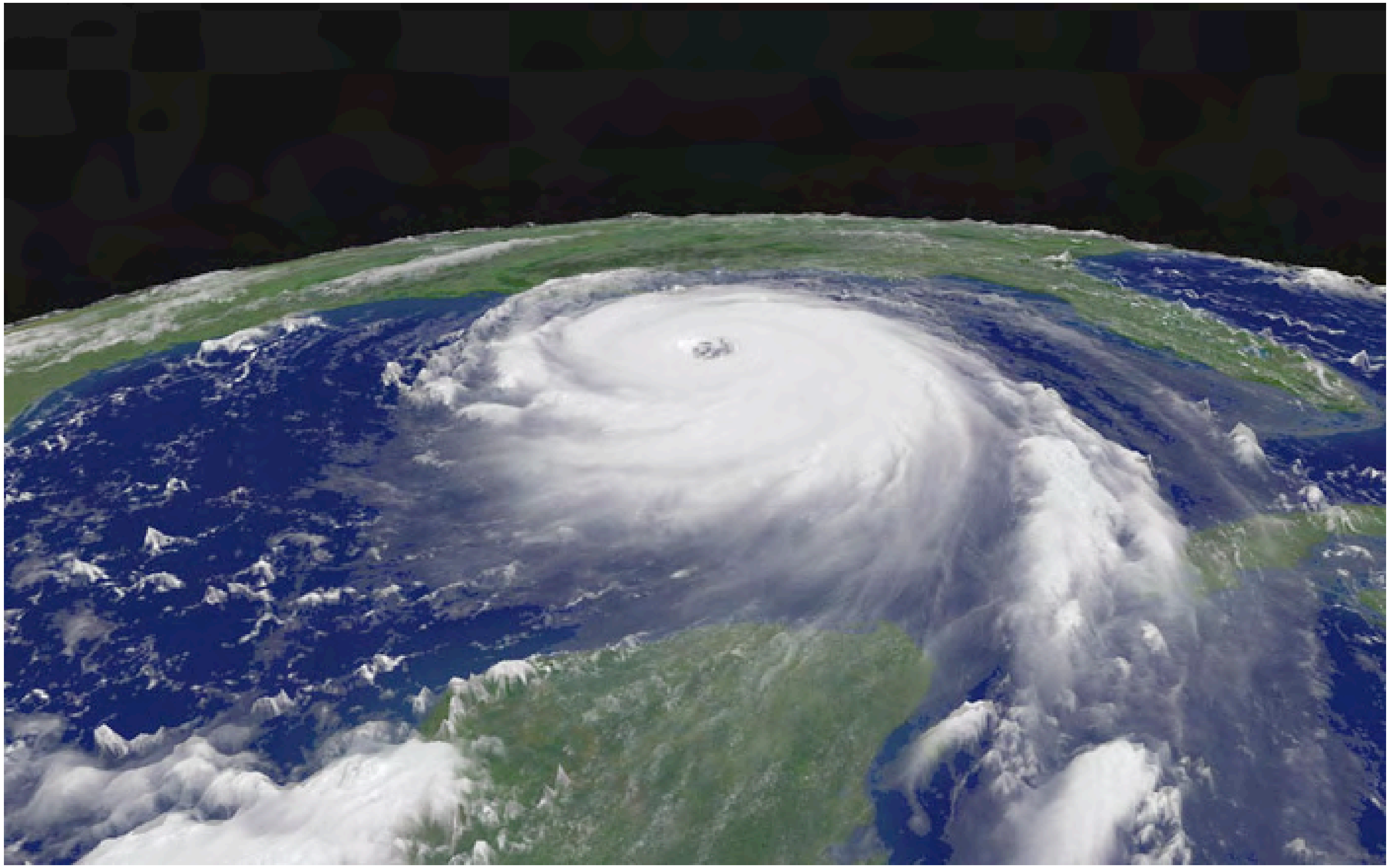
Lagrangian Coherent Structures in Geophysical Flows



Philip Du Toit
California Institute of Technology

Movies can be viewed at http://www.cds.caltech.edu/~marsden/wiki/projects/LCSProjects

Collaborators

- Francois Lekien, Ecole Polytechnique, Brussels
- Shawn Shadden, Stanford
- Benjamin Bastide, Ecole des Mines
- Evan Gawlik, Caltech
- Chad Couliette
- George Haller, MIT
- Jerrold Marsden, Caltech
- Philip Du Toit, Caltech

Coherent Structures in Flows

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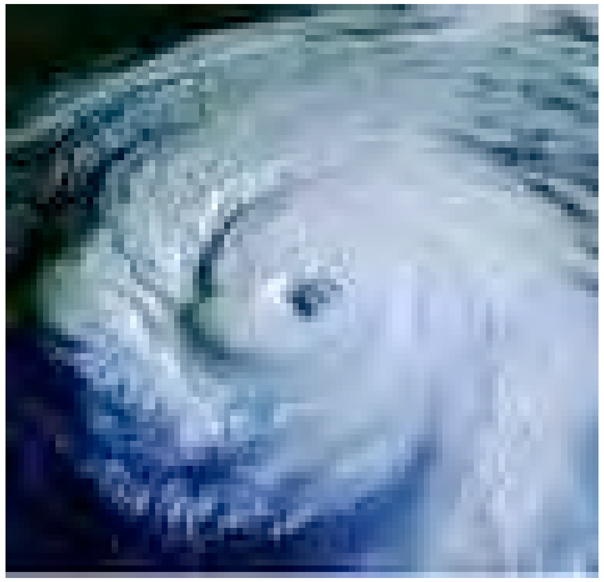
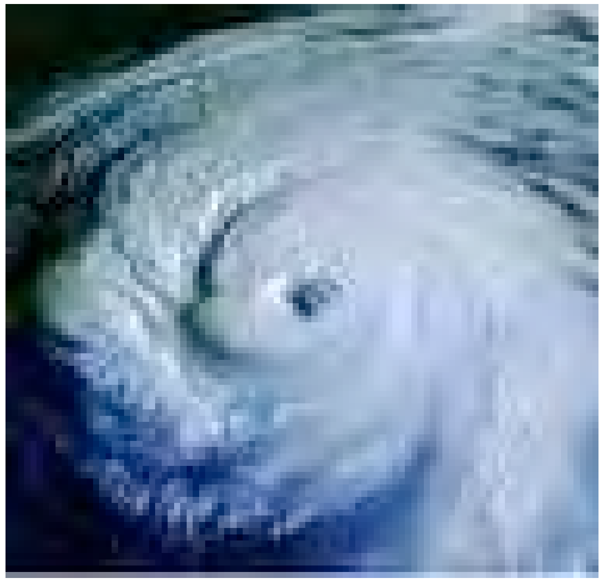


Hurricanes

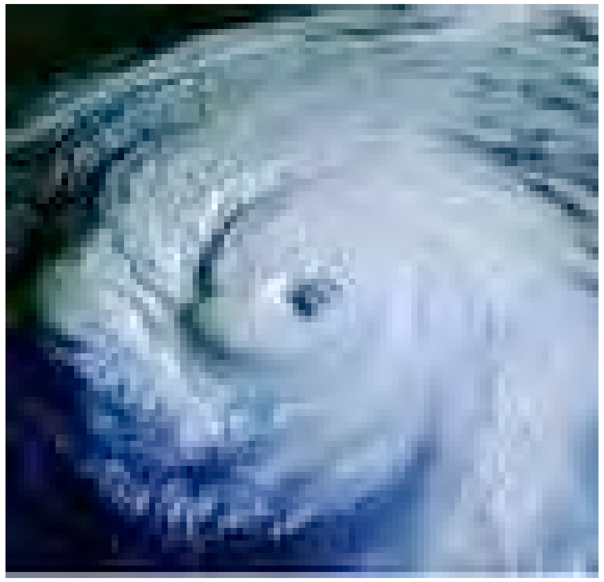
Coherent Structures in Flows



Hurricanes

Jupiter's red spot

Coherent Structures in Flows



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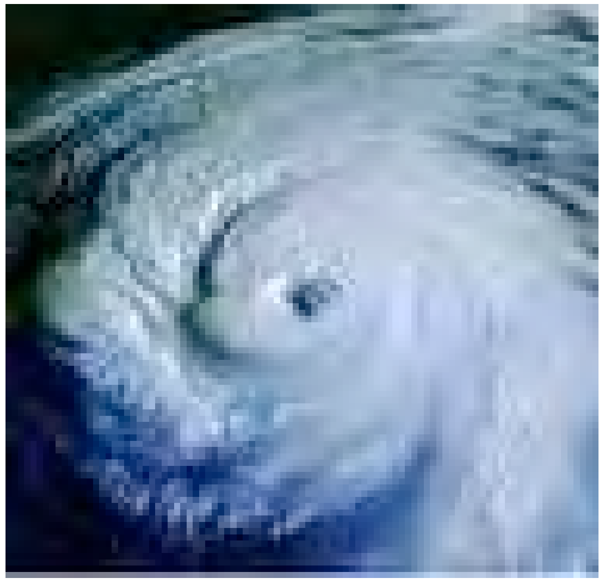

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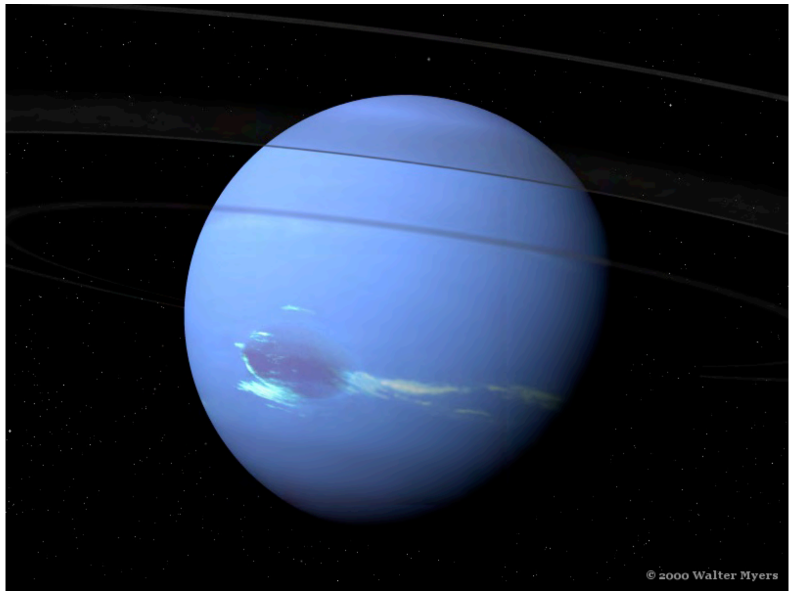

Neptune's great dark spot

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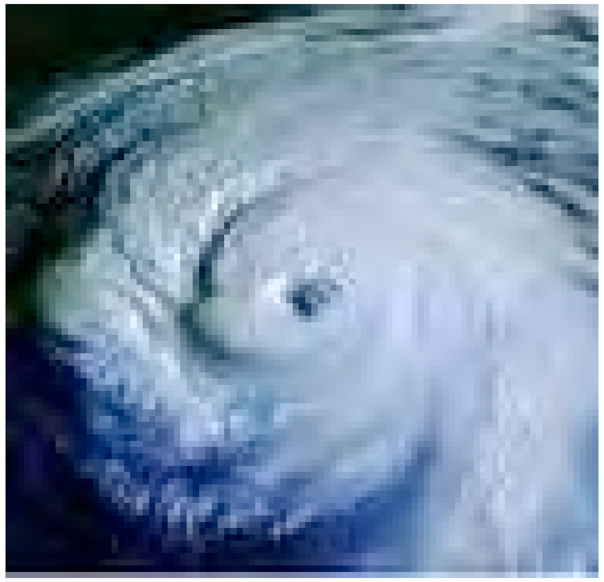

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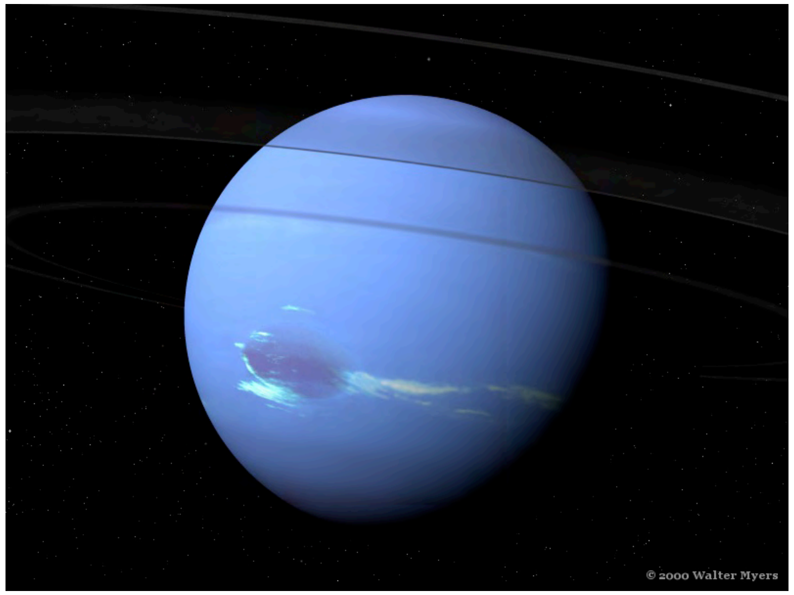

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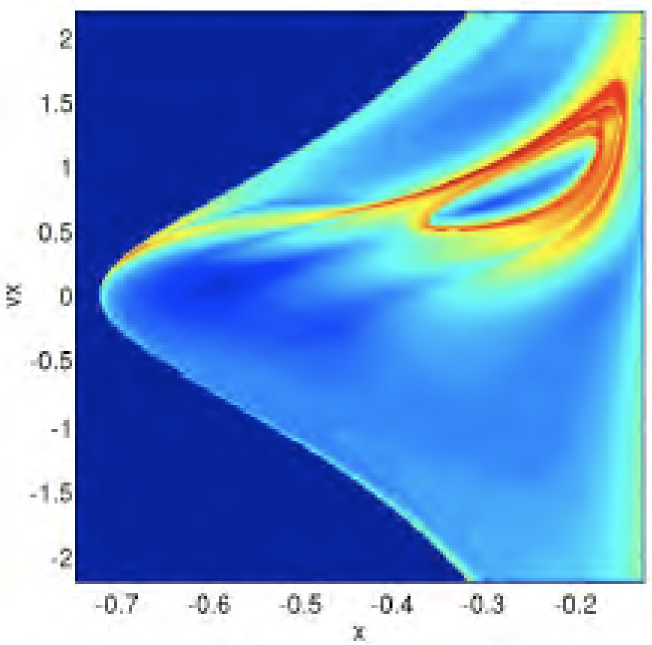
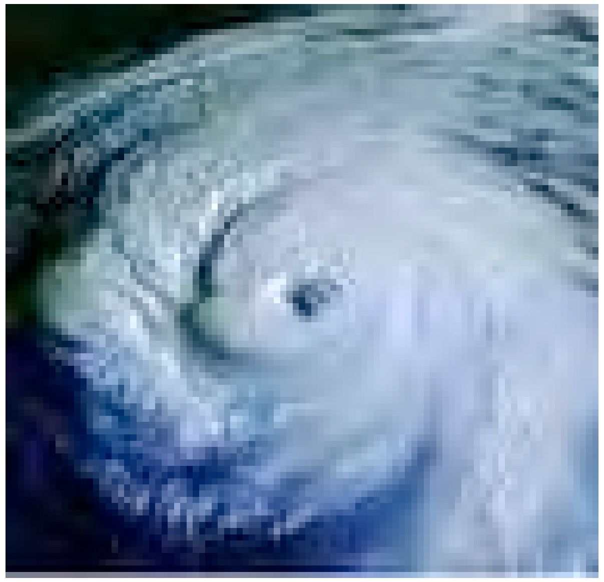


Elliptic 3-body problem

Coherent Structures in Flows



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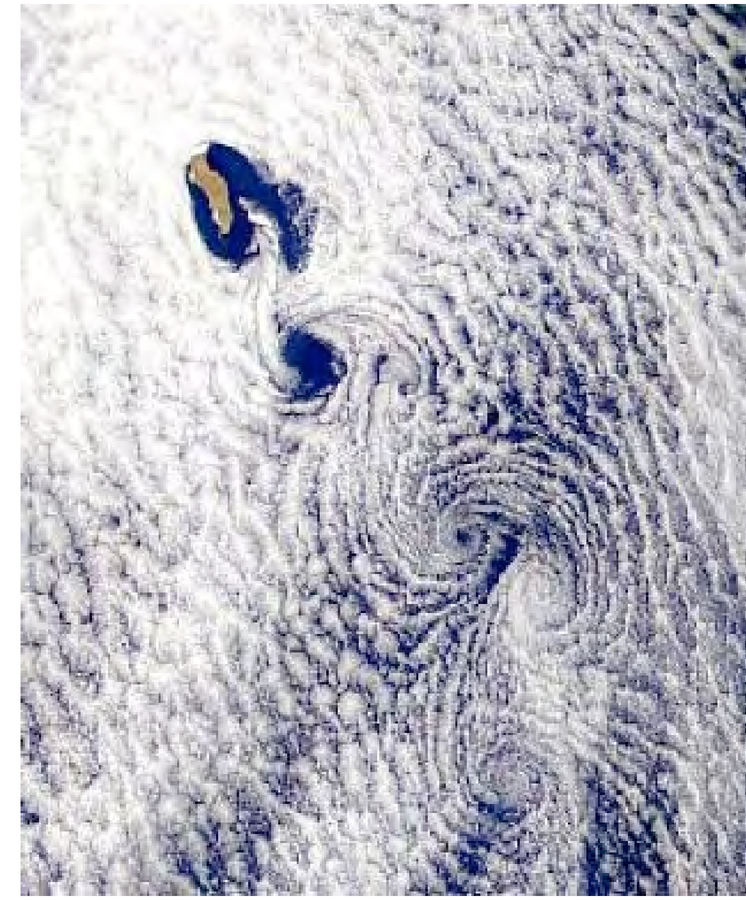

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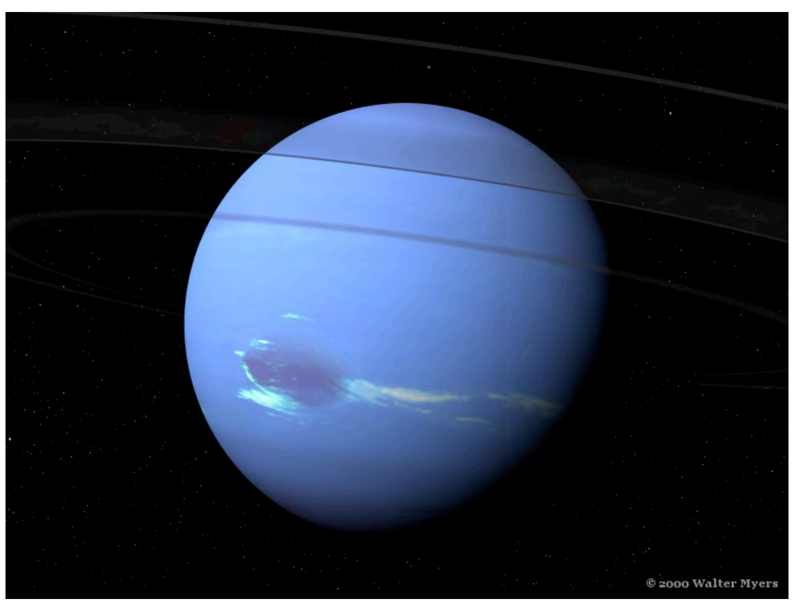

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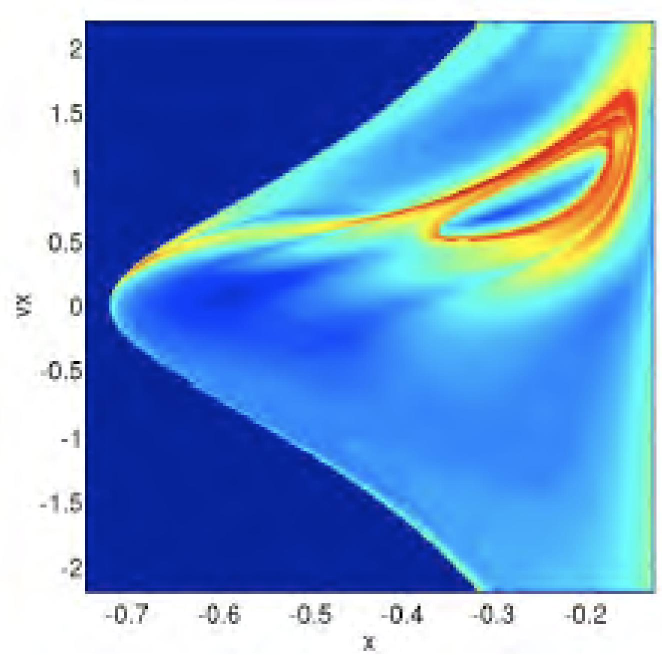
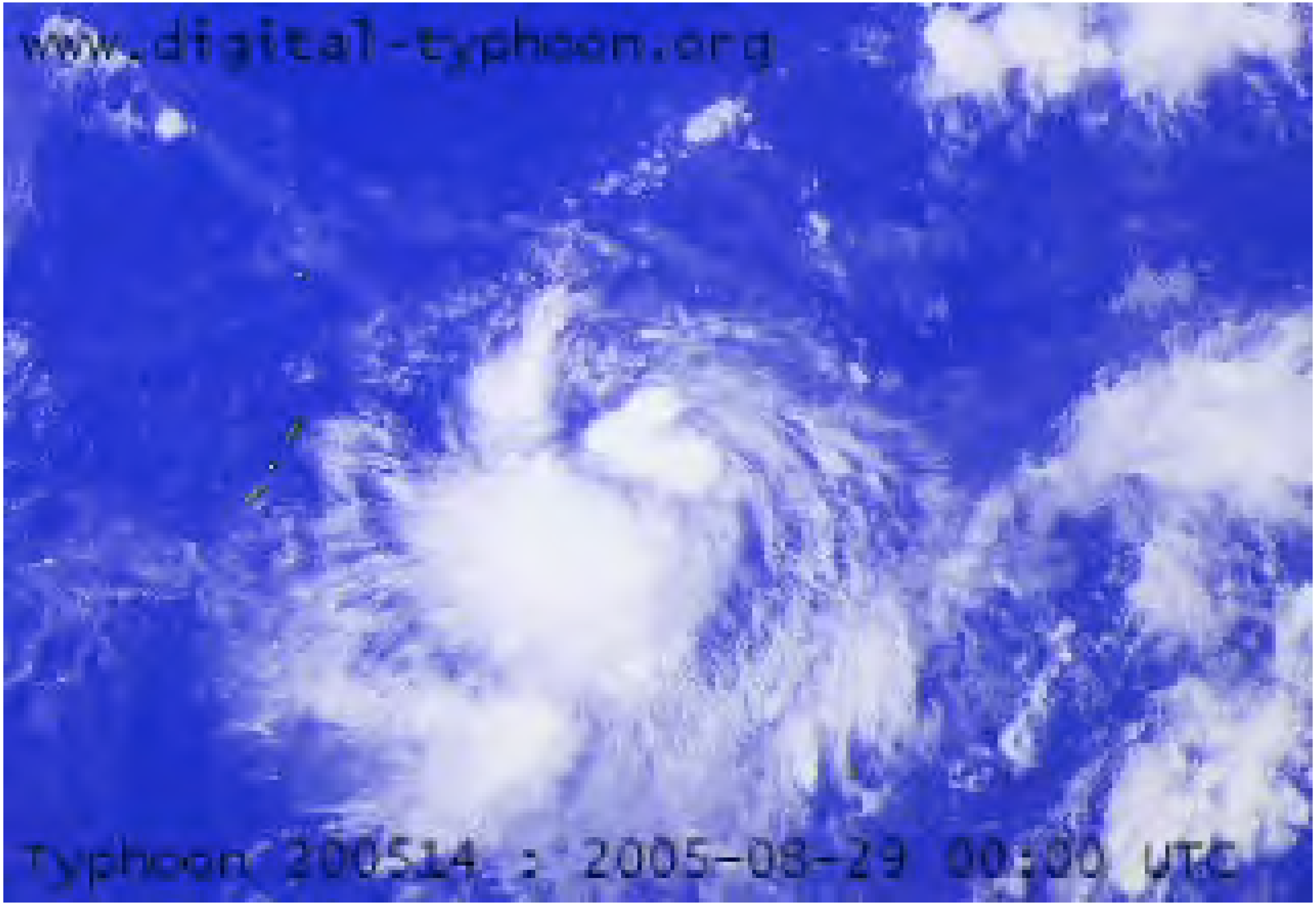


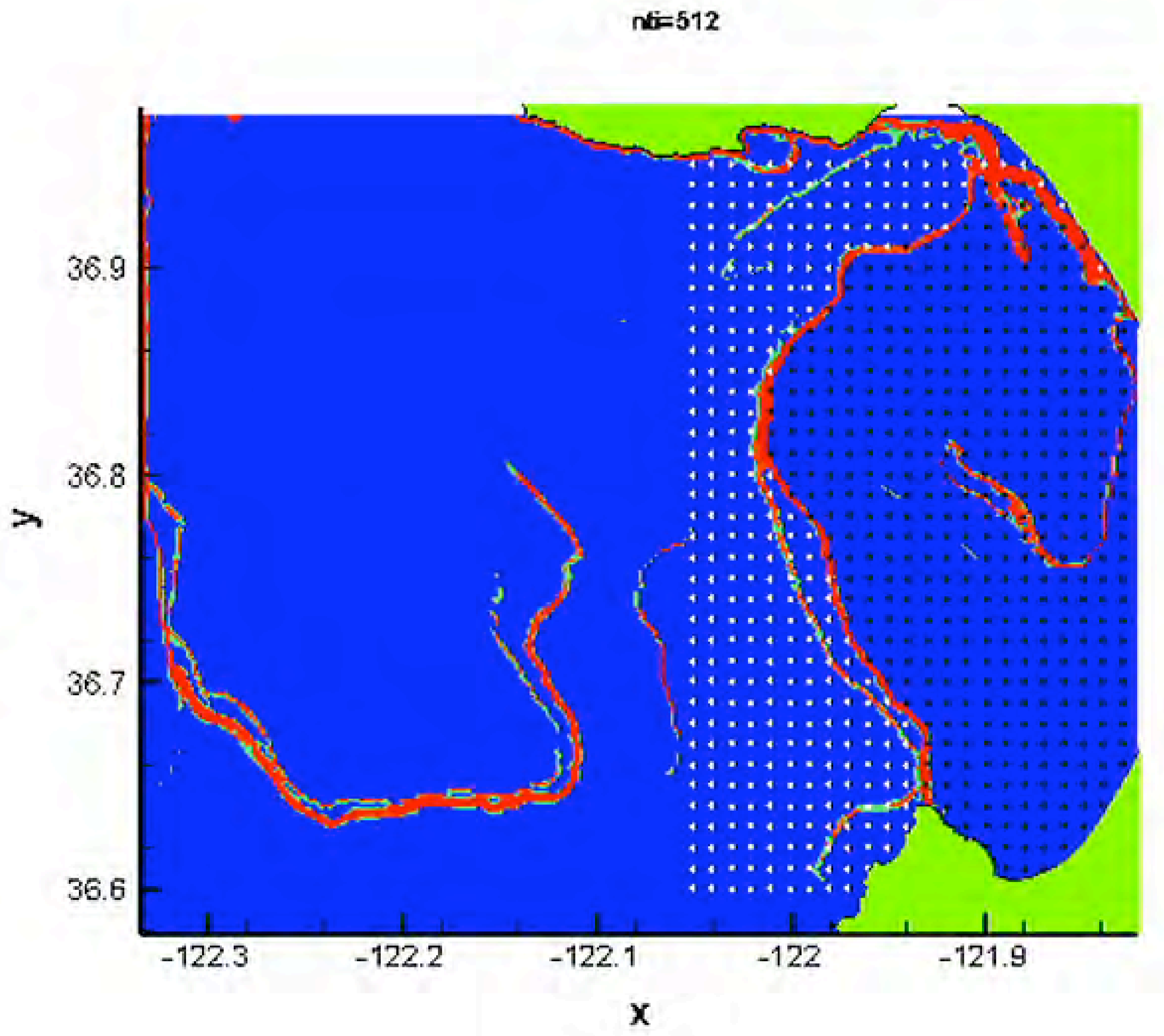
Elliptic 3-body problem

Bio-locomotion

Aperiodic flows



Finding structures in flows



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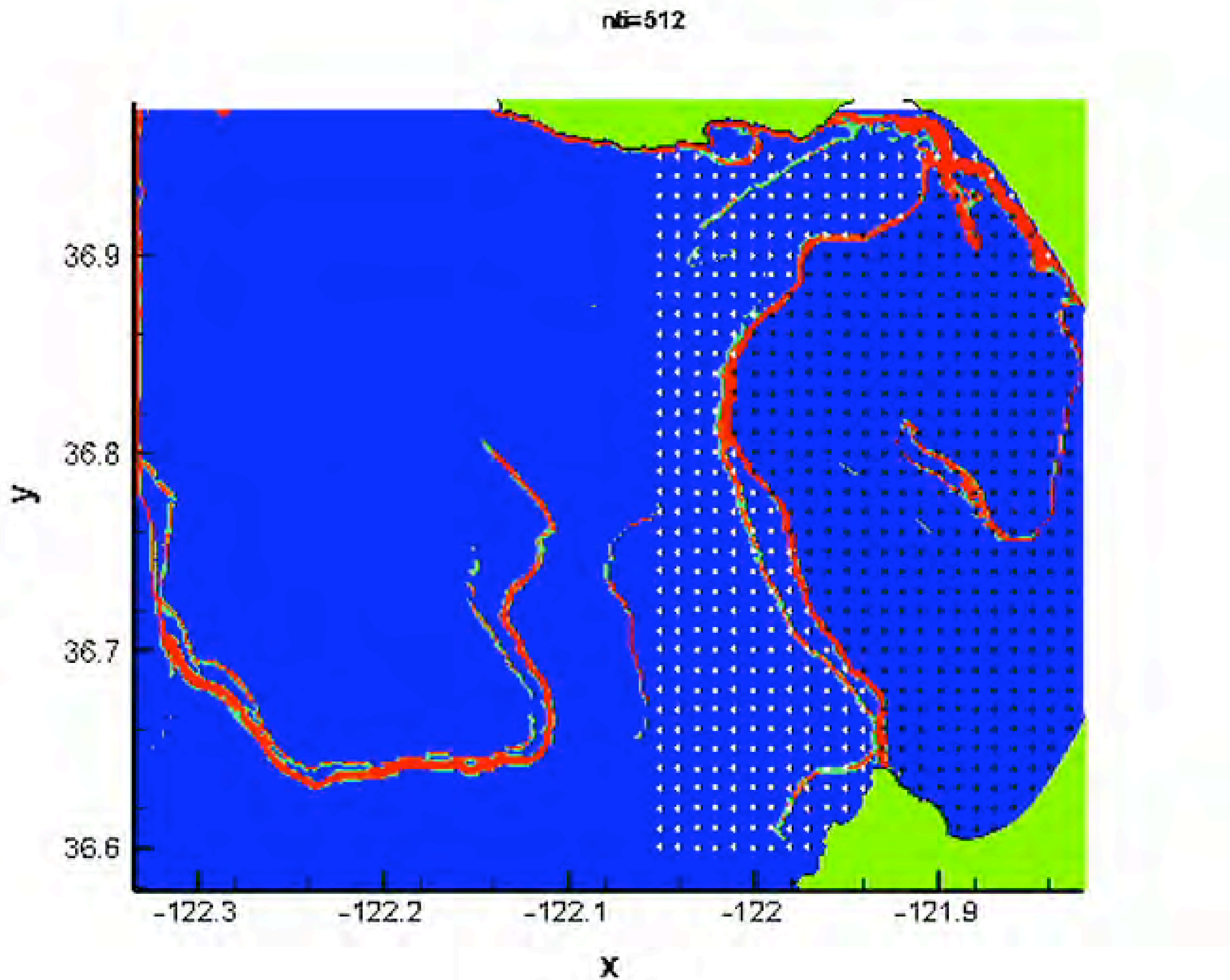
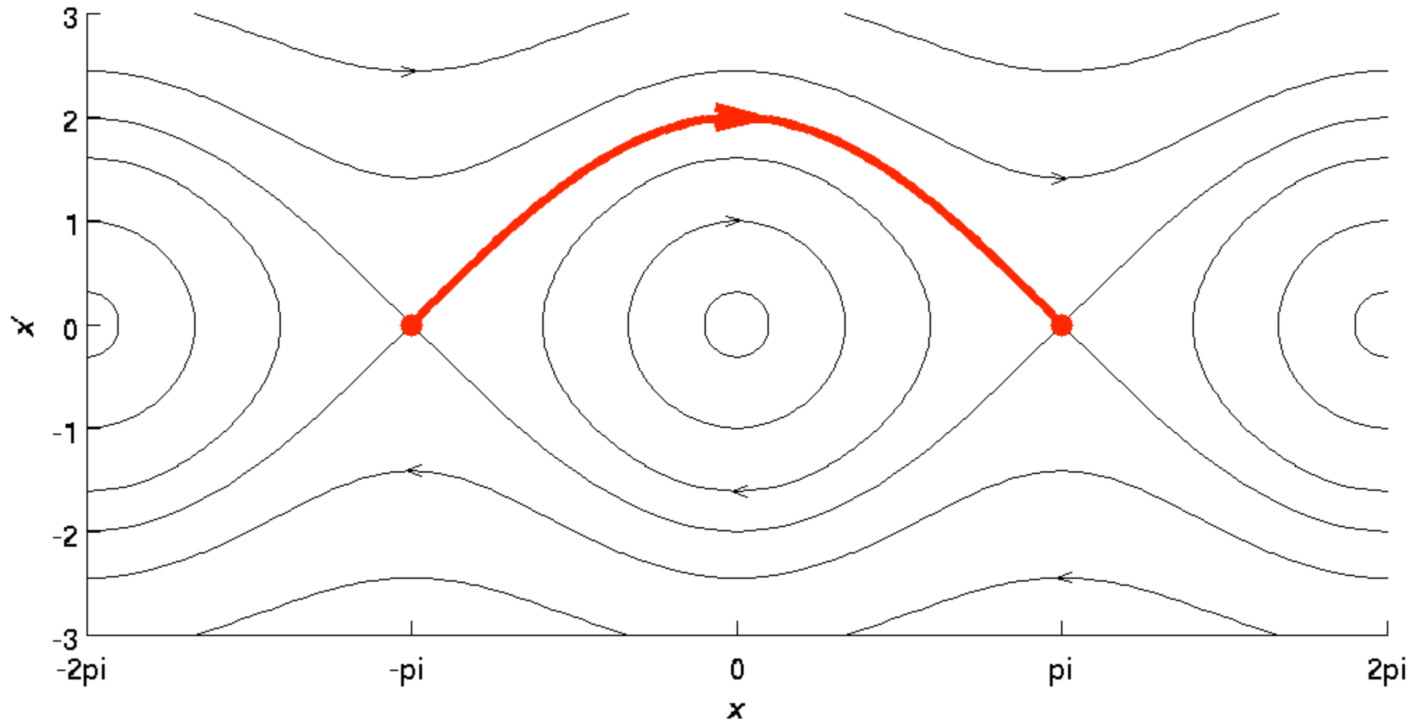


Finding structures in flows

Invariant manifolds in the simple pendulum

$$\ddot{x} + \sin x = 0$$



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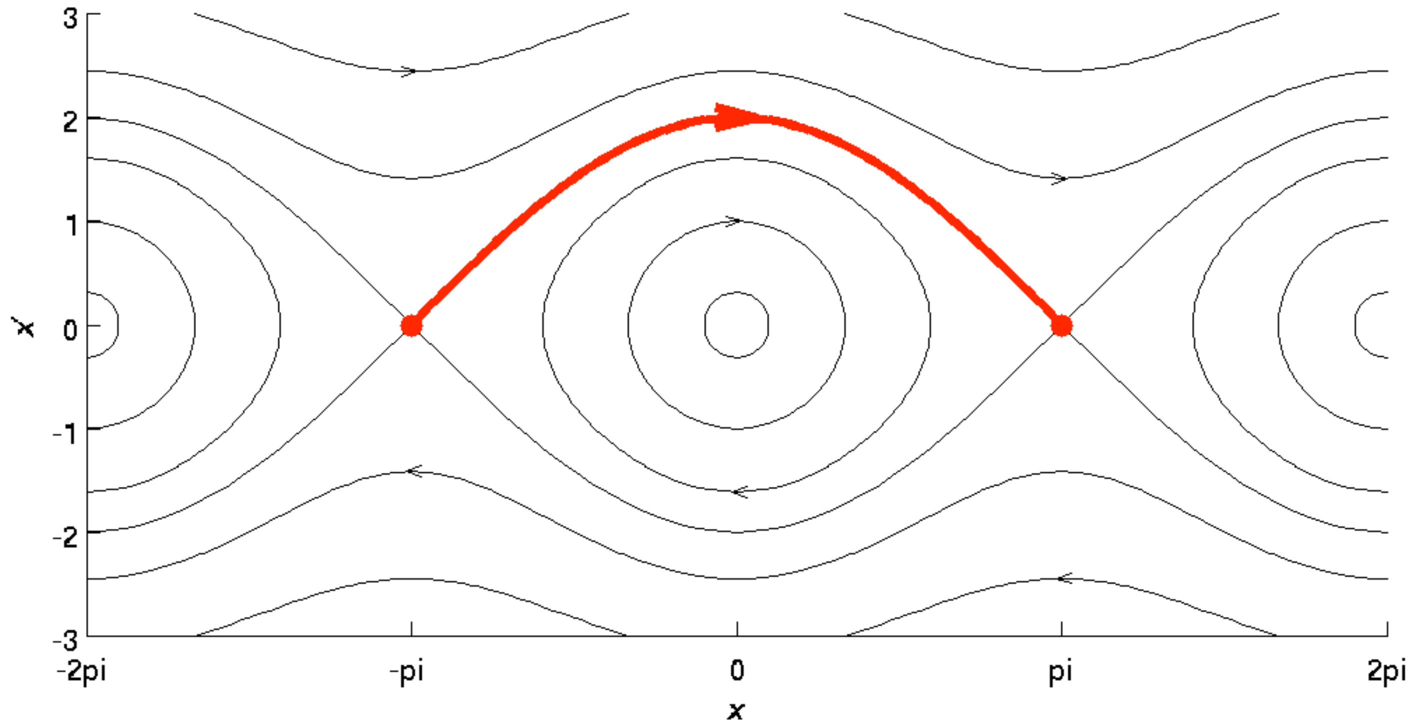

The stable and unstable manifolds:

Invariant manifolds in the simple pendulum

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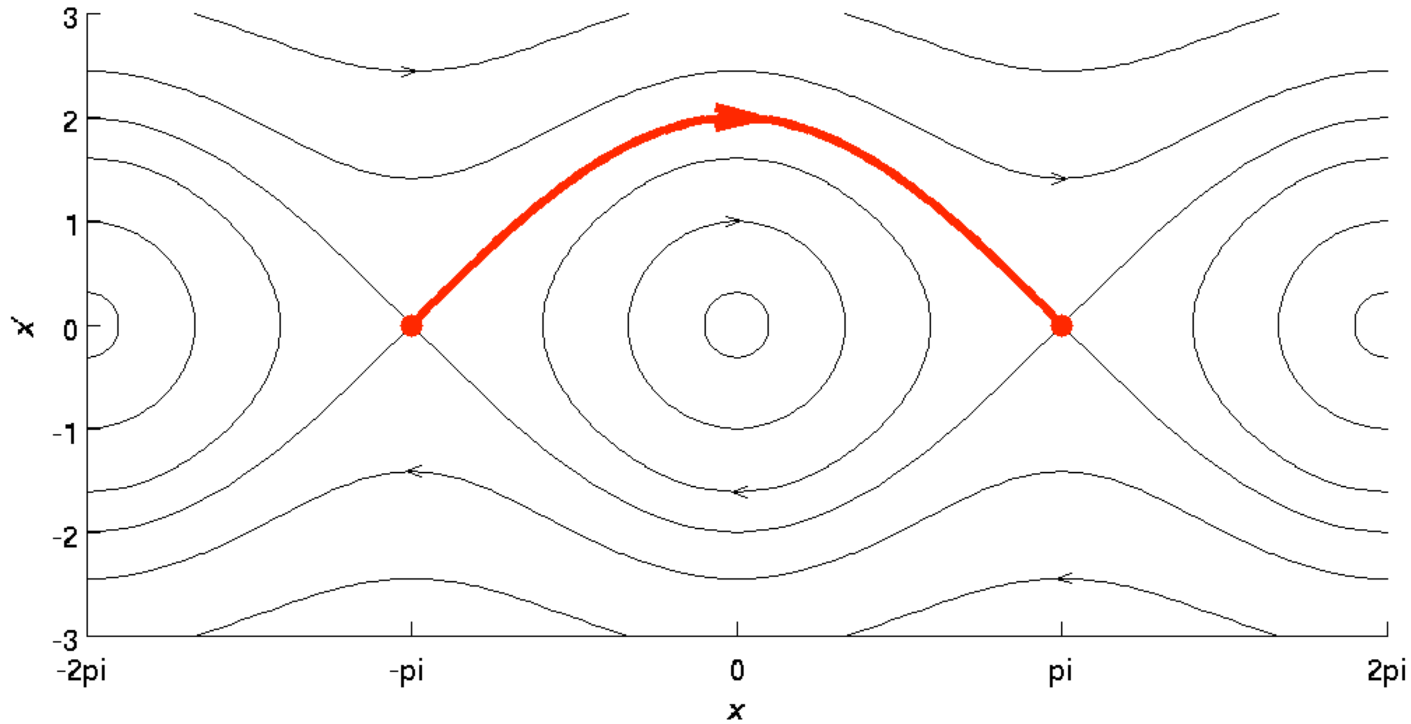

