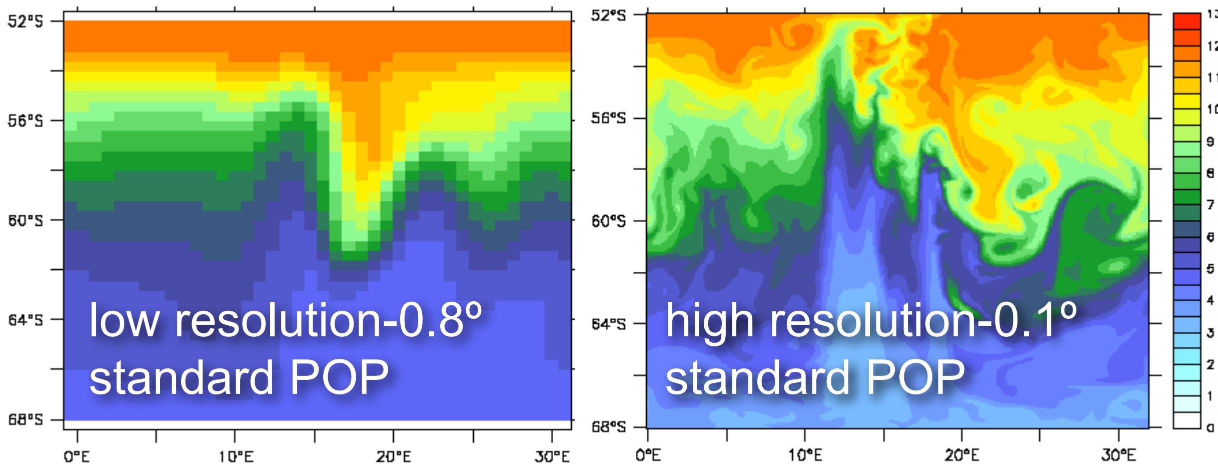
Baroclinic Instability:

1. Eastward zonal wind causes northward Ekman transport
2. Circulation tilts isotherms (lines of constant temperature)
3. Potential energy of baroclinic instability converted to kinetic energy.
4. Small scale turbulence (eddies) transport heat and kinetic energy

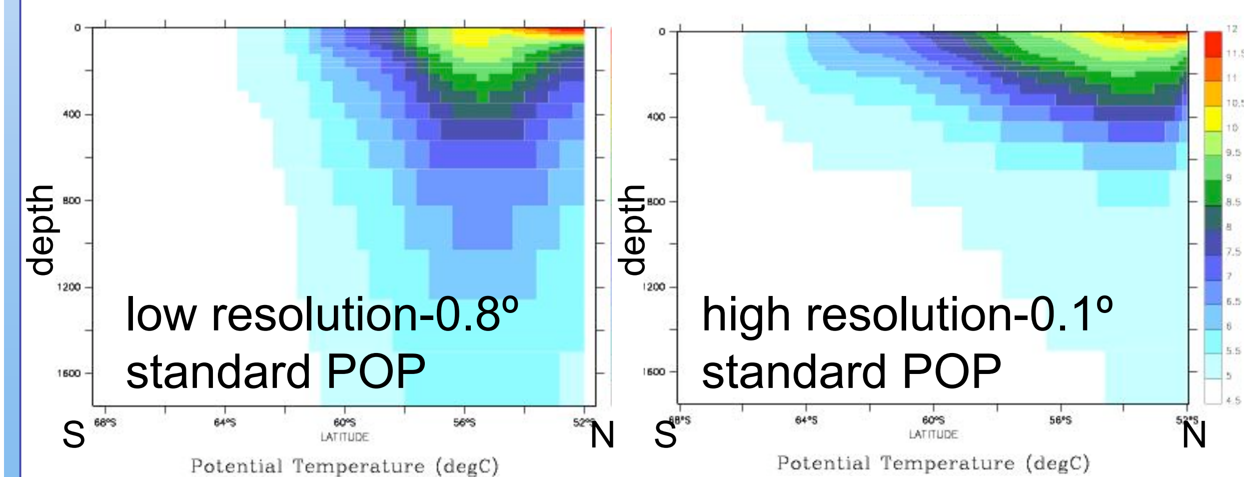
Thermocline depth is determined by the eddy transport quantities.

The Baroclinic Instability

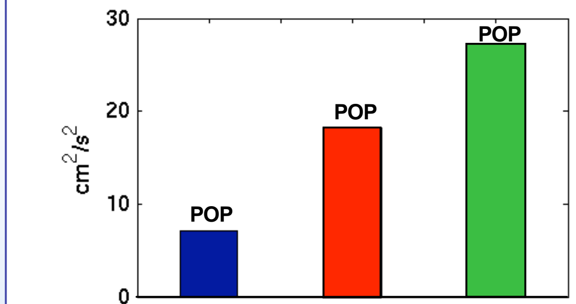
Surface temperature



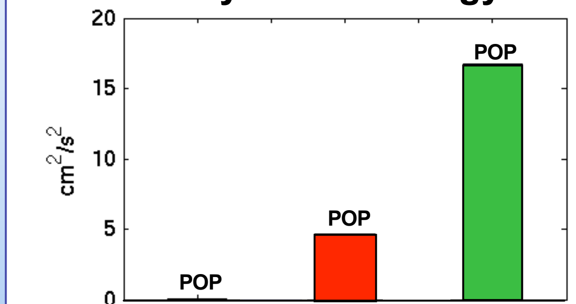
Potential temperature - vertical cross section



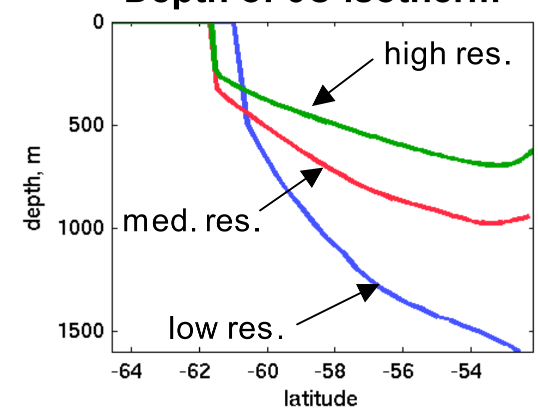
Kinetic energy



Eddy kinetic energy

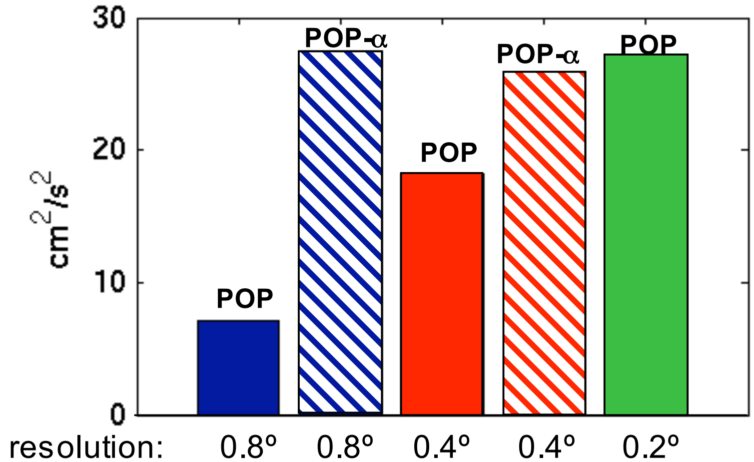


Depth of 6C isotherm

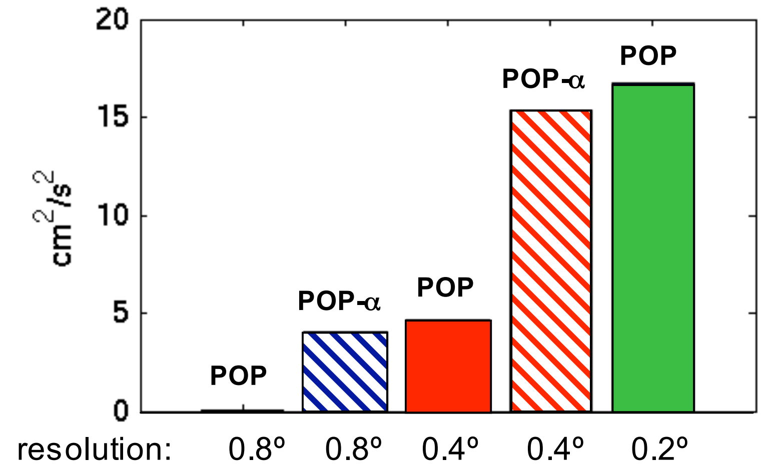


POP-alpha Results

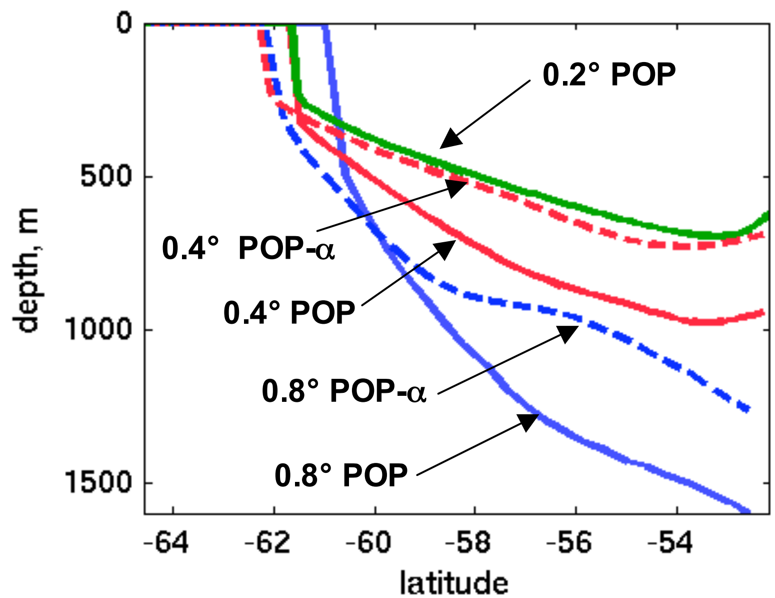
Kinetic energy



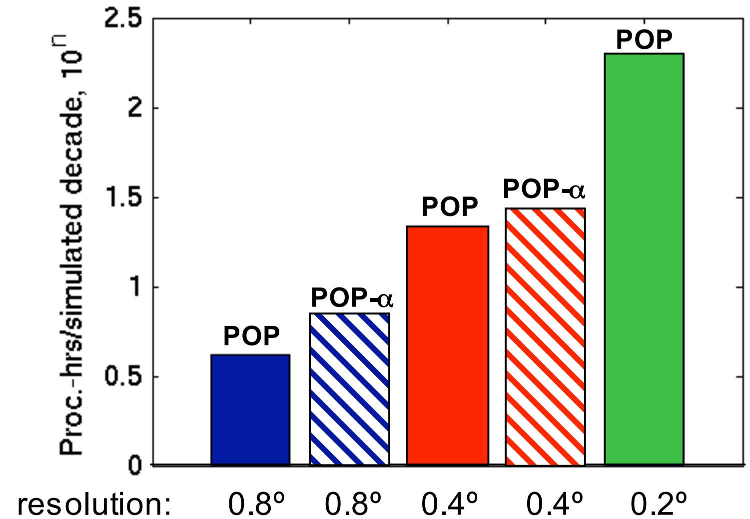
Eddy kinetic energy



Depth of 6C isotherm



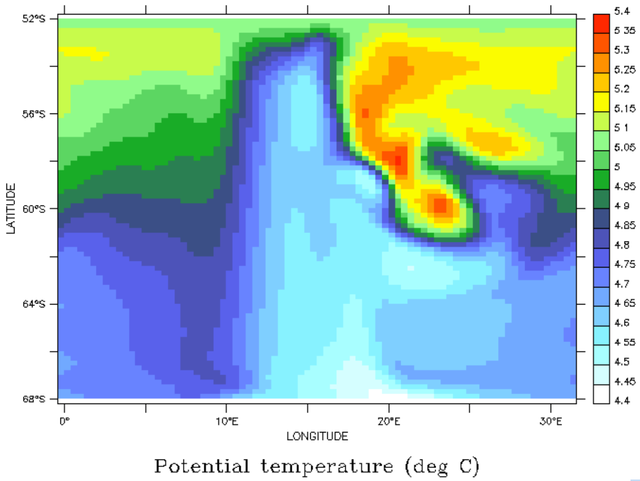
Computation time (log scale)



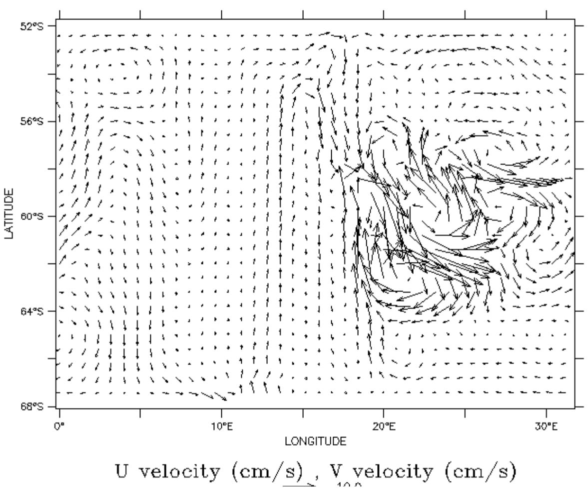
Can you see more eddies with LANS-alpha?

