Lagrangian Coherent Structures in Geophysical Flows

Movies can be viewed at http://www.cds.caltech.edu/~marsden/wiki/projects/LCSProjects
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Coherent Structures in Flows
Coherent Structures in Flows

Hurricanes
Coherent Structures in Flows

Hurricanes

Jupiter’s red spot
Coherent Structures in Flows

Hurricanes

Jupiter’s red spot

Atmosphere
Coherent Structures in Flows

Hurricanes

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Neptune’s great dark spot
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Elliptic 3-body problem
Coherent Structures in Flows

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Elliptic 3-body problem

Bio-locomotion
Aperiodic flows
Finding structures in flows
Finding structures in flows
Finding structures in flows
Invariant manifolds in the simple pendulum

\[ \ddot{x} + \sin x = 0 \]
Invariant manifolds in the simple pendulum

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The stable and unstable manifolds:
Invariant manifolds in the simple pendulum

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The stable and unstable manifolds:

- are **invariant**.
Invariant manifolds in the simple pendulum

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The stable and unstable manifolds:
- are invariant.
- are separatrices between regions of different dynamical behavior.
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- are **separatrices** between regions of different dynamical behavior.
- form the **boundaries of invariant sets**.
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The stable and unstable manifolds:

- are **invariant**.
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- form the **boundaries of invariant sets**.
- dictate structure and **transport** in the flow.
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- form the boundaries of invariant sets.
- dictate structure and transport in the flow.
- are surfaces of greatest stretching.
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The stable and unstable manifolds:

- are invariant.
- are separatrices between regions of different dynamical behavior.
- form the boundaries of invariant sets.
- dictate structure and transport in the flow.
- are surfaces of greatest stretching.
- are attached to fixed points, periodic orbits, etc.
Separatrices for aperiodic systems

\[ \ddot{x} + \sin x = f(t) \]
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Define the LCS to be ridges in the FTLE field.

\[ \sigma_T(t, x) := \frac{1}{|T|} \ln \left\| \frac{d\phi_t^{t+T}}{dx} \right\|_2 \]
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The LCS
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\]

The LCS

• are defined as surfaces of greatest stretching.

We will see that LCS

• are (almost) invariant.
• are separatrices between regions of different dynamical behavior.
• form the boundaries of invariant sets.
• dictate structure and transport in the flow.
• are NOT attached to fixed points, periodic orbits, etc.
A quick test

\[ \ddot{x} + \sin x = 0 \]
A quick test

\[ \ddot{x} + \sin x = 0 \]
Now add periodic forcing ...
Now add periodic forcing ... 

$$\ddot{x} + \sin x + \epsilon \dot{x} \sin t = 0$$
Now add periodic forcing ...

\[ \ddot{x} + \sin x + \epsilon \dot{x} \sin t = 0 \]
Now add periodic forcing ...

\[ \ddot{x} + \sin x + \epsilon \dot{x} \sin t = 0 \]
The Poincare map
Homoclinic tangles
Lobe Dynamics
Lobe Dynamics
Lobe Dynamics
Lobe Dynamics
Lobe Dynamics
Lobe Dynamics
Lobe Dynamics
LCS for periodic forcing
LCS for periodic forcing
Lobes dynamics with periodic forcing
Lobes dynamics with periodic forcing
LCS for aperiodic forcing

\[ \ddot{x} + \sin x = f(t) \]
LCS for aperiodic forcing
LCS for aperiodic forcing
LCS for aperiodic forcing

Captures transient chaos
(Aside on tangles)
(Aside on tangles)
(Aside on tangles)
(Aside on tangles)
(Aside on tangles)
(Aside on tangles)
Transport in hurricanes

Typhoon 200514 : 2005-08-29 00:00 UTC

www.digital-typhoon.org
Will Warming Lead to a Rise in Hurricanes?

By CORNELIA DEAN
Published: May 29, 2007

When people worry about the effects of global warming, they worry more about hurricanes than anything else. In surveys, almost three-quarters of Americans say there will be more and stronger hurricanes in a warming world. By contrast, fewer than one-quarter worry about increased coastal flooding.
Entrainment and Hurricane Intensity

\[ V_p^2 = \frac{T_s - T_0}{T_0} \left( F_\downarrow - F_\uparrow \right) + \frac{F_{\text{entrainment}}}{C_D \rho |V|} \]

Dr. Kerry Emanuel, MIT,
Environmental Factors Affecting Tropical Cyclone Power Dissipation (In Press).
Stability of Hurricanes

The New York Times

(29 May 2007)
LCS in typhoon Nabi
LCS in typhoon Nabi
Smale’s Horseshoe Map
Smale’s Horseshoe Map
Horshoes in Typhoon Banyan
Horshoes in Typhoon Banyan
Horshoes in Typhoon Banyan
2005 Atlantic Hurricane season

[Map of Atlantic Hurricane season with track lines and labels for hurricanes and tropical storms, showing their paths and dates.]