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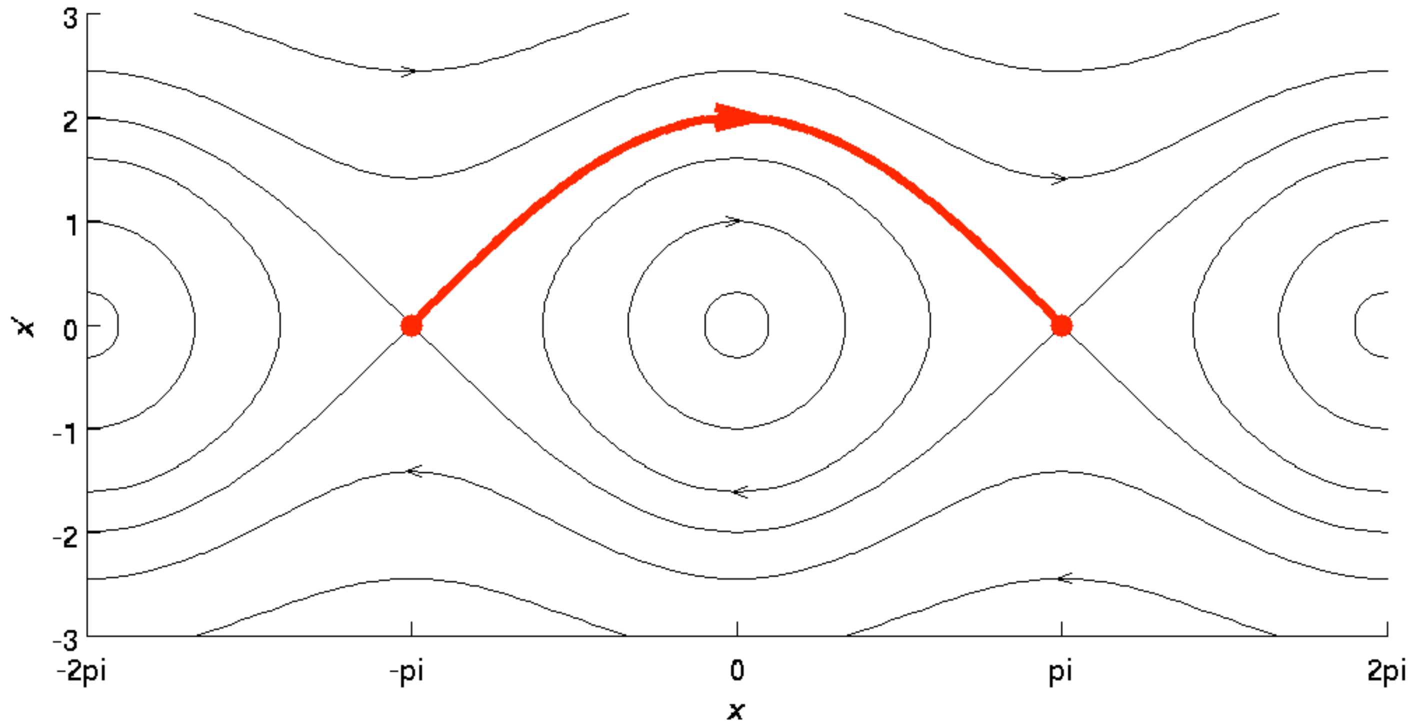

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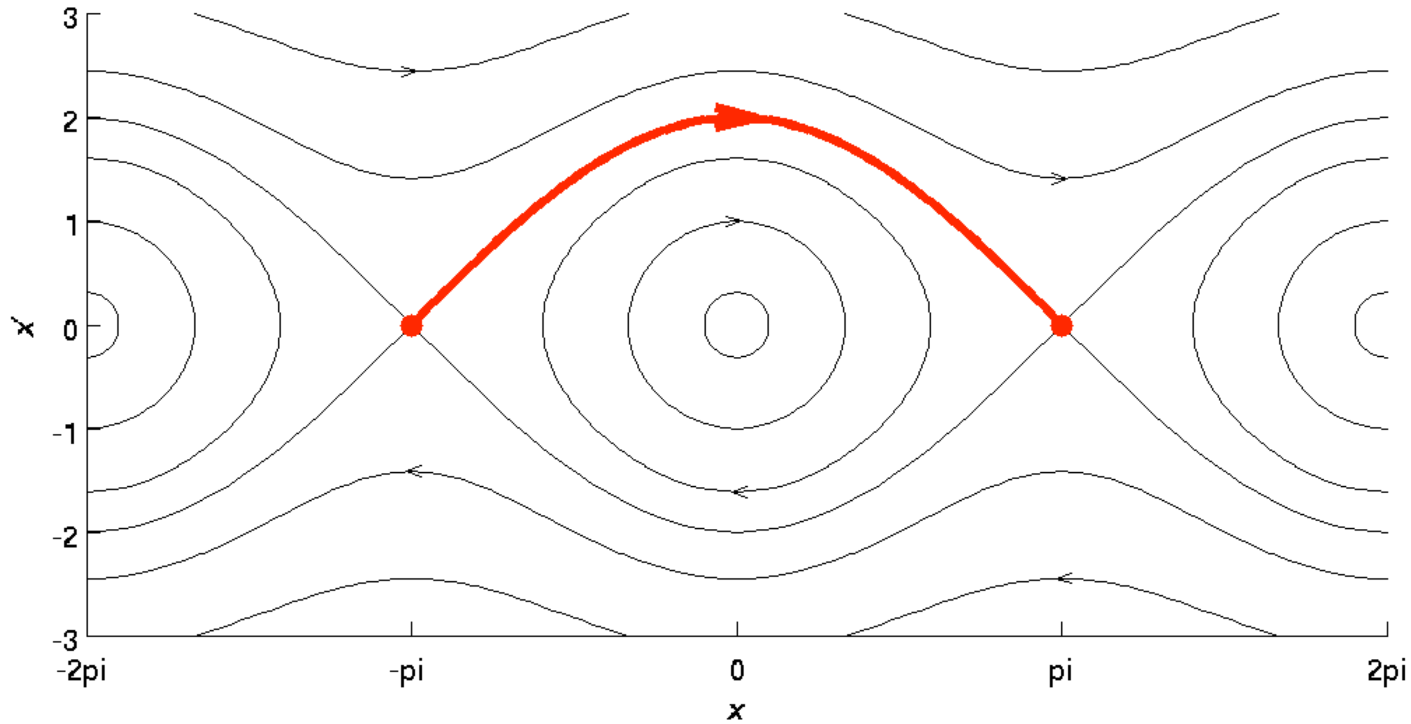

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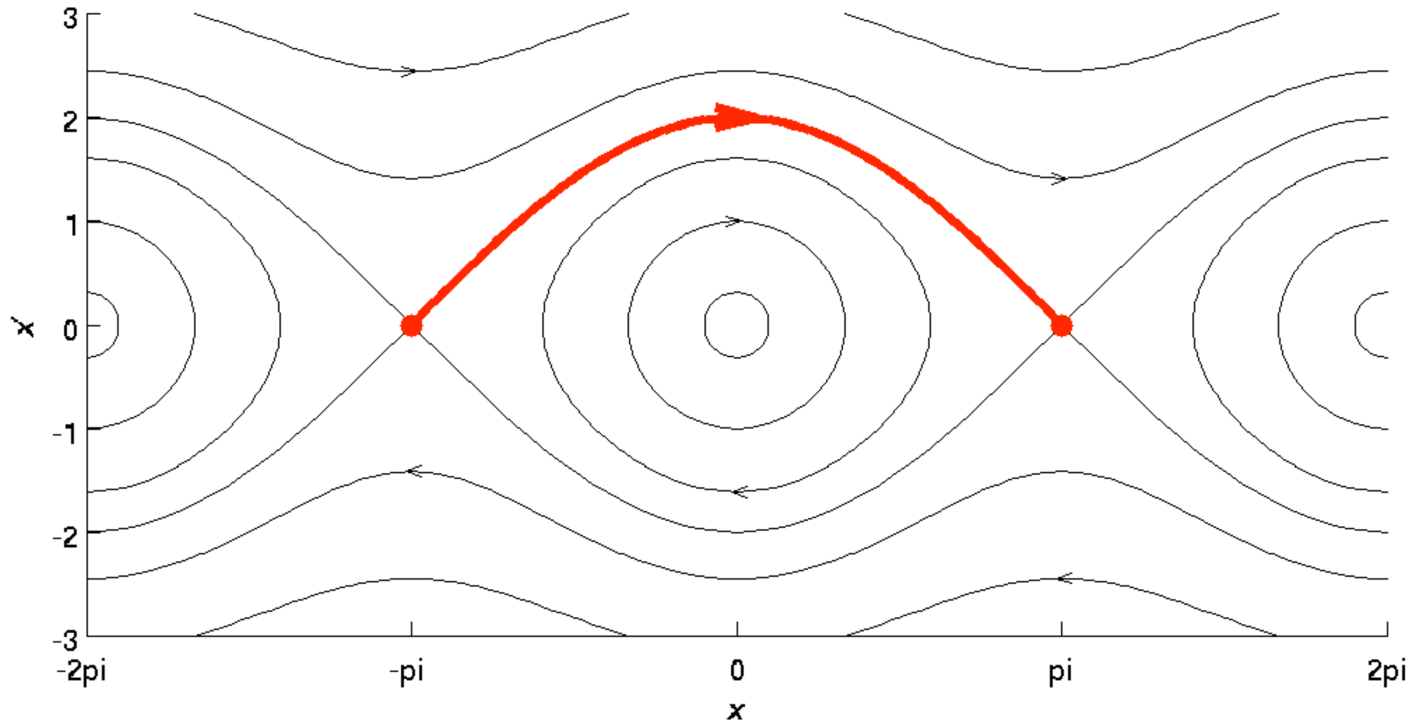

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Invariant manifolds in the simple pendulum

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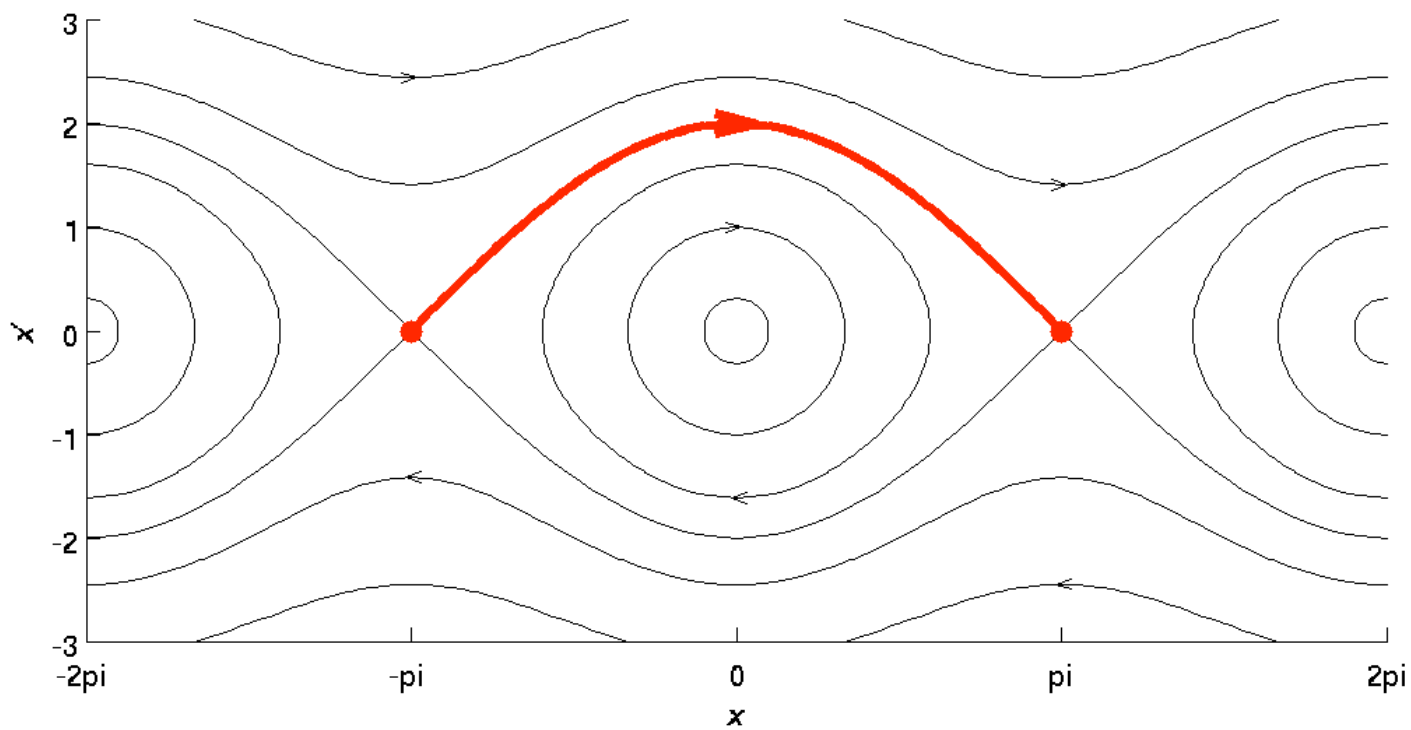

The stable and unstable manifolds:

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- dictate structure and transport in the flow.
- are surfaces of greatest stretching.

Invariant manifolds in the simple pendulum

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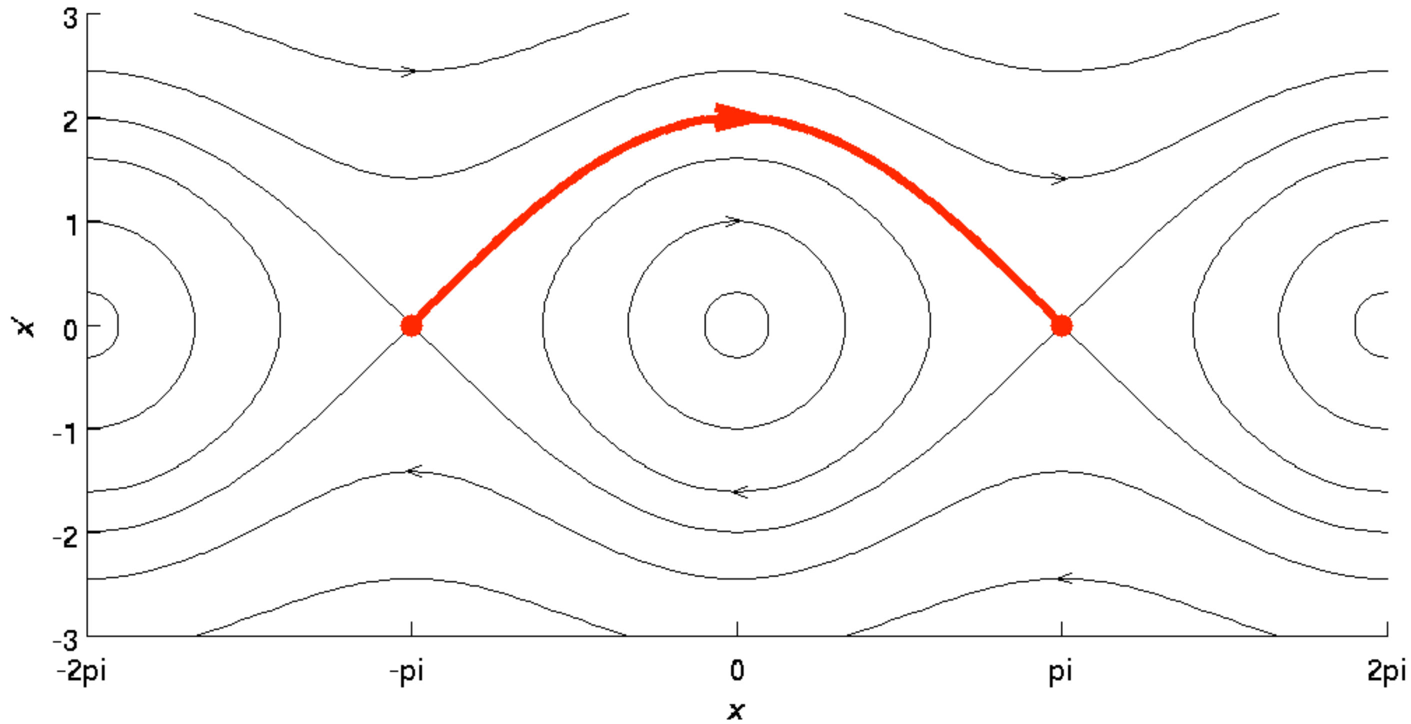

The stable and unstable manifolds:

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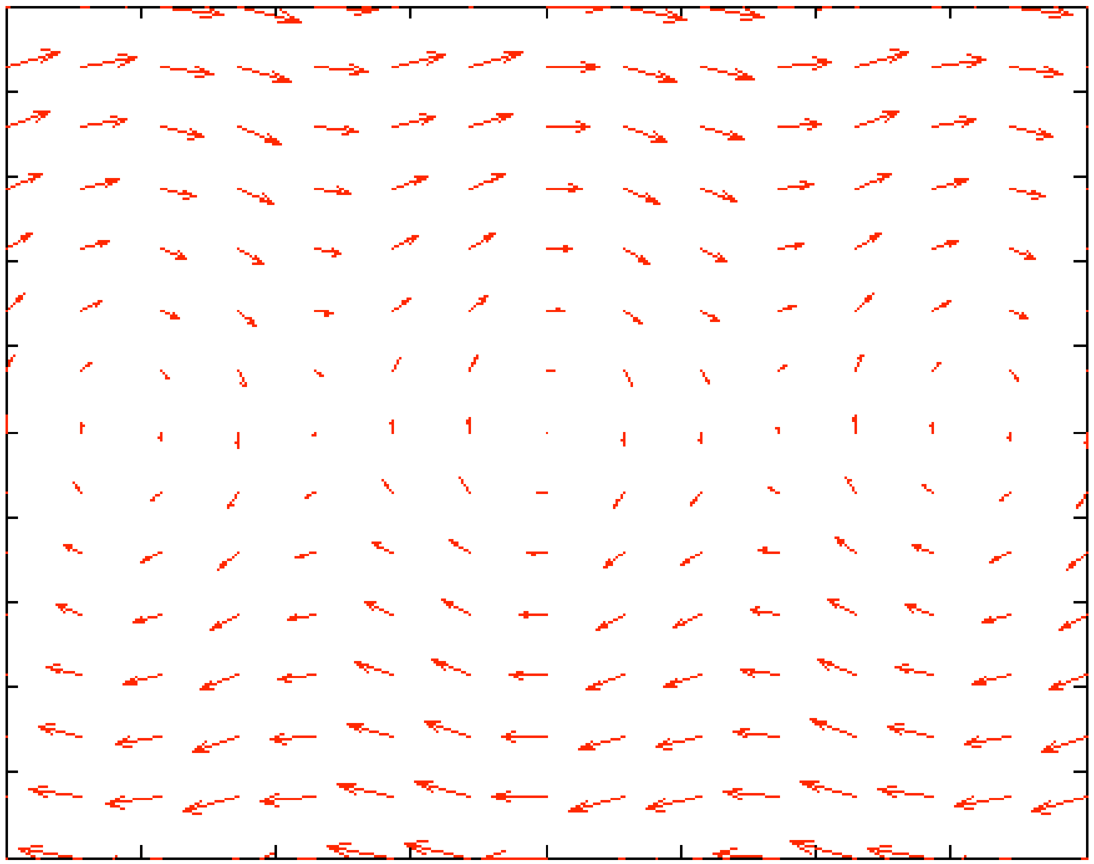
- form the boundaries of invariant sets.
- dictate structure and transport in the flow.
- are surfaces of greatest stretching.
- are attached to fixed points, periodic orbits, etc.

Separatrices for aperiodic systems

$$\ddot{x} + \sin x = f(t)$$



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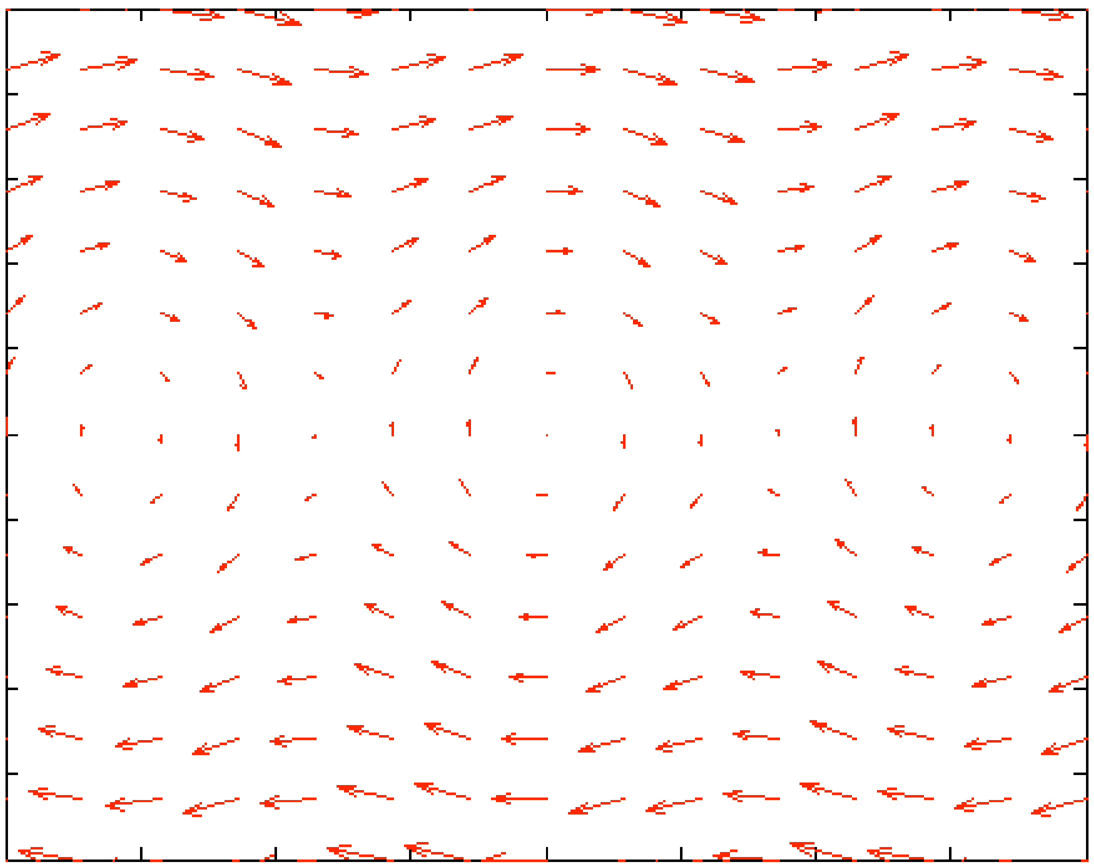
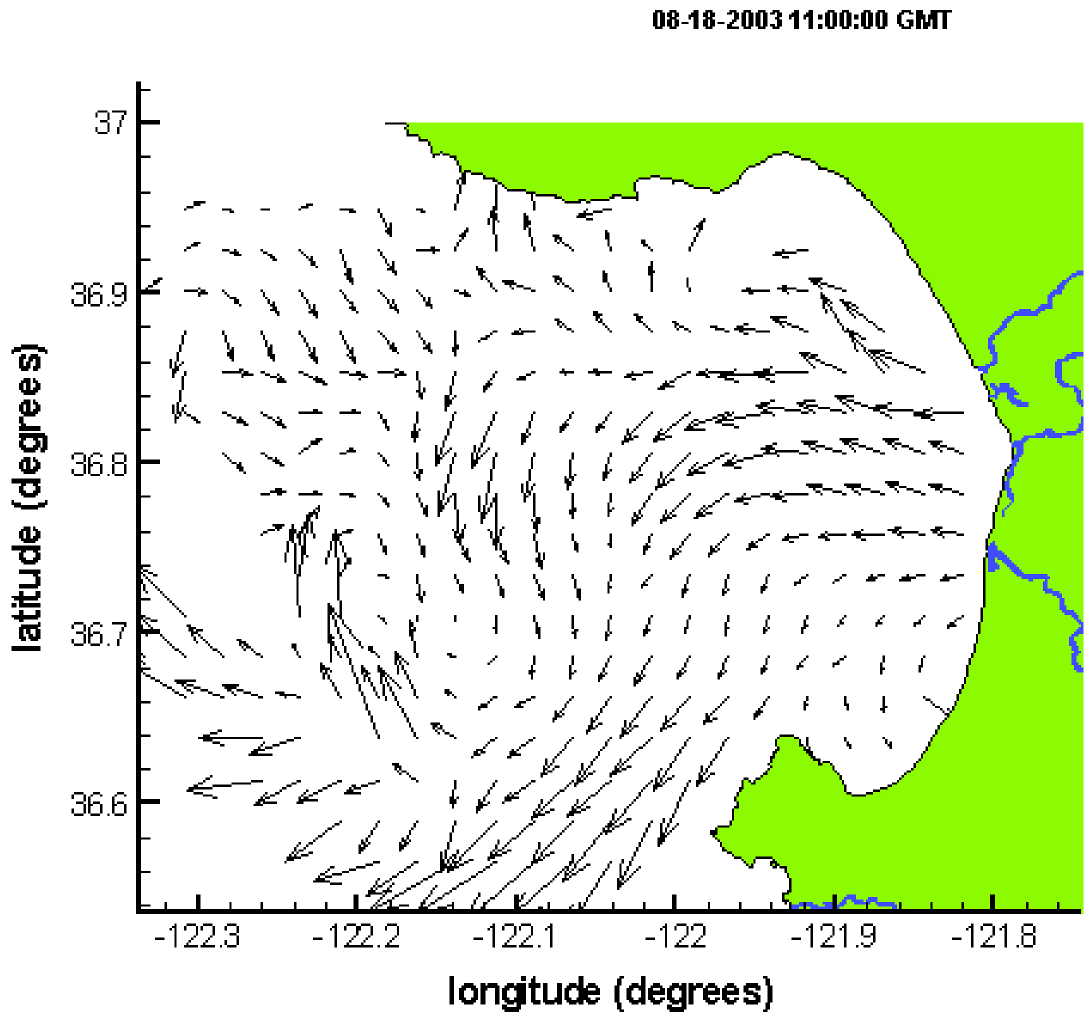


Finite data set



Define the LCS to be ridges in the FTLE field.

$$\sigma_T(t, \mathbf{x}) := \frac{1}{|T|} \ln \left\| \frac{d\phi_t^{t+T}}{d\mathbf{x}} \right\|_2$$

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The LCS

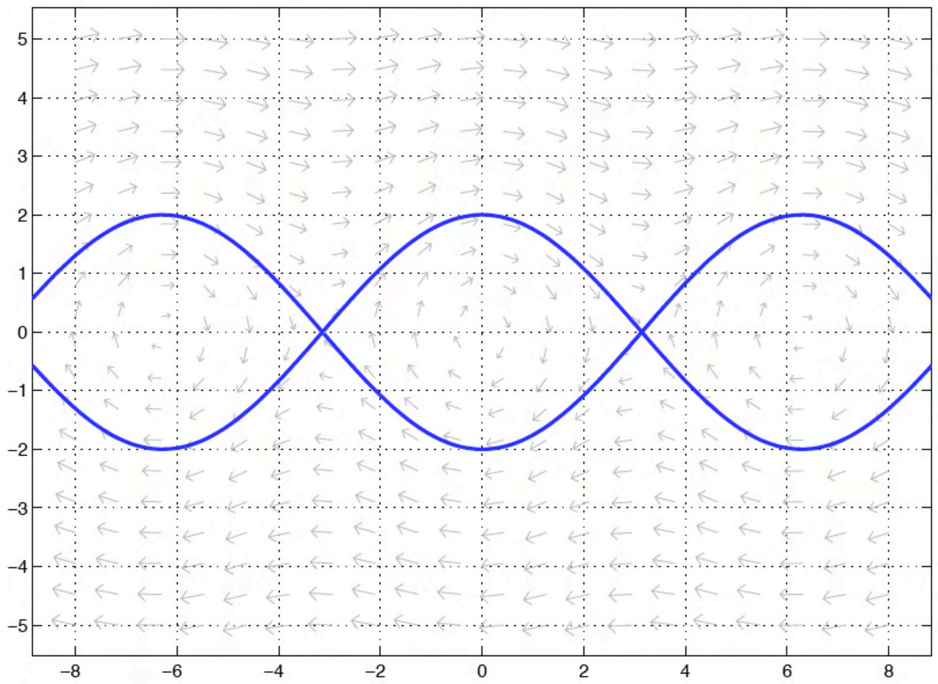
- are defined as surfaces of greatest stretching.

We will see that LCS

- are (almost) invariant.
- are separatrices between regions of different dynamical behavior.
- form the boundaries of invariant sets.
- dictate structure and transport in the flow.
- are NOT attached to fixed points, periodic orbits, etc.

A quick test

$$\ddot{x} + \sin x = 0$$



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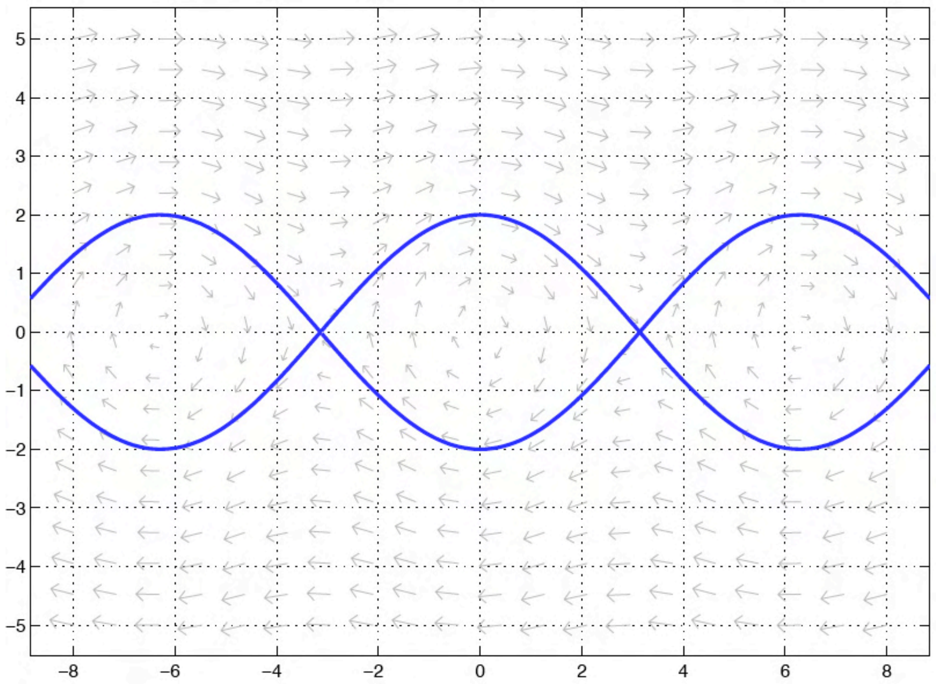

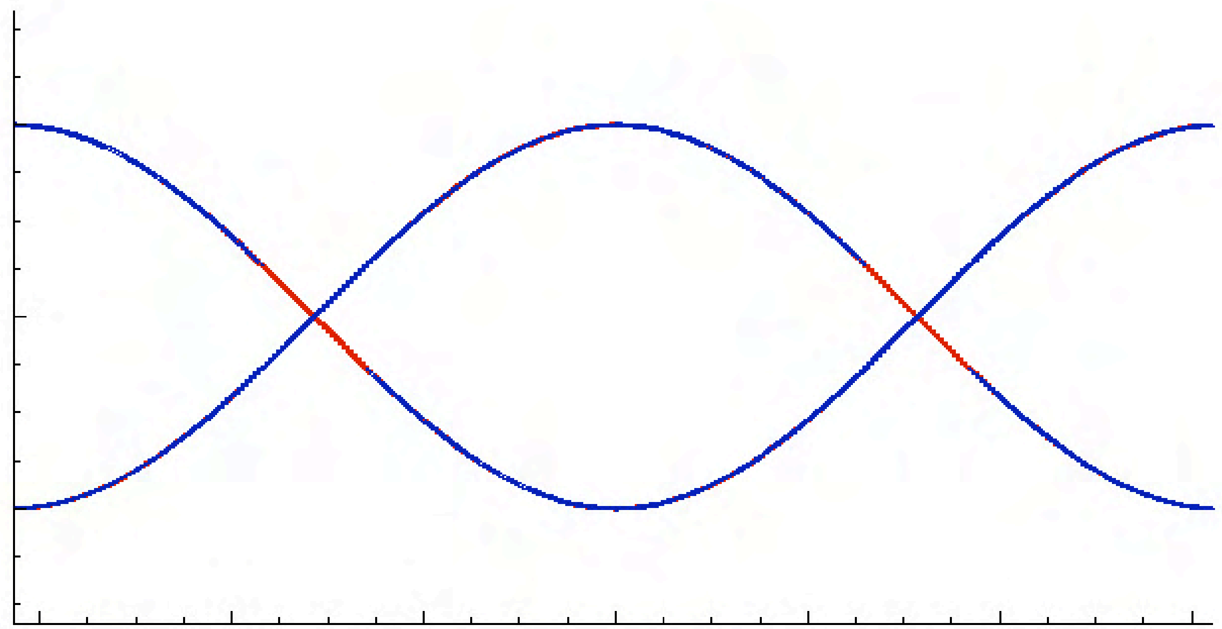

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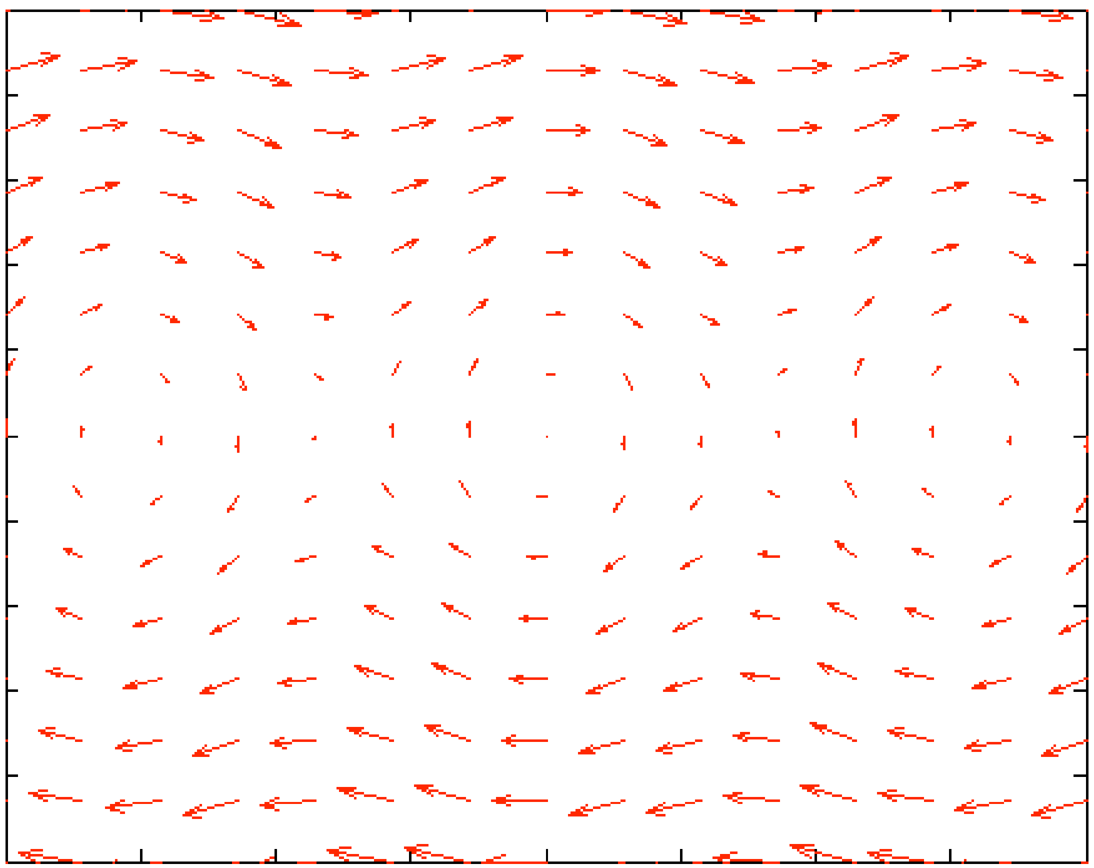