POP, 0.4° resolution

Pot. Temperature

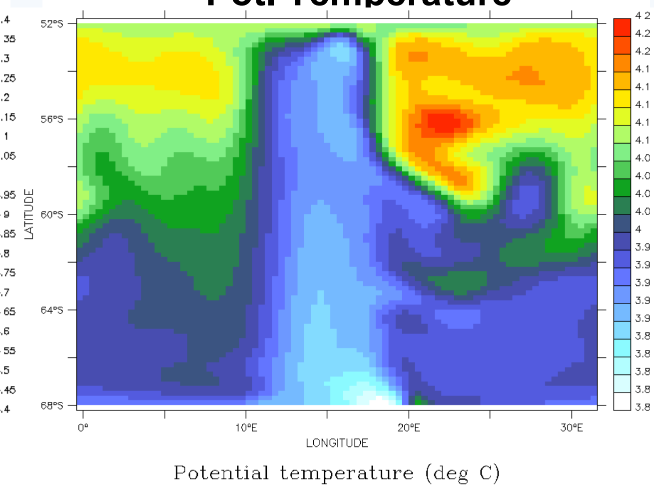


Velocity

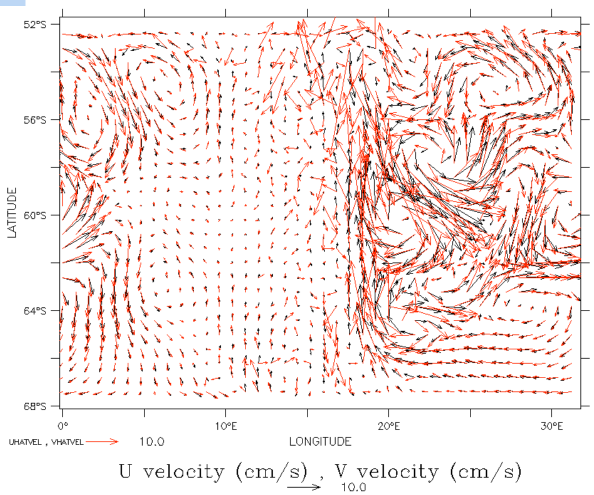


POP-α, 0.4° resolution

Pot. Temperature

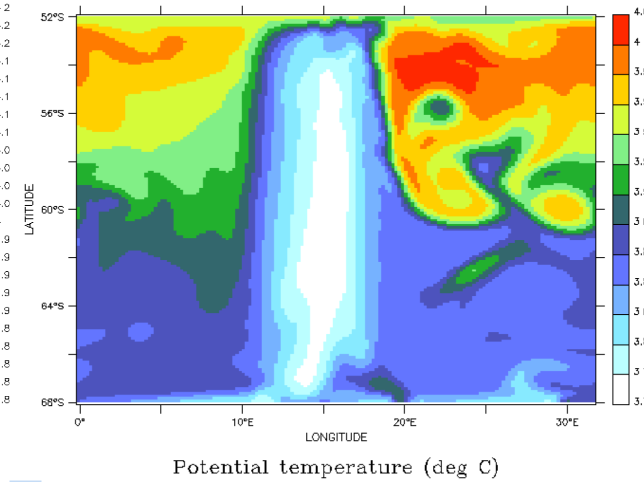


Velocity

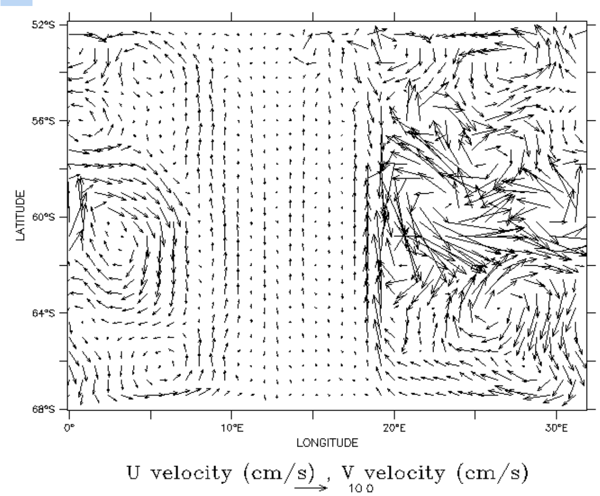


POP, 0.2° resolution

Pot. Temperature



Velocity



All sections at a depth of 1600m

rough velocity: red, smooth: black

Dispersion Relation for LANS-α using linearized shallow water equations

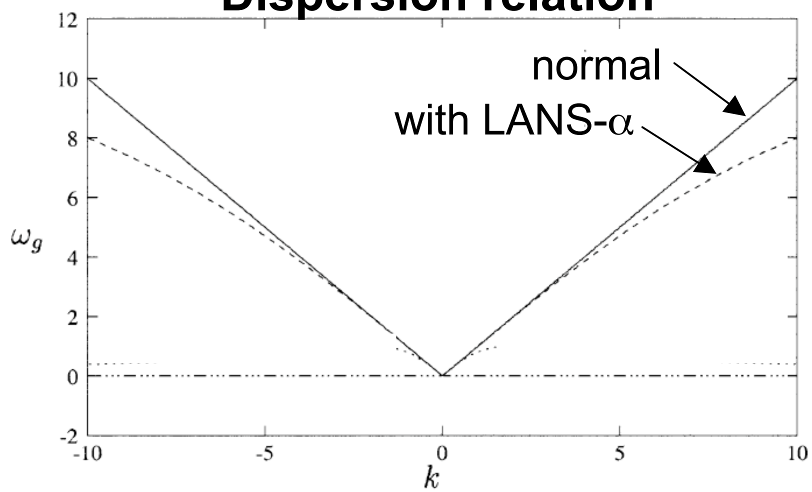
Gravity waves

normally: $$\omega^2 = k^2 gH$$

with LANS-α: $$\omega^2 = \frac{k^2 gH}{1+\alpha^2 k^2}$$

frequency ω, wavenumber *k*, gravity *g*, height *H*

Dispersion relation



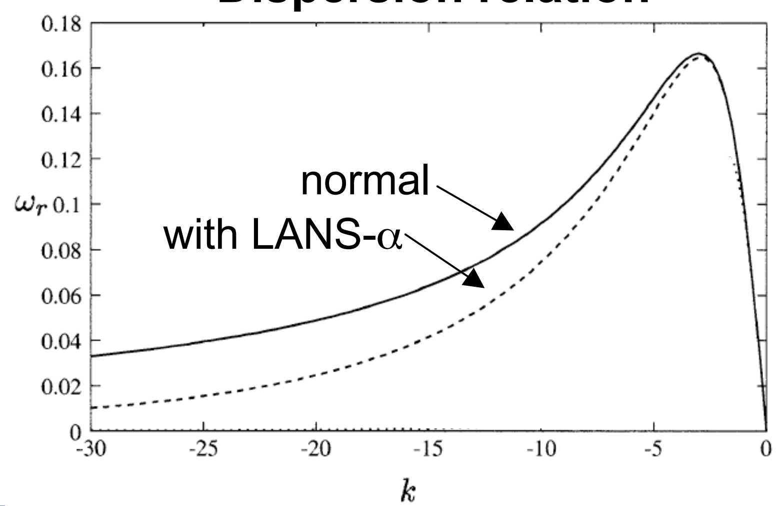
Rossby waves

normally: $$\omega = \frac{-k\beta}{k^2 + 1/R^2}$$

LANS-α: $$\omega = \frac{-k\beta}{k^2\left(1+\alpha^2 k^2\right) + 1/R^2}$$

Rossby radius $R = \sqrt{gH}/f_0$ beta $\beta = \partial_y f$

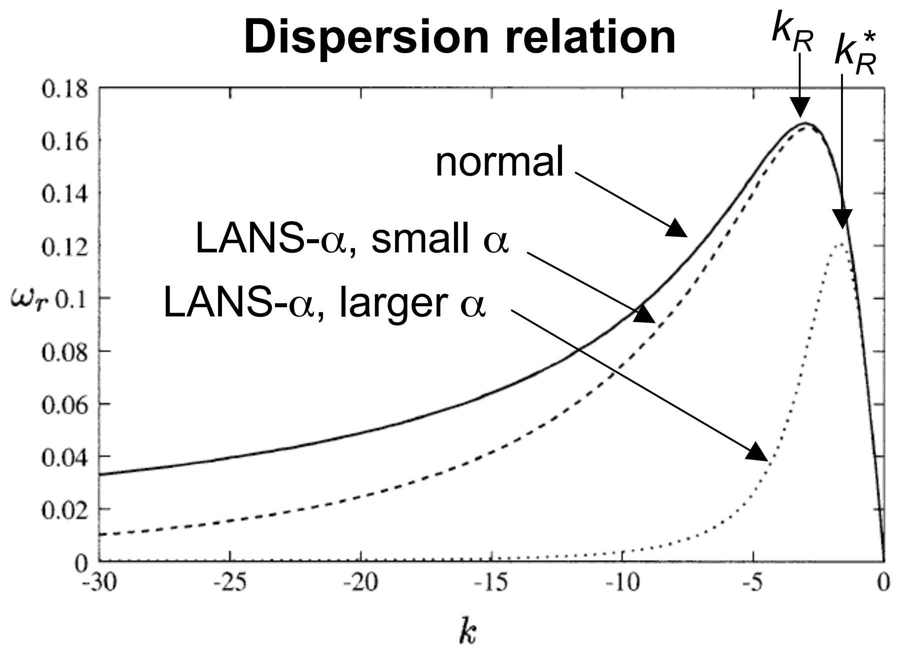
Dispersion relation



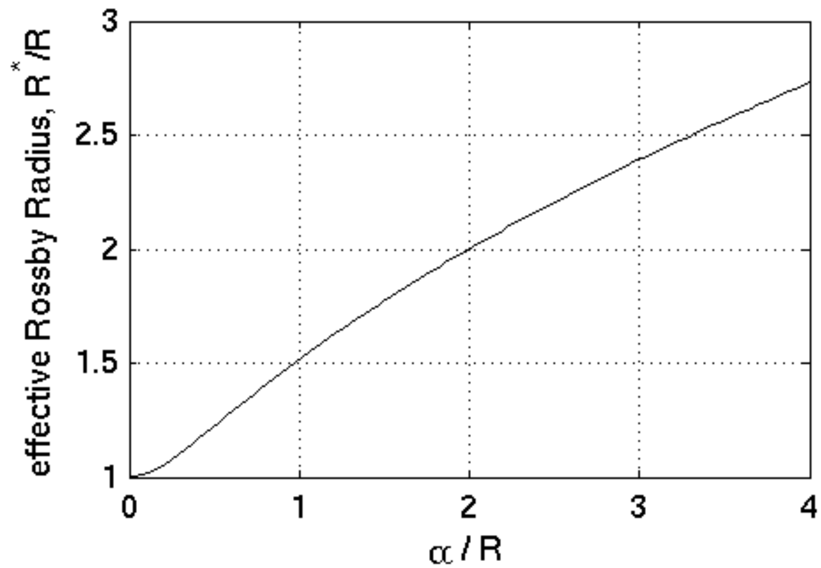
LANS-α slows down gravity and Rossby waves at high wave number.

What does LANS-α do to the Rossby Radius?

Solve for k_R, the wavenumber of the Rossby Radius:



Use that to find
R^*, the effective Rossby Radius,
as a function of α:

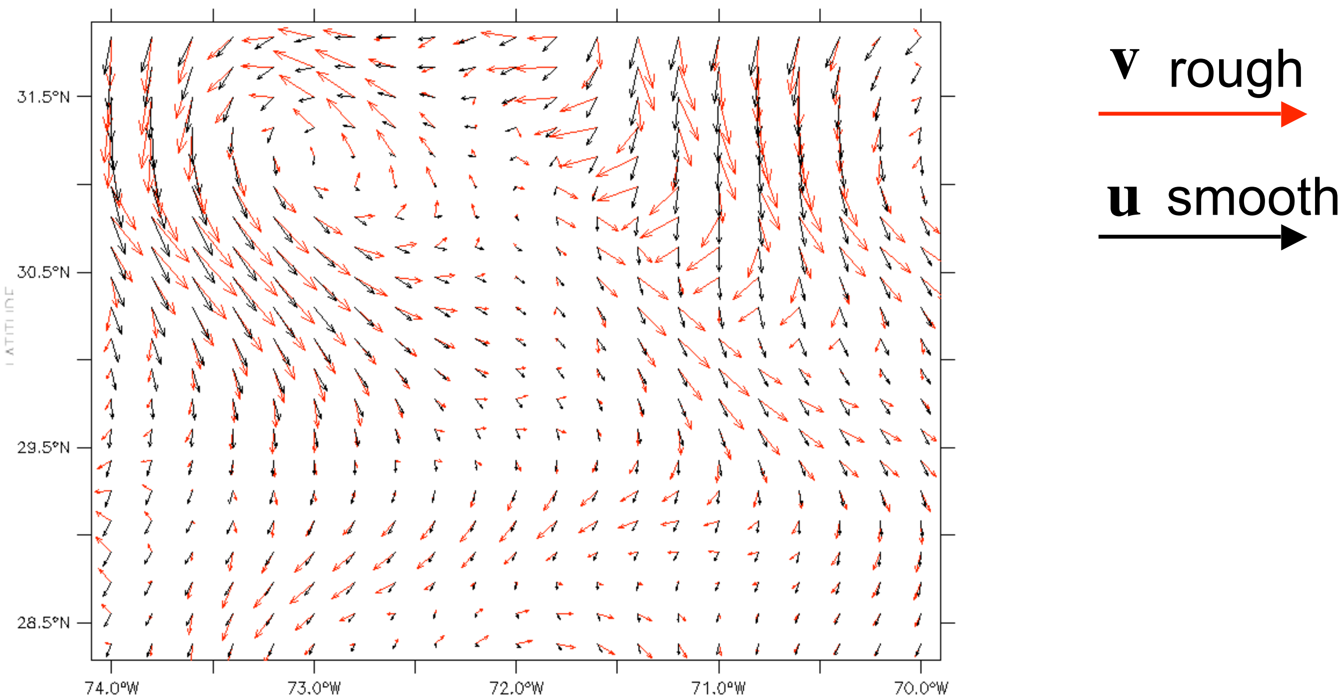


LANS-α makes the Rossby Radius effectively larger.

