Summary

The Navier-Stokes-$\alpha$ model of turbulence is a modification of the Navier-Stokes equations in which the vorticity is advected and smoothed by a kernel associated with the Green's function of the Helmholtz operator scaled by a parameter $\alpha$. The statistical properties of the smoothed velocity field are expected to match those of the Navier-Stokes turbulence for scales larger than $\alpha$, thus providing a more computable model for those scales.

For wavenumbers $k$ such that $k_0 > 1$, corresponding to spatial scales smaller than $\alpha$, there are three candidate power laws for the energy spectrum, corresponding to three possible characteristic time scales in the model equations: one from the smoothed field, the second from the rough field and the third from a special combination of the two. In two dimensions, the second time scale may be understood to characterize the dynamics of the conserved quantity, namely the enstrophy of the rough field.

We measure the scaling of the energy spectra from high-resolution simulations of the two-dimensional Navier-Stokes-$\alpha$ model, in the limit as $\alpha \to 0$. The energy spectrum of the smoothed velocity field scales as $k^{-1}$ in the direct enstrophy cascade regime, consistent with dynamics dominated by the timescale associated with the rough field. We are thus able to deduce that the dynamics of the conserved quantity, namely the enstrophy of the rough field, governs the scaling of all derived statistical quantities.

For the two-dimensional Navier-Stokes-$\alpha$ model, the timescale coming from the special combination of the rough and smoothed field is understood to characterize the dynamics of the conserved enstrophy.

We measure the scaling of the energy spectra from high-resolution simulations of the two-dimensional Navier-Stokes-$\alpha$ model, in the limit as $\alpha \to 0$. The energy spectrum of the smoothed velocity field scales as $k^{-1}$ in the direct enstrophy cascade regime, consistent with dynamics dominated by the timescale associated with the special combination of the rough and smoothed field.

This result is consistent with our result above, on the behavior of the spectral slope of the two-dimensional Navier-Stokes-$\alpha$ model in the wavenumber regime $k_0 > 1$.

Analytical and Numerical Study of the Two-dimensional Navier-Stokes-$\alpha$ and Leray-$\alpha$ models of turbulence

Evelyn M. Lunasin, Susan Kurien, Mark A. Taylor, and Edriss S. Titi

Journal of Turbulence 2007