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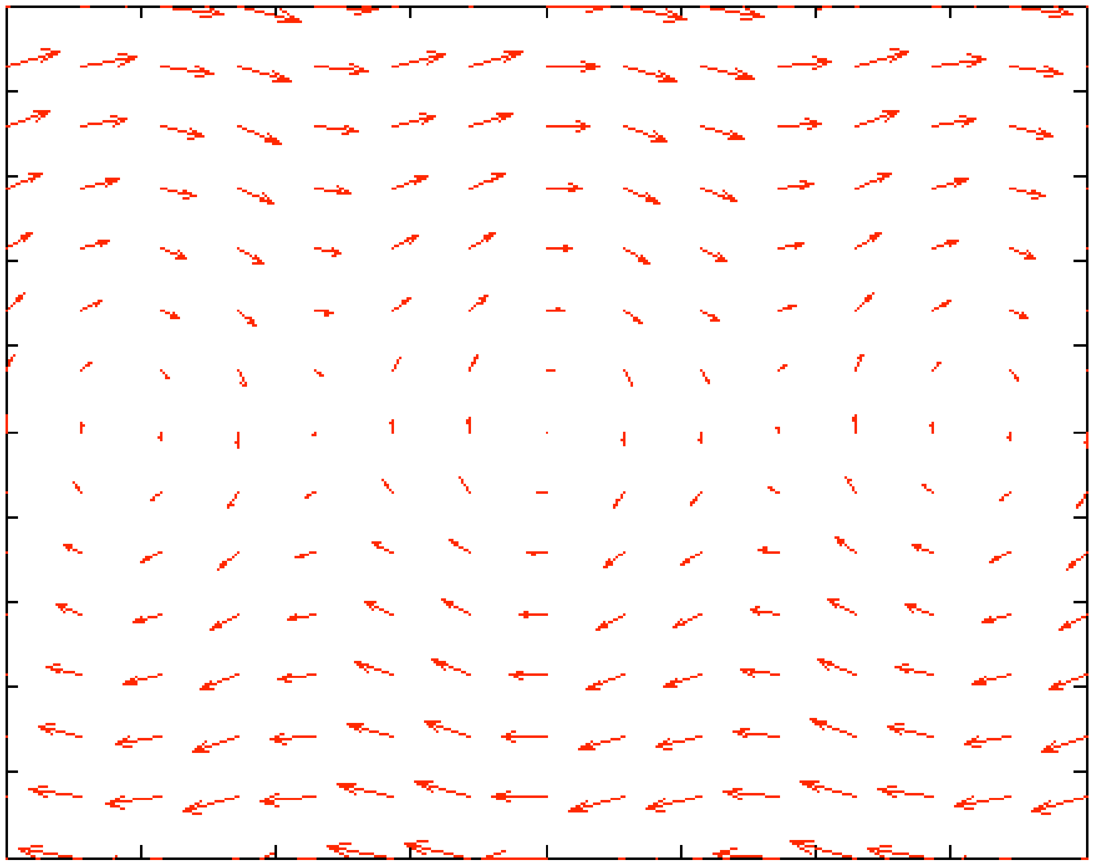
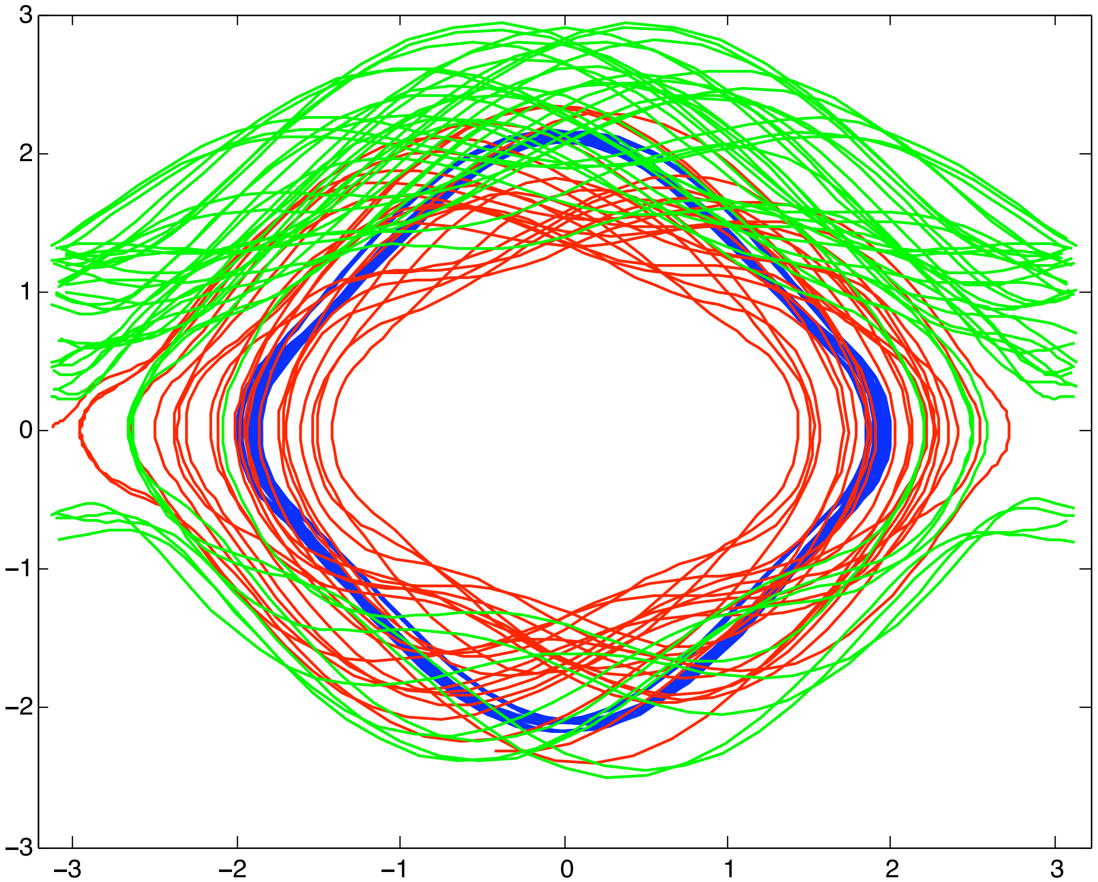
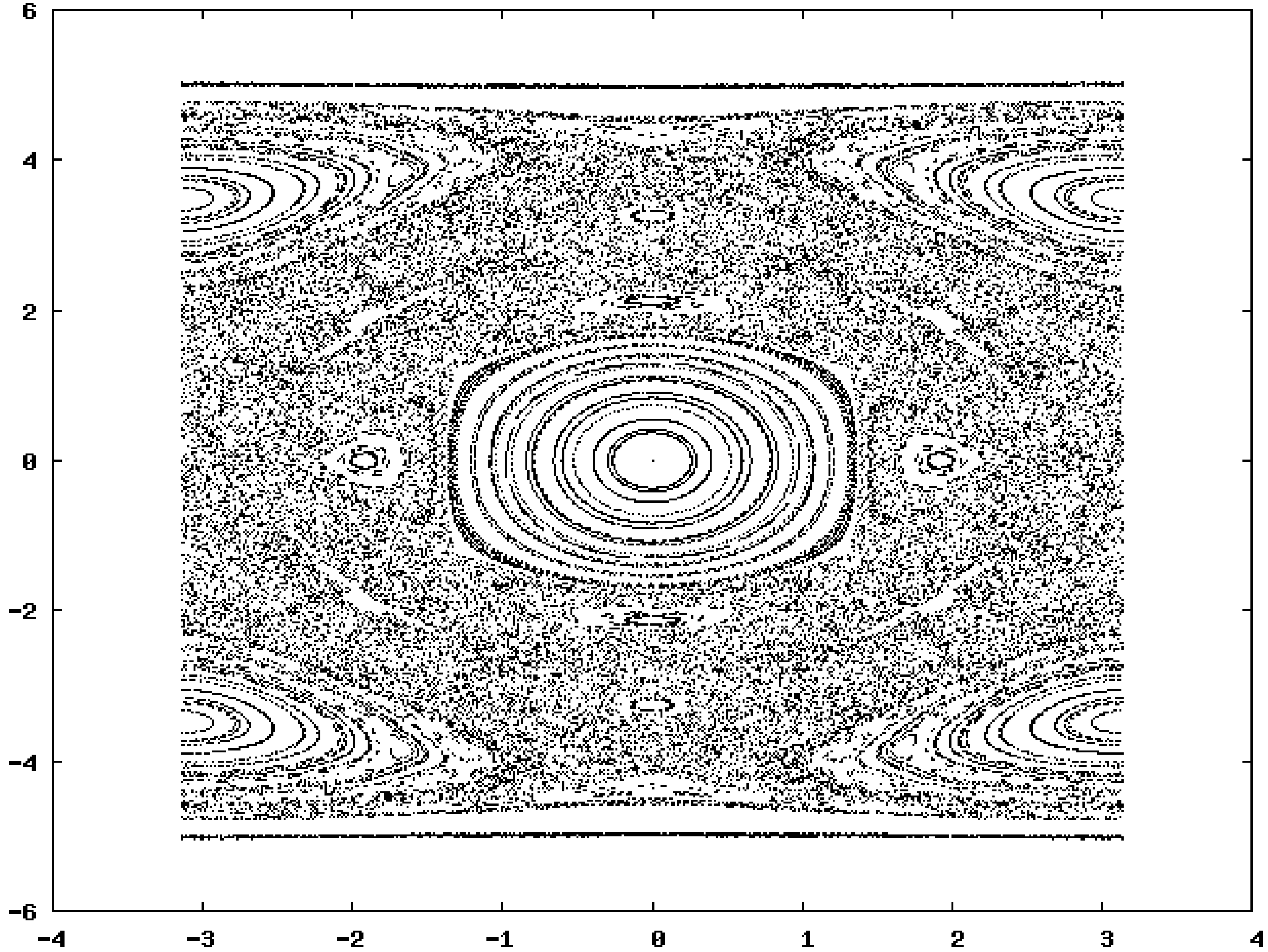
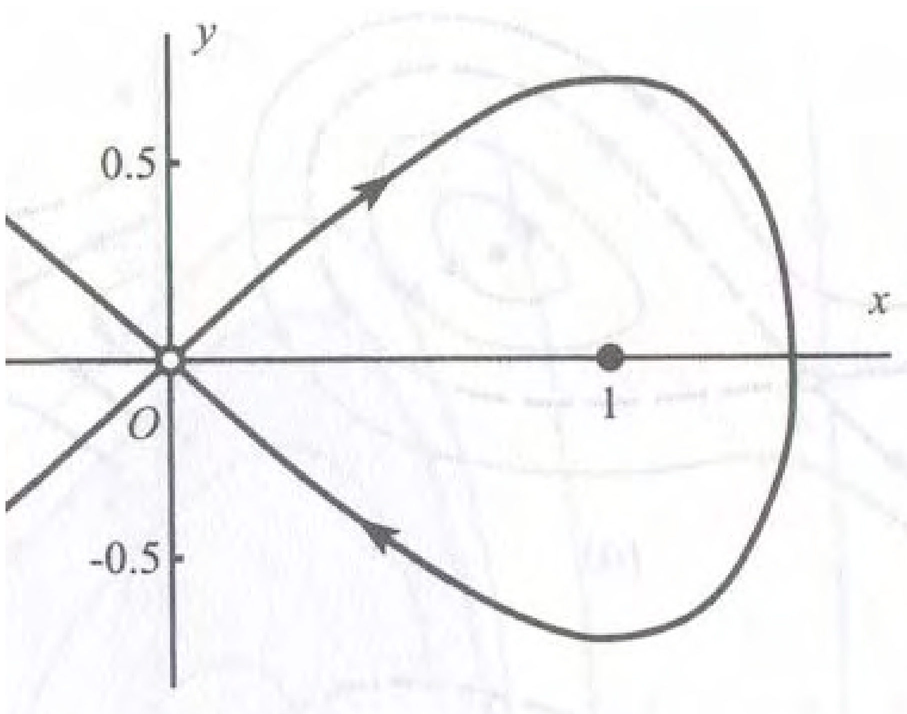




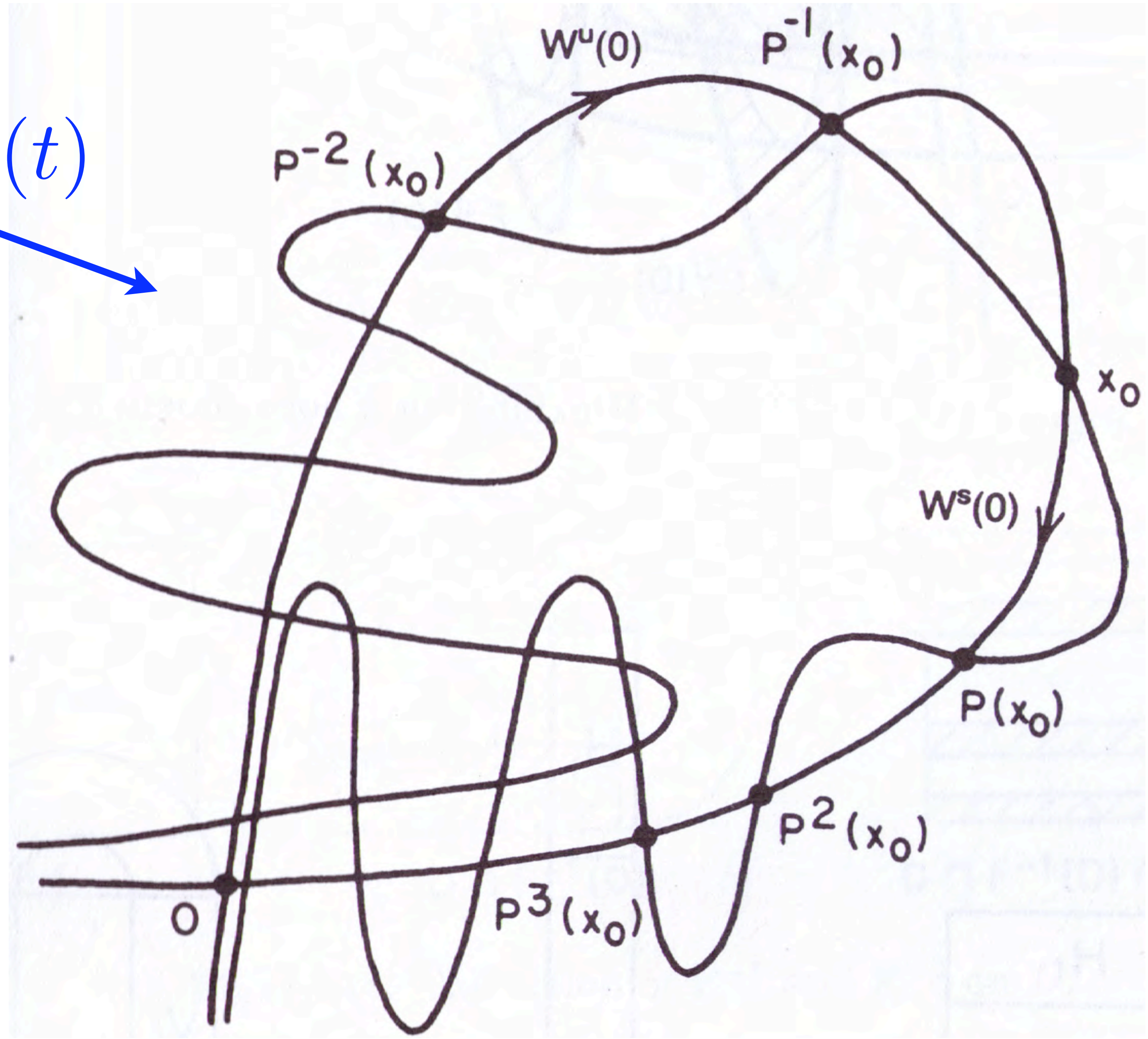
The Poincare map



Homoclinic tangles



$f(t)$



Lobe Dynamics

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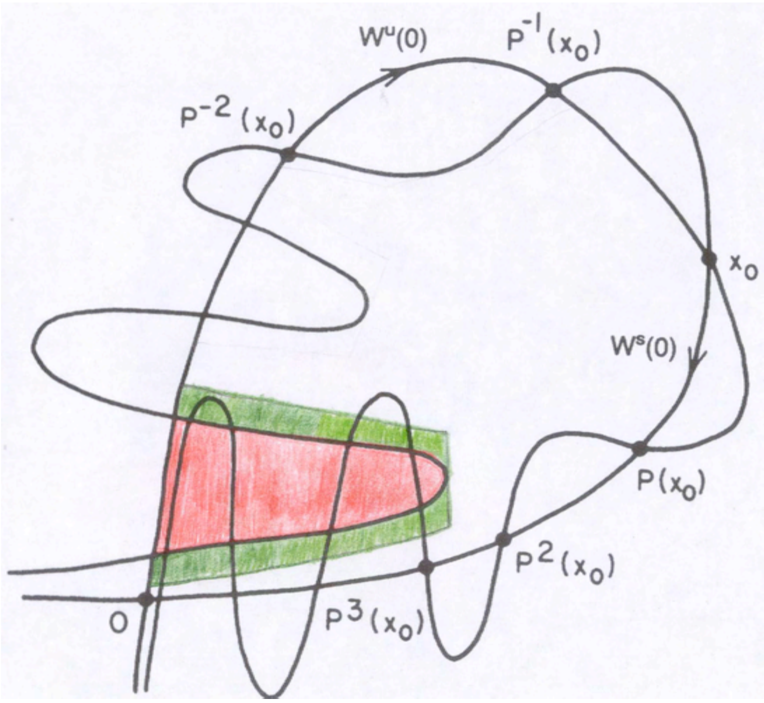

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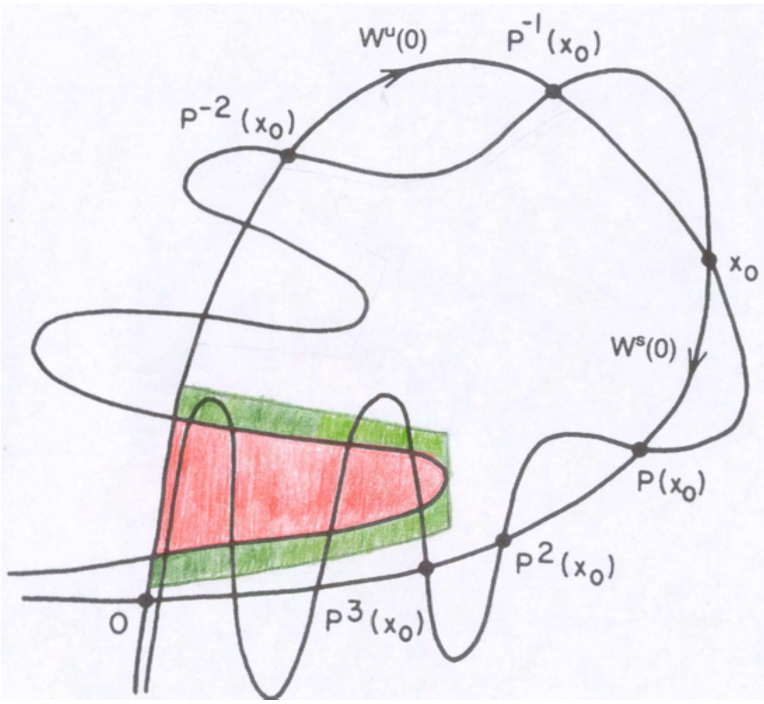

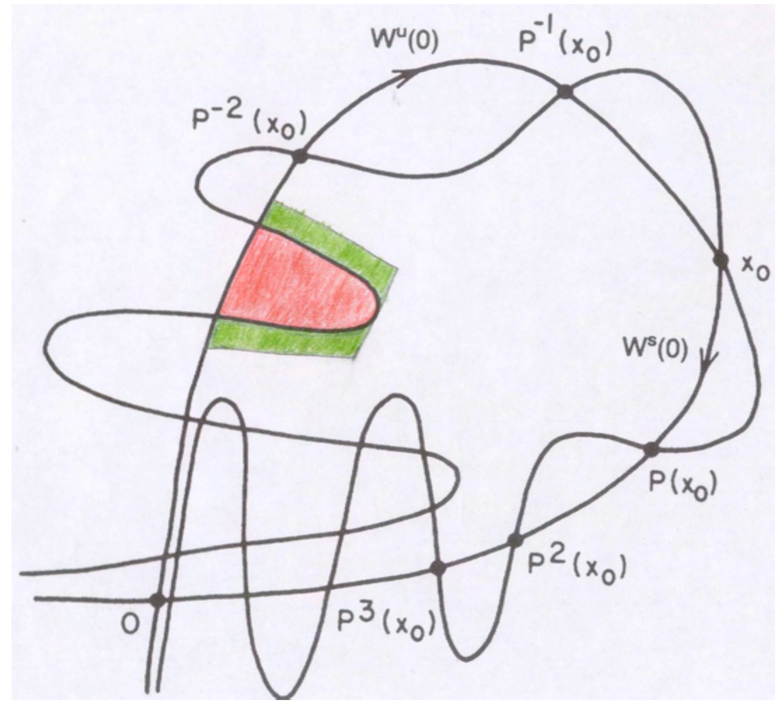

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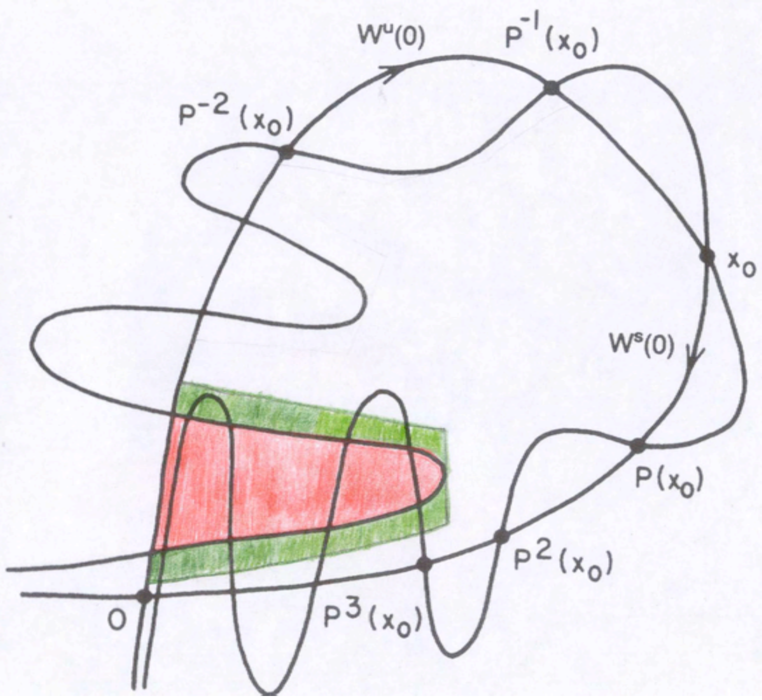

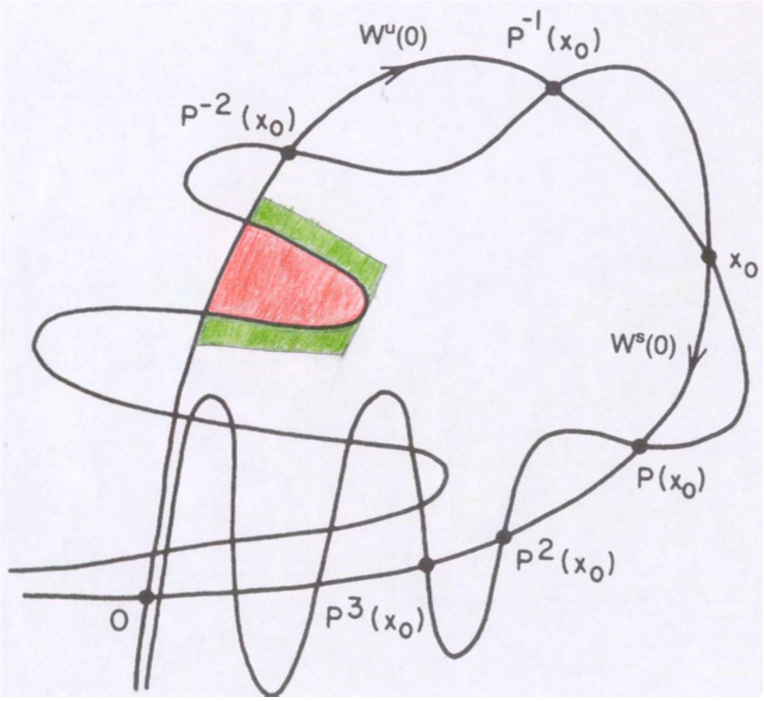

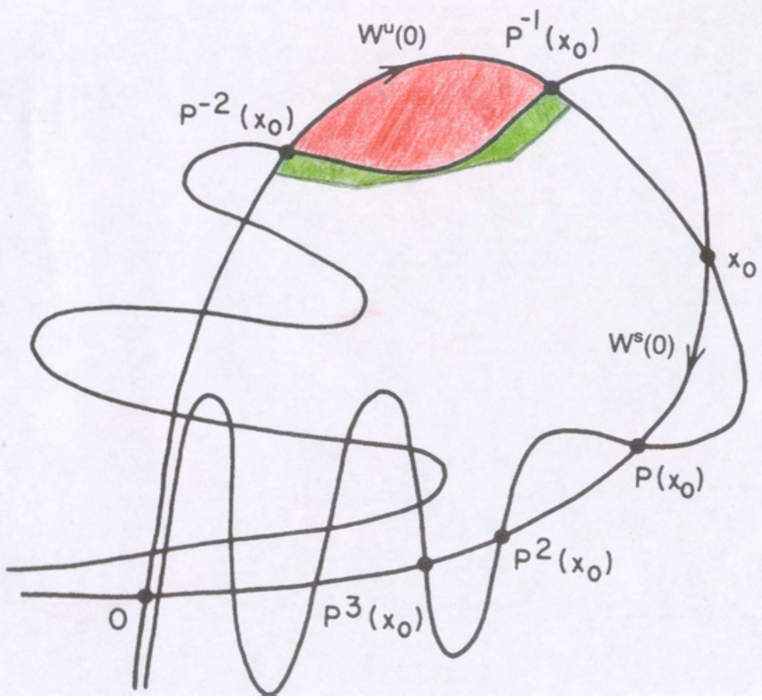

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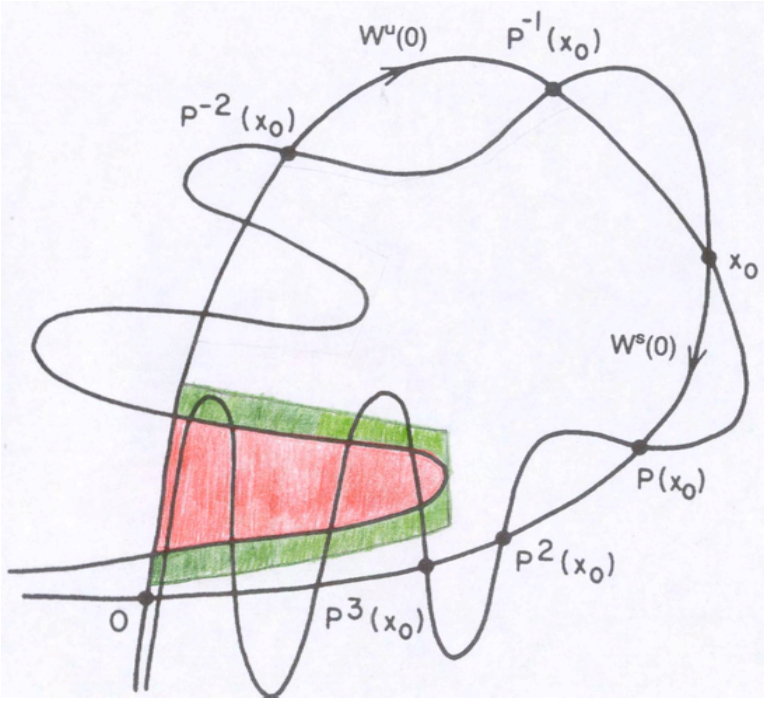

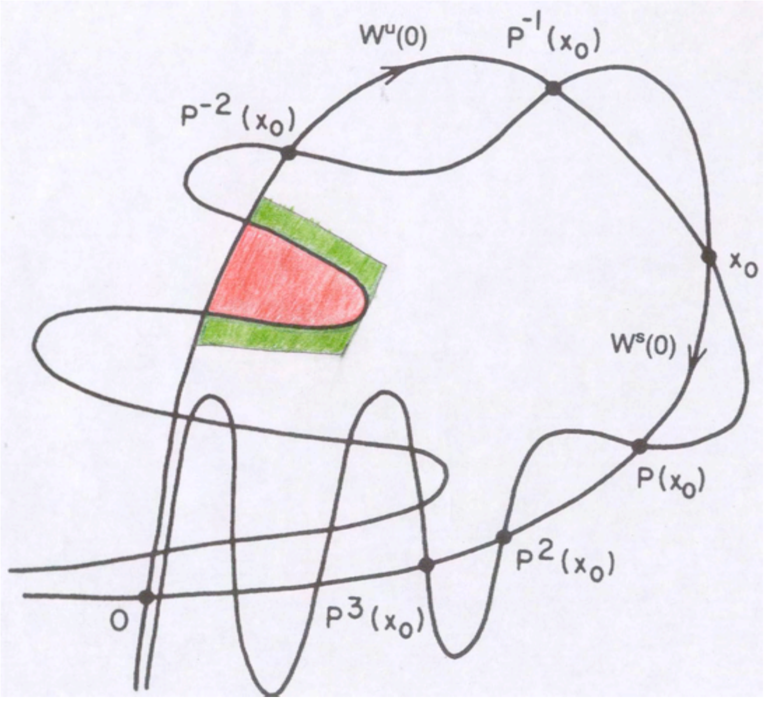

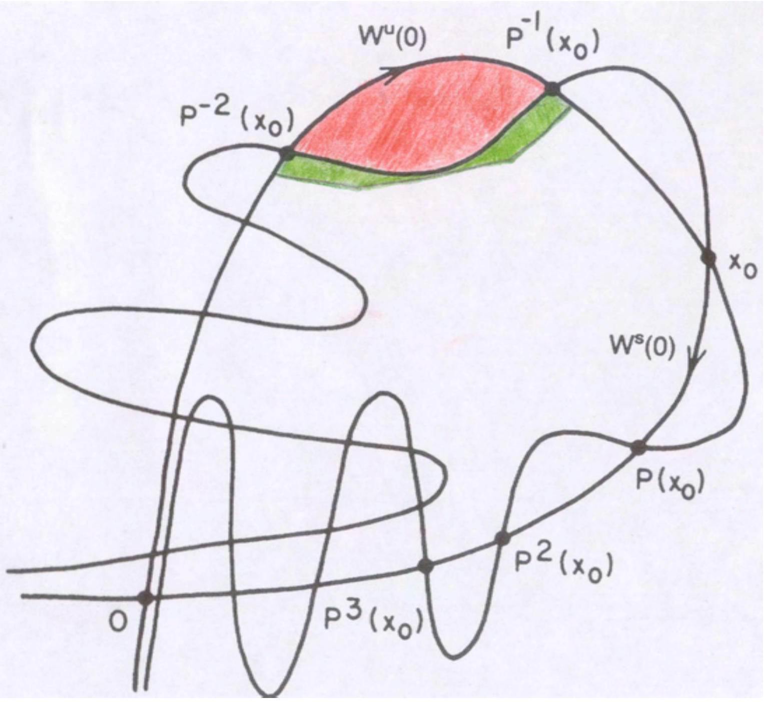

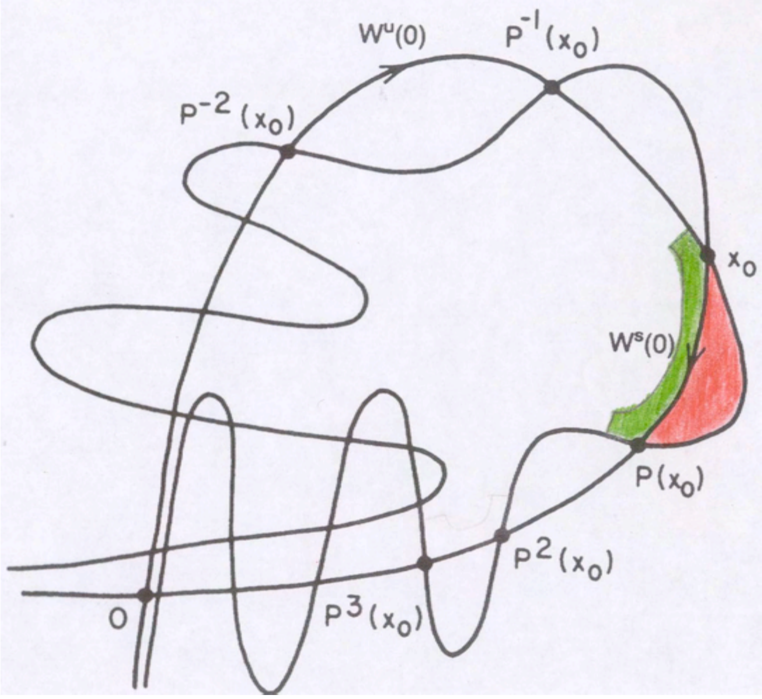

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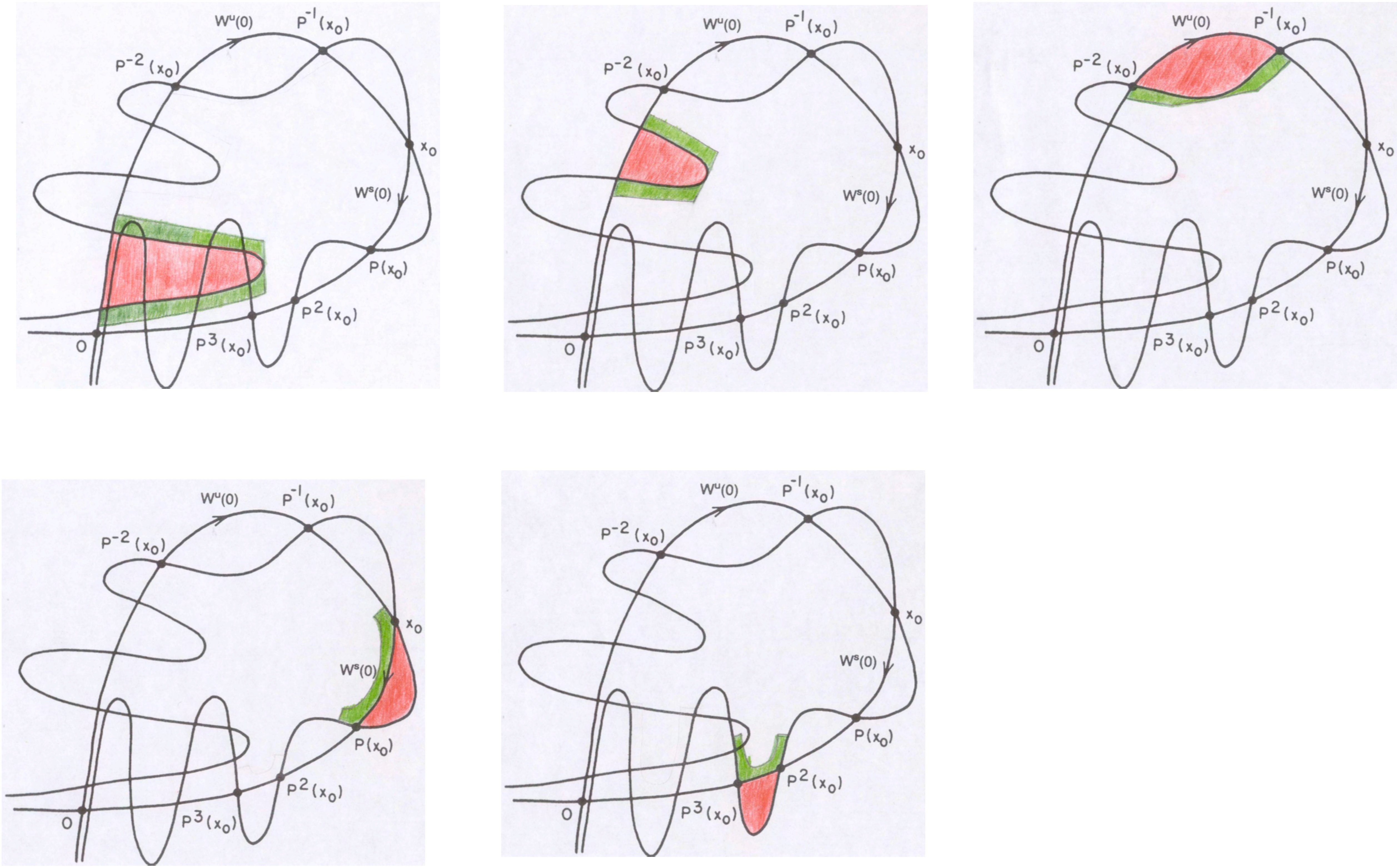

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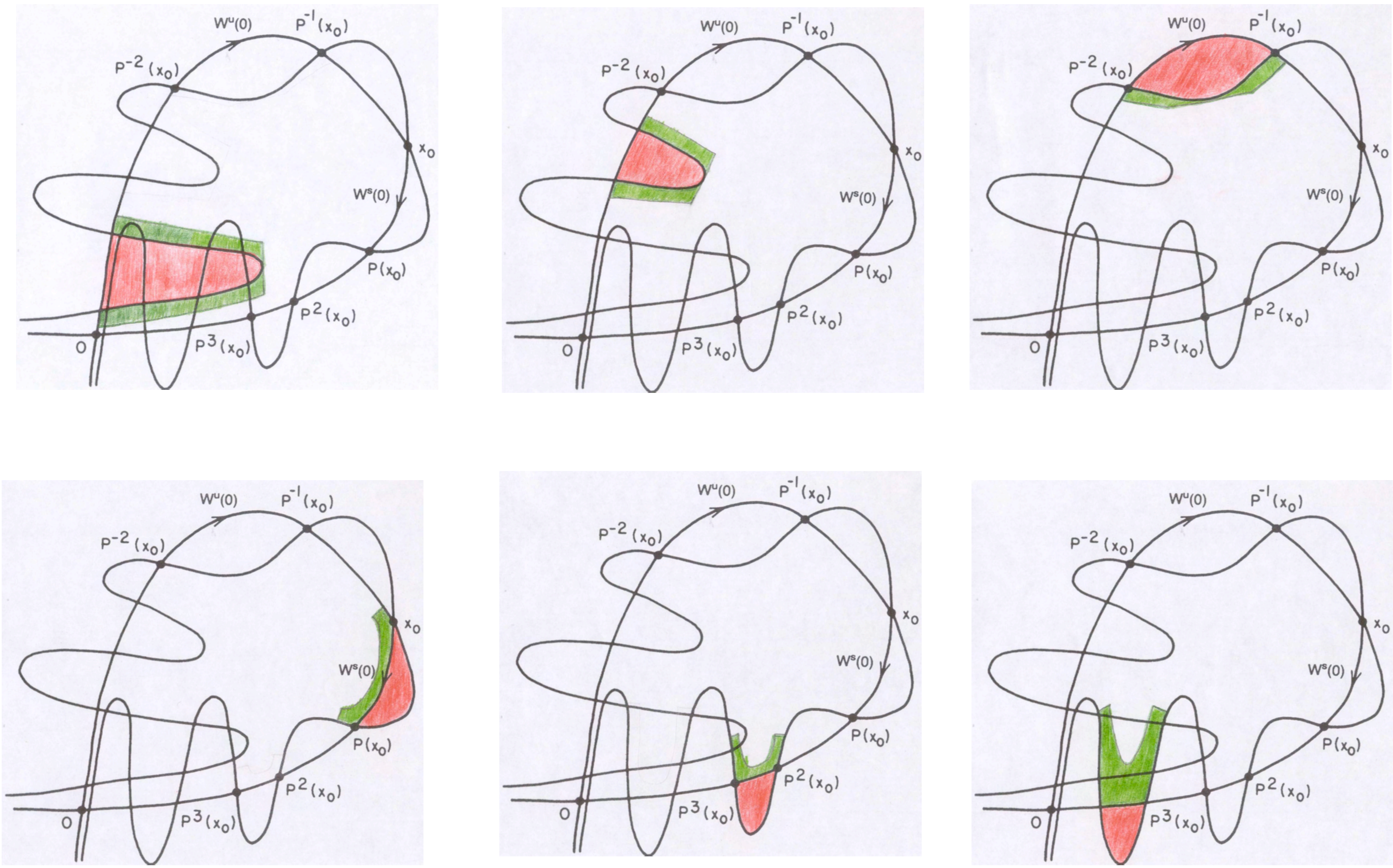
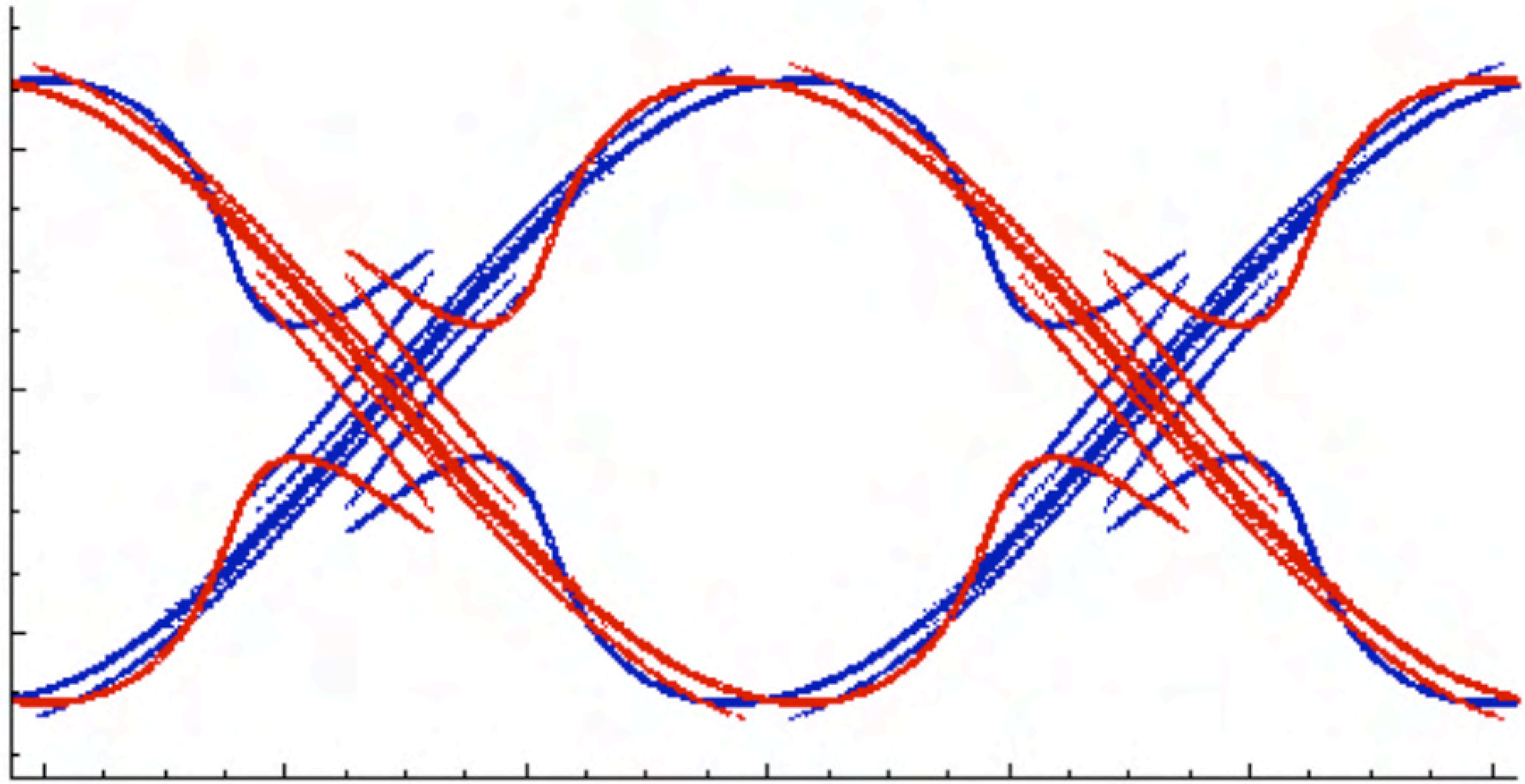