The POP-alpha model

Issues:

3. How do we smooth the velocity in an Ocean General Circulation Model?



Helmholtz inversion

$$\mathbf{u} = \left(1 - \alpha^2 \nabla^2\right)^{-1} \mathbf{v}$$

is costly!

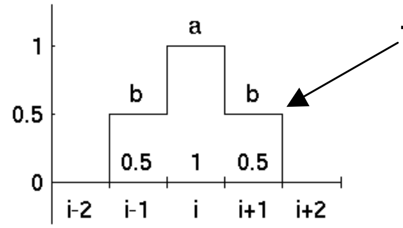
or: use a filter

$$\mathbf{u}(\mathbf{x}) = \int G(\mathbf{r})\mathbf{v}(\mathbf{x}-\mathbf{r})d\mathbf{r}$$

for example, a top-hat filter

Filter Instabilities

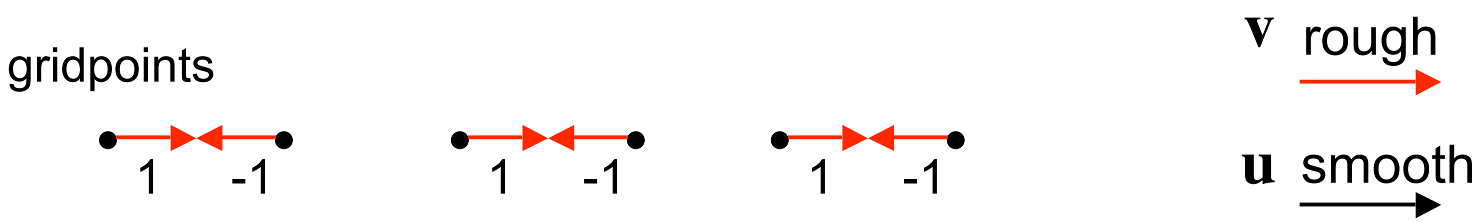
1D



filter, $G(\mathbf{r})$

$$\mathbf{u}_i = \frac{b\mathbf{v}_{i-1} + \mathbf{v}_i + b\mathbf{v}_{i+1}}{1+2b}$$

If $b = 0.5$, smoothing filters out the Nyquist frequency.

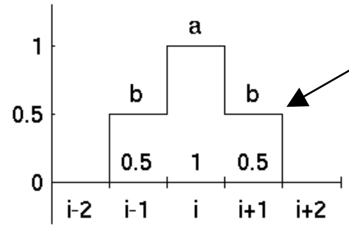


$$\mathbf{u}_i = \frac{b\mathbf{v}_{i-1} + \mathbf{v}_i + b\mathbf{v}_{i+1}}{1+2b} = \frac{\frac{1}{2}(-1) + 1 + \frac{1}{2}(-1)}{1 + 2\frac{1}{2}} = 0$$

The smooth velocity $\mathbf{u}$ is blind to this oscillation.
Therefore, the free surface height η cannot counter it!

Filter Instabilities

1D



filter, $G(\mathbf{r})$

$$\mathbf{u}_i = \frac{b\mathbf{v}_{i-1} + \mathbf{v}_i + b\mathbf{v}_{i+1}}{1+2b}$$

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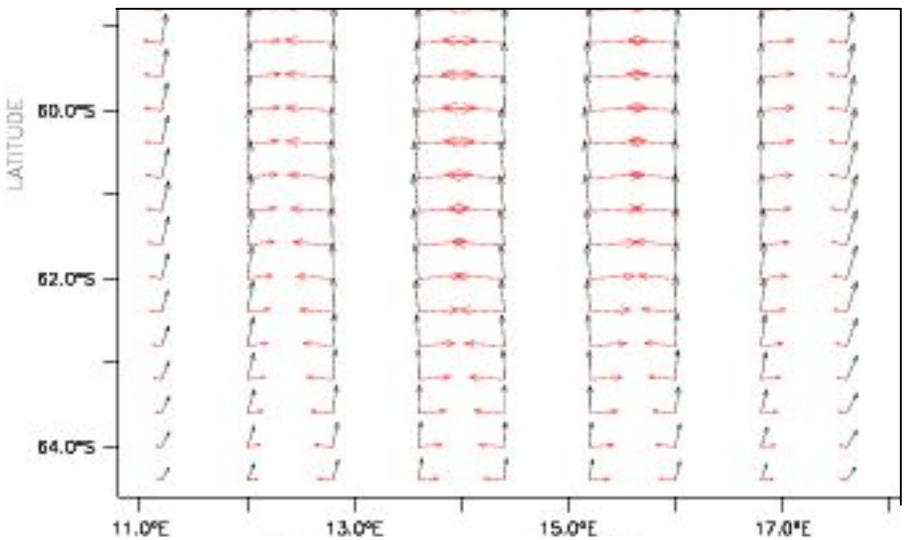


$\mathbf{v}$ rough

370 m/s

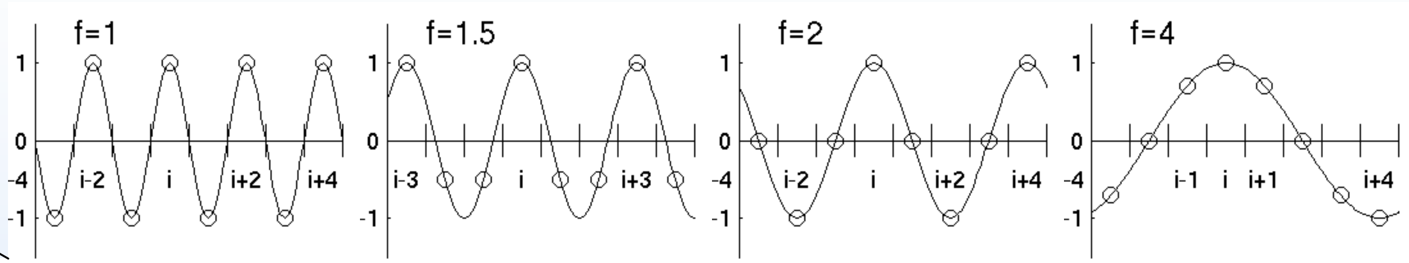
$\mathbf{u}$ smooth

30 m/s

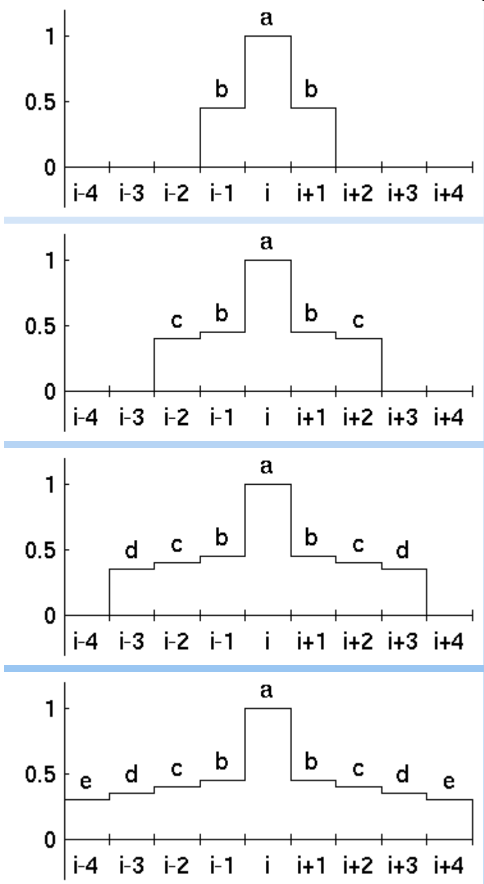
Condition for stability is $b < 0.5$

Filter: Conditions for stability

rough velocity $\mathbf{v}$



filter:



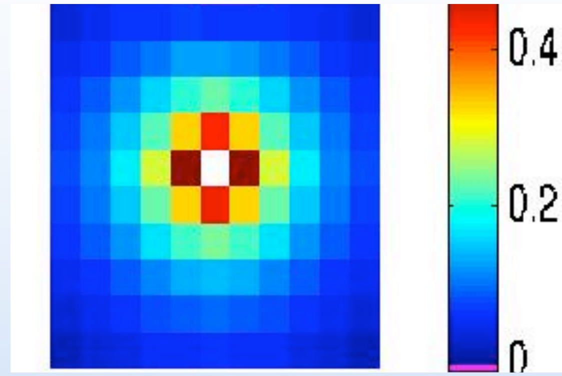
$b < 1/2$	stable ∀ b	stable ∀ b	stable ∀ b
$b < 1/2+c$	$b+c < 1$	$c < 1/2$	stable ∀ b,c
$b+d < 1/2+c$	$b+c < 1+2d$	$c < 1/2$	$d < 1/2+b$
$b+d < 1/2+c+e$	$b+c+e < 1+2d$	$c < 1/2+e$	$d+2e < 1+b$

Filter analysis: Helmholtz inversion Green's function

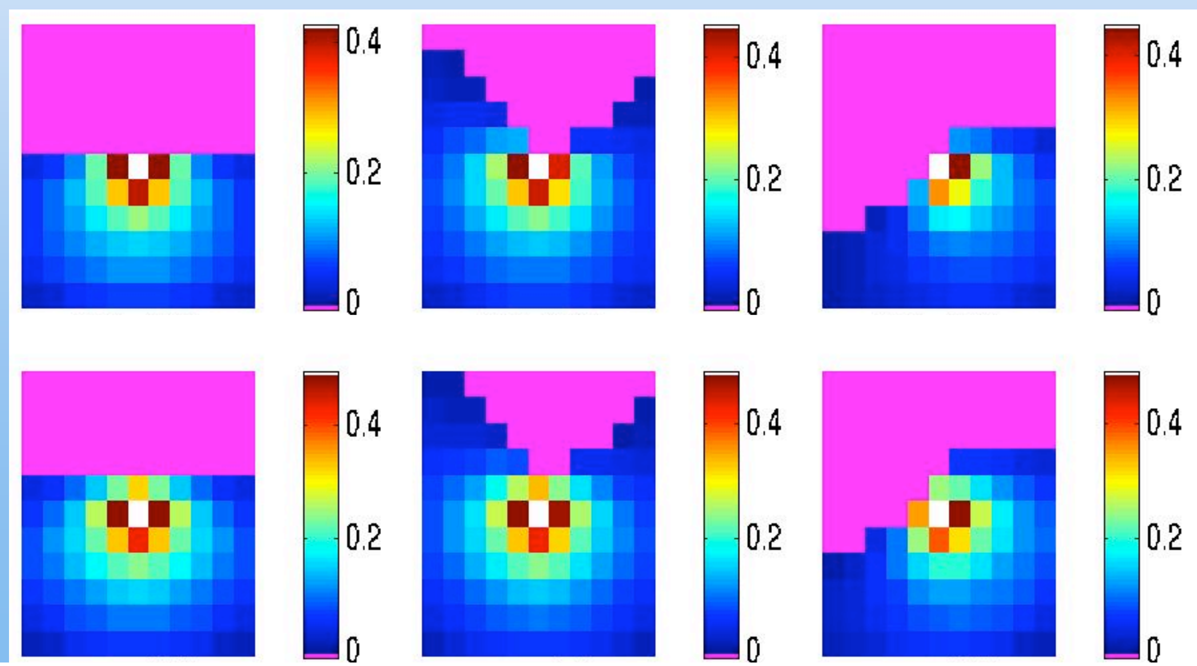
Take **v** to be a point source:

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

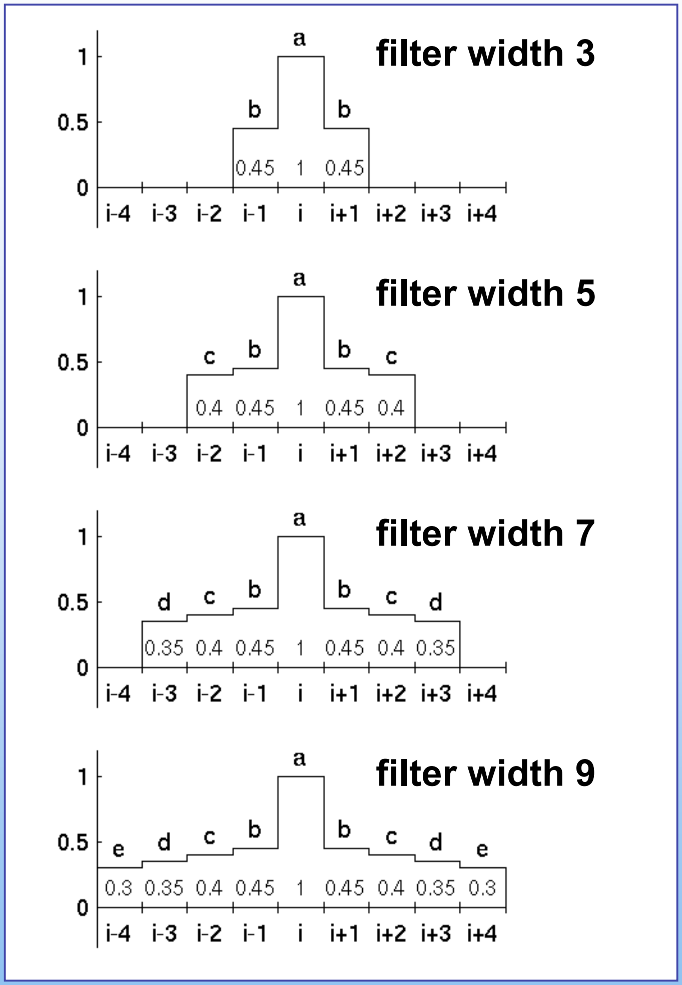
Then compute $\mathbf{u} = \left(1 - \alpha^2 \nabla^2\right)^{-1} \mathbf{v}$