- Hurricanes (H)
- Tropical Storm (T)
- Tropical Depression
- Extra-tropical
- Wave/Low
- Subtropical Depression
- Subtropical Storm (ST)

Legend:
- ● Position at 0000 UTC
- ■ Position/date at 1200 UTC

Dates and locations of hurricanes and tropical storms are marked on the map.
Tracking Easterly waves

Drifsonde trajectories and dropsonde locations on September 16th 2006

Atlantic Ocean

Western Africa

Latitud (°N)

Longitud (°E)
2005 Atlantic Hurricane Season
2005 Atlantic Hurricane Season
Vortex Rings
Vortex Rings
Vortex Rings
Vortex Rings
Vortex Rings
Vortex rings in the laboratory
LCS for the vortex ring

Vortex velocity field
LCS for the vortex ring

Vortex velocity field
LCS for the vortex ring
LCS and vortex boundaries
LCS and vortex boundaries

\[ n_{\text{hi}} = 52 \]
Lobes in the vortex ring
Jellyfish
Jellyfish
Jellyfish
LCS for ozone hole break up
LCS and mixing in Monterey bay
LCS and mixing in Monterey bay

repelling LCS

Attracting LCS
LCS and mixing in Monterey bay

Repelling LCS

Attracting LCS
LCS and mixing in Monterey bay

Repelling LCS

Attracting LCS
Mixing in Monterey Bay
Mixing in Monterey Bay
Mixing in Monterey Bay
Mixing in Monterey Bay
Neptune’s Great Dark Spot
Modelling the Dark spot

Eccentricity

Orientation
Kida vortex velocities
In the present paper I have integrated the equations of motion and plotted curves for the paths of particles in the well-known simple cases of two-dimensional motion, viz., for liquid contained in a rotating elliptic cylinder and in a rotating equilateral triangular prism, and for liquid extending to infinity and disturbed by the translation or rotation of an elliptic cylinder.
Trajectories in the Kida vortex
LCS in the Kida vortex
Lobes in the kida vortex
Mediterranean Salt Lenses (Meddies)
Mediterranean Salt Lenses (Meddies)
Lobes in Meddies.
Interacting vortices
Interacting vortices
LCS in interacting vortices
LCS in interacting vortices
LCS in interacting vortices
LCS in interacting vortices
LCS in interacting vortices
LCS in interacting vortices
LCS in interacting vortices 3D
LCS in interacting vortices 3D
3D LCS in atmosphere of Titan
3D LCS in atmosphere of Titan
3D LCS in atmosphere of Titan
3D LCS in a convection cell
3D LCS in a convection cell
Hairpin vortices in near wall turbulent flow

Melissa Green,
Clancy Rowley,
George Haller.
Hairpin vortices in near wall turbulent flow

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Hairpin vortices in near wall turbulent flow

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Clancy Rowley,
George Haller.
LCS in blood flow
Invariant manifolds in space travel
Invariant manifolds in space travel
Invariant manifolds in space travel
Numerical considerations
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• Choosing integration time, T.
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• Choosing integration time, $T$.  
• Resolution and robustness.
Numerical considerations

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- Resolution and robustness.
Numerical considerations

- Choosing *integration time*, $T$.
- **Resolution** and robustness.

- **Memory**: huge data in; huge data out.
Numerical considerations

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• Straightforward to **parallelize**.
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- Computations are now “**bandwidth limited**.”
Numerical considerations

- Choosing **integration time**, $T$.
- **Resolution** and robustness.

- **Memory**: huge data in; huge data out.
- Straightforward to **parallelize**.
- Computations are now “**bandwidth limited**.”
- Computed online in near **real time**.
Near Real-Time Computation of LCS

http://ourocean.jpl.nasa.gov/MB06/

LCS

The LCS (Lagrangian Coherent Structure) analysis was generated by MANGEN (Manifold Generator) using ROMS forecast model output. ROMS generate 2-day forecast daily and MANGEN performs a 48 hour forward integration of the model surface velocity fields to find the LCS structure in the Monterey Bay area. Currently, we only run Mangen over a subdomain of the ROMS model. The subdomain covers a rectangular area from (-122.74, 35.62) to (-121.10, 37.48).

Animation

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Near Real-Time Computation of LCS

http://ourocean.jpl.nasa.gov/MB06/
Upcoming LCS projects ...

Numerics:

• Visualization in 3D.
• Automatic extraction of LCS.
• Fast computation methods.

Theory:

• What are the properties of LCS?
• Relationship with transfer operator methods.

Applications:

• 3D Hurricanes.
• 3D Oceans.
• Quantified transport and mixing in geophysical flows.