LCS for periodic forcing



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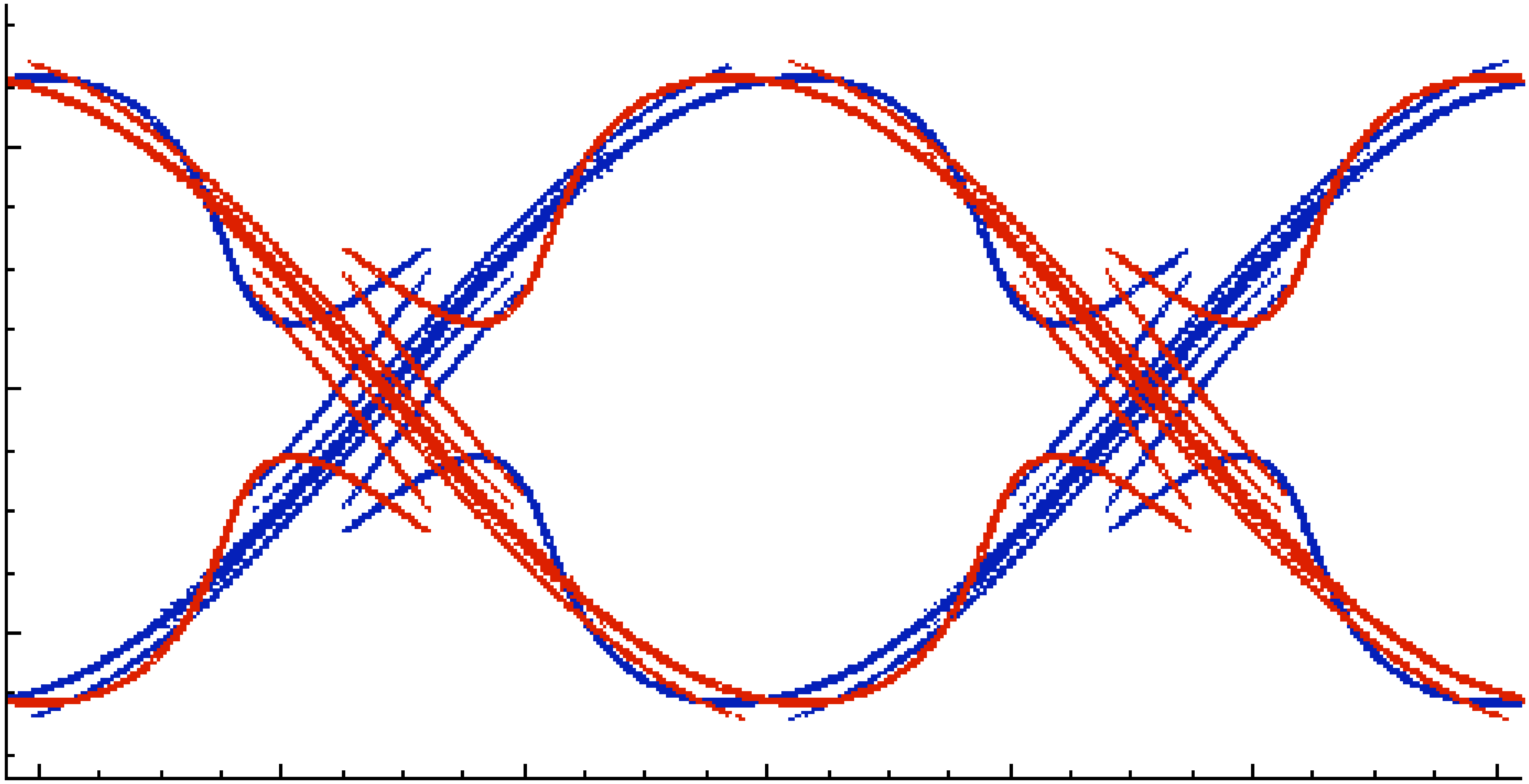
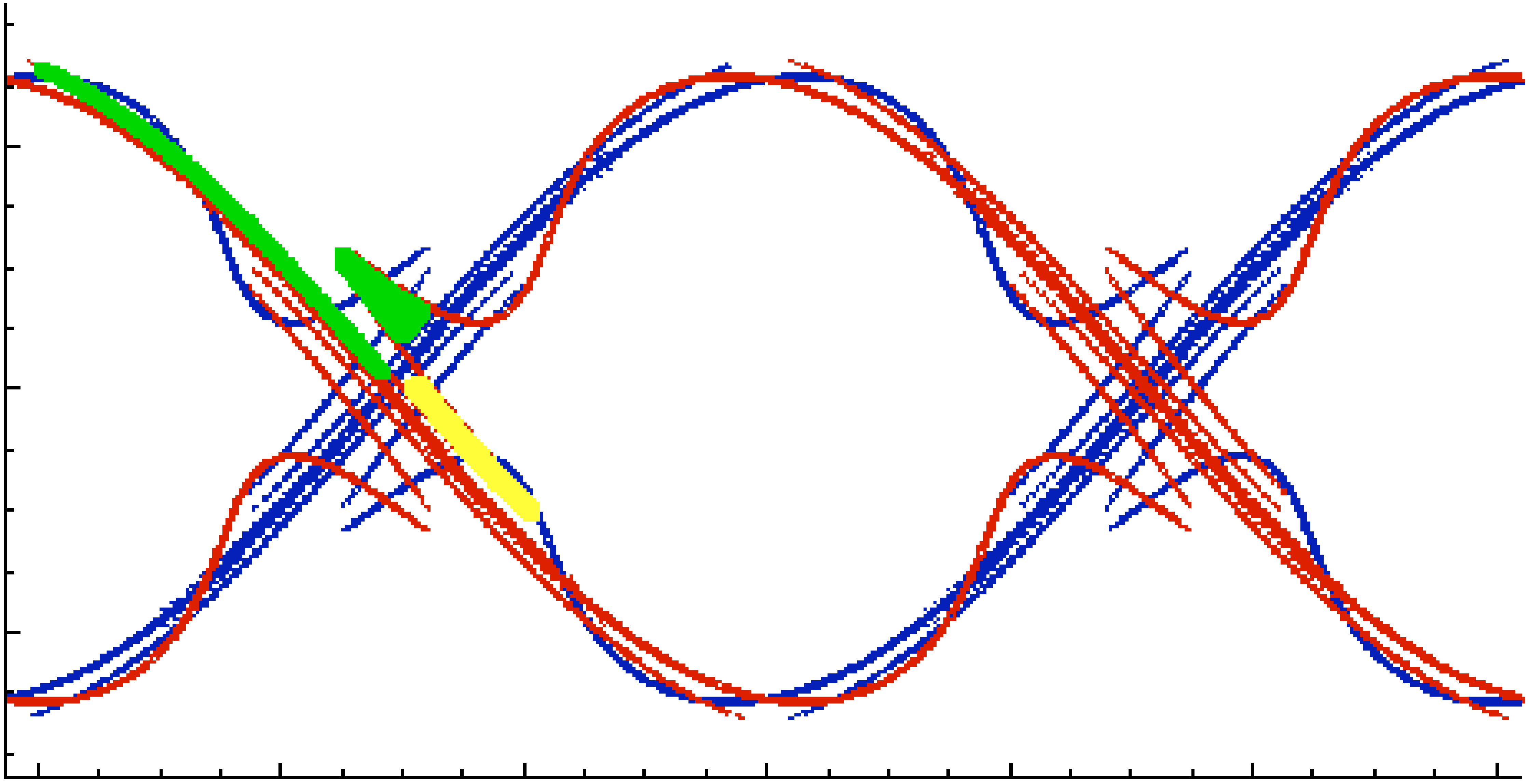


Lobes dynamics with periodic forcing



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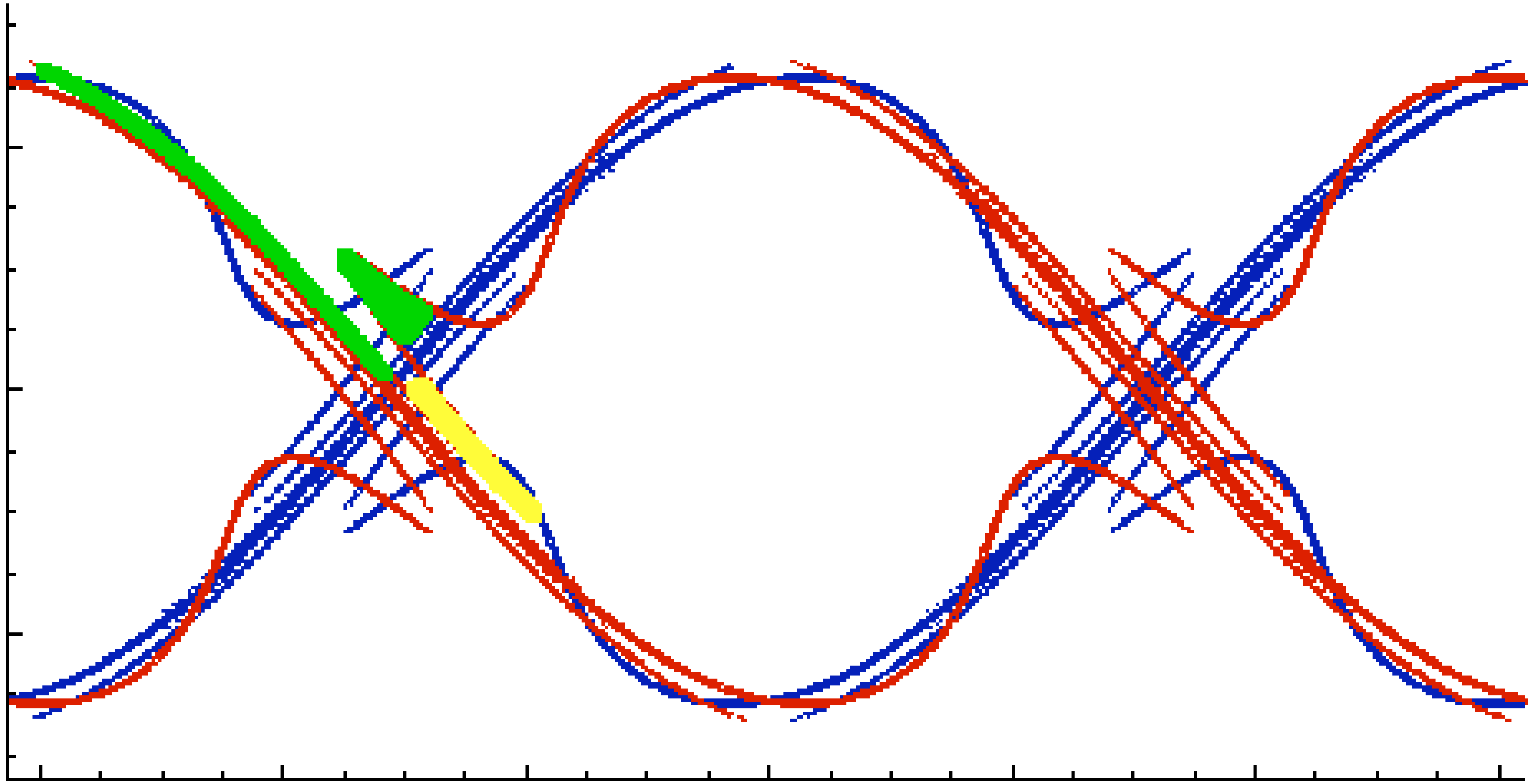

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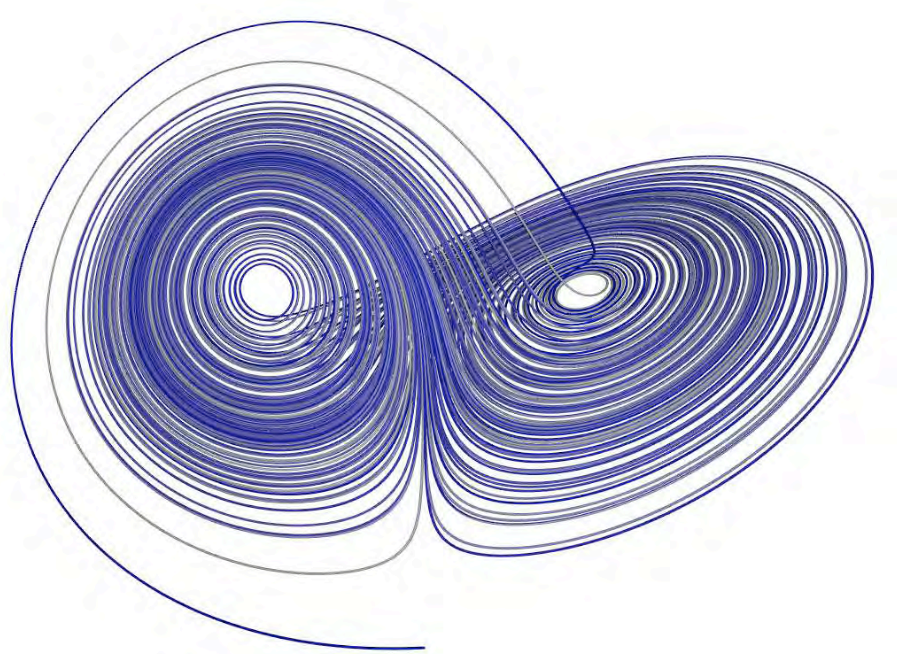

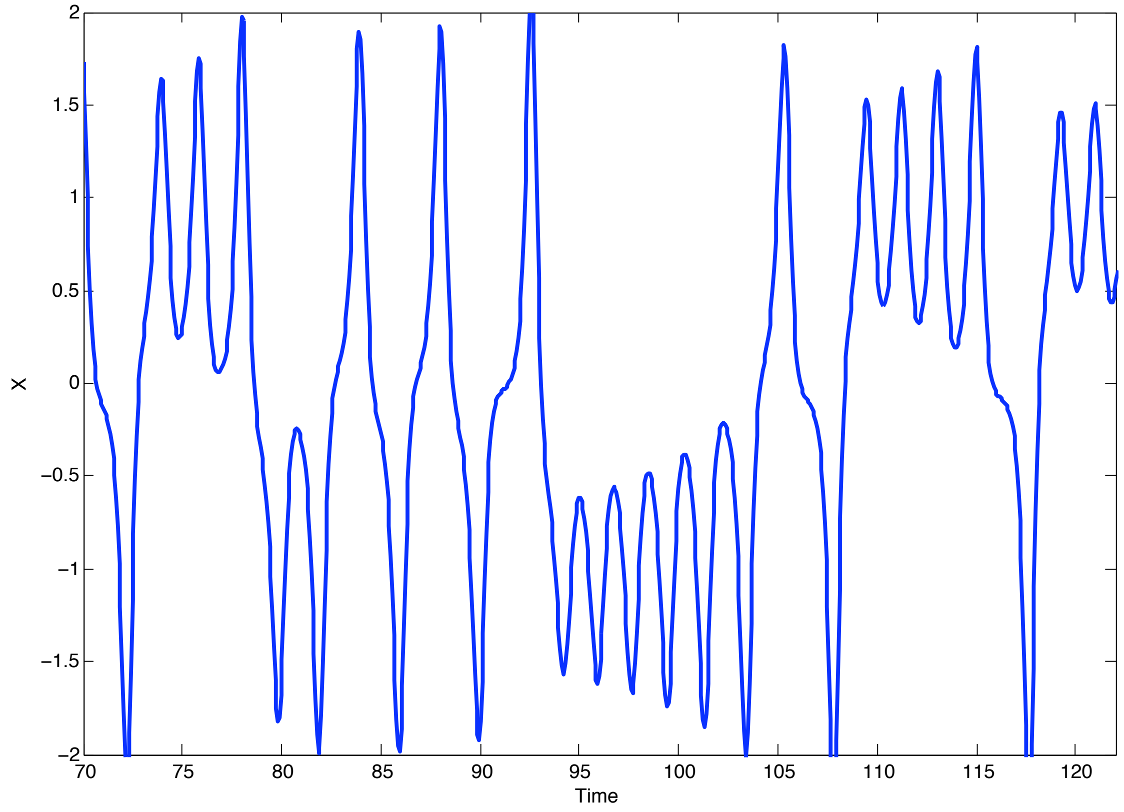

$f(t)$

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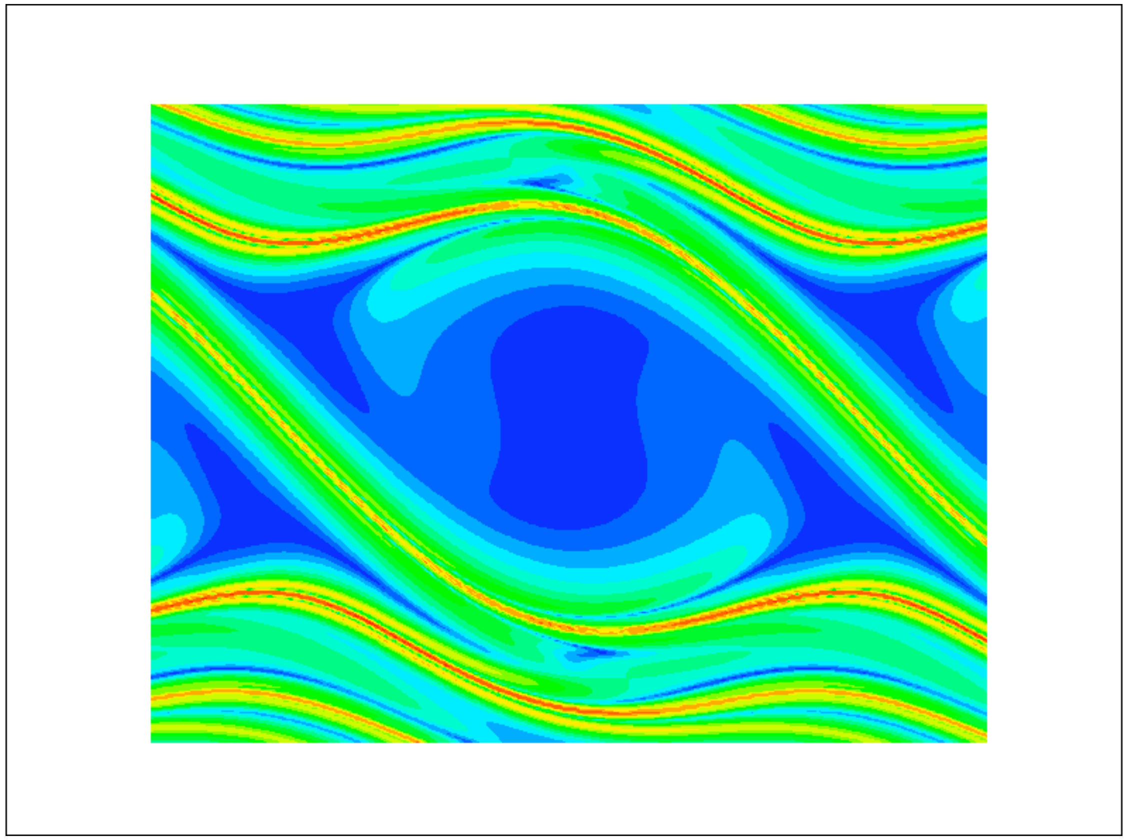

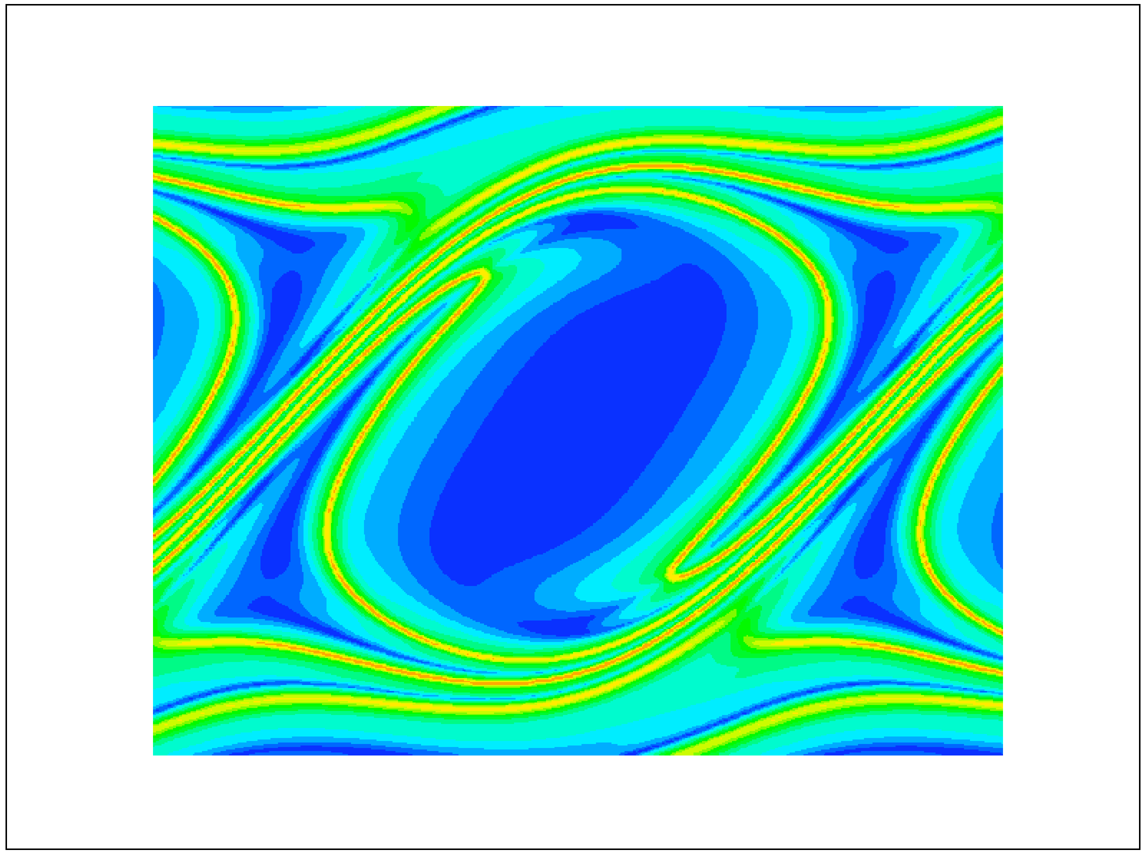

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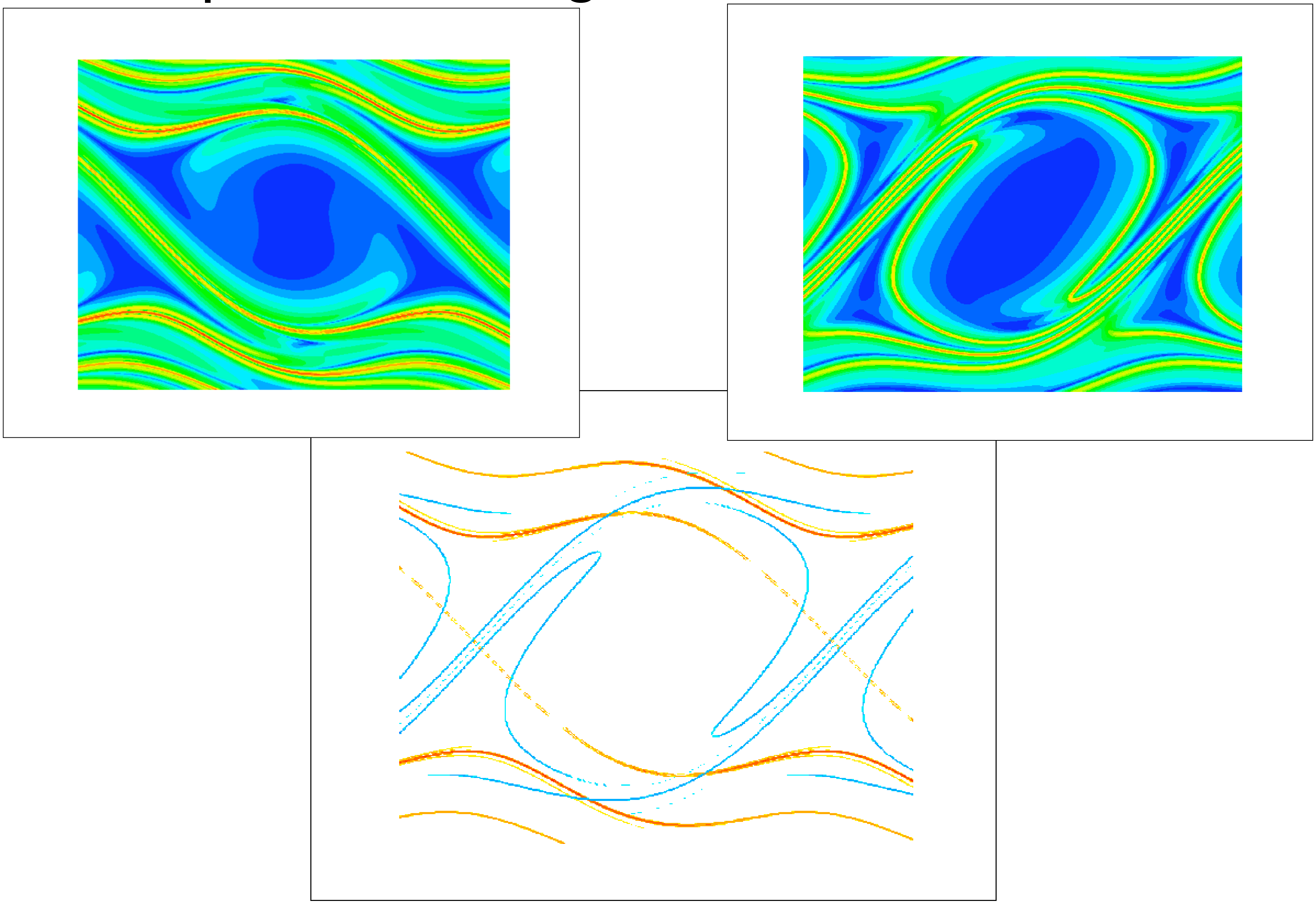

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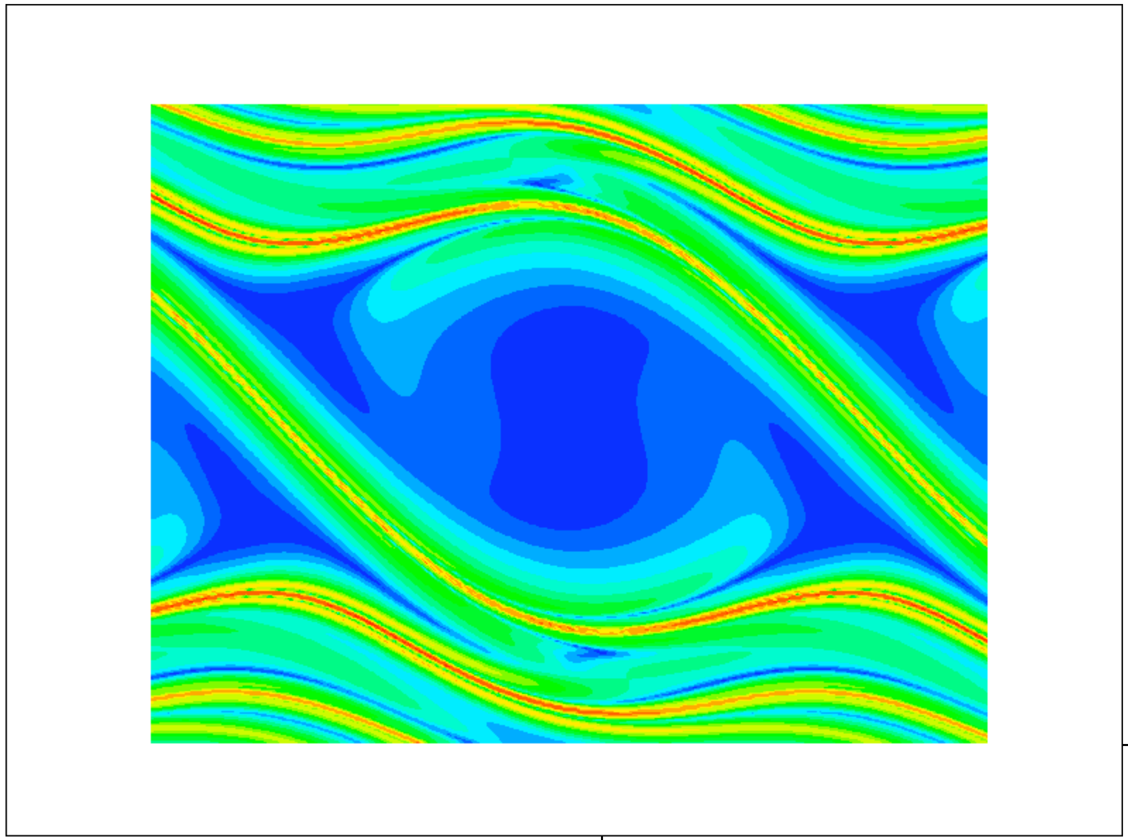
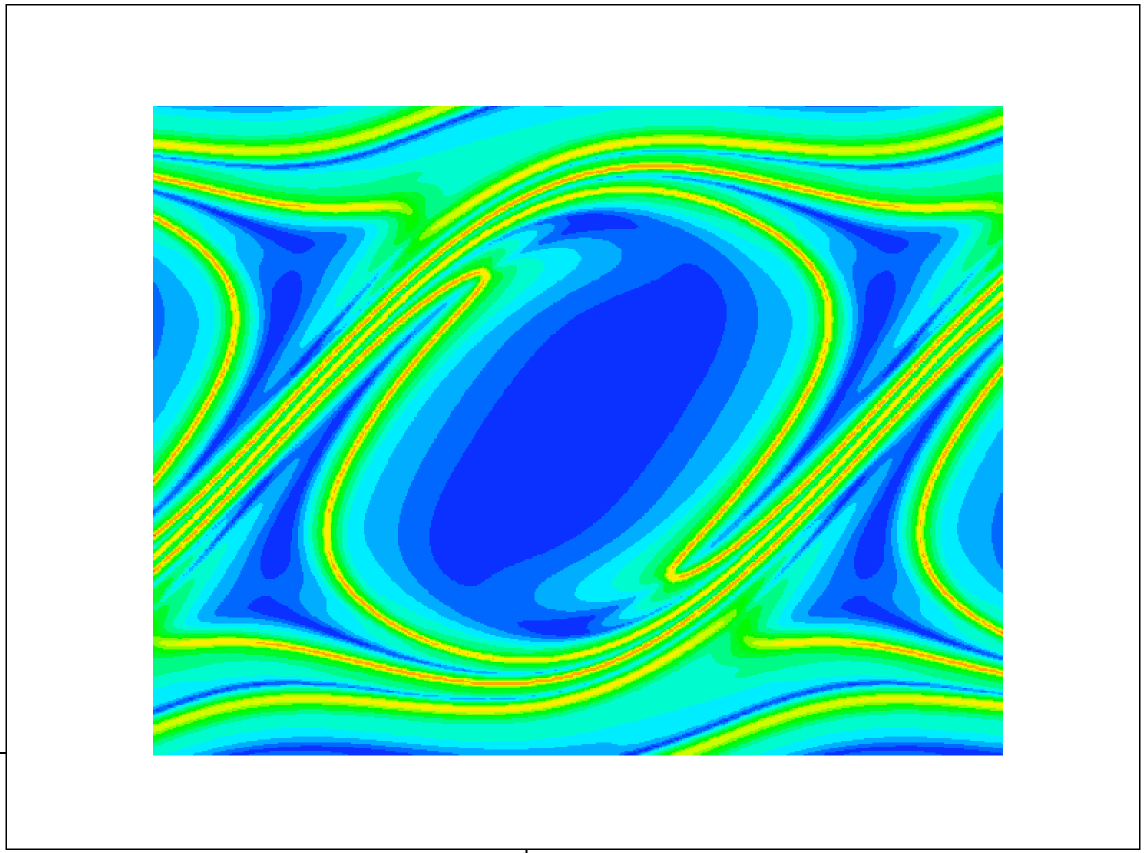
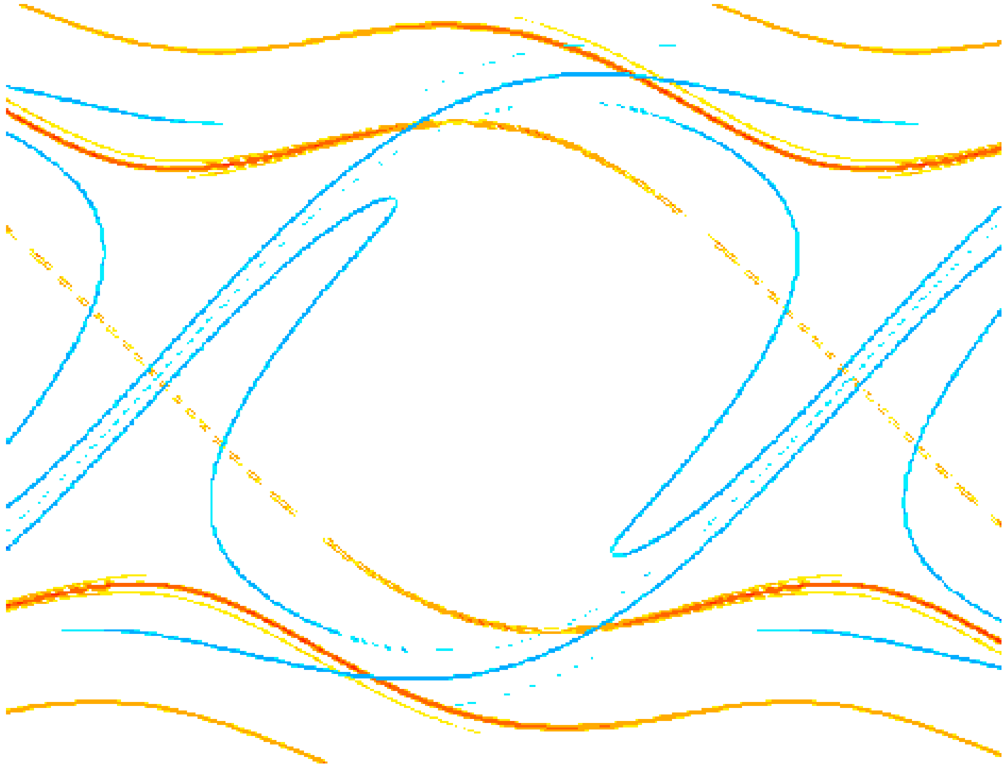
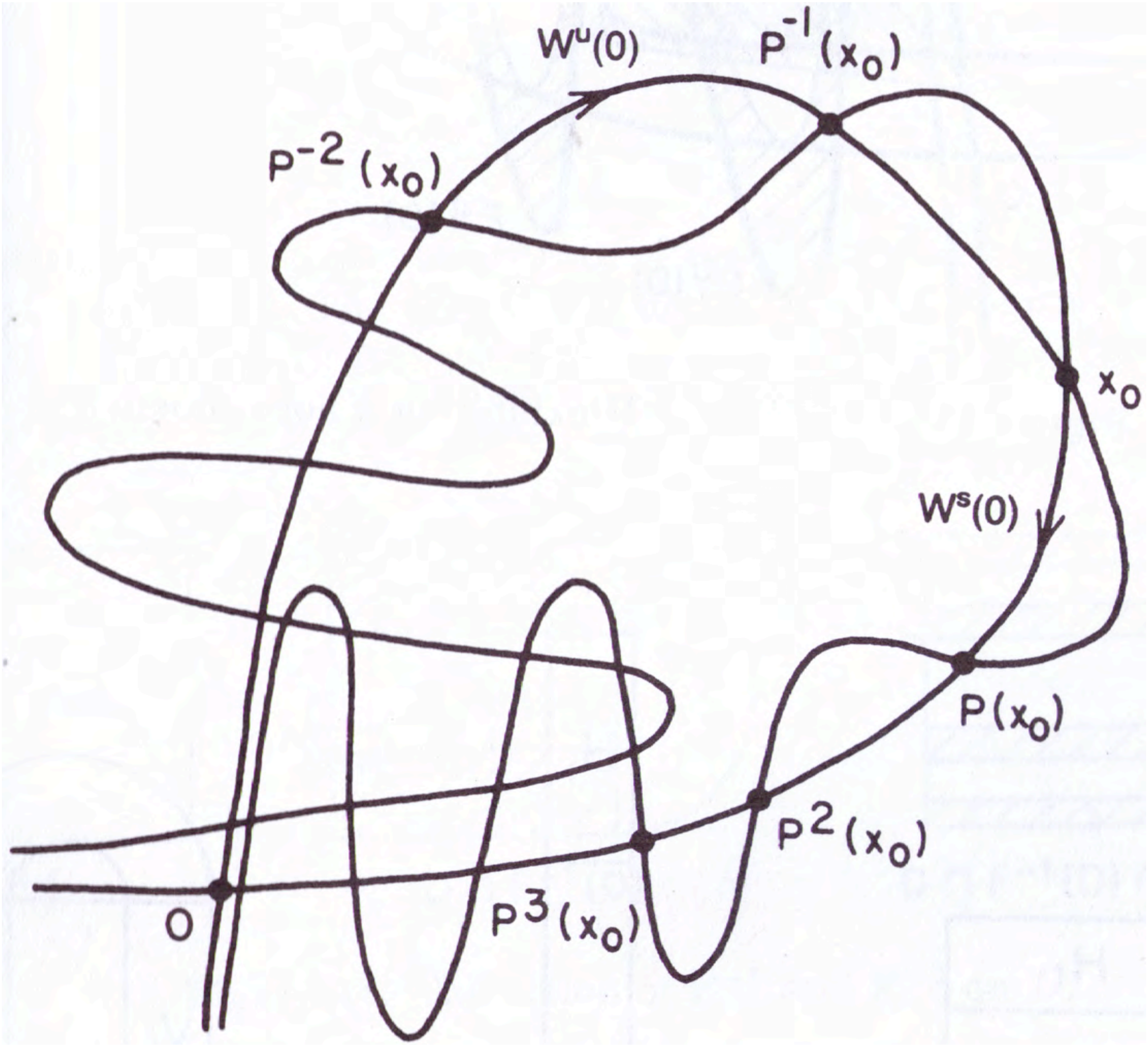






Captures transient chaos

(Aside on tangles)