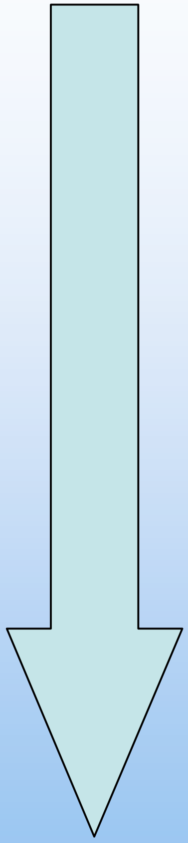


Can use this to understand filter near boundaries:



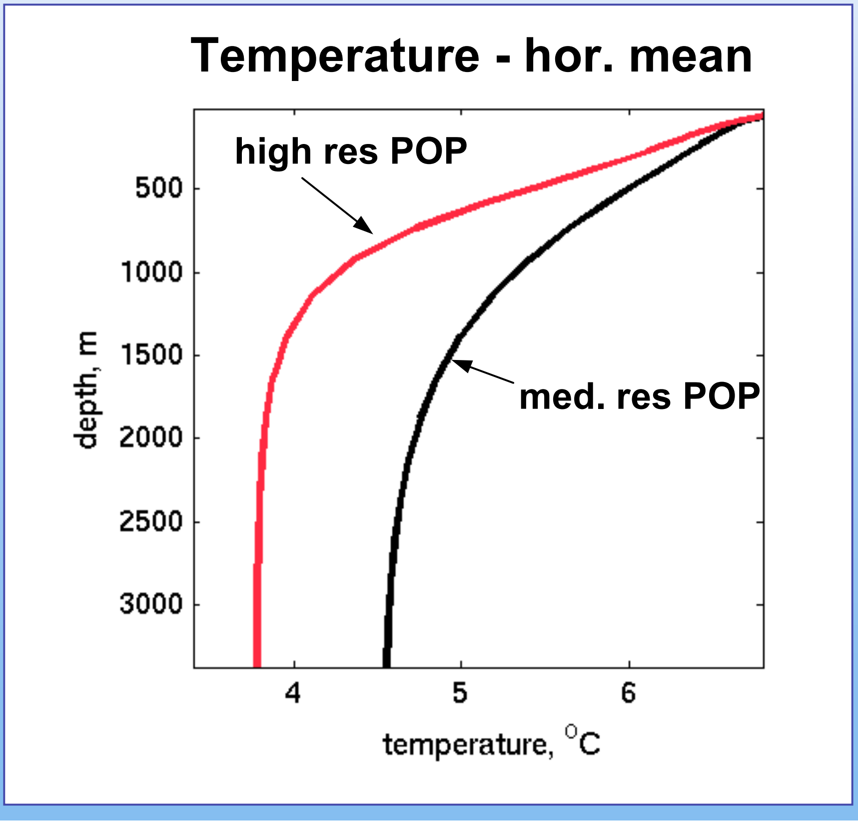
Filters



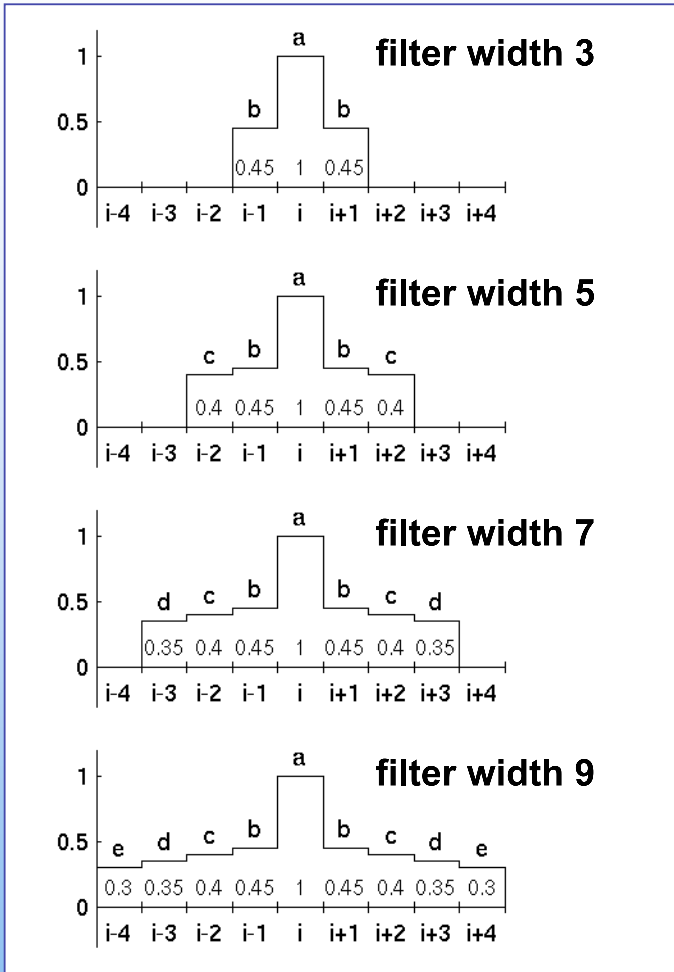


Wider filters result in:

- Stronger smoothing
- Effects are like larger α
- More computation
- More ghostcells



Filters



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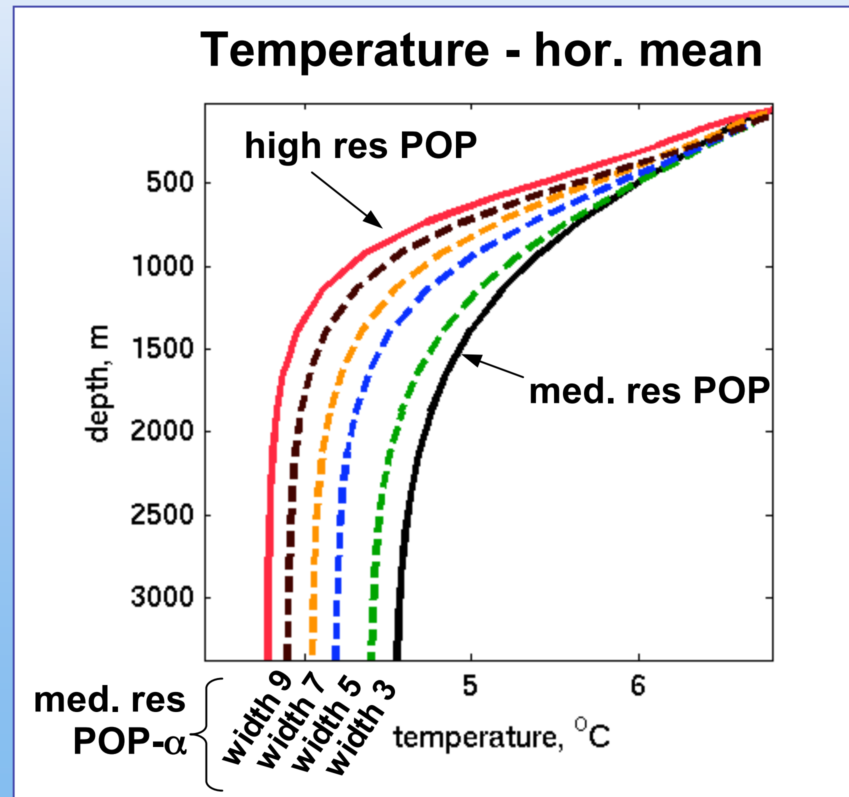
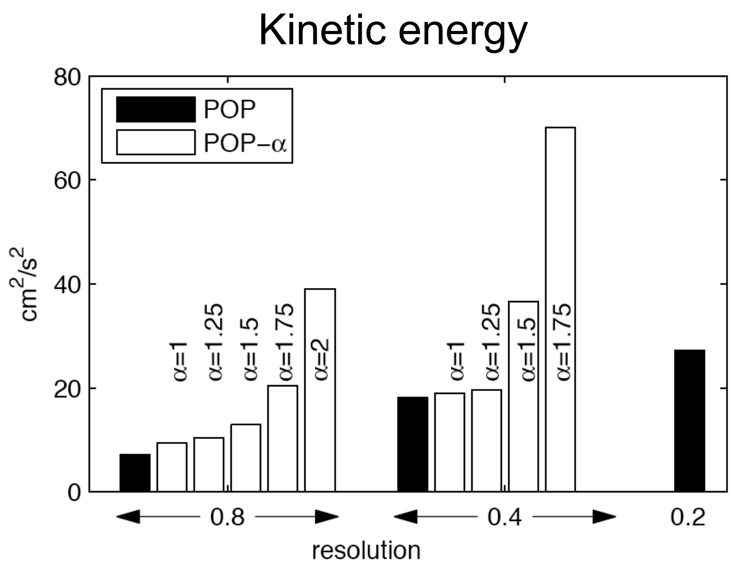
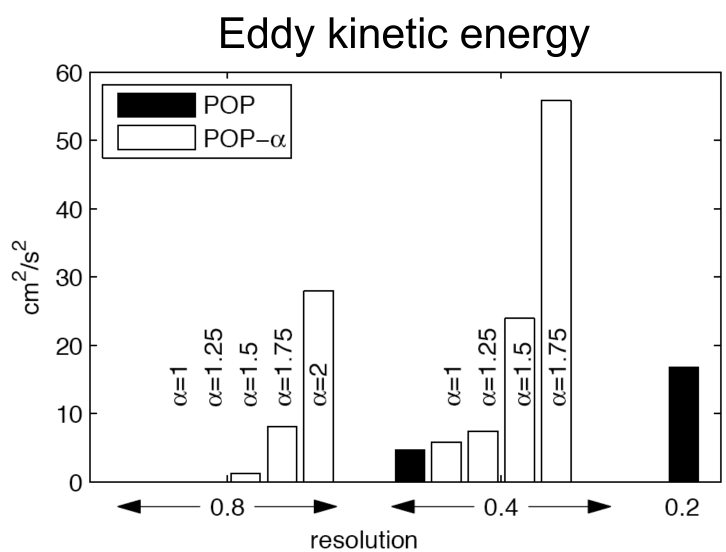
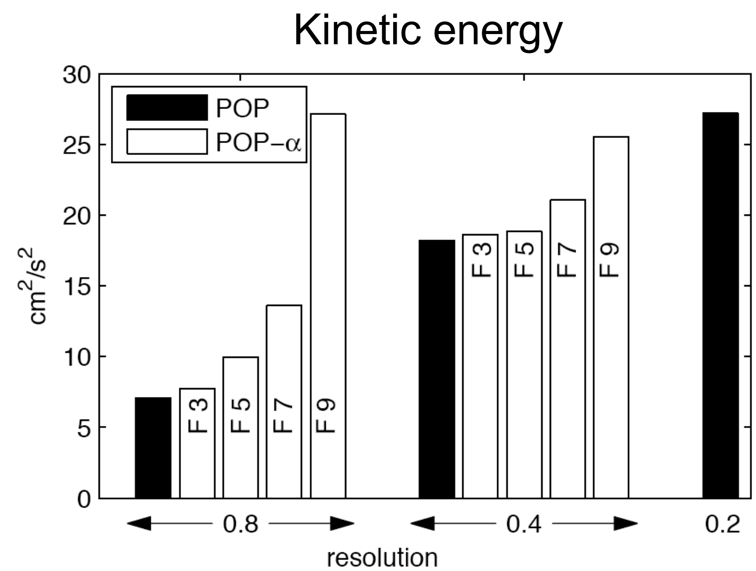


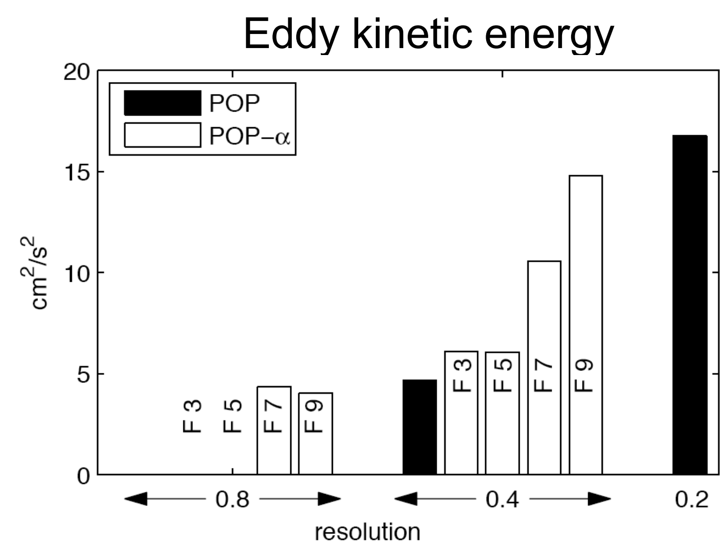
Helmholtz inversion: vary alpha…



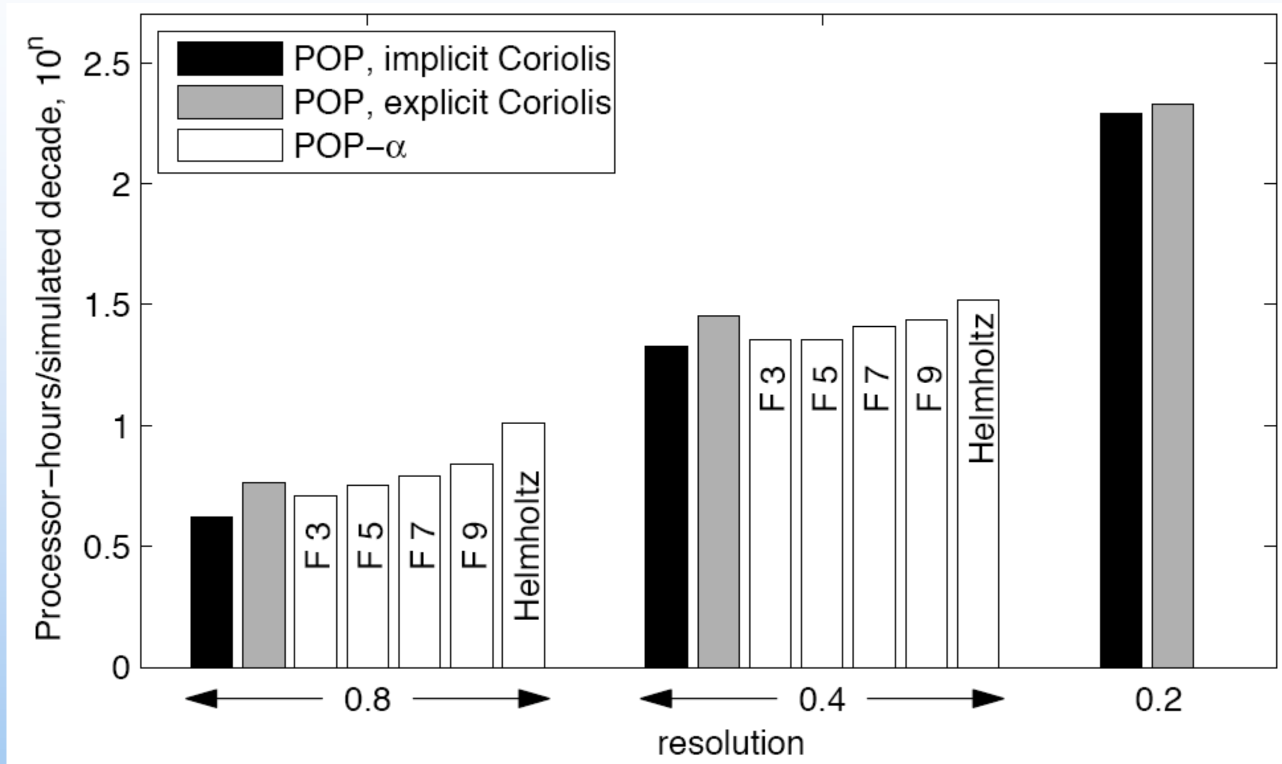


Filters: vary the filter width…





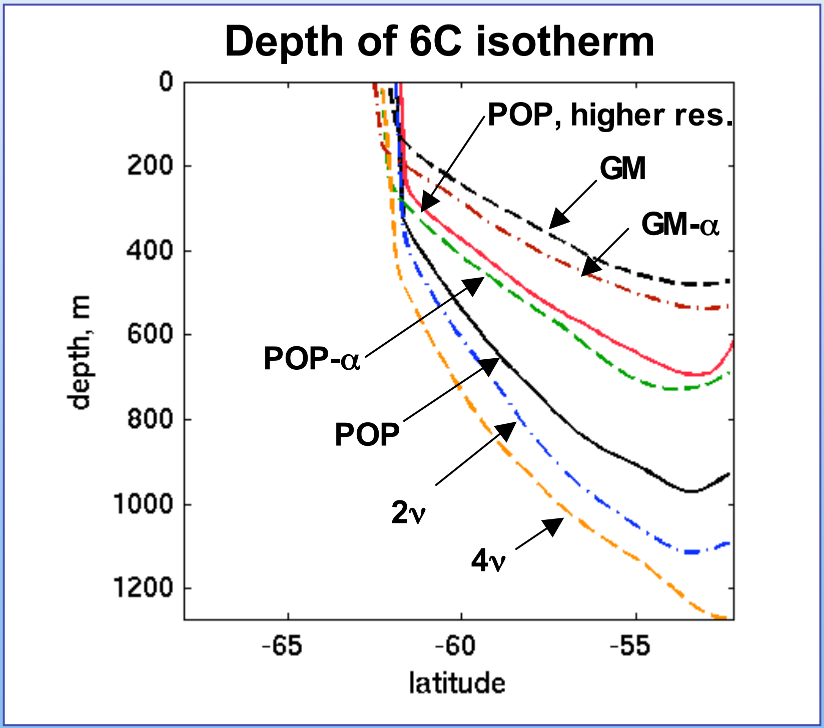
Adding LANS-α increases computation time by <30%

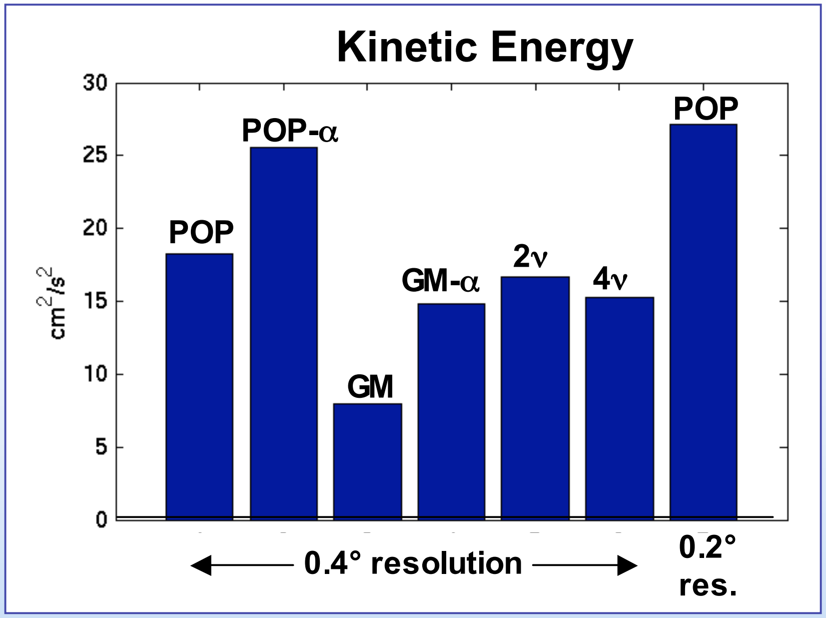


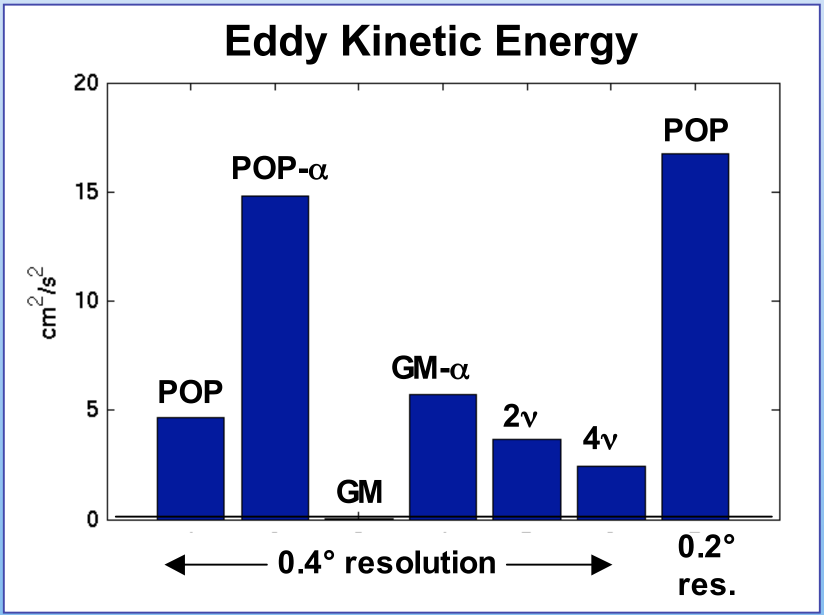
We can take *smaller* timesteps with LANS-α

How does LANS-α compare with other turbulence models?

POP	Hypervisc. only
POP-α	Hypervisc, LANS- α
GM	Hypervisc, Gent-McW.
GM-α	Hypervisc, Gent-McW, LANS-α
2ν	Hypervisc, 2x viscosity coef.
4ν	Hypervisc, 4x viscosity coef.

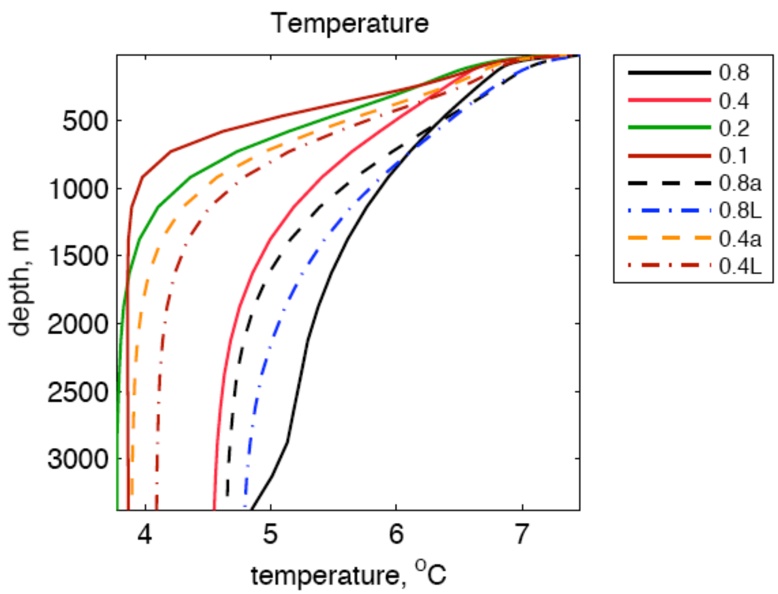




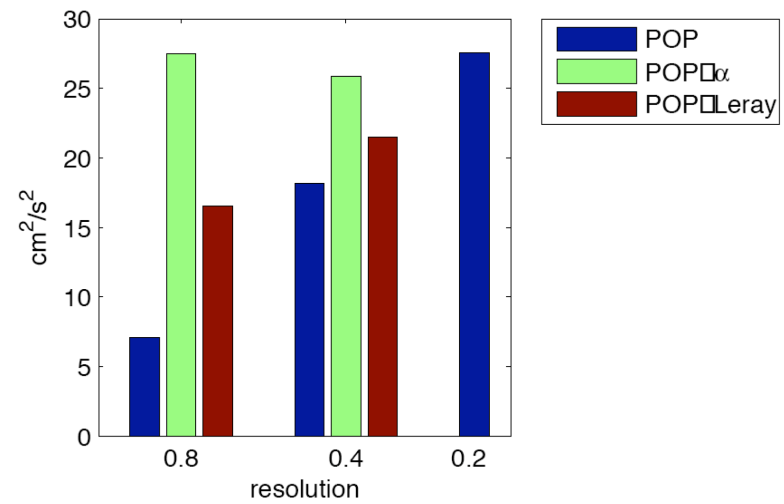


Leray Model is about half as strong as LANS-α

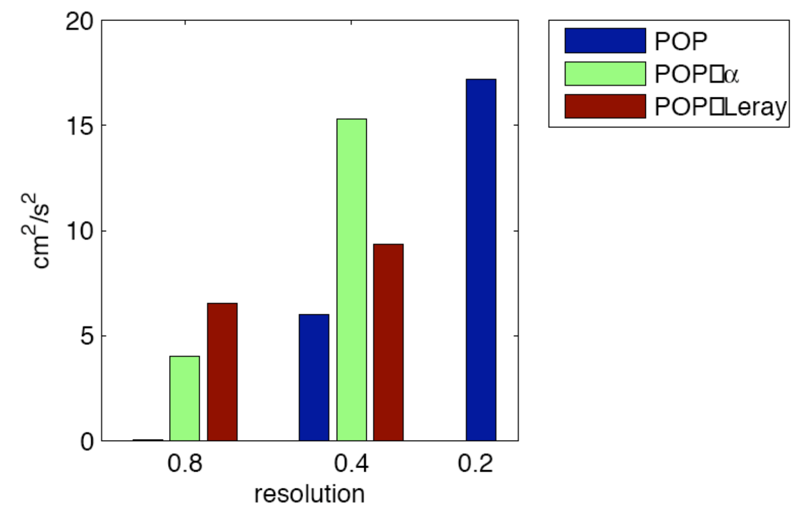
Depth of 6C isotherm



Kinetic Energy



Eddy Kinetic Energy



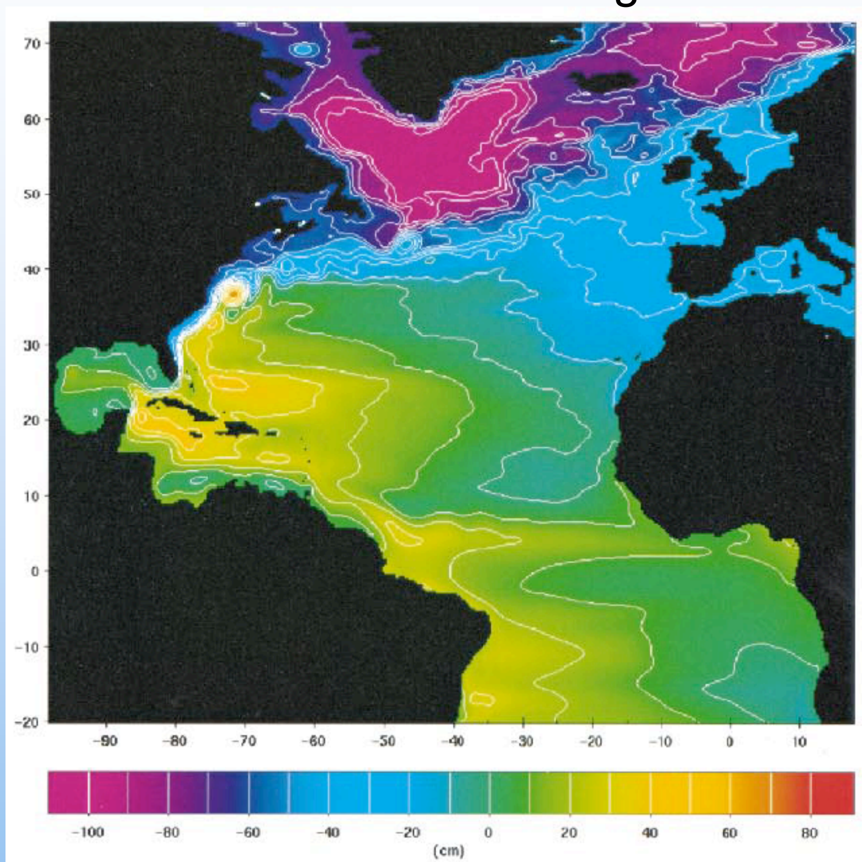
Outline

- POP ocean model & climate change assessment
- LANS-α implementation in POP
- Idealized test case: the channel domain
- The real thing: the North Atlantic