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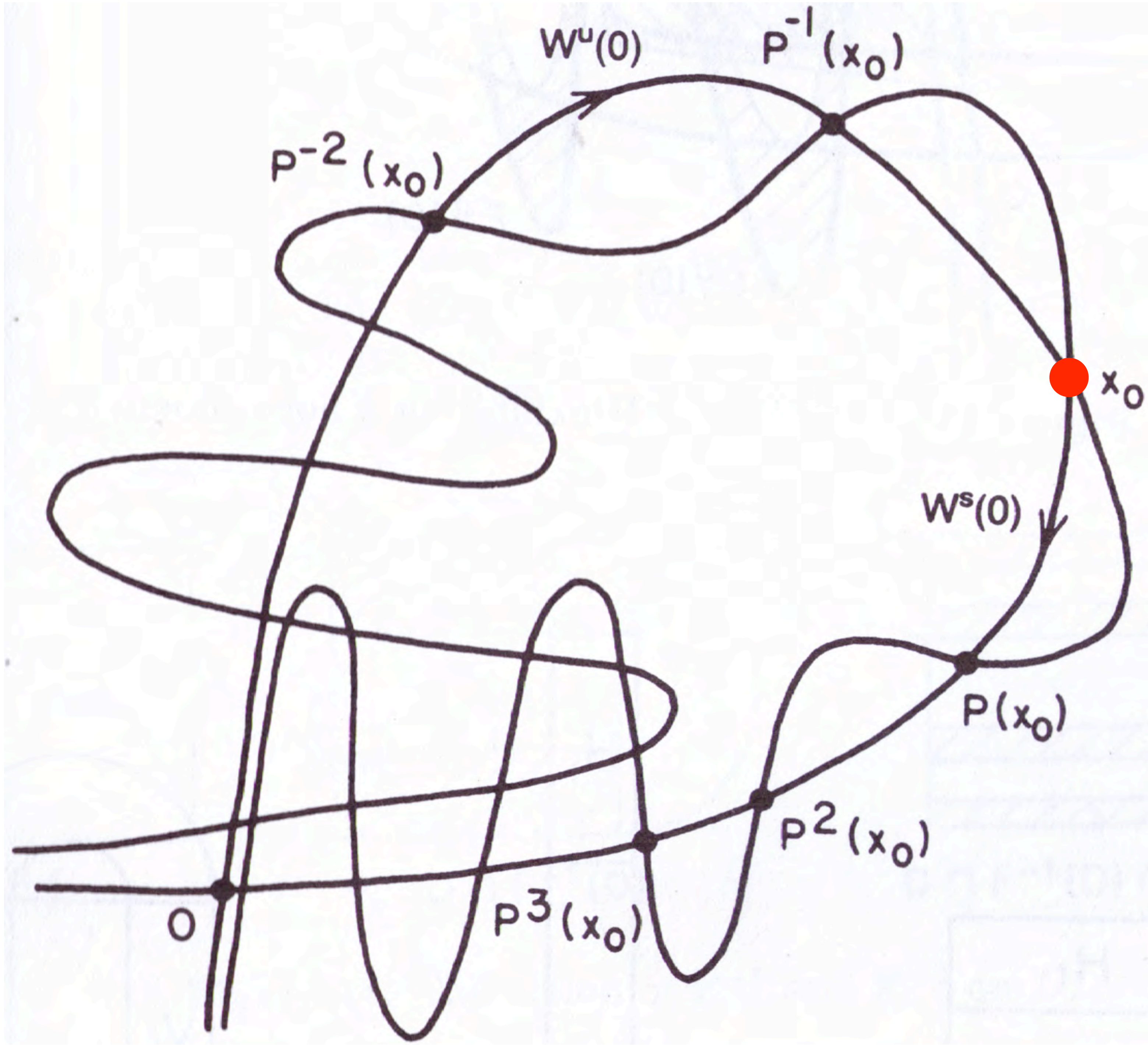

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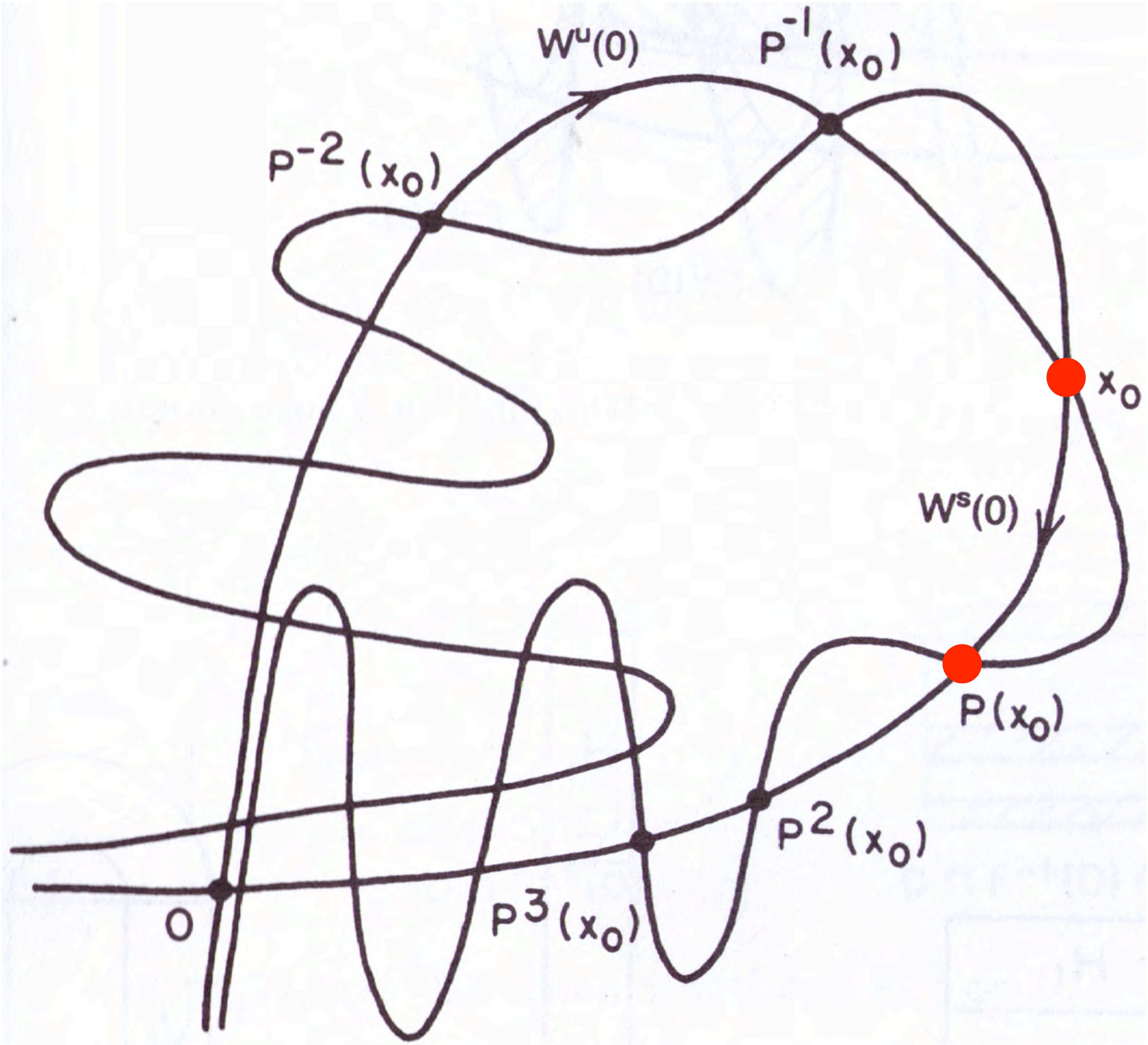

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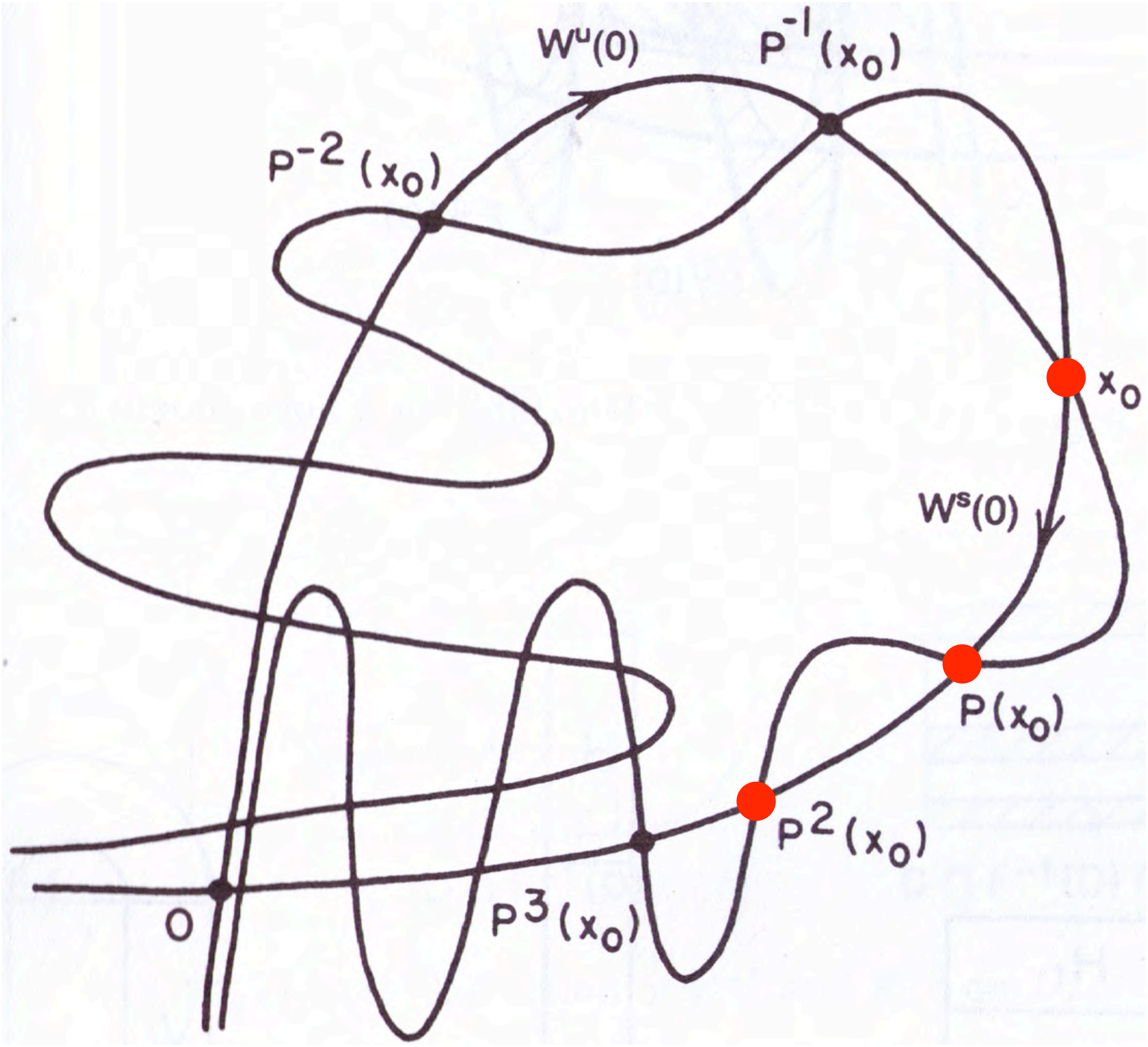
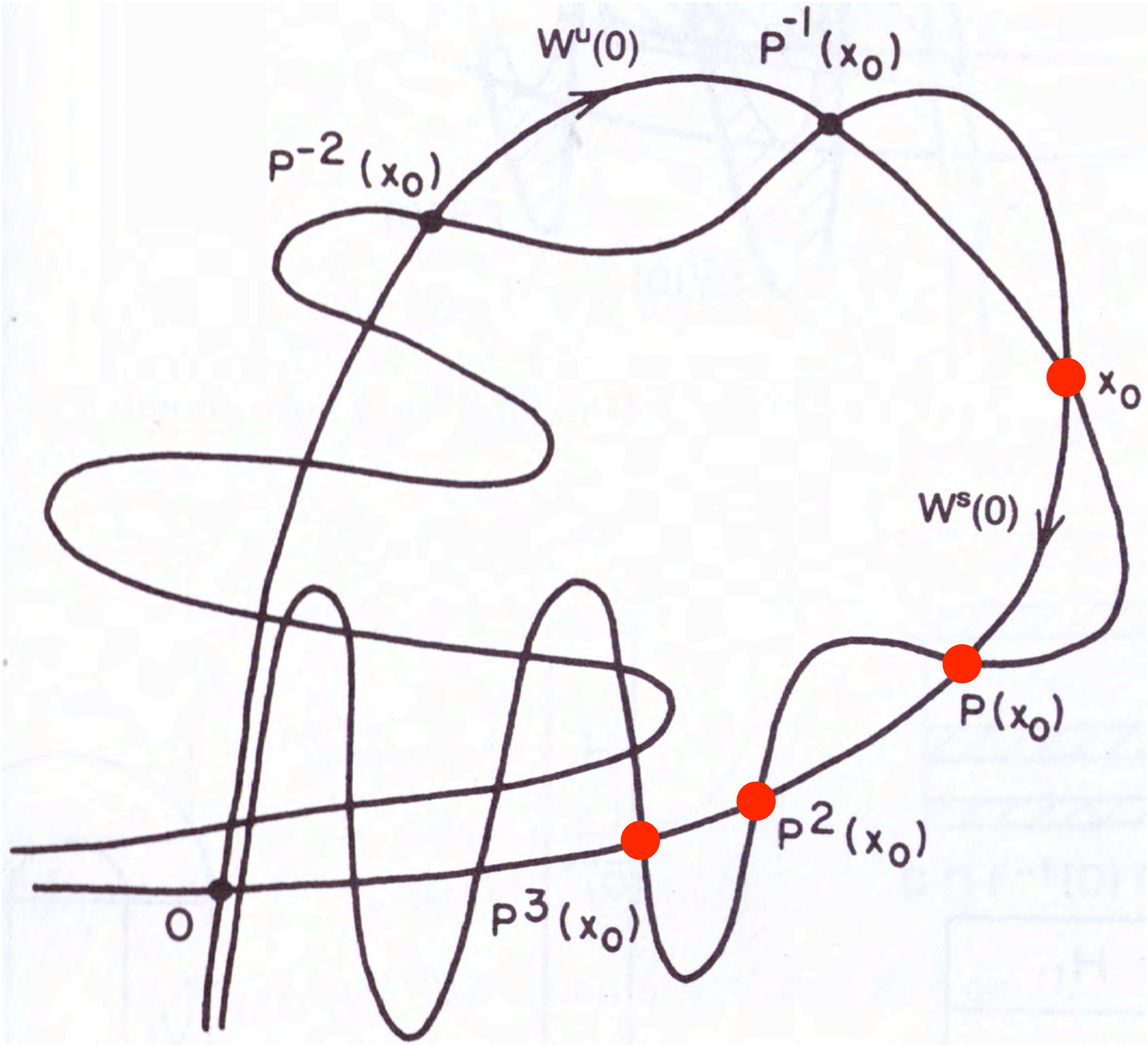


(Aside on tangles)



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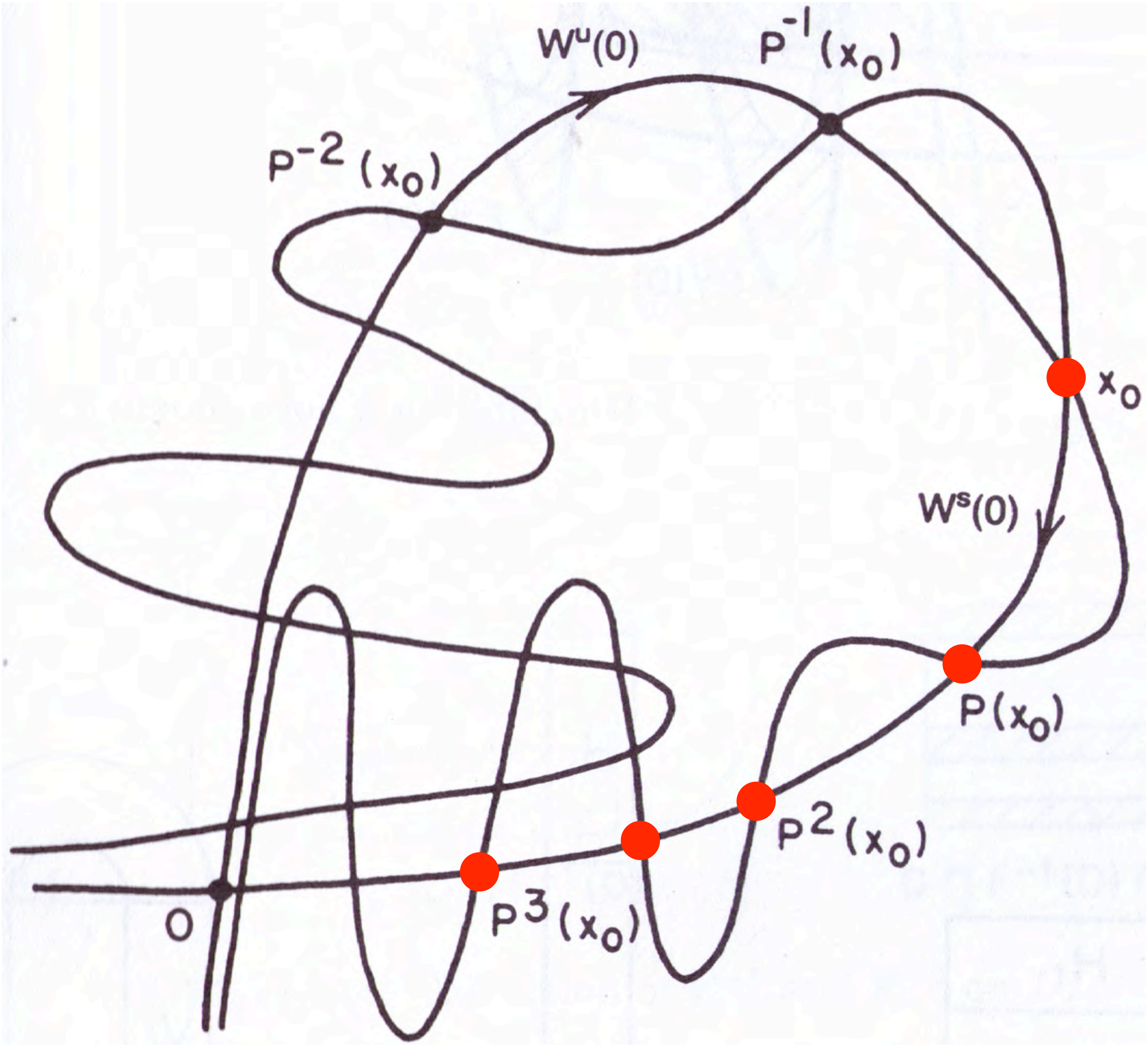
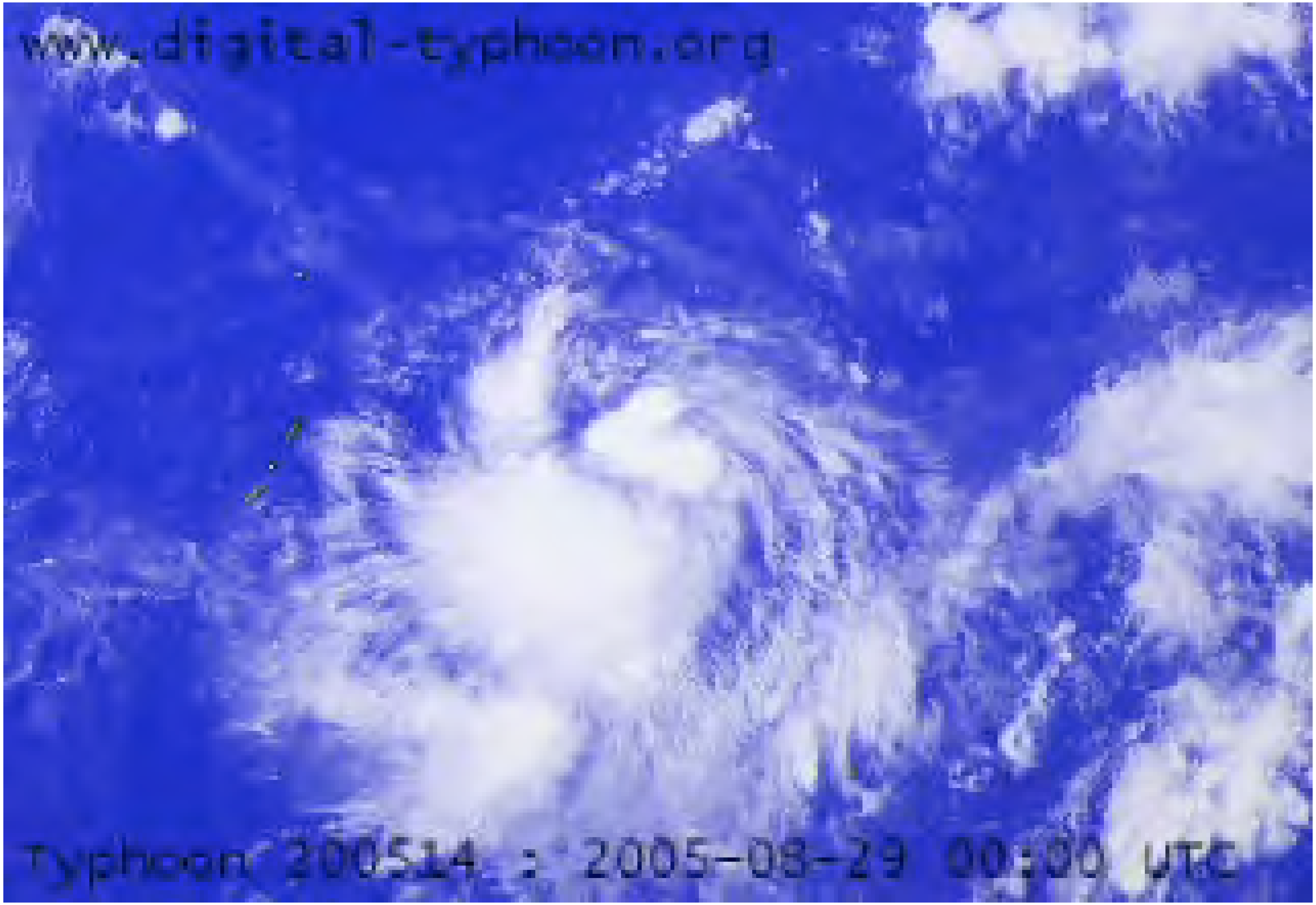
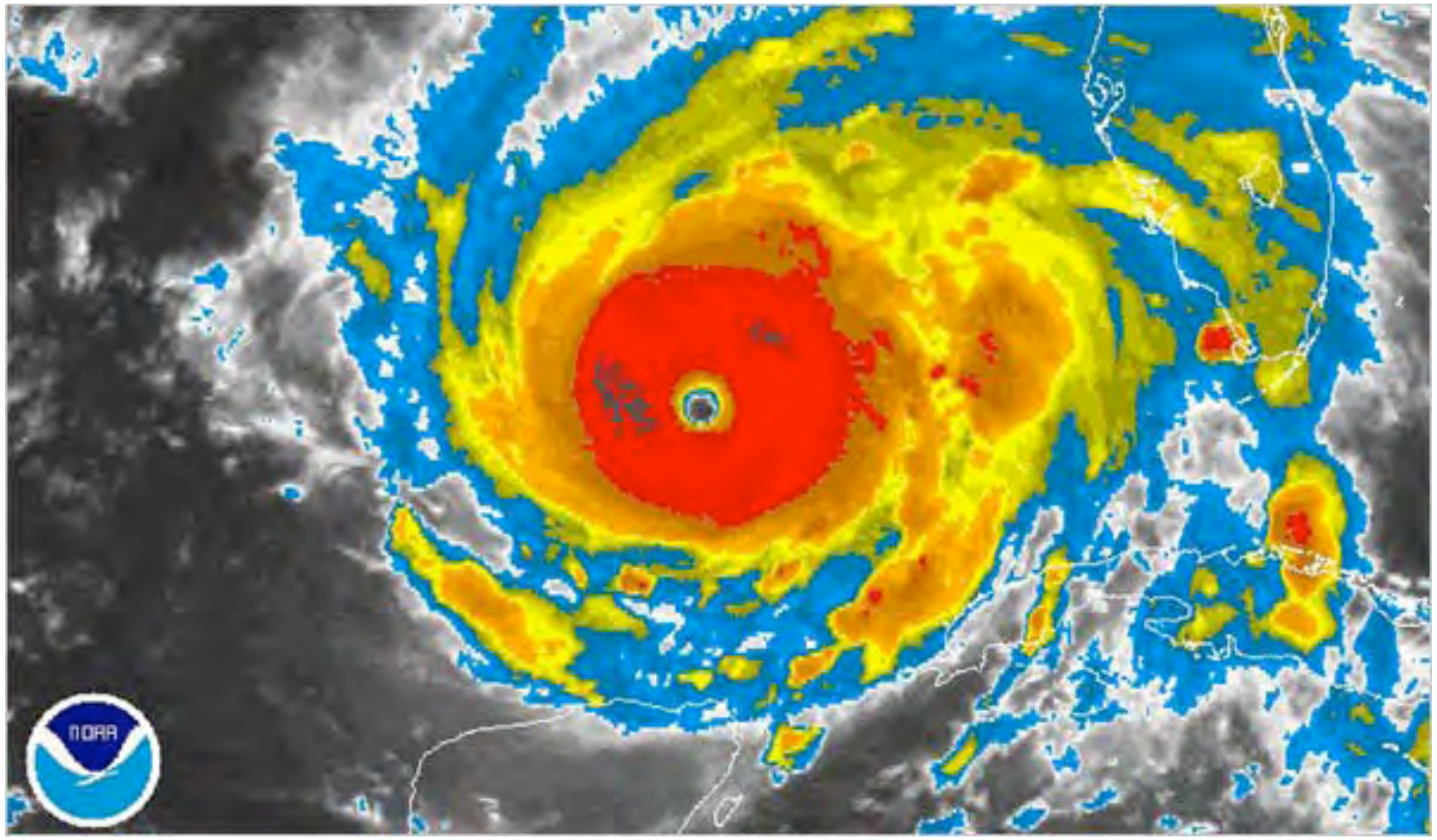


Transport in hurricanes



29 May 2007

Will Warming Lead to a Rise in Hurricanes?



National Oceanic and Atmospheric Administration

By CORNELIA DEAN
Published: May 29, 2007

When people worry about the effects of global warming, they worry more about hurricanes than anything else. In surveys, almost three-quarters of Americans say there will be more and stronger hurricanes in a warming world. By contrast, fewer than one-quarter worry about increased coastal flooding.

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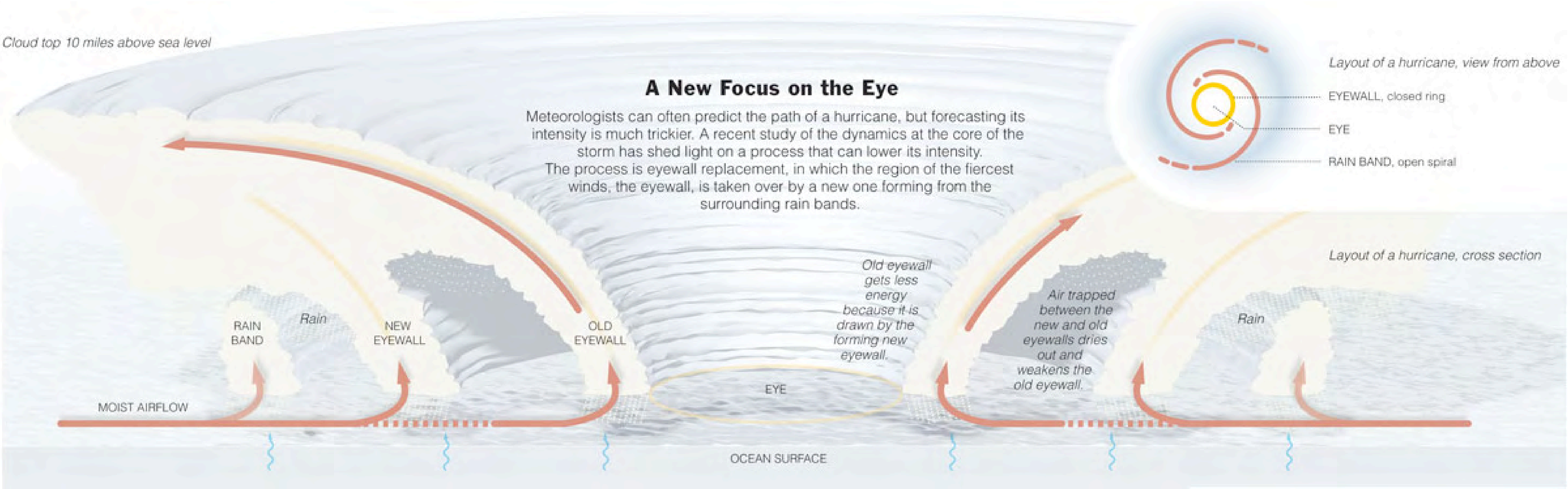
Entrainment and Hurricane Intensity

$$V_p^2 = \frac{T_s - T_0}{T_0} \frac{F_\downarrow - F_\uparrow + \boxed{F_{\mathrm{entrainment}}}}{C_D \rho |\mathbf{V}|}$$

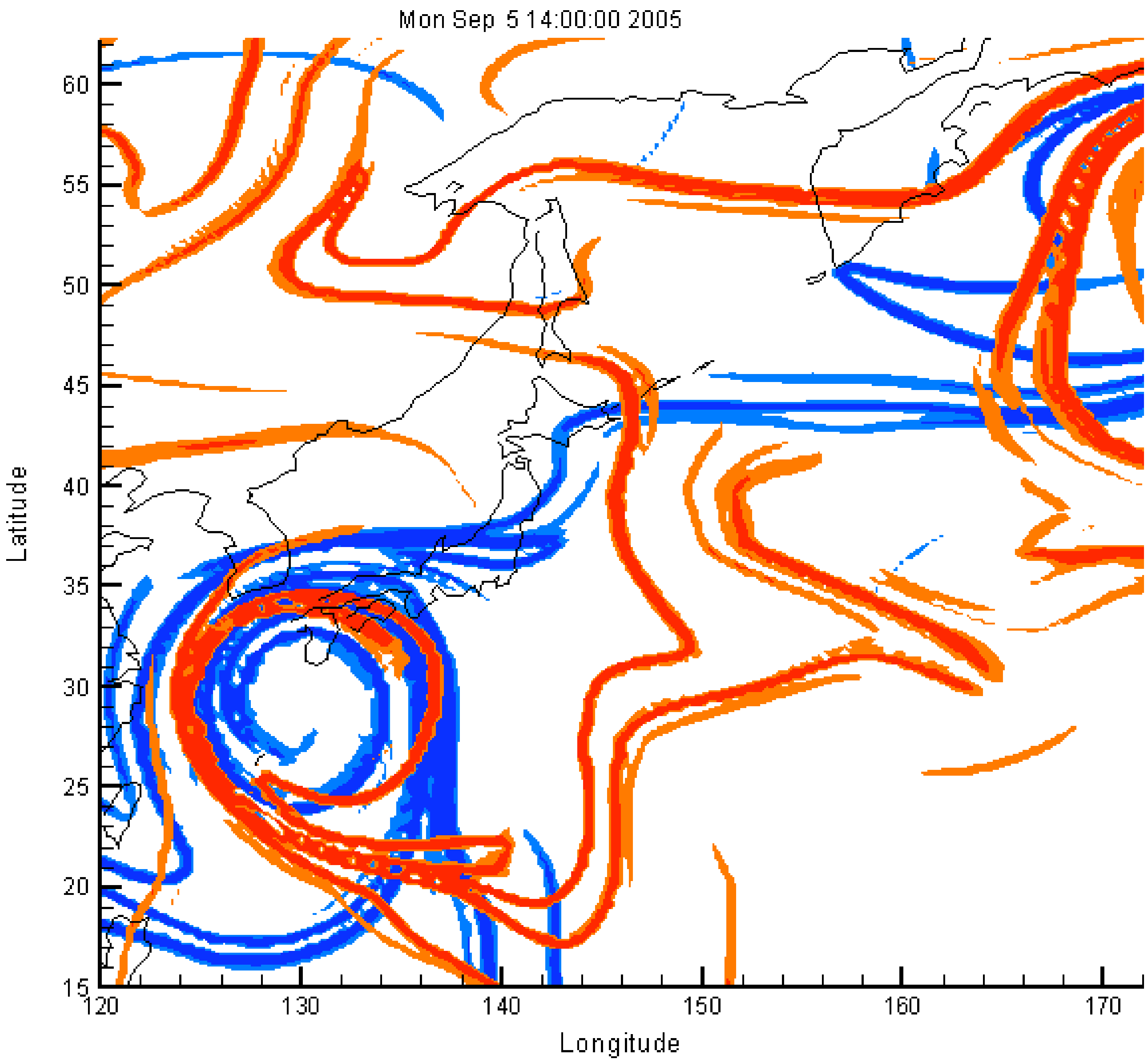
Dr. Kerry Emanuel, MIT,
Environmental Factors Affecting Tropical Cyclone Power Dissipation (In Press).

Stability of Hurricanes

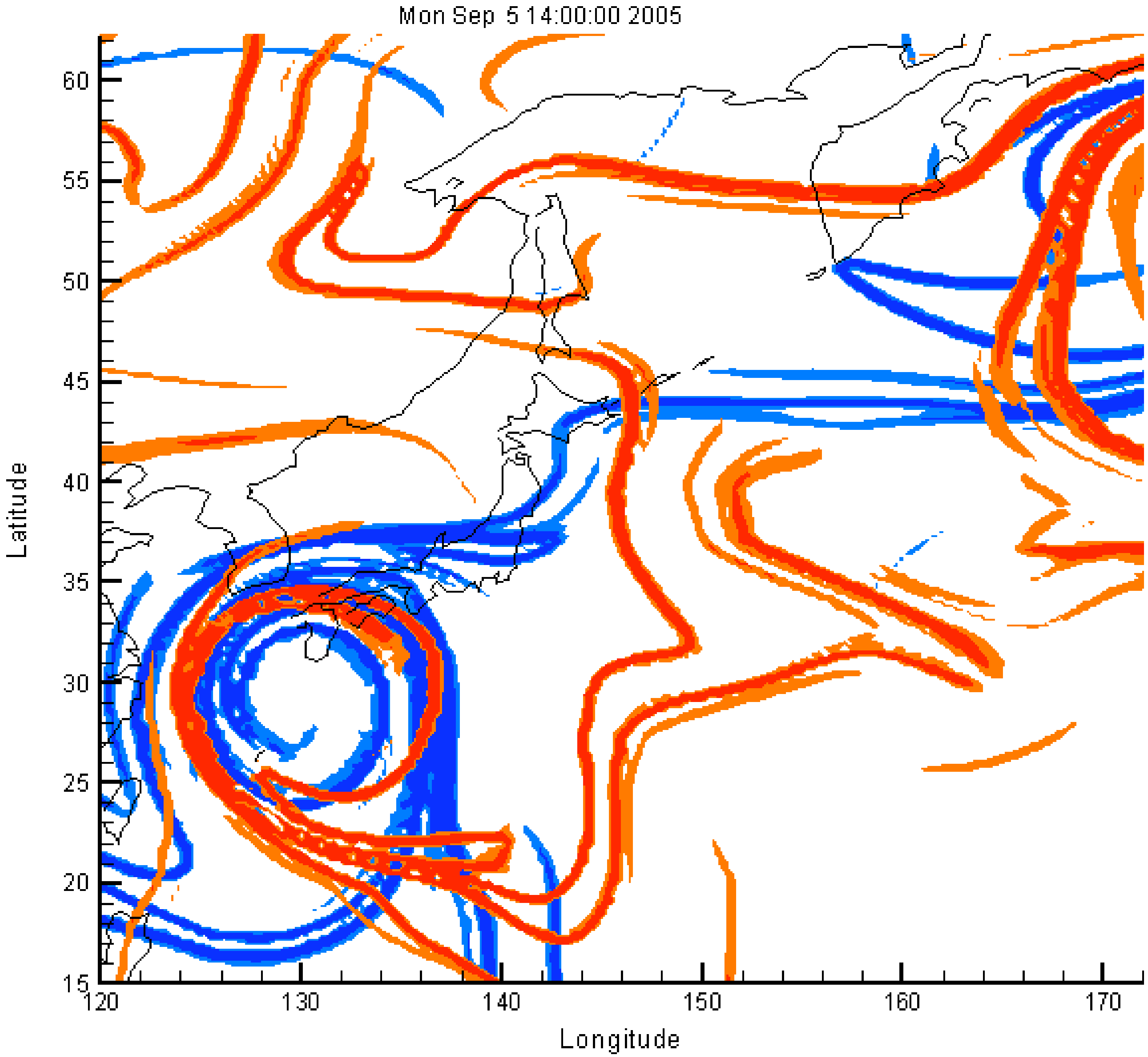
The New York Times (29 May 2007)