POP simulations of the North Atlantic

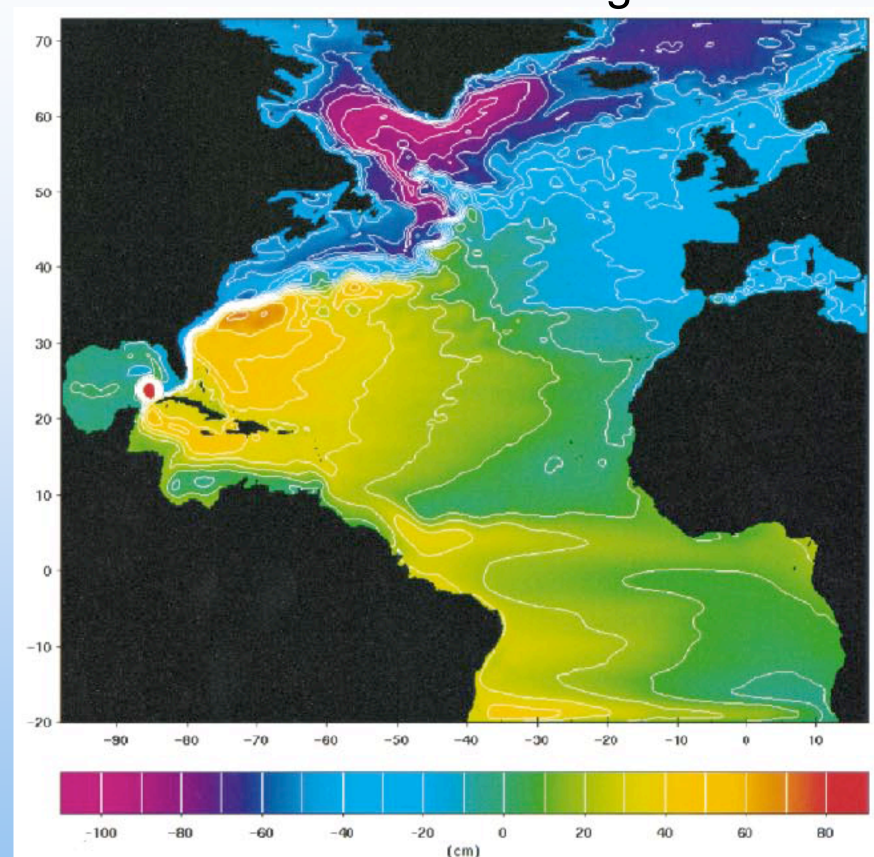
POP, 0.28º resolution

Sea surface height



POP, 0.1º resolution

Sea surface height

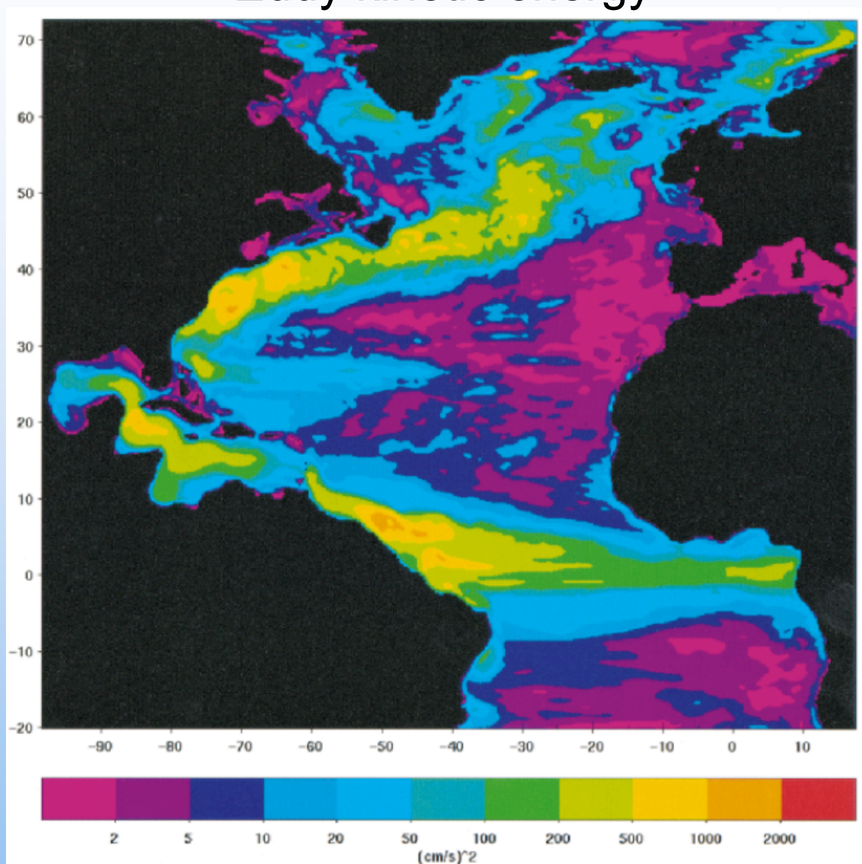


More realistic Gulf Stream and
Northwest corner at high resolution

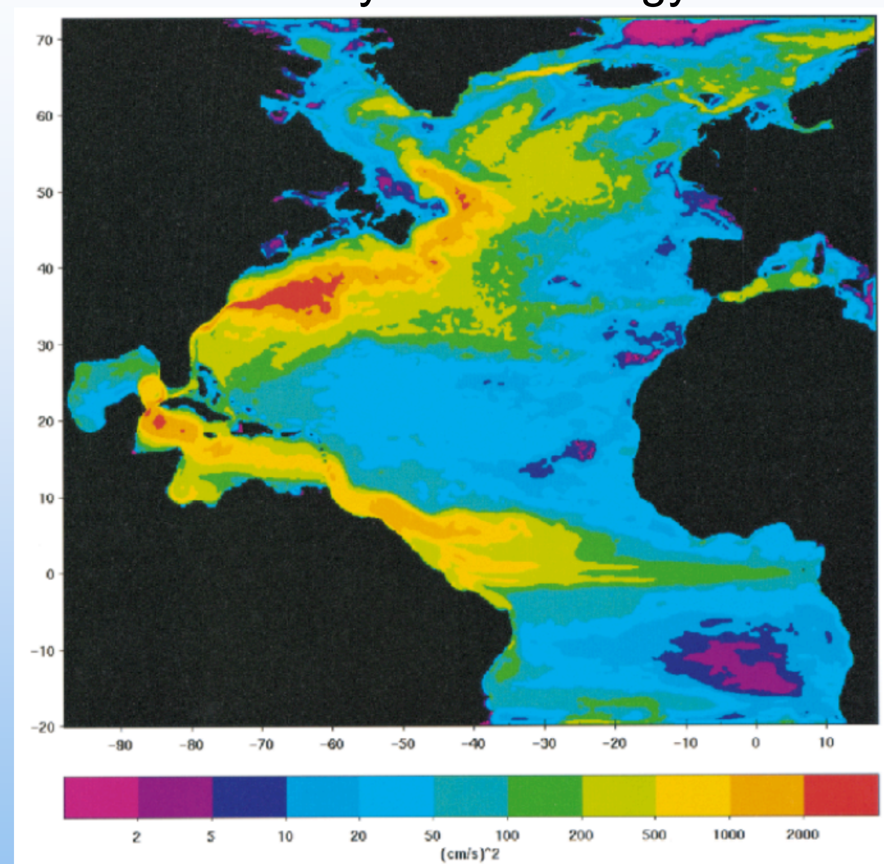
Smith et. al. 2000 jpo

POP simulations of the North Atlantic

POP, 0.28º resolution
Eddy kinetic energy



POP, 0.1º resolution
Eddy kinetic energy



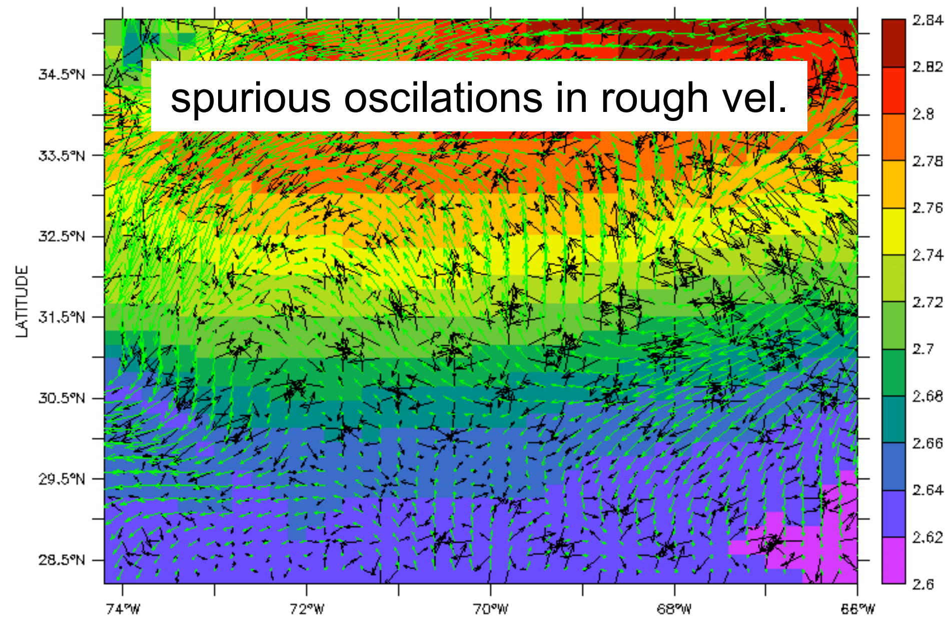
Higher eddy kinetic energy at high res.

Smith et. al. 2000 jpo

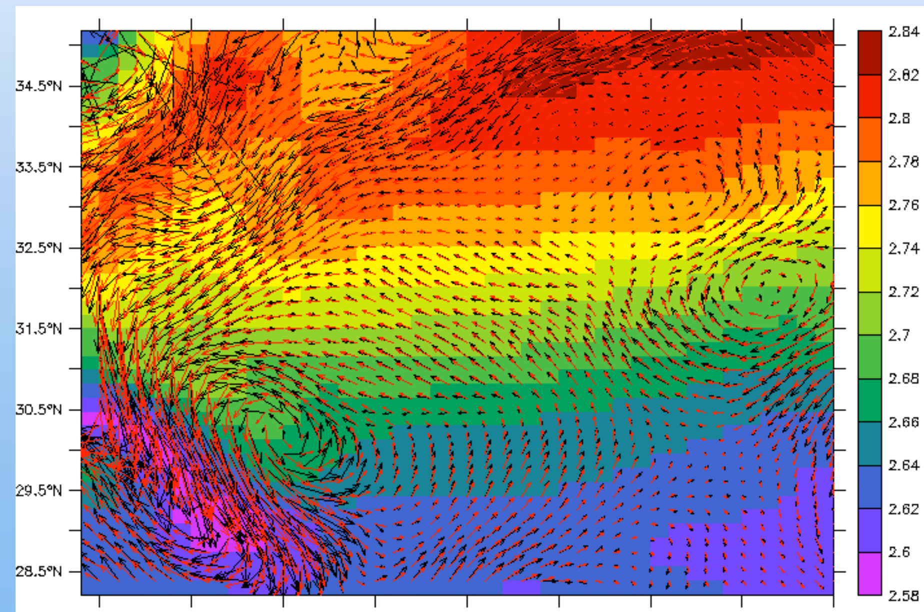
LANS-α in POP: North Atlantic simulations

The punch lines:

1. Due to rough boundaries and fast jets in realistic domains, both POP-α and Leray have numerical instabilities in the North Atlantic, particularly at boundaries.
2. The Leray Model runs longer in the North Atlantic, and has higher EKE and visibly more vortices (but still boundary issues).
3. The key is to use the right boundary conditions for the smoothing step (Helmholtz or filter).