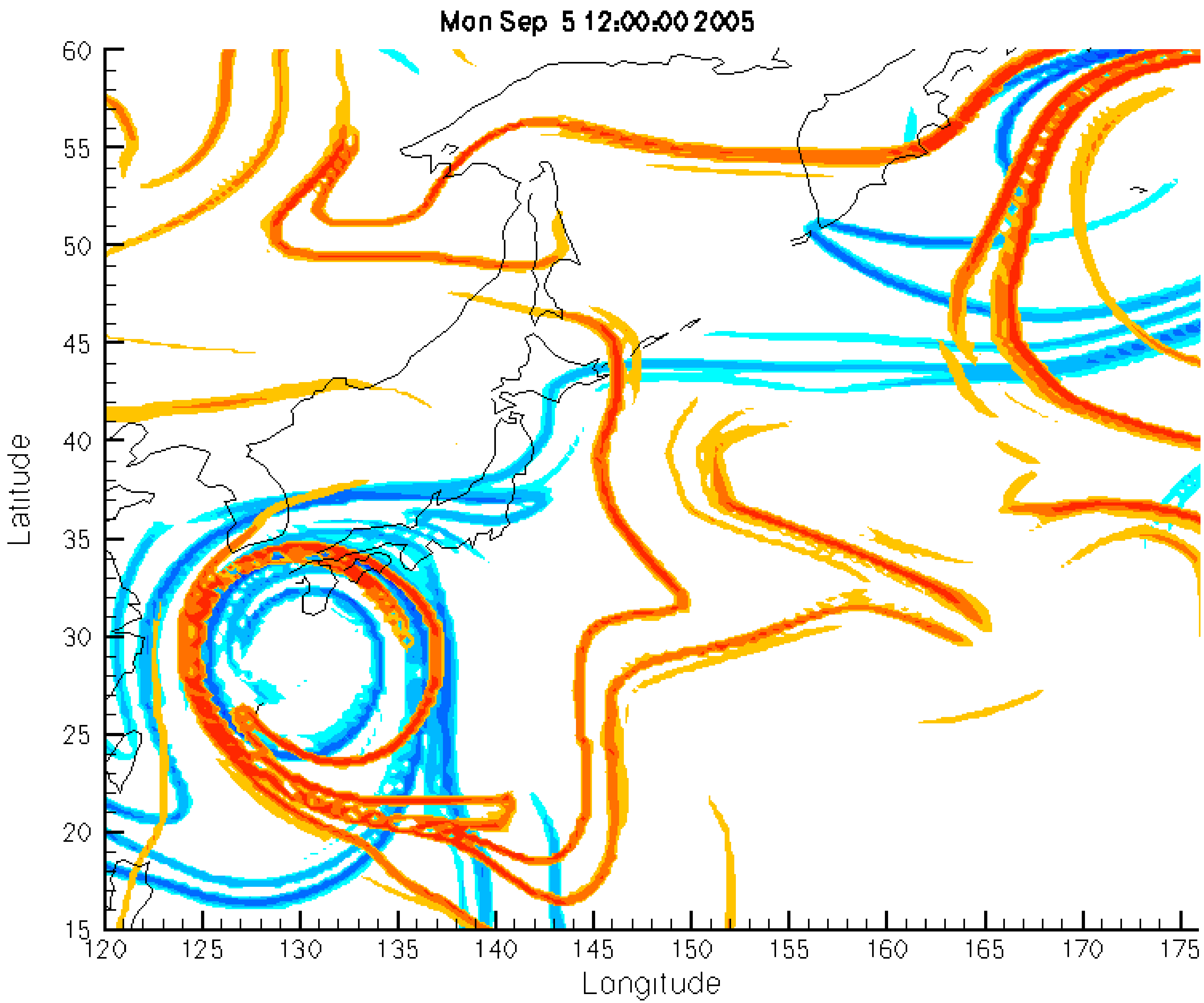


LCS in typhoon Nabi



LCS in typhoon Nabi

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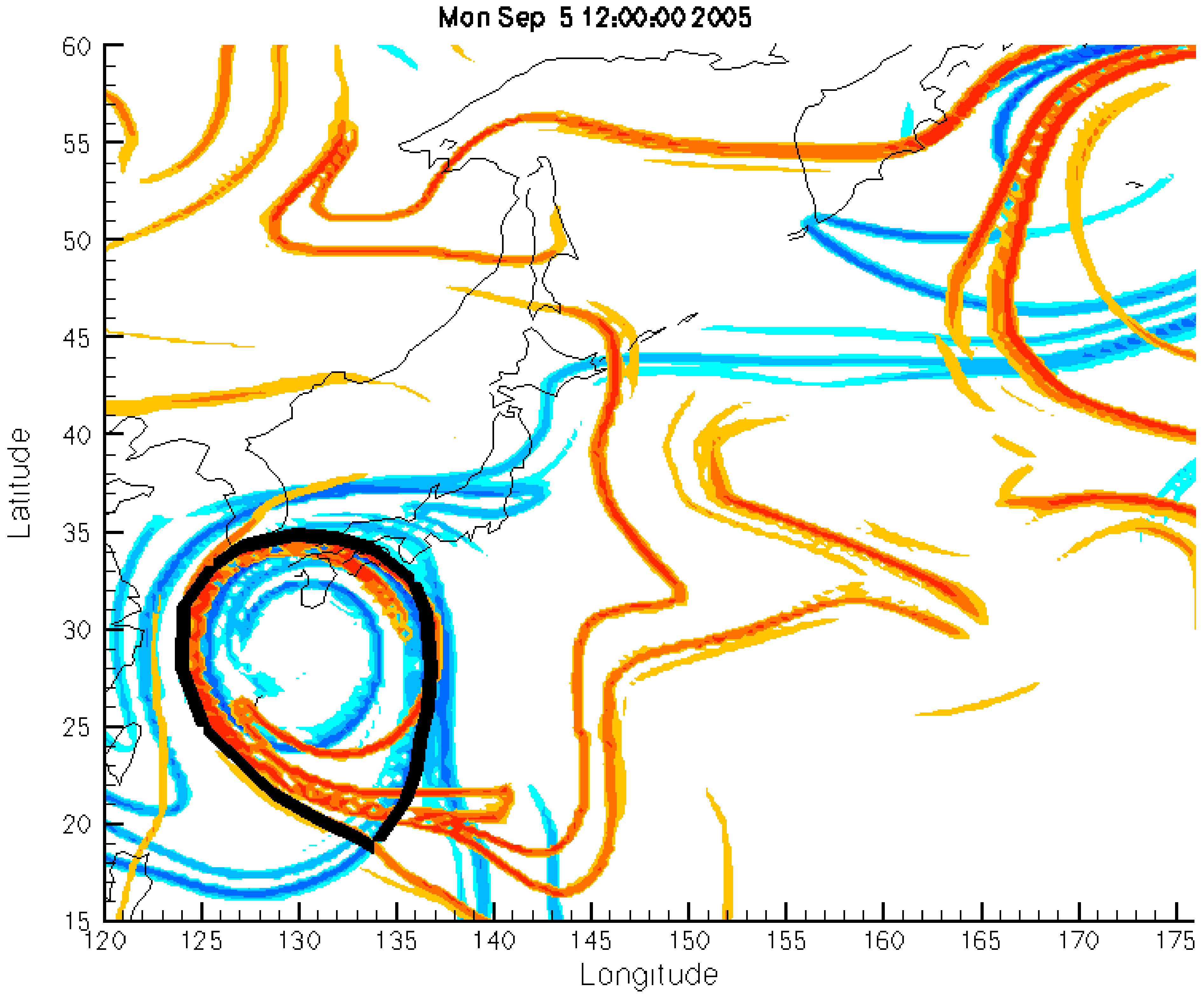
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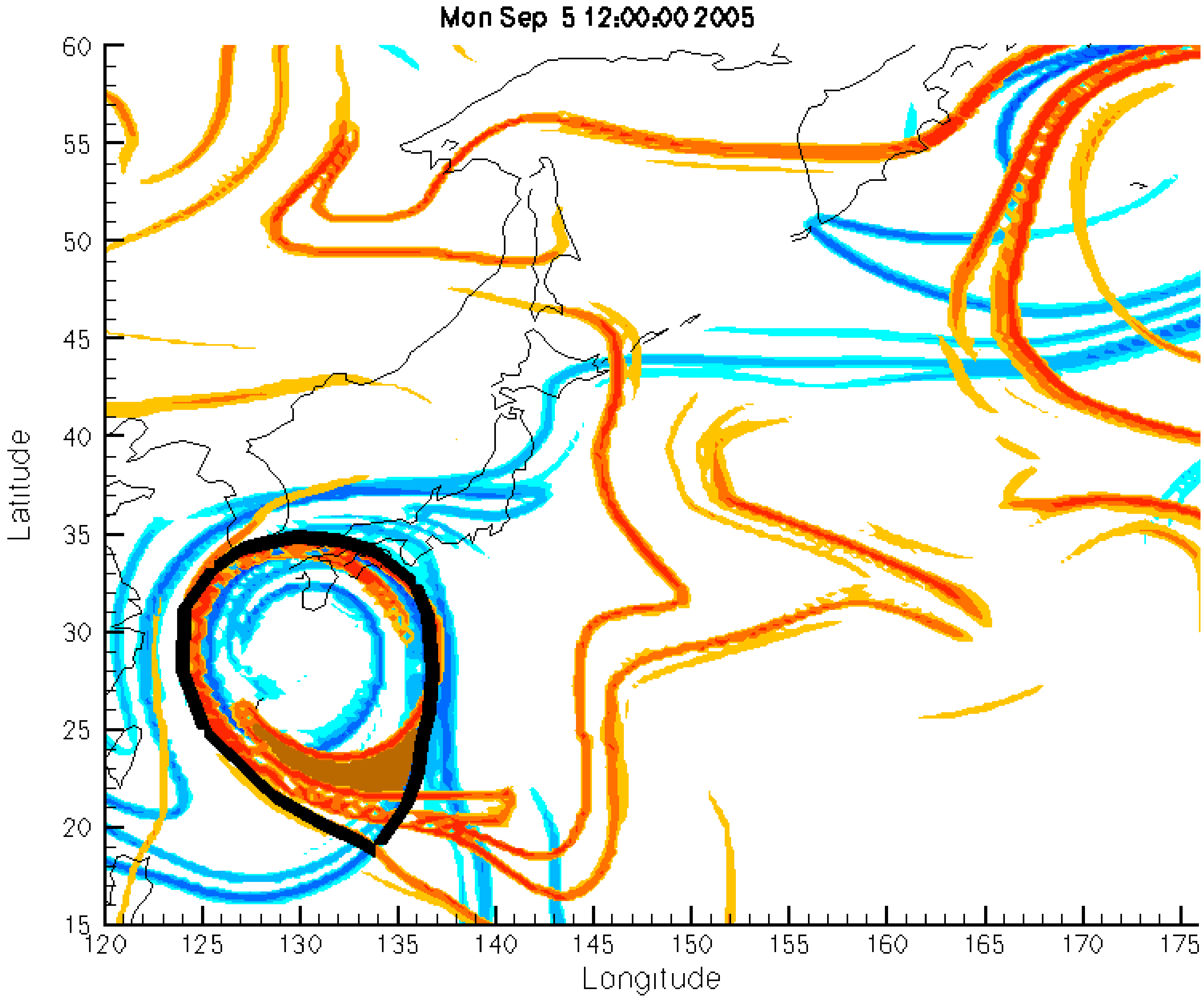
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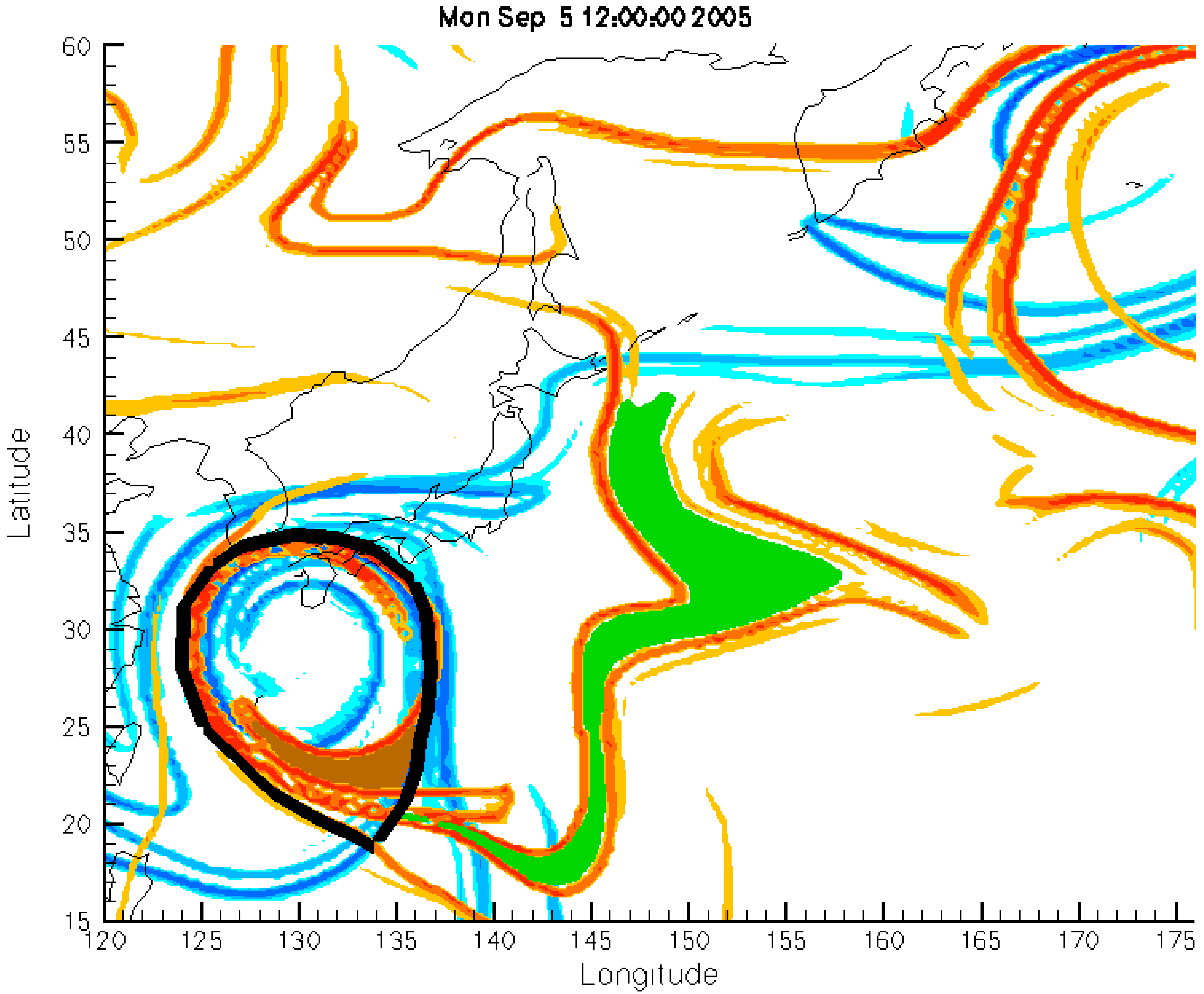
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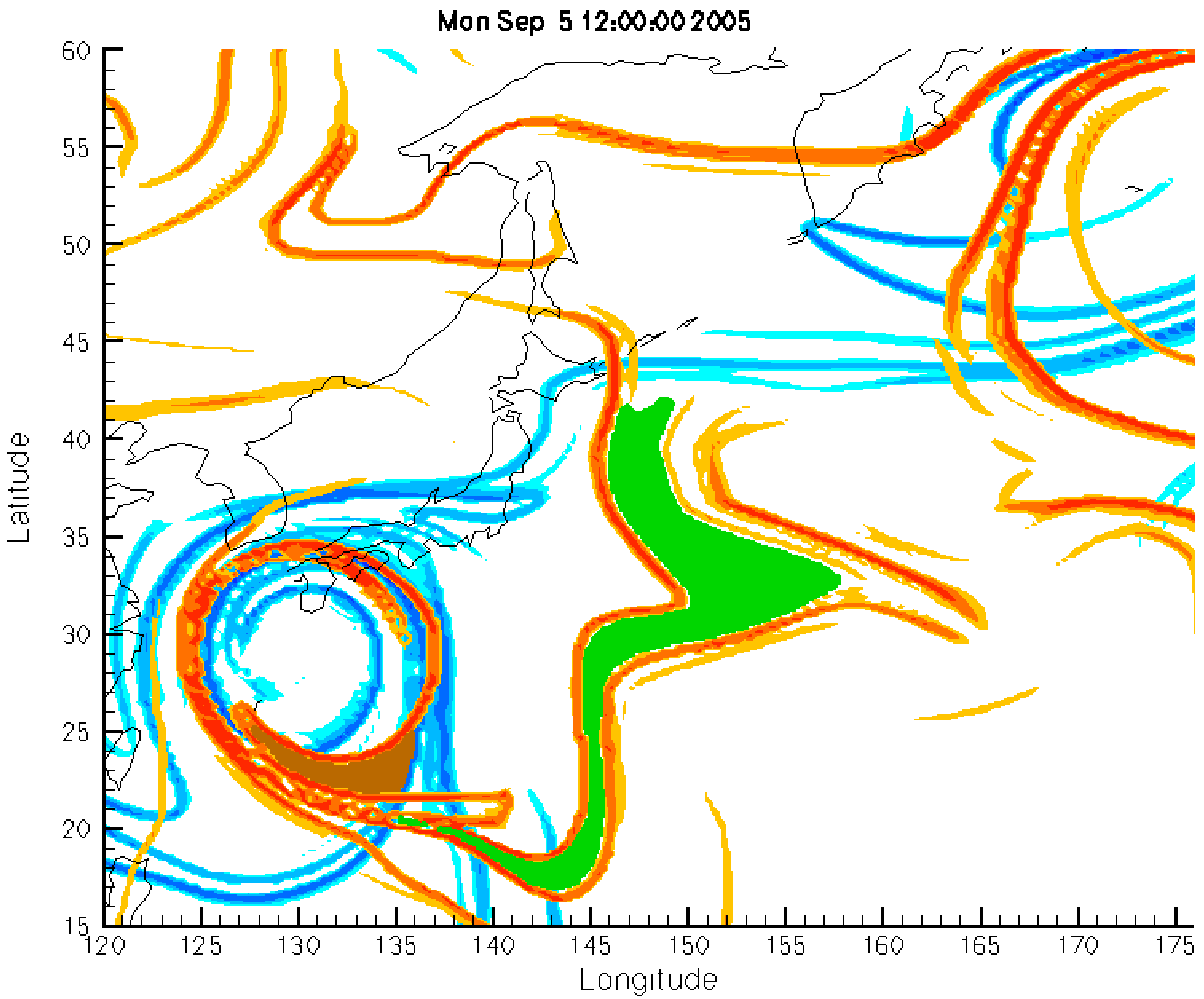

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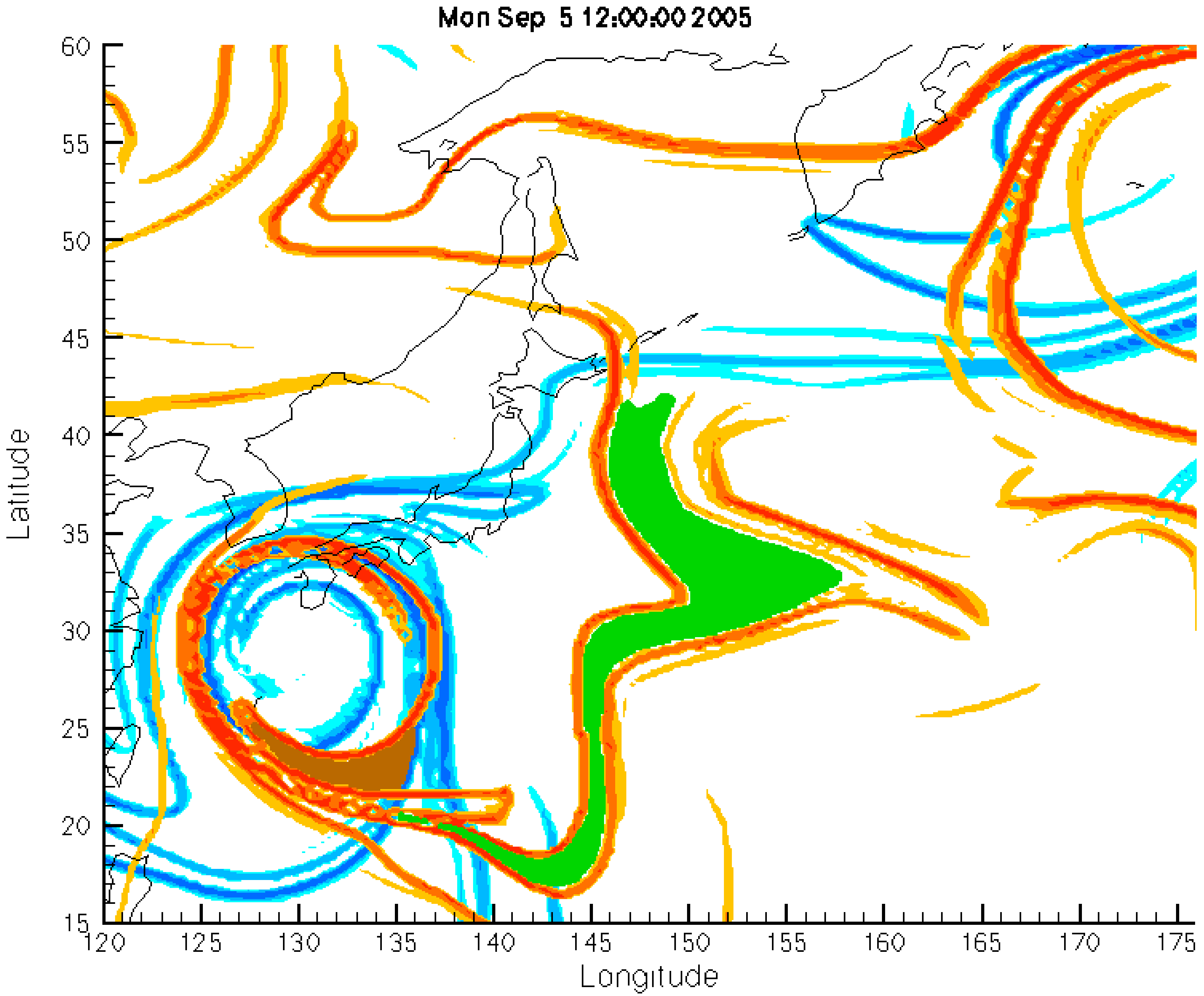
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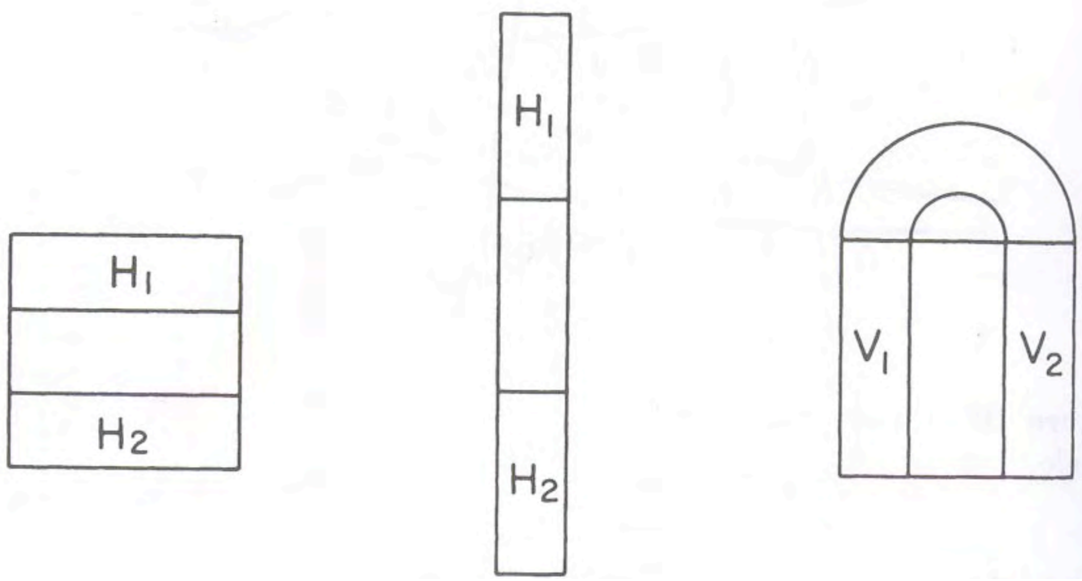
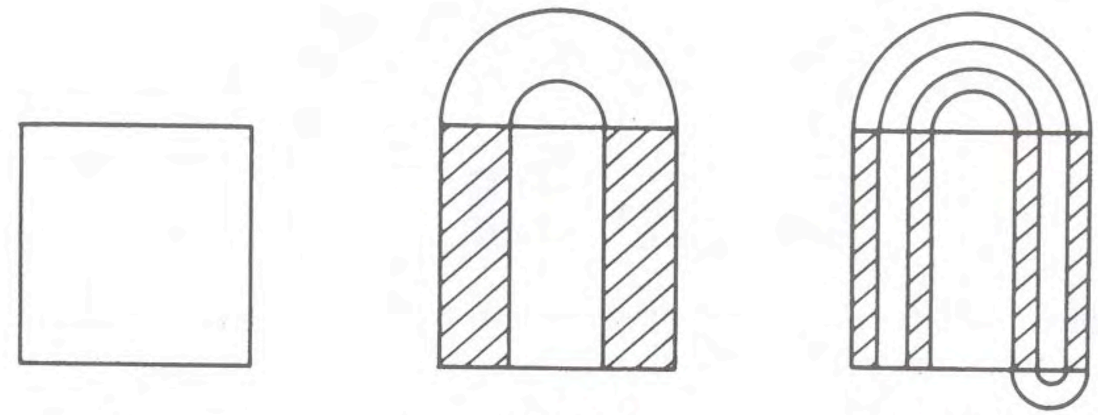
Smale’s Horseshoe Map

Smale’s Horseshoe Map





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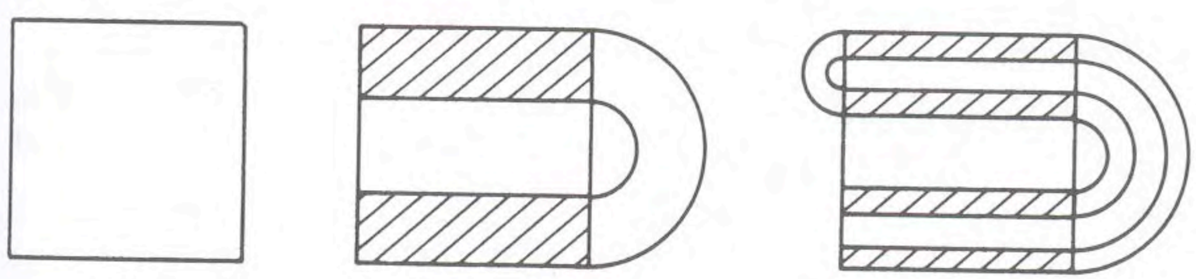

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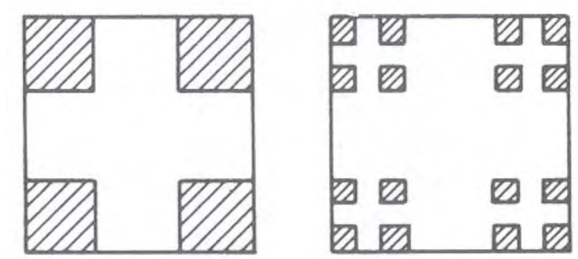
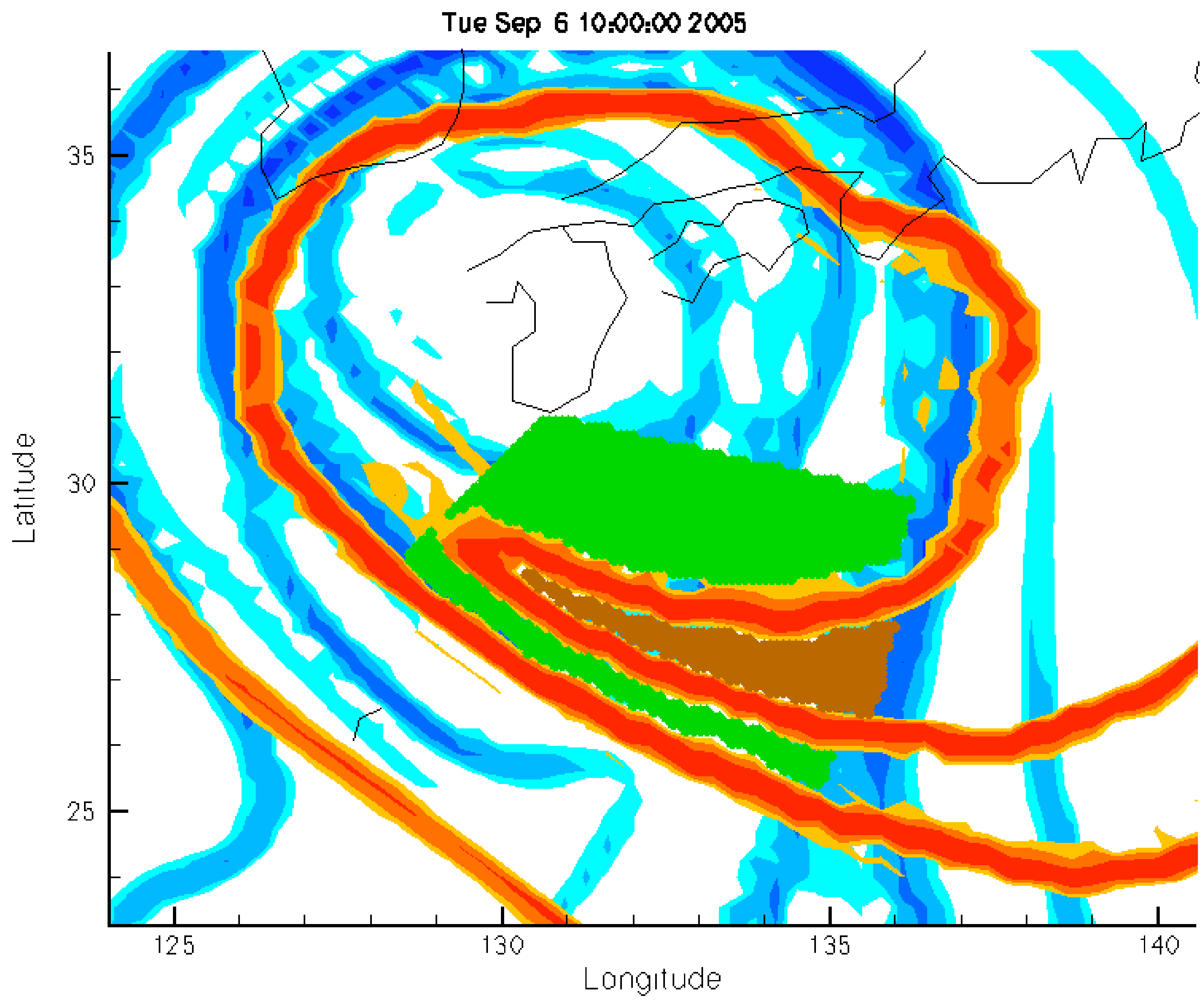


Horshoes in Typhoon Banyan



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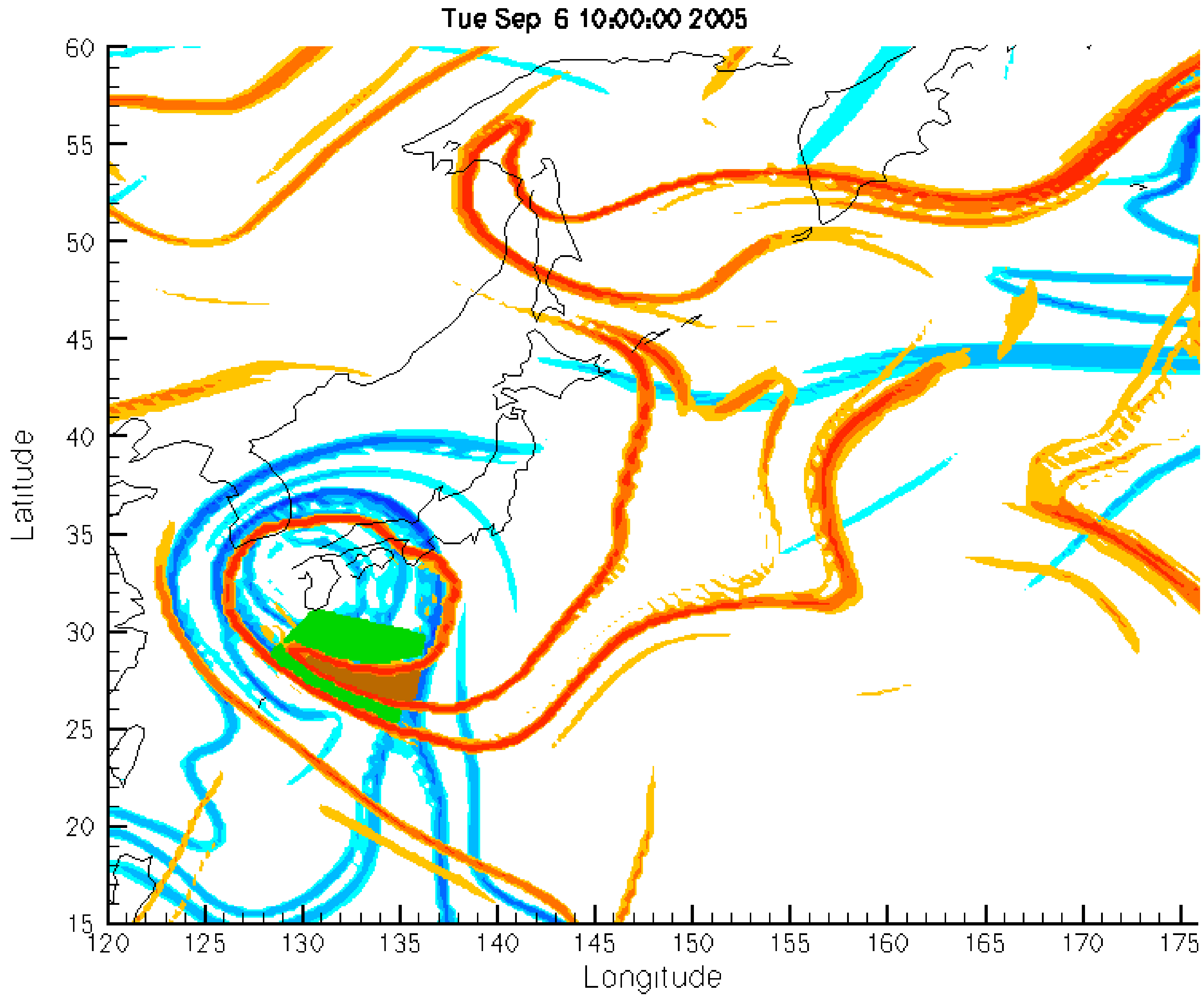
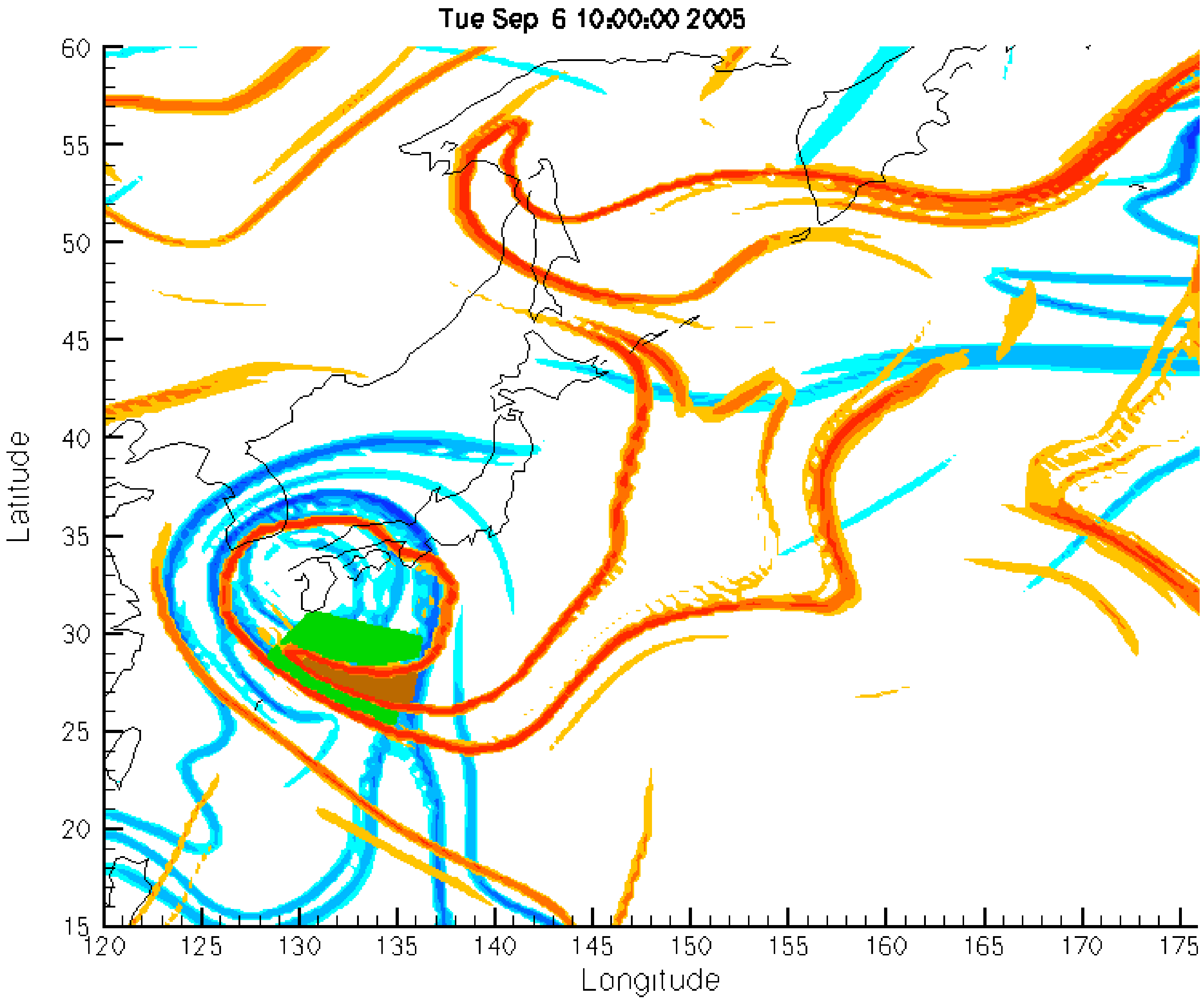
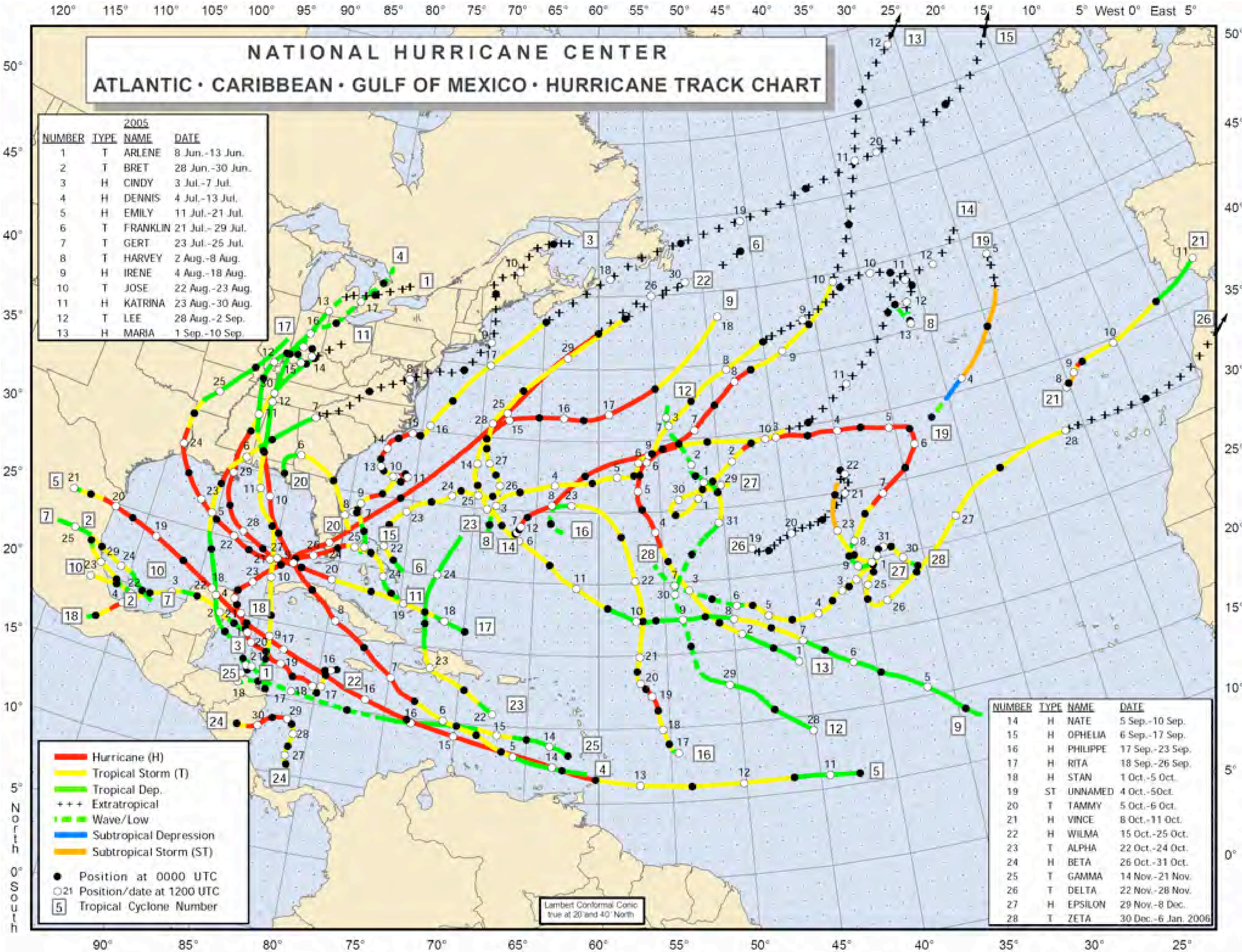


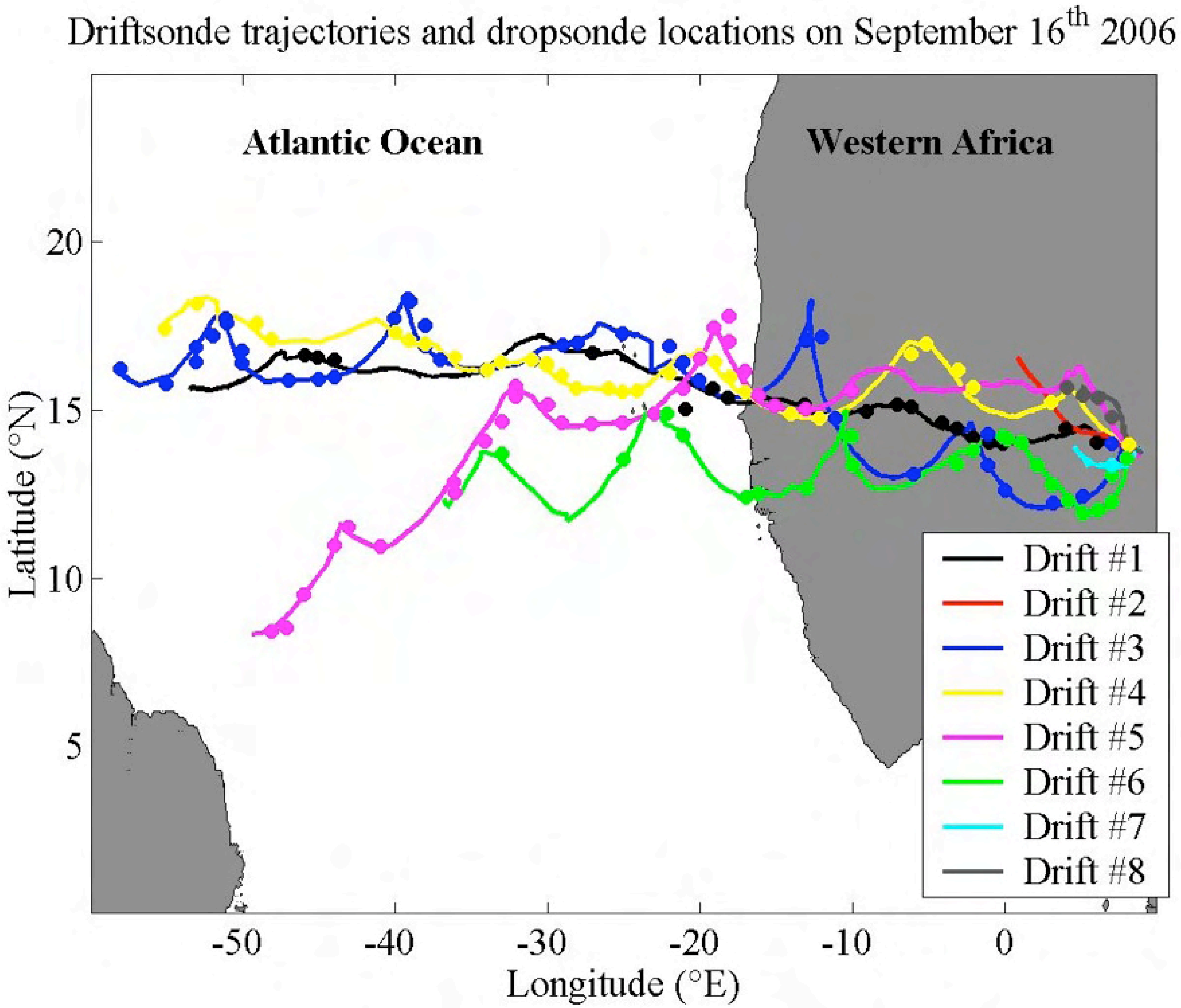
Horshoes in Typhoon Banyan



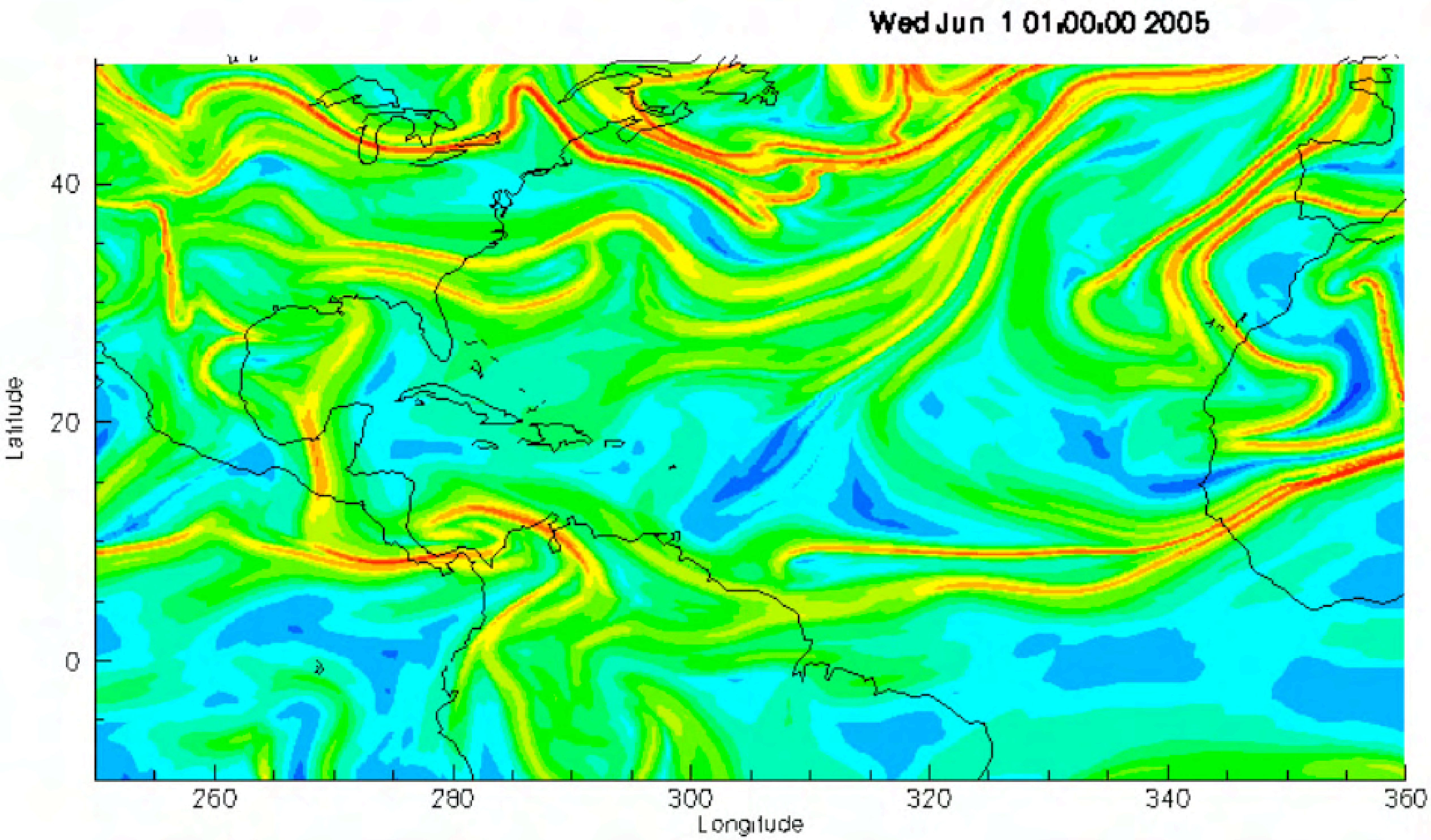
2005 Atlantic Hurricane season



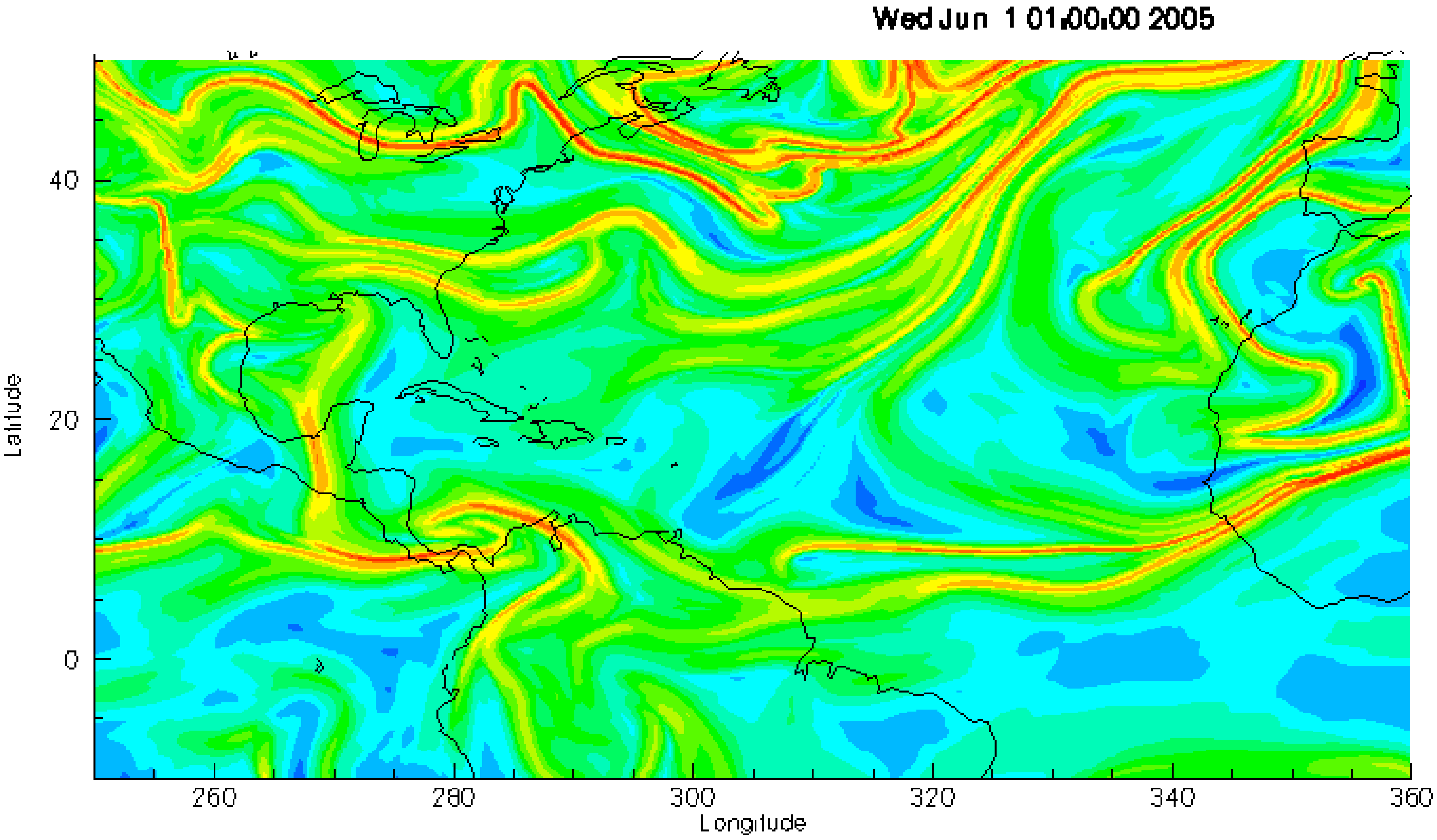
Tracking Easterly waves



2005 Atlantic Hurricane Season

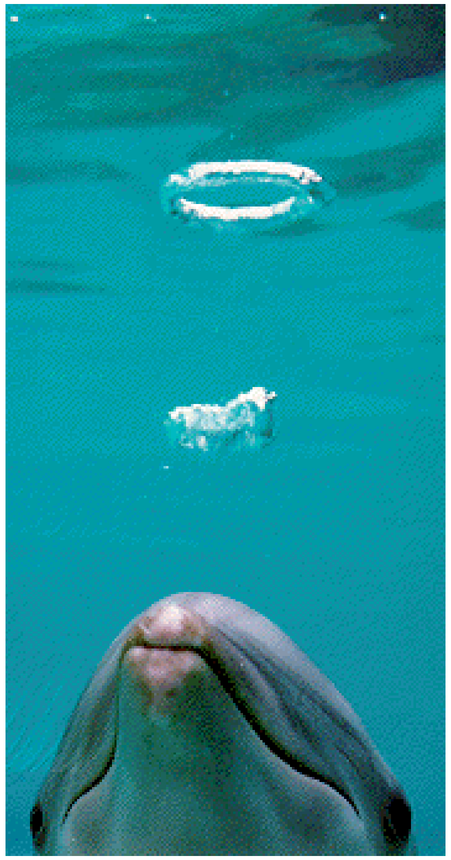


2005 Atlantic Hurricane Season



Vortex Rings

Vortex Rings



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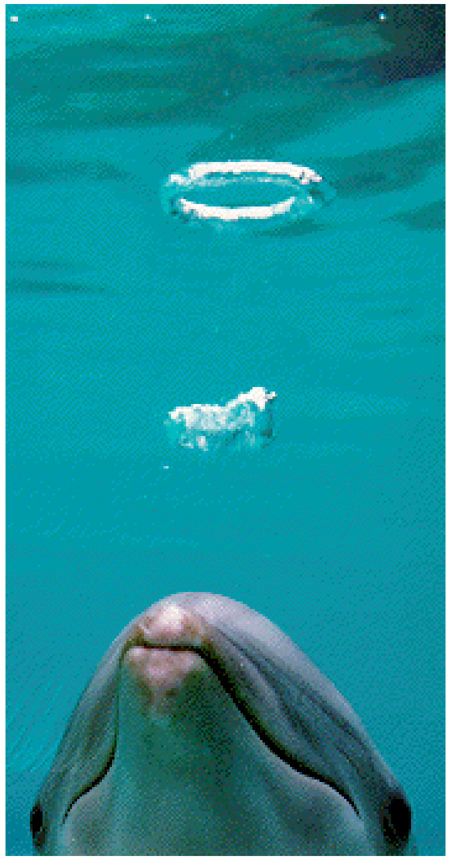

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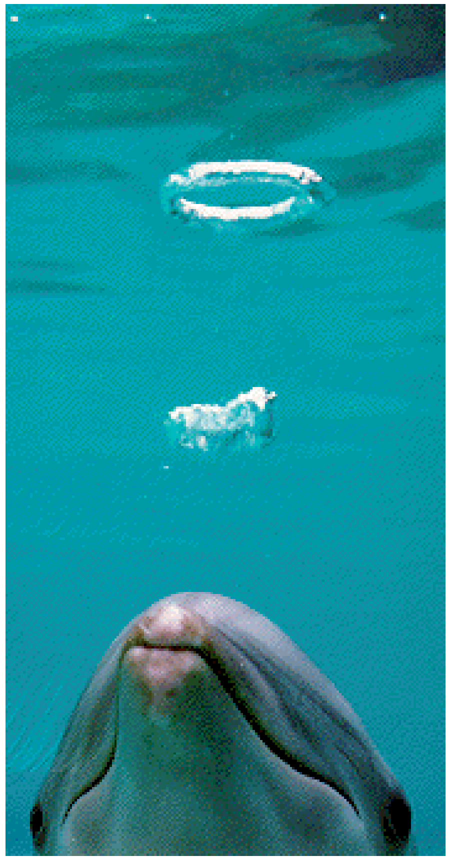

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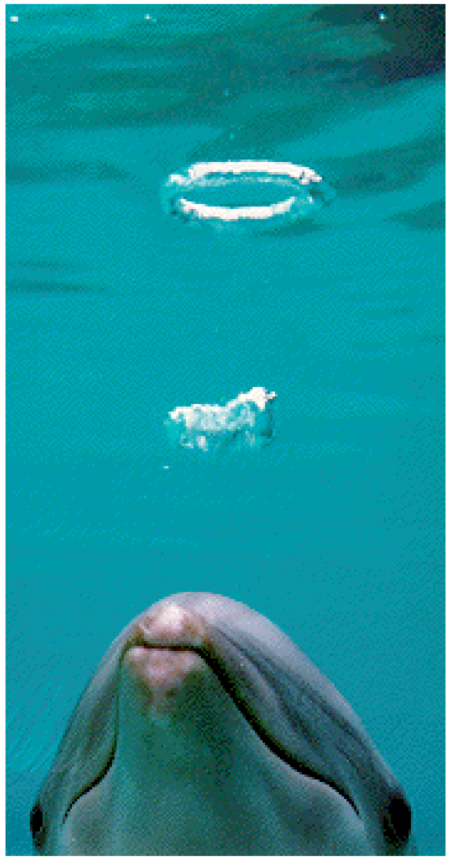
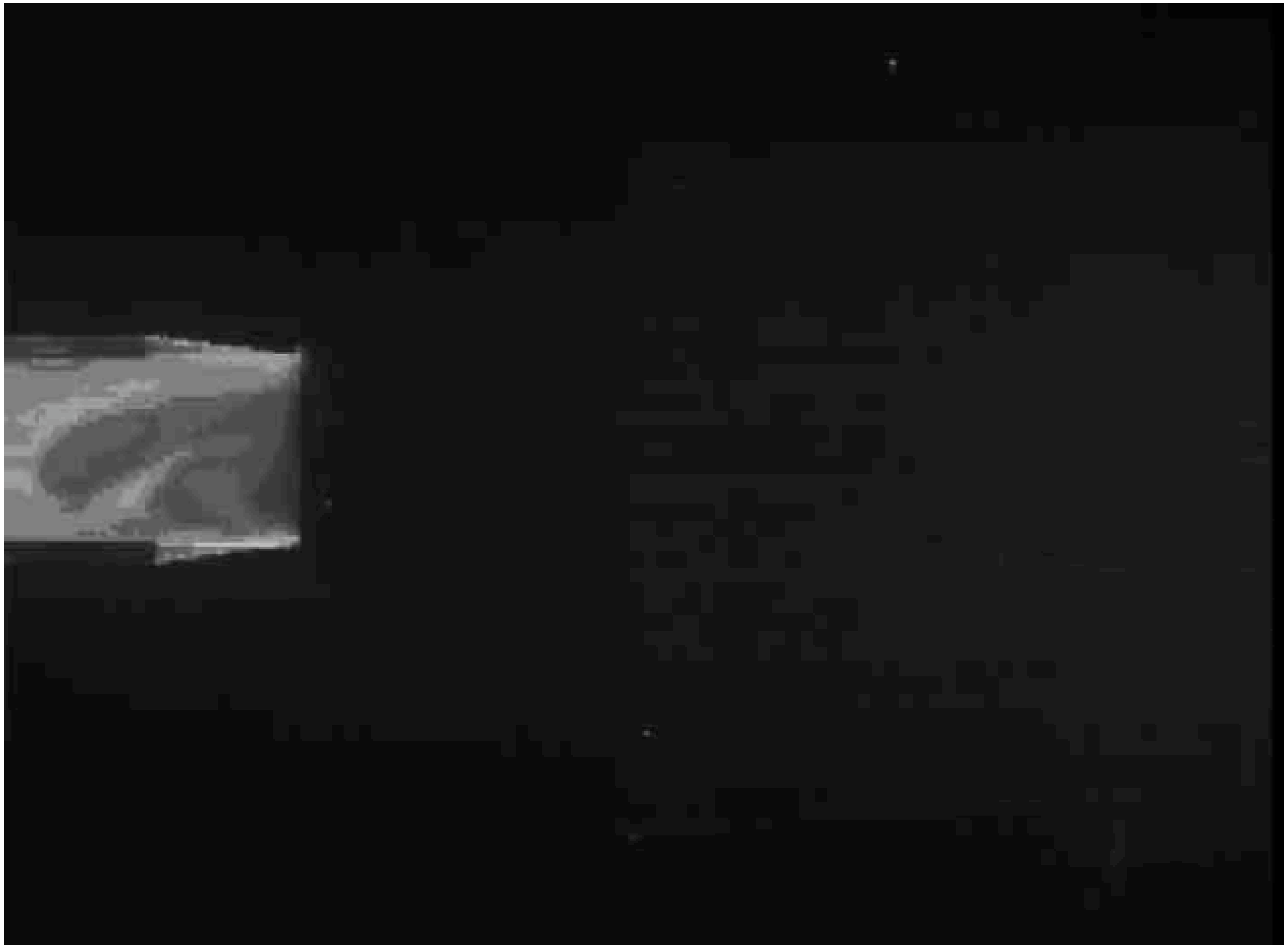
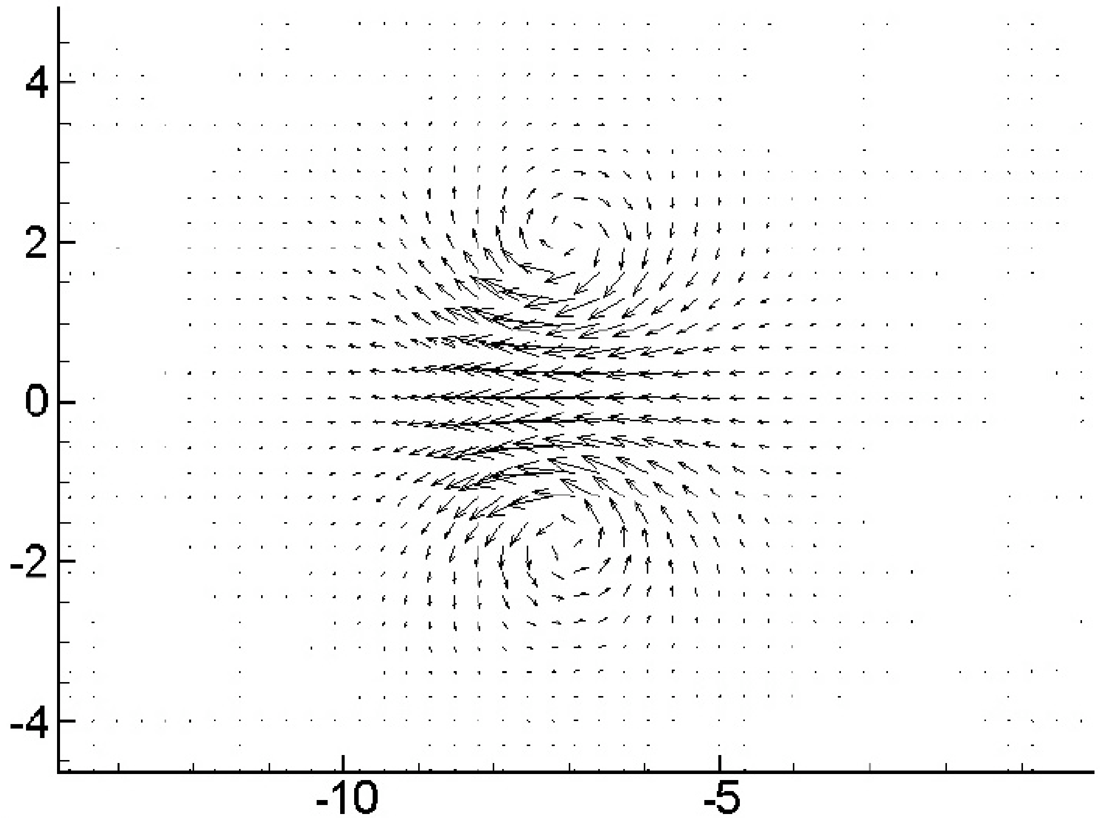


Vortex rings in the laboratory

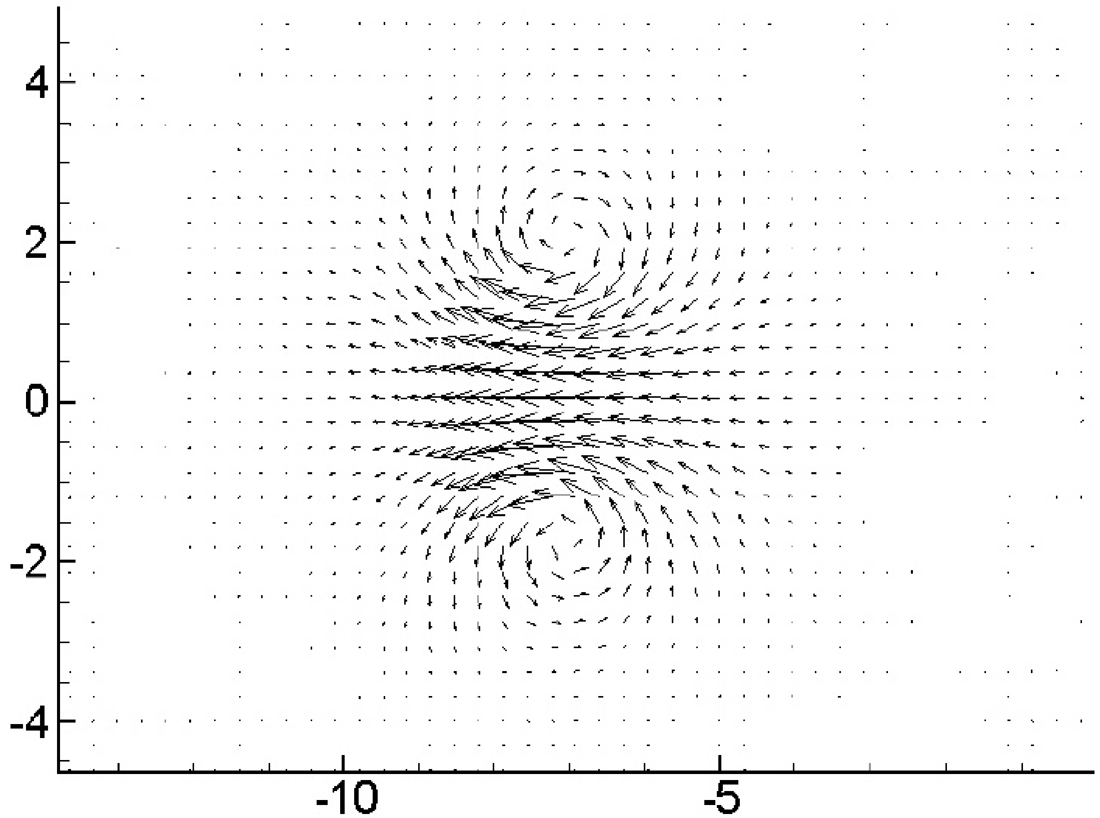


LCS for the vortex ring



Vortex velocity field

LCS for the vortex ring



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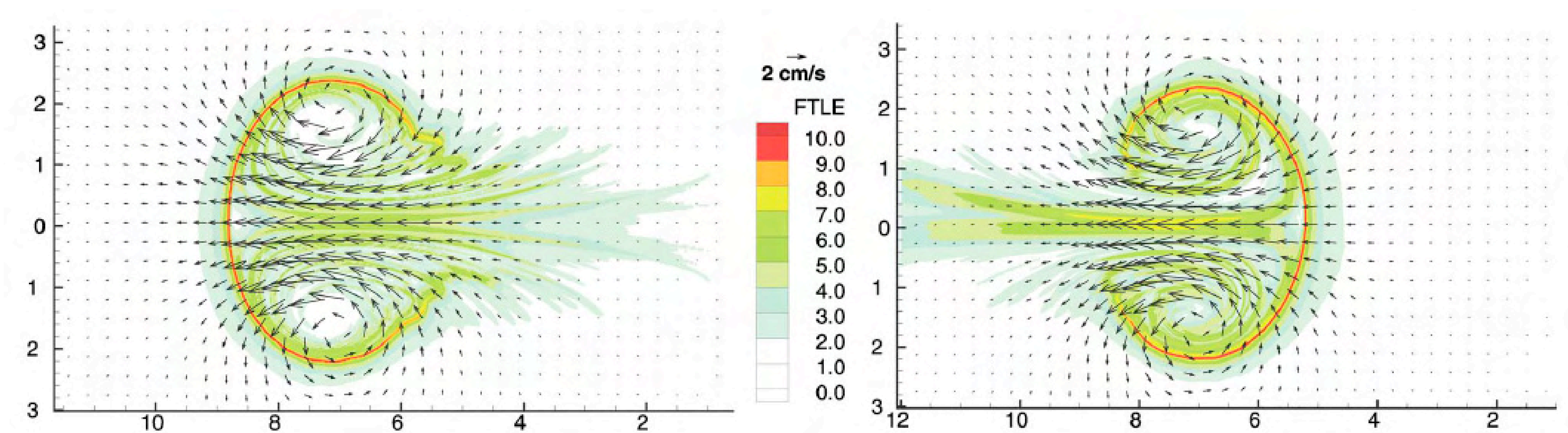

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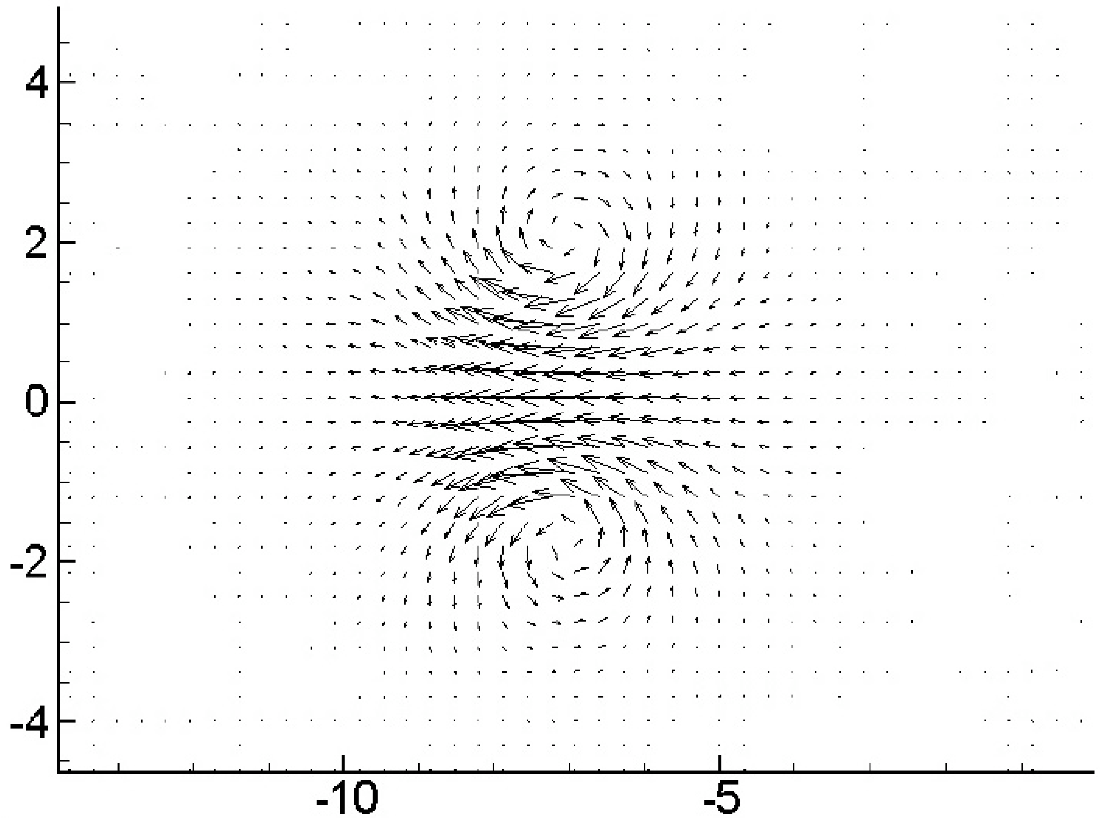

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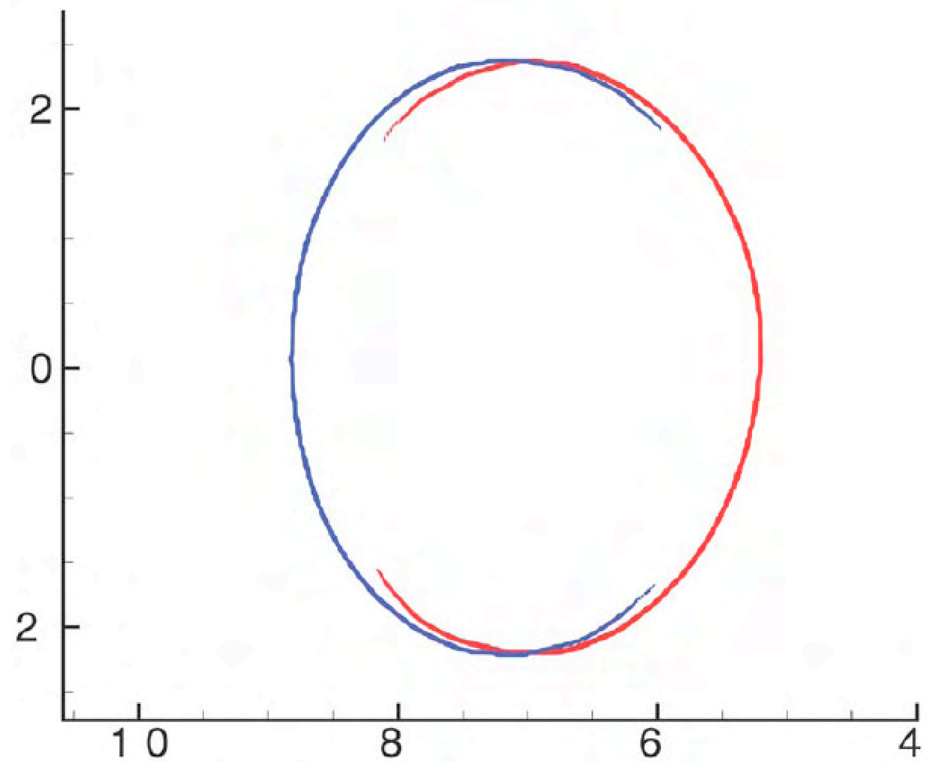
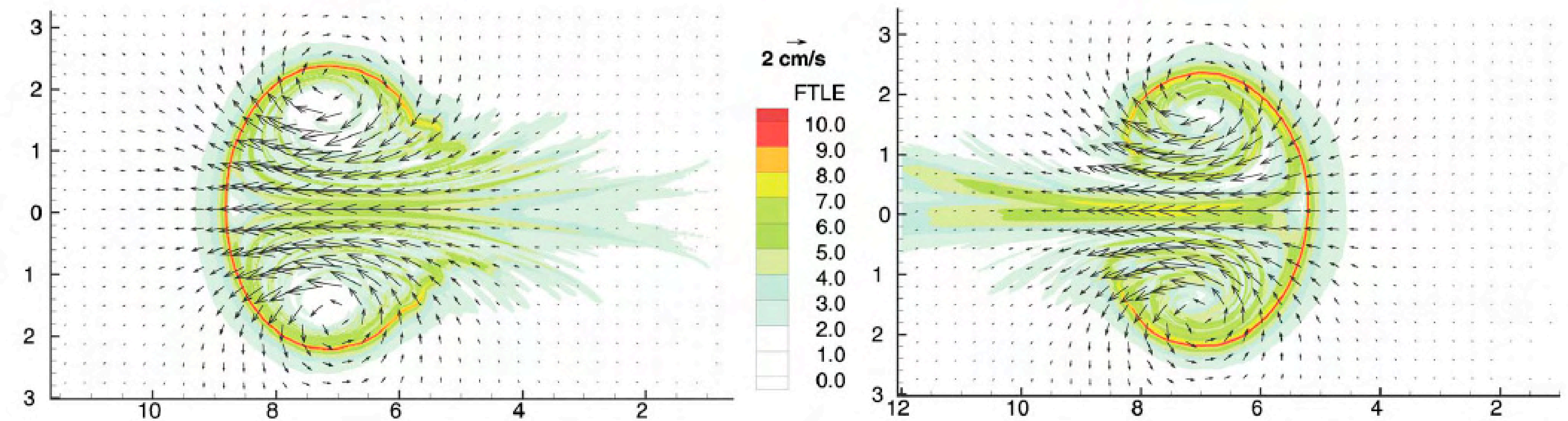
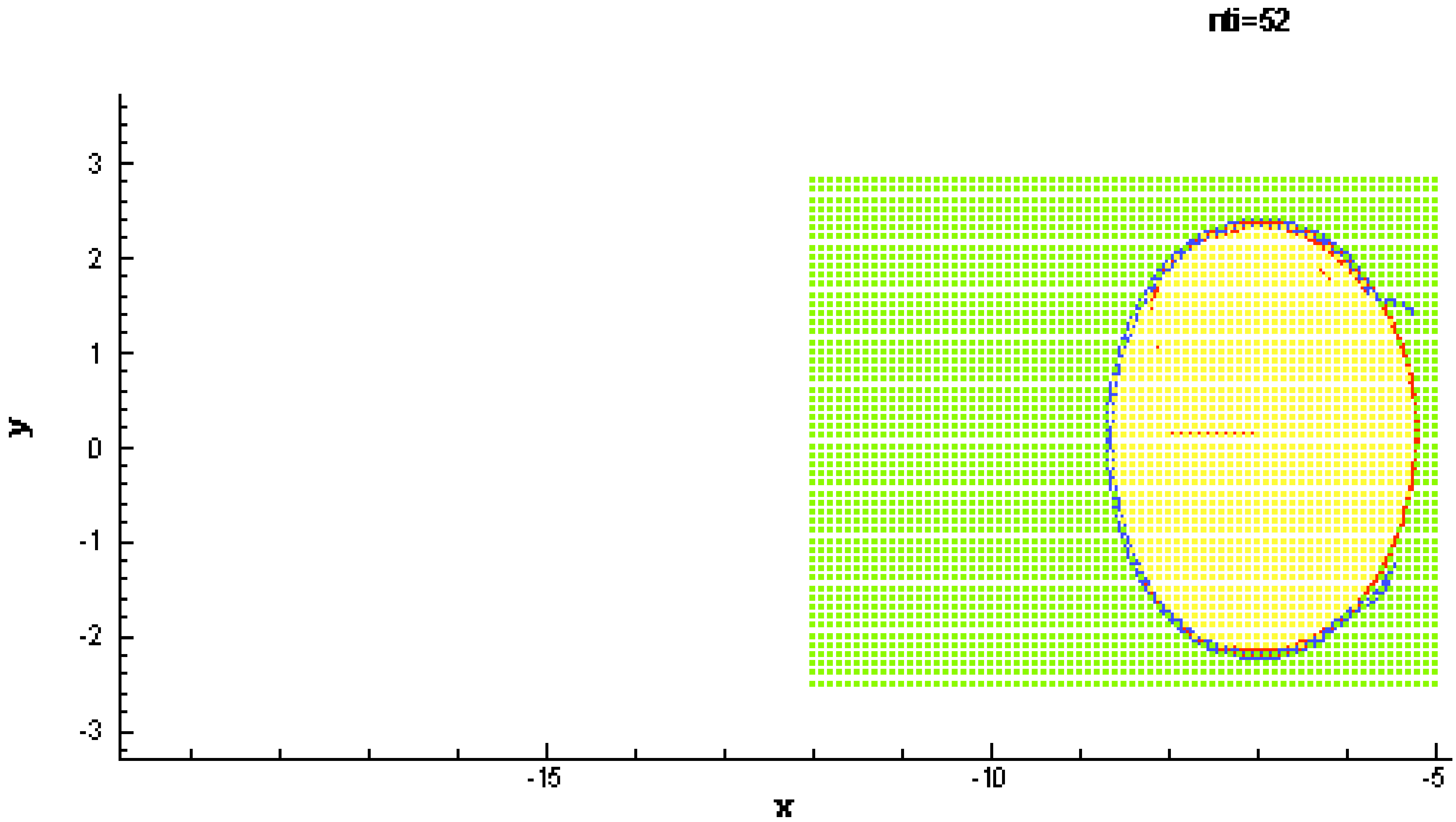