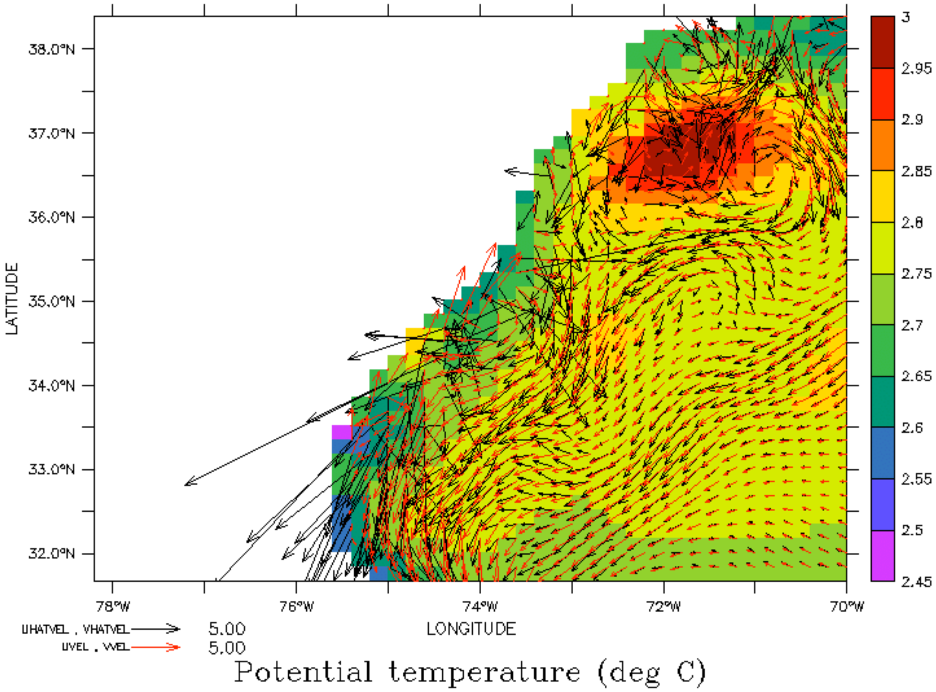
filter weights: 0.45 0.4 0.35 0.3

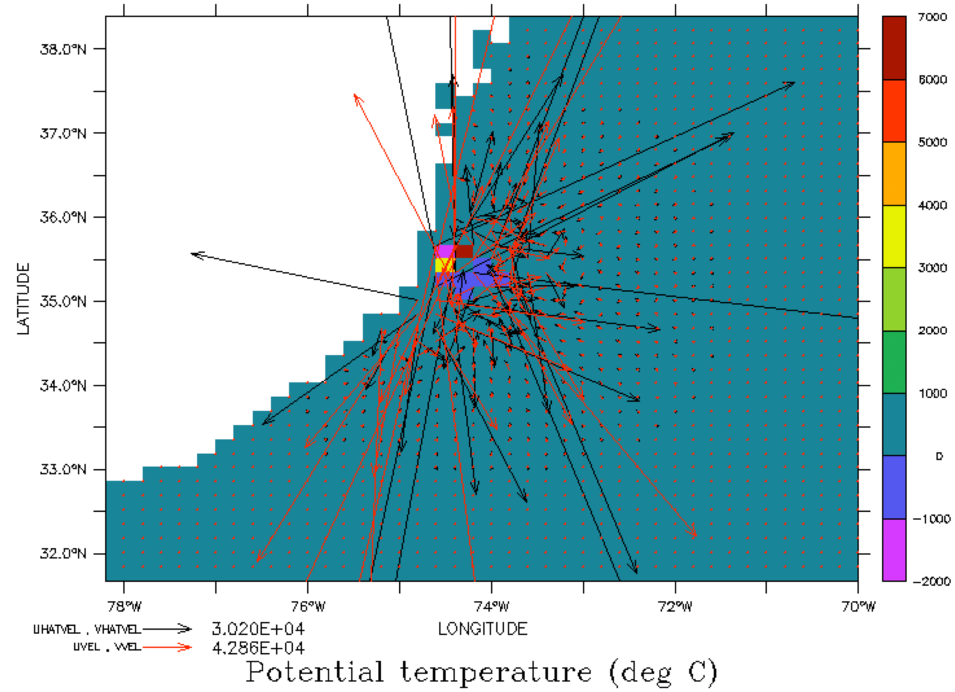


filter weights: 0.35 0.3 0.25 0.2

LANS-α in POP: North Atlantic simulations

Even with lower filter weights, numerical instabilities at boundary stop code after 6000 time steps (70 days):



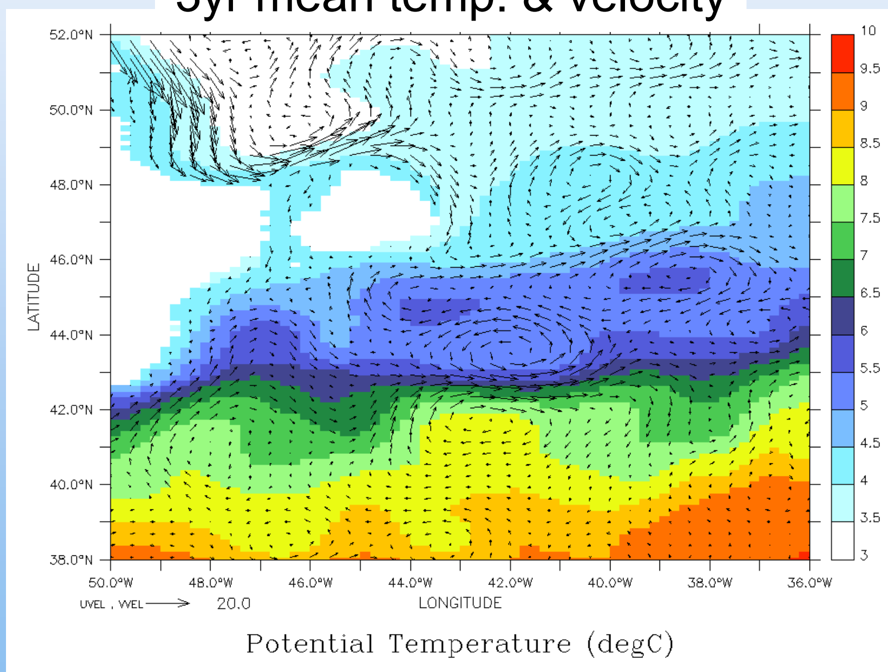


Leray Model runs stably in North Atlantic

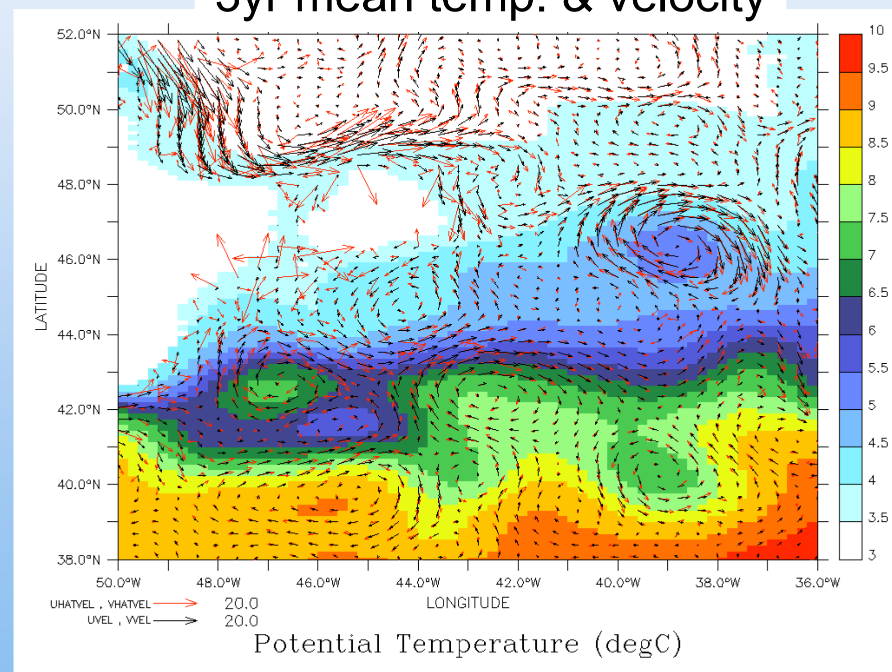
$$\frac{\partial \mathbf{v}}{dt} + \mathbf{u}\cdot\nabla\mathbf{v} + \underbrace{v_j \nabla u_j}_{} - \mathbf{f}\times\mathbf{u} = -\frac{1}{\rho_0}\nabla p + \nu\nabla^2\mathbf{v} + \mathbf{F}$$

extra nonlinear term
not in Leray model

POP 0.2º
3yr mean temp. & velocity



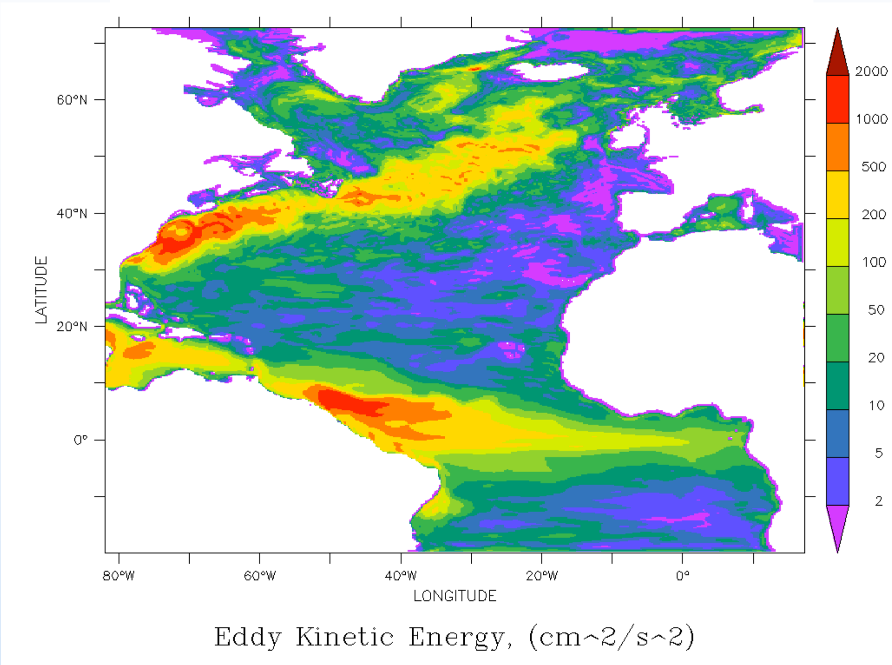
POP-Leray 0.2º
3yr mean temp. & velocity



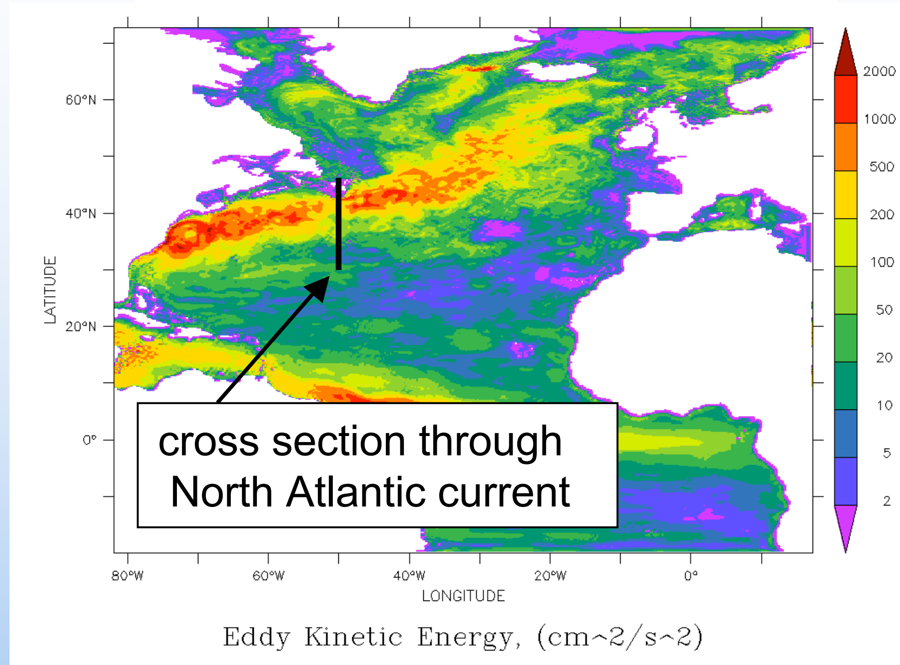
Leray Model has visibly more vortices, but still boundary issues.

POP-Leray has higher KE and EKE than POP

POP 0.2º
Eddy kinetic energy (3yr mean)

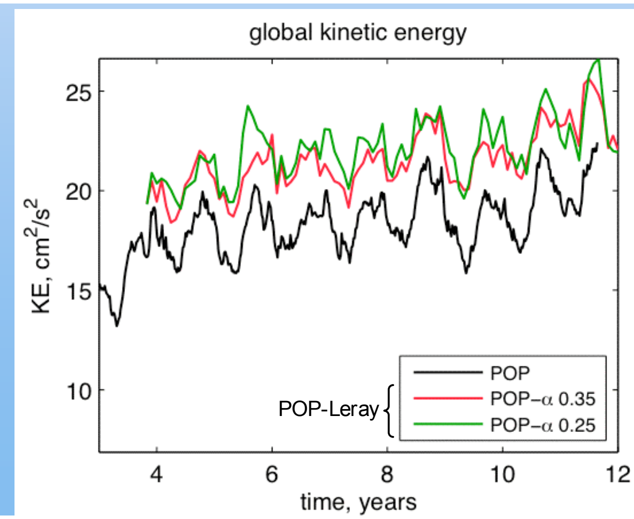


POP-Leray 0.2º
Eddy kinetic energy (3yr mean)



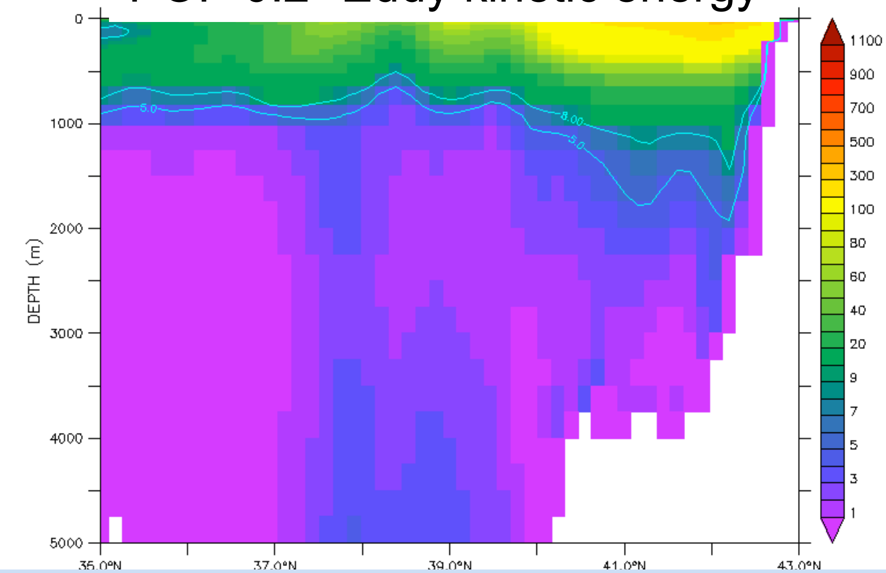
Globally averaged EKE (3 yr mean):