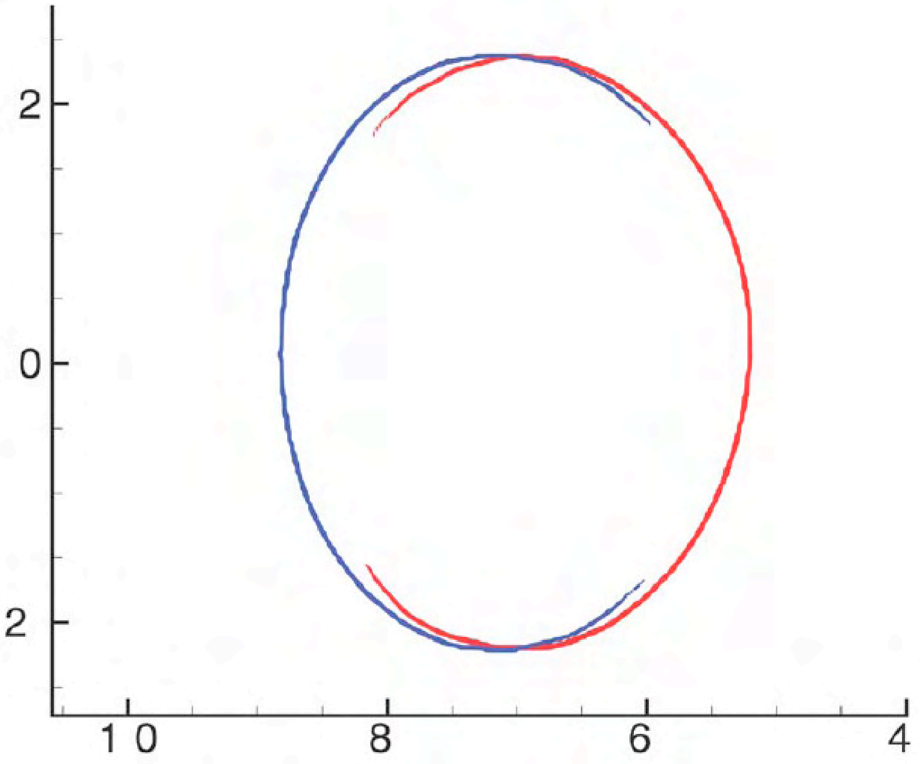


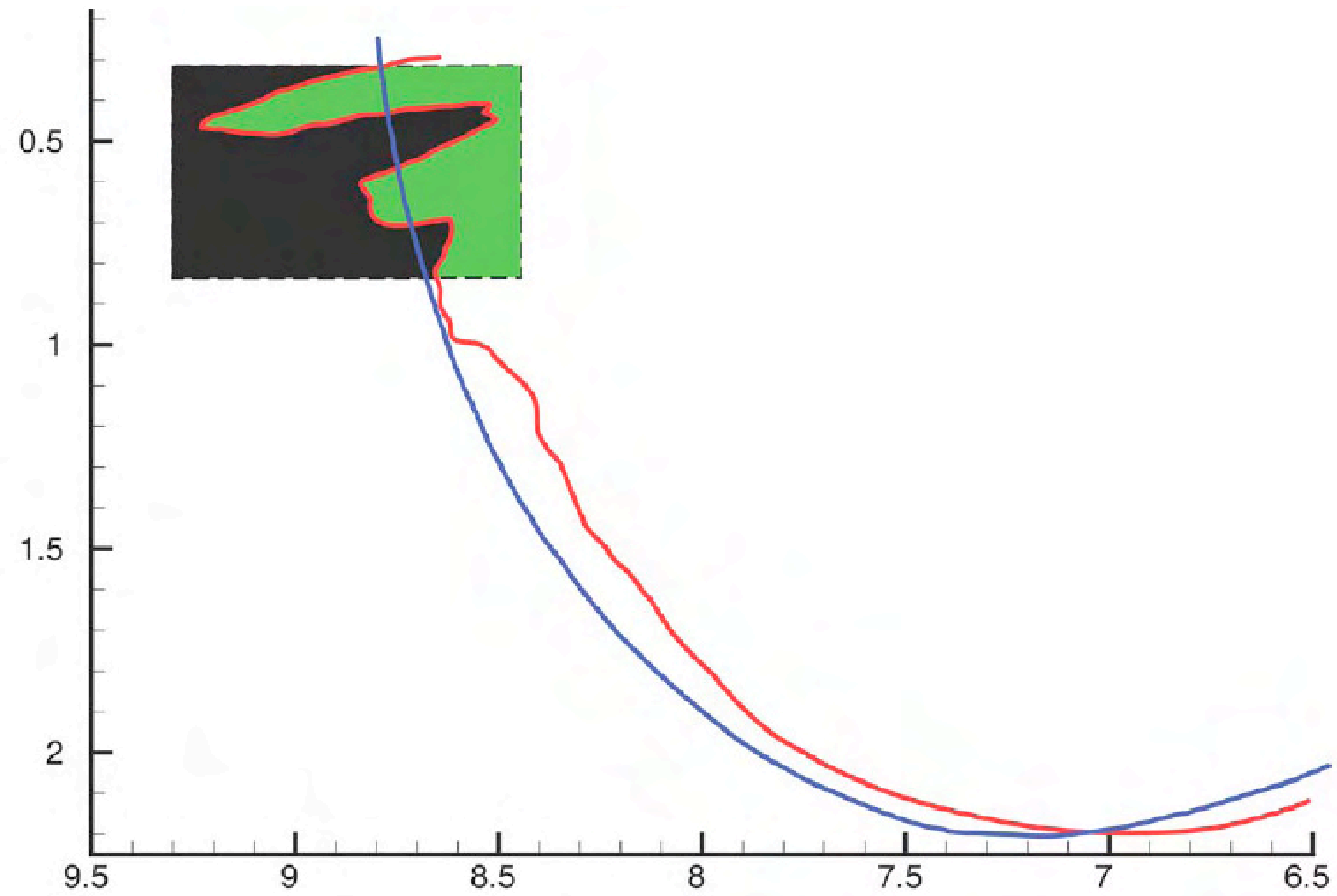
LCS and vortex boundaries

LCS and vortex boundaries

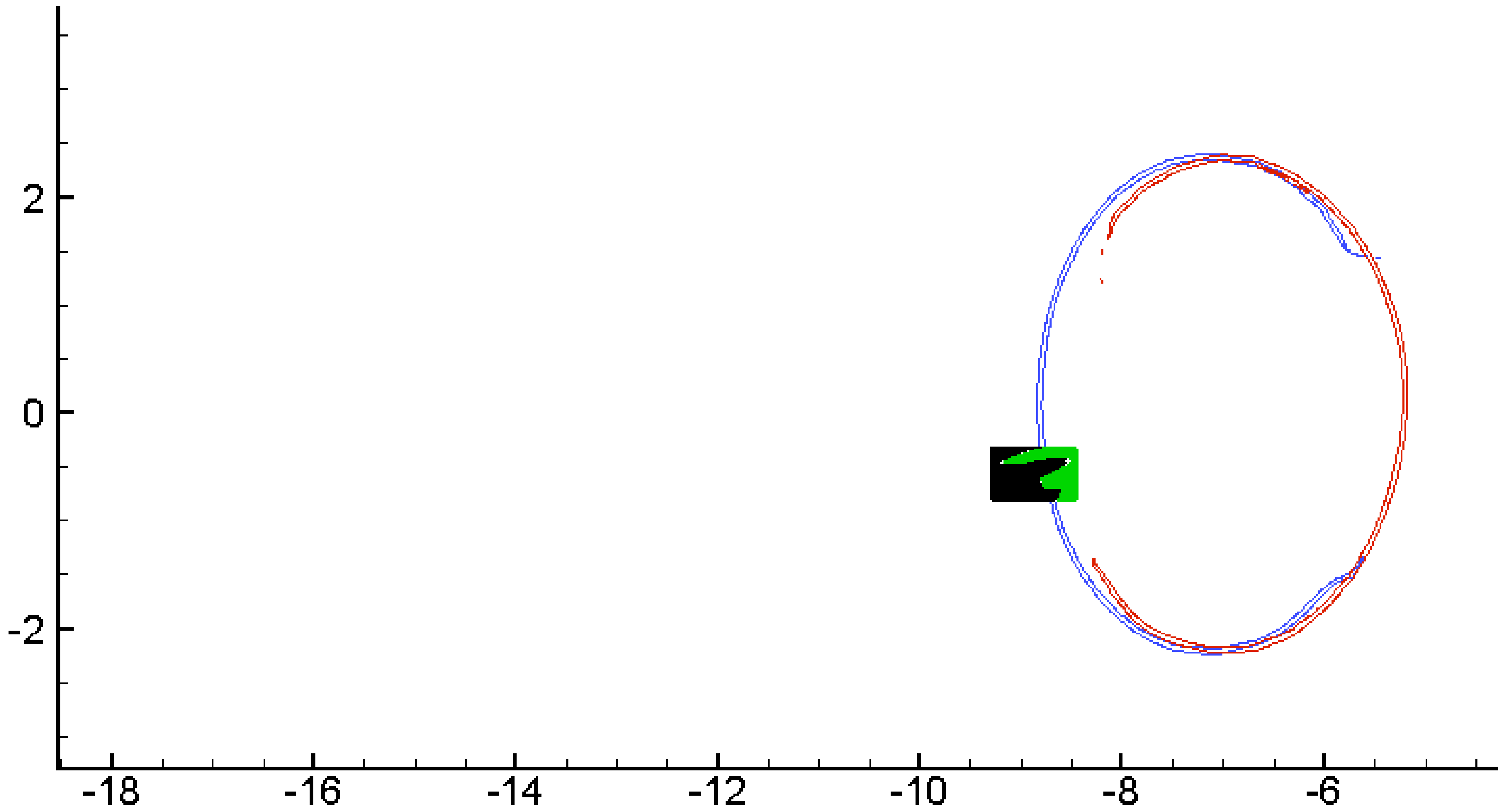


Lobes in the vortex ring





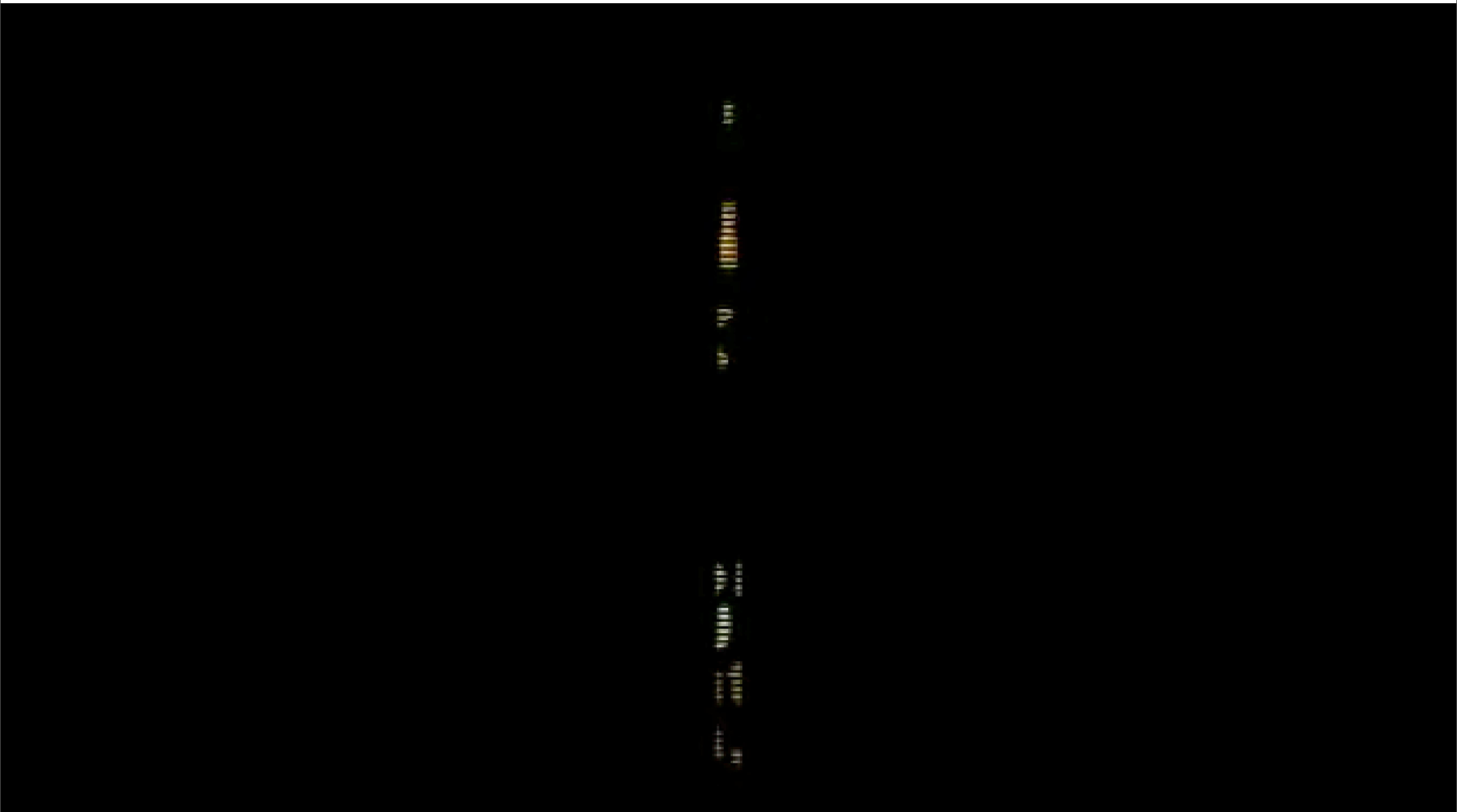
Vortex



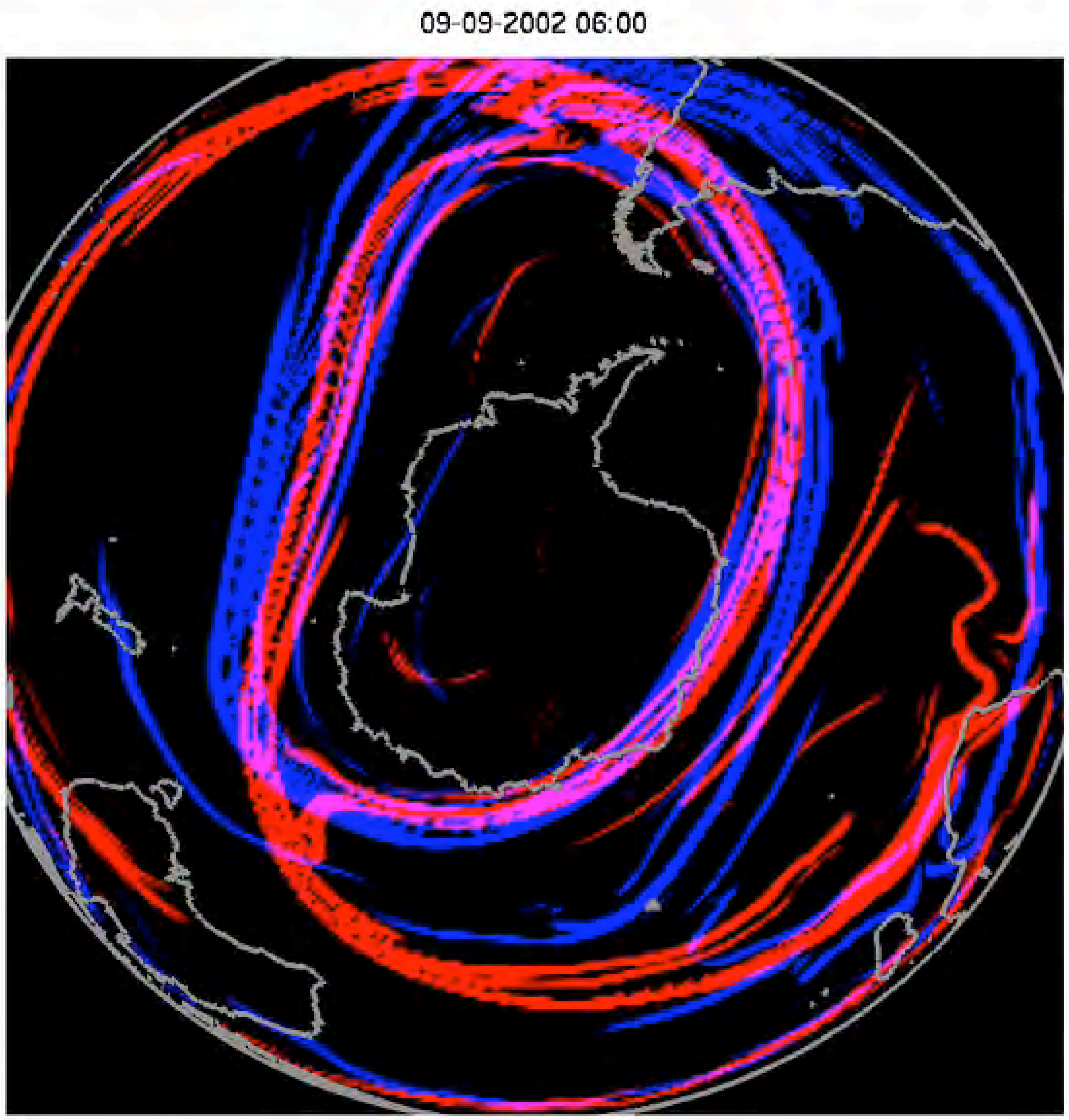
Jellyfish

Jellyfish

Jellyfish

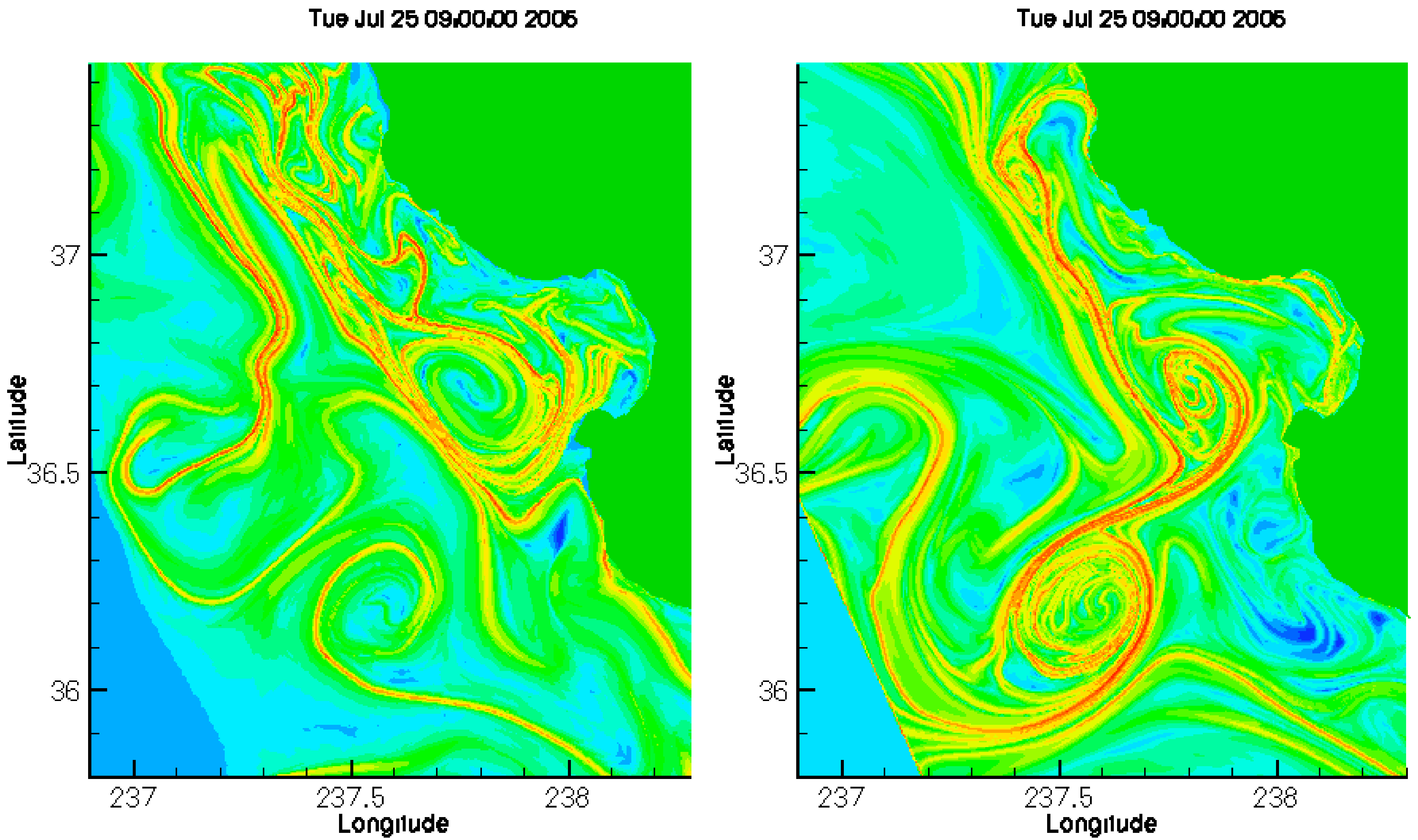


LCS for ozone hole break up



LCS and mixing in Monterey bay

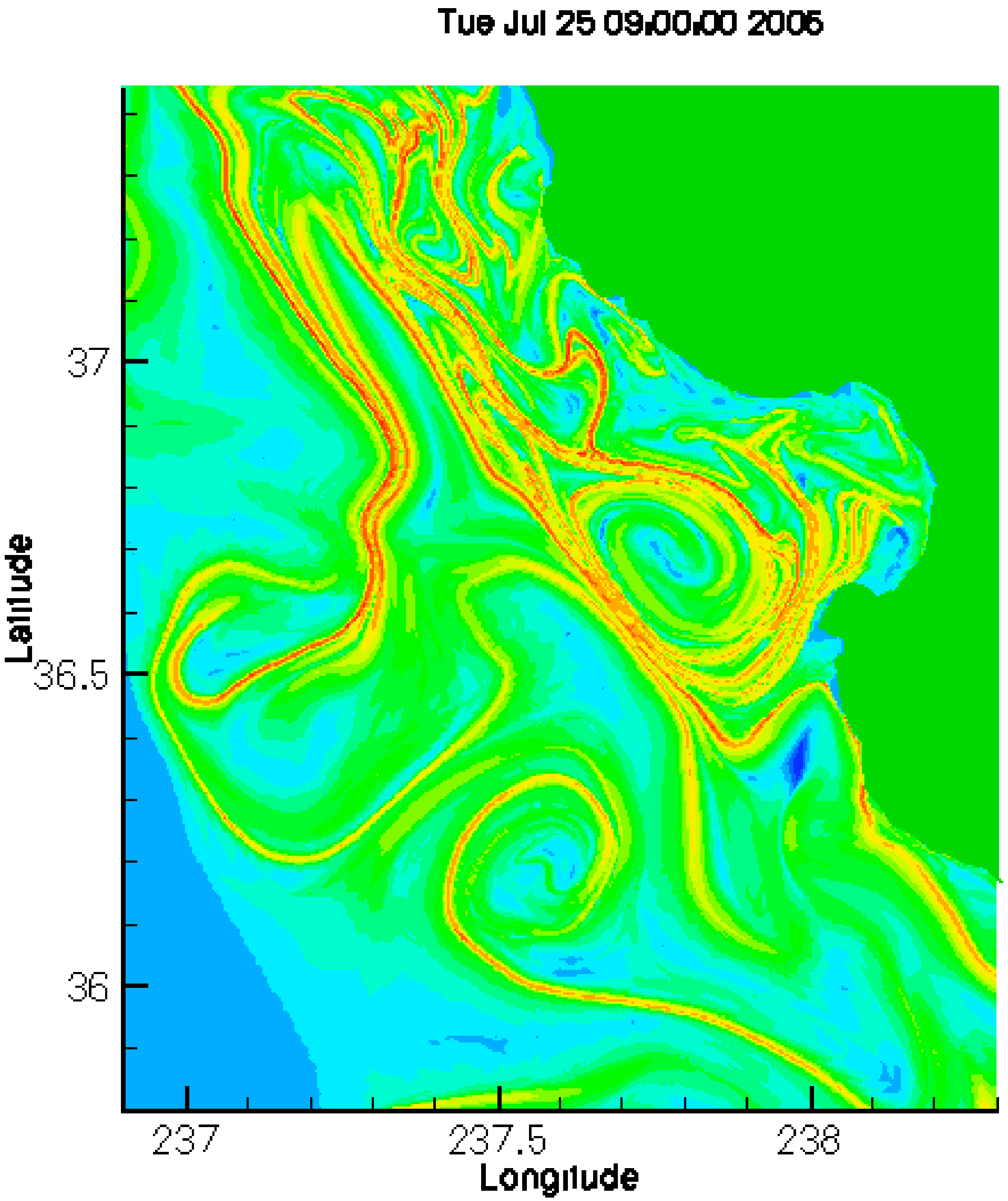
LCS and mixing in Monterey bay



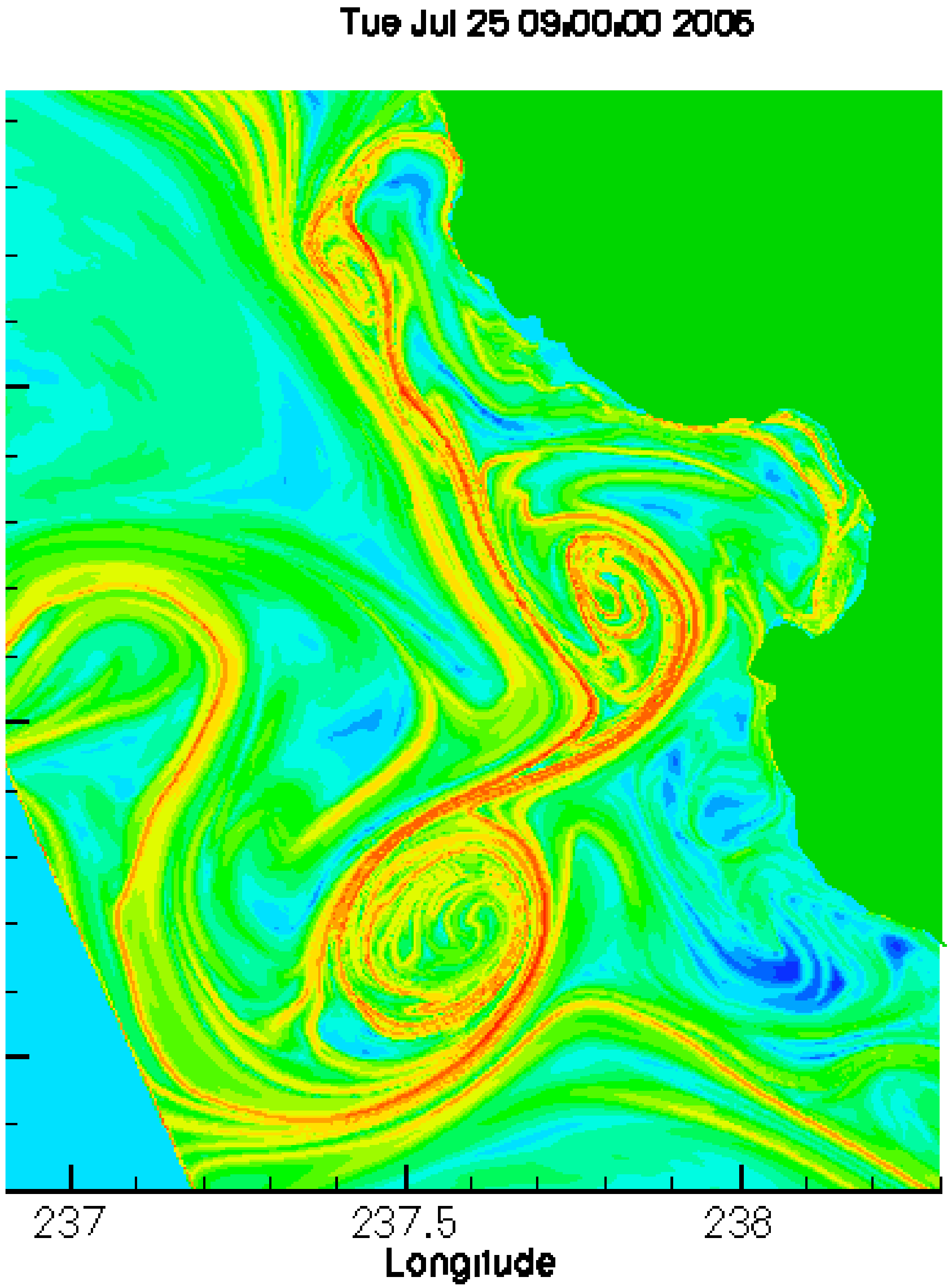
Repelling LCS

Attracting LCS

LCS and mixing in Monterey bay

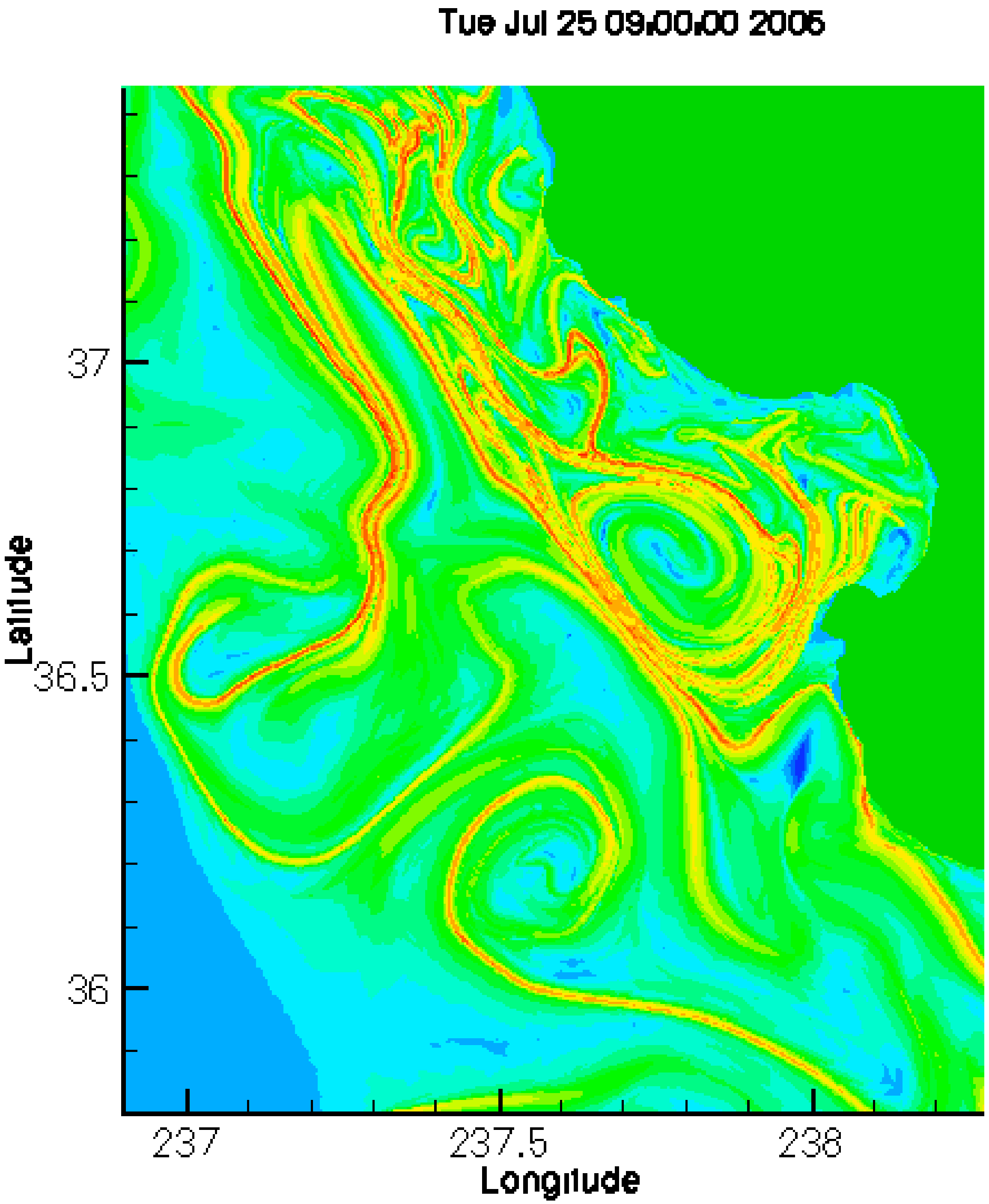


Repelling LCS

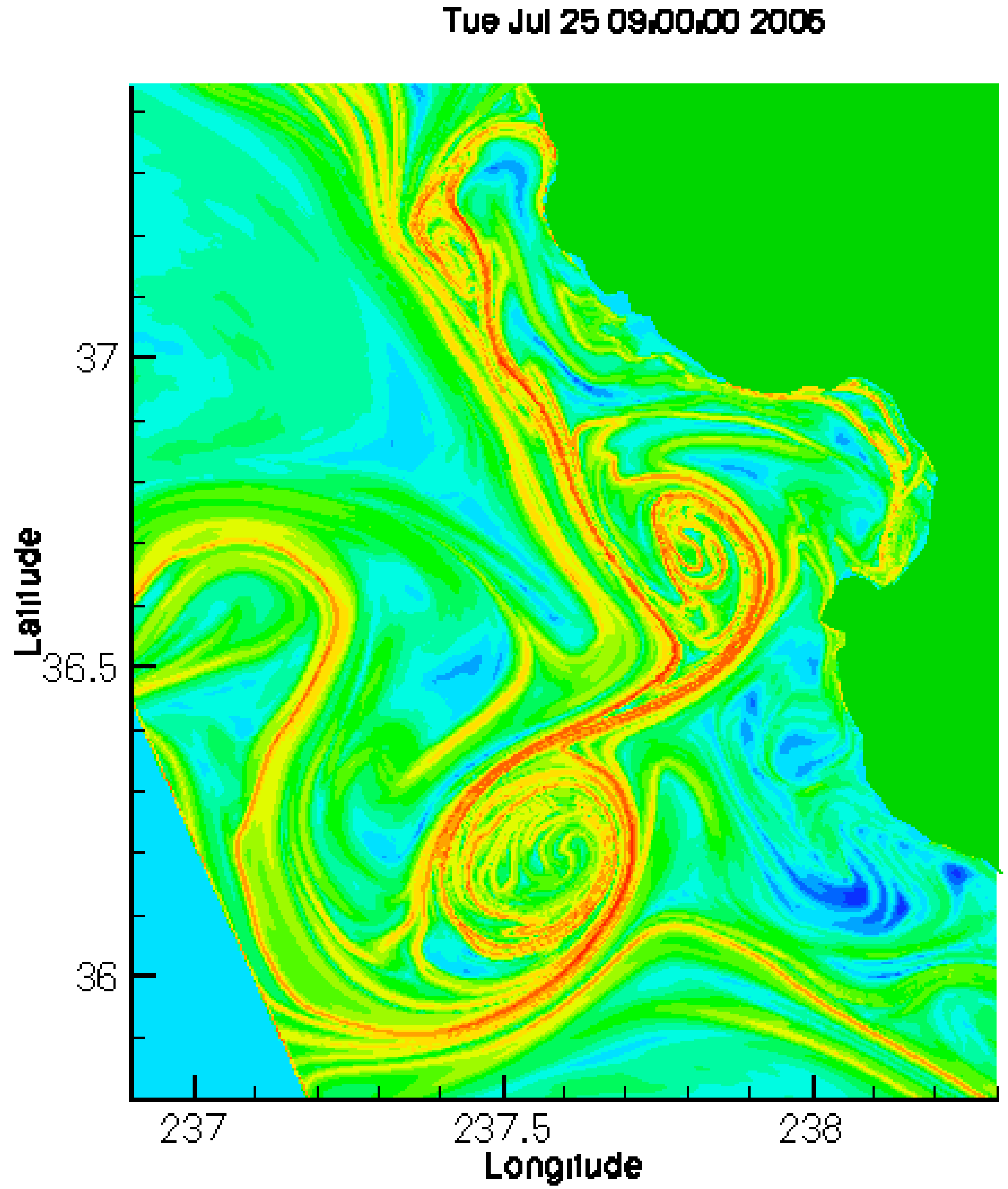


Attracting LCS

LCS and mixing in Monterey bay



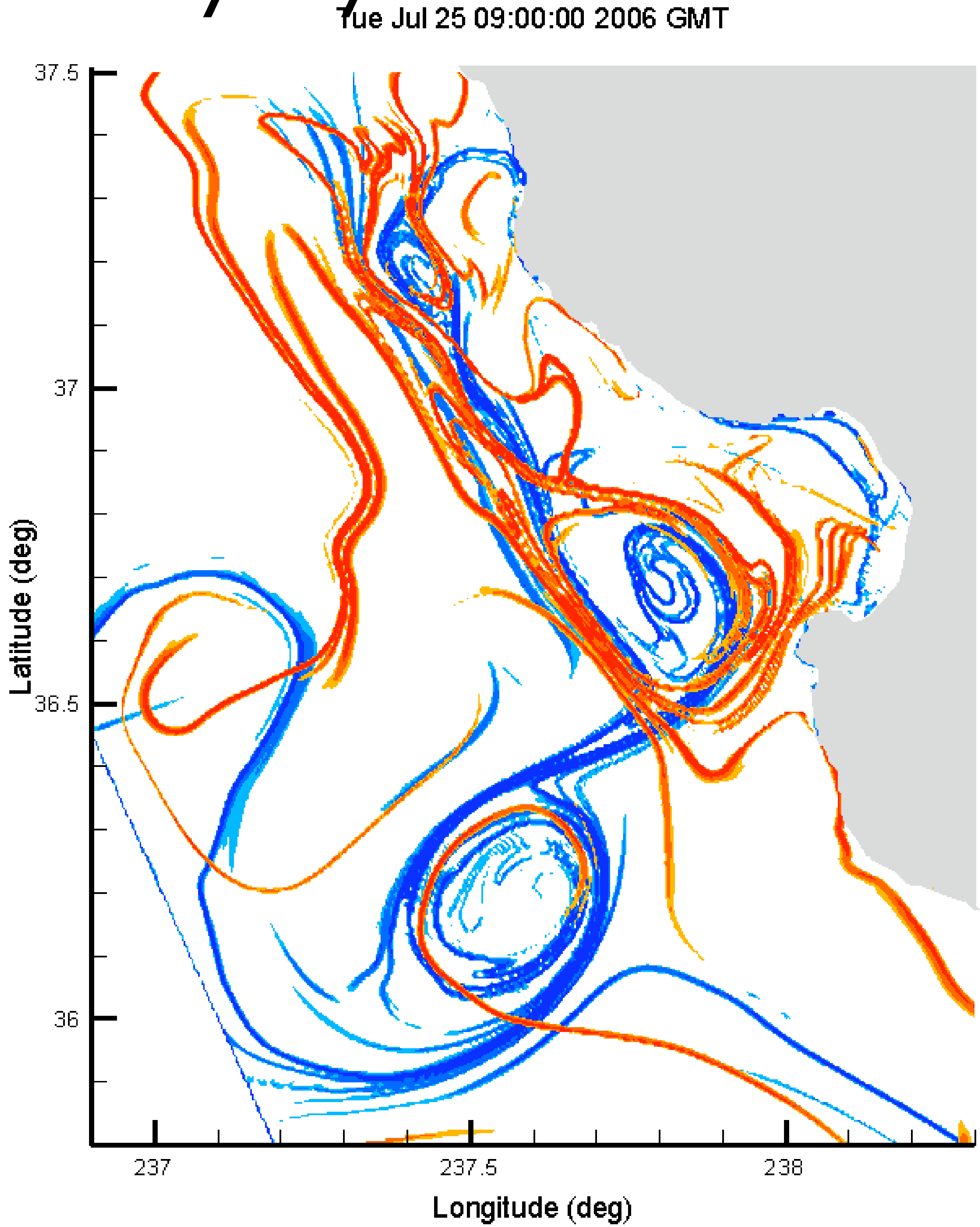
Repelling LCS



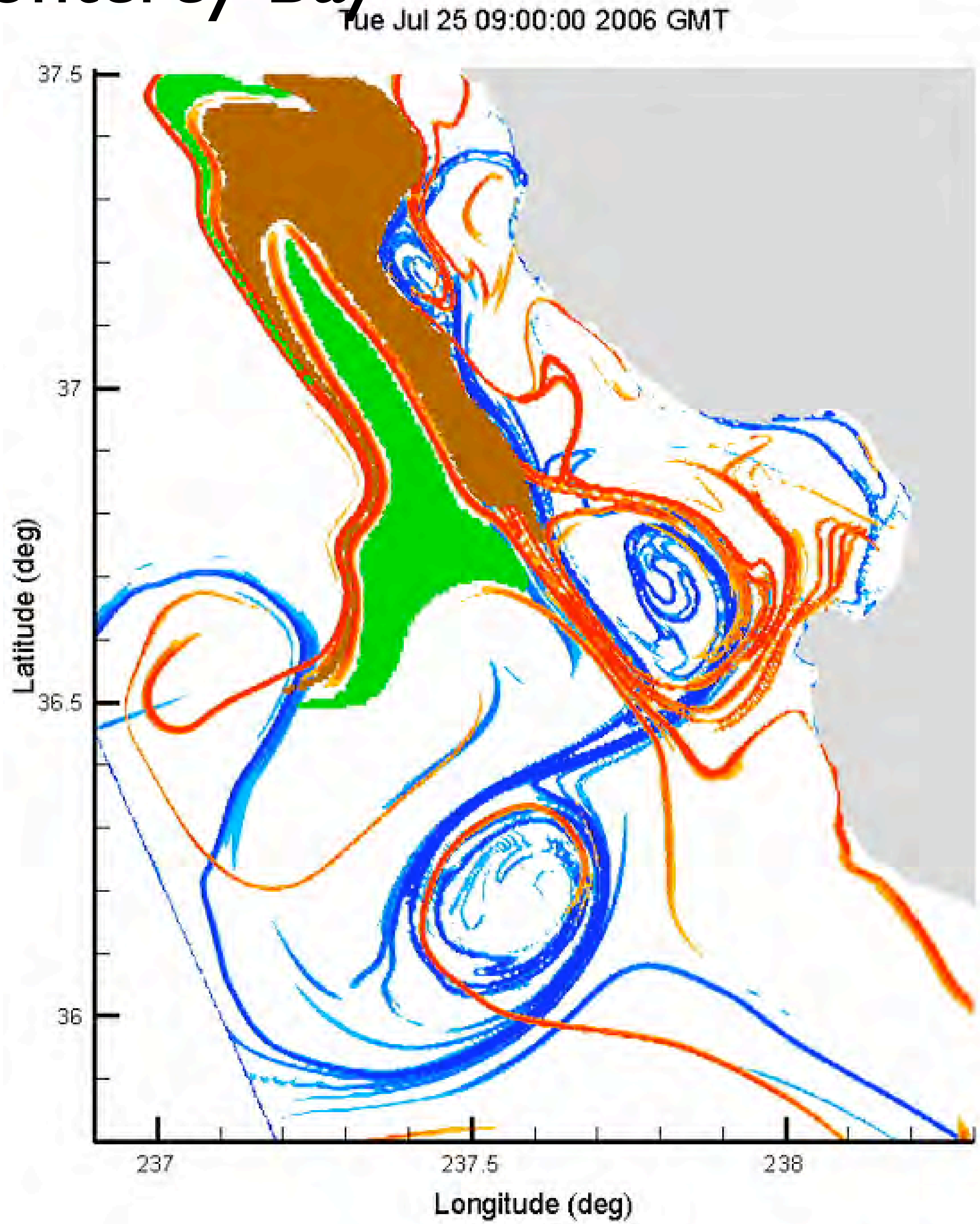
Attracting LCS

Mixing in Monterey Bay