POP 0.2º:	11.1	
POP-Leray 0.2º:	13.1	
POP 0.1º:	29.4	(Smith et. al. 2000)

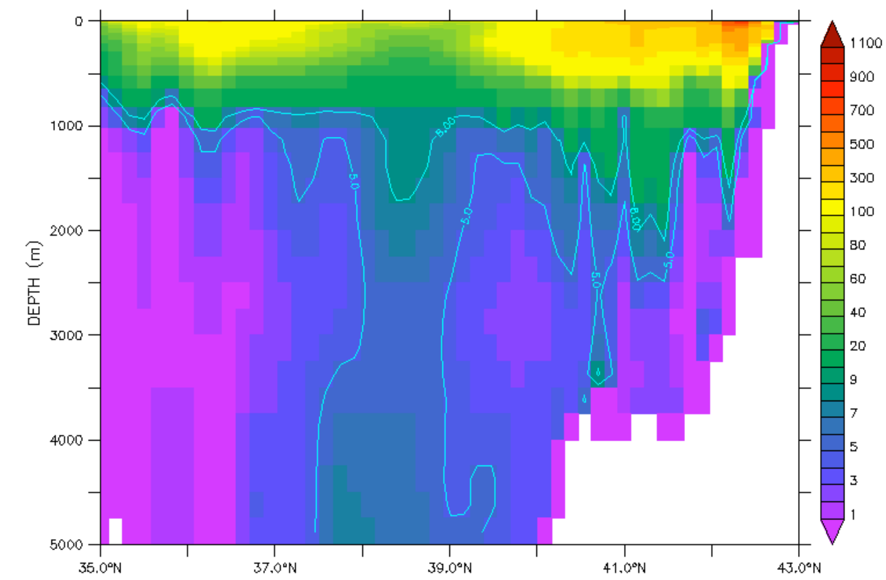


POP-Leray has higher KE and EKE that POP

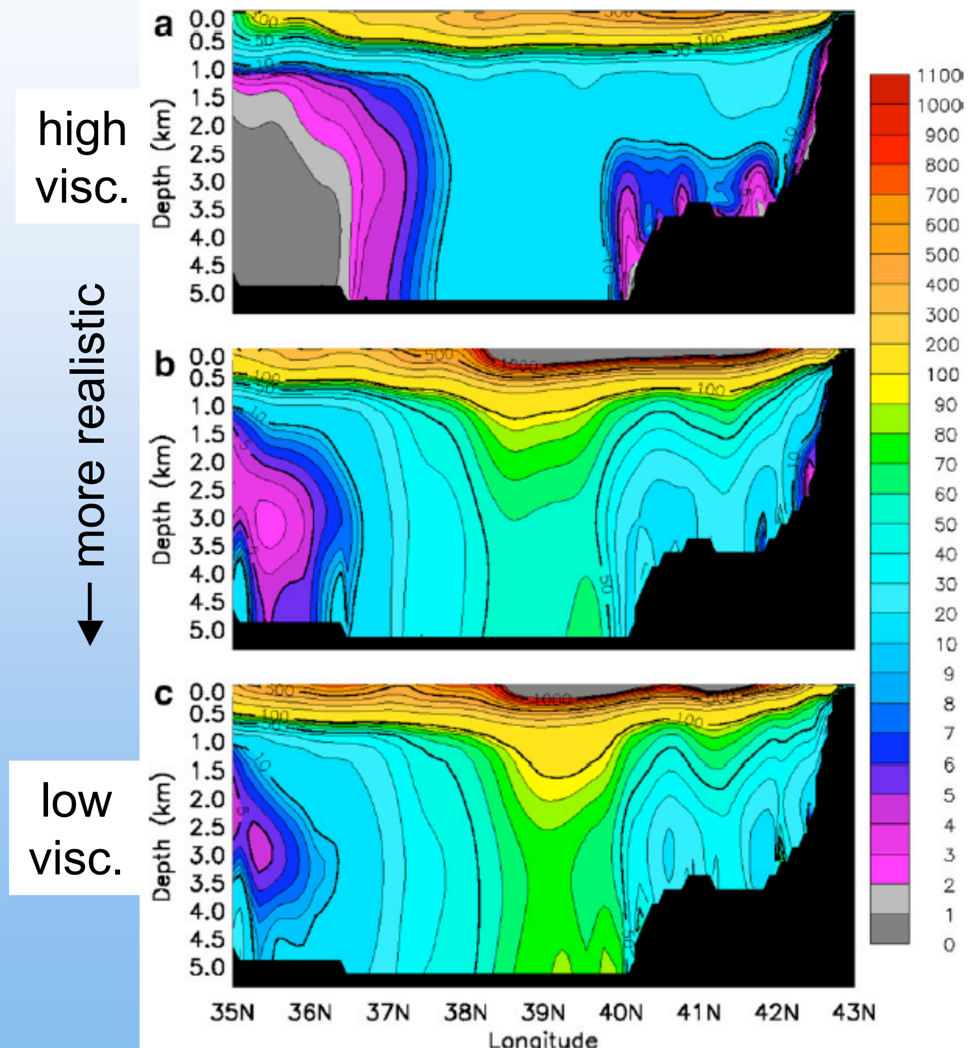
POP 0.2º Eddy kinetic energy



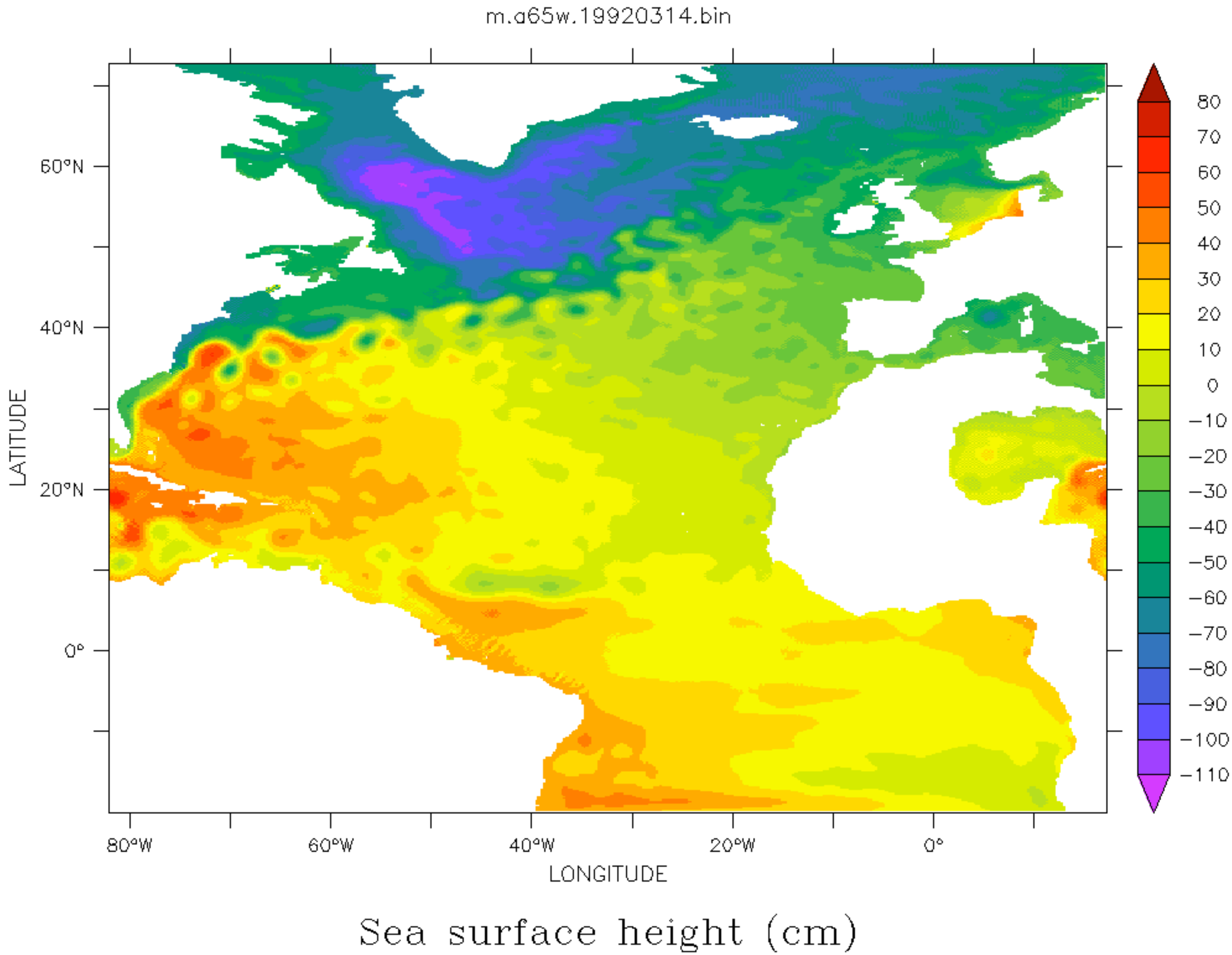
POP-Leray 0.2º Eddy kinetic energy



POP 0.1º EKE,
varying viscosity



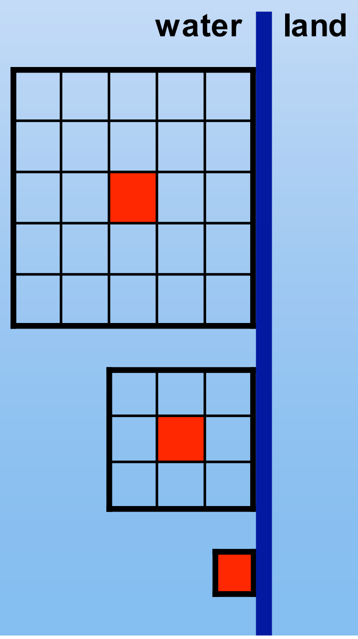
(Bryan et. al. 2007)


m.a65w.19920314.bin
60°N
40°N
20°N
0°
LATITUDE
80°W
60°W
40°W
20°W
0°
LONGITUDE
80
70
60
50
40
30
20
10
0
−10
−20
−30
−40
−50
−60
−70
−80
−90
−100
−110
Sea surface height (cm)

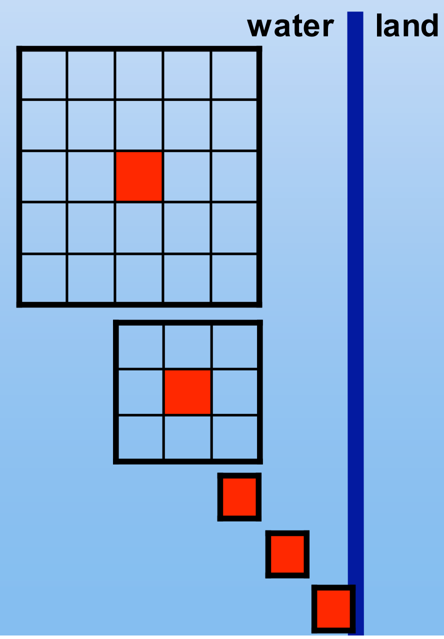
What are my boundary conditions?

B.C.	Equation
$\mathbf{v}=0$	$\partial_t \mathbf{v} + \mathbf{u}\cdot\nabla\mathbf{v} + v_j \nabla u_j - \mathbf{f}\times\mathbf{u} = -\frac{1}{\rho_0}\nabla p + \nu\nabla^2\mathbf{v} + \mathbf{F}$
$\mathbf{u}=0$	$\mathbf{u} = \left(1-\alpha^2\nabla^2\right)^{-1}\mathbf{v}$
I've tried many!	$\mathbf{u} = \mathrm{filter}(\mathbf{v})$

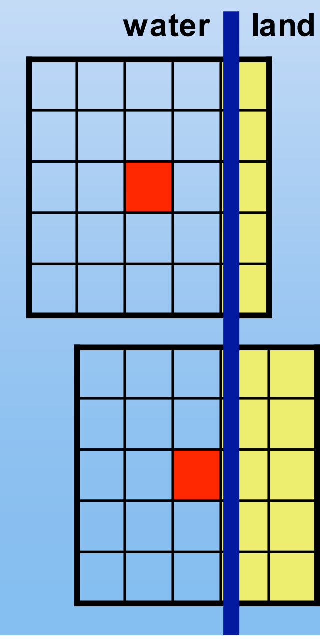
Option 1: shrink filter at boundary



Option 2: shrink filter *near* boundary



Option 3: make filter weights=0 on land



A Possibility: Use variable alpha

$$\begin{cases} \partial_t \mathbf{v} + \mathbf{u}\cdot\nabla\mathbf{v} \boxed{+ \left|\nabla\mathbf{u}\right|^2 \nabla\alpha^2(\mathbf{x})} + v_j \nabla u_j - \mathbf{f}\times\mathbf{u} = -\frac{1}{\rho_0}\nabla p + \nu\nabla^2\mathbf{v} + \mathbf{F} \\ \mathbf{u} = \left(1 \boxed{- 2\alpha(\mathbf{x})\nabla\alpha(\mathbf{x})\cdot\nabla} - \alpha^2\nabla^2\right)^{-1}\mathbf{v} \end{cases}$$

We are thinking about this…

Summary

- Higher resolution can solve all of your problems.
- You can't possibly have high enough resolution to solve your problems.

- The LANS-alpha model captures higher-resolution effects in our test problem, where eddies near the grid-scale are important.
- POP-Leray runs longer than POP- α in the North Atlantic domain, and shows promising signs, like higher eddy activity
- Both POP-α and POP-Leray have problems with the rough boundaries and topography of the North Atlantic.
- Further work on boundary conditions for LANS-alpha, with Helmholtz or filter smoothing, is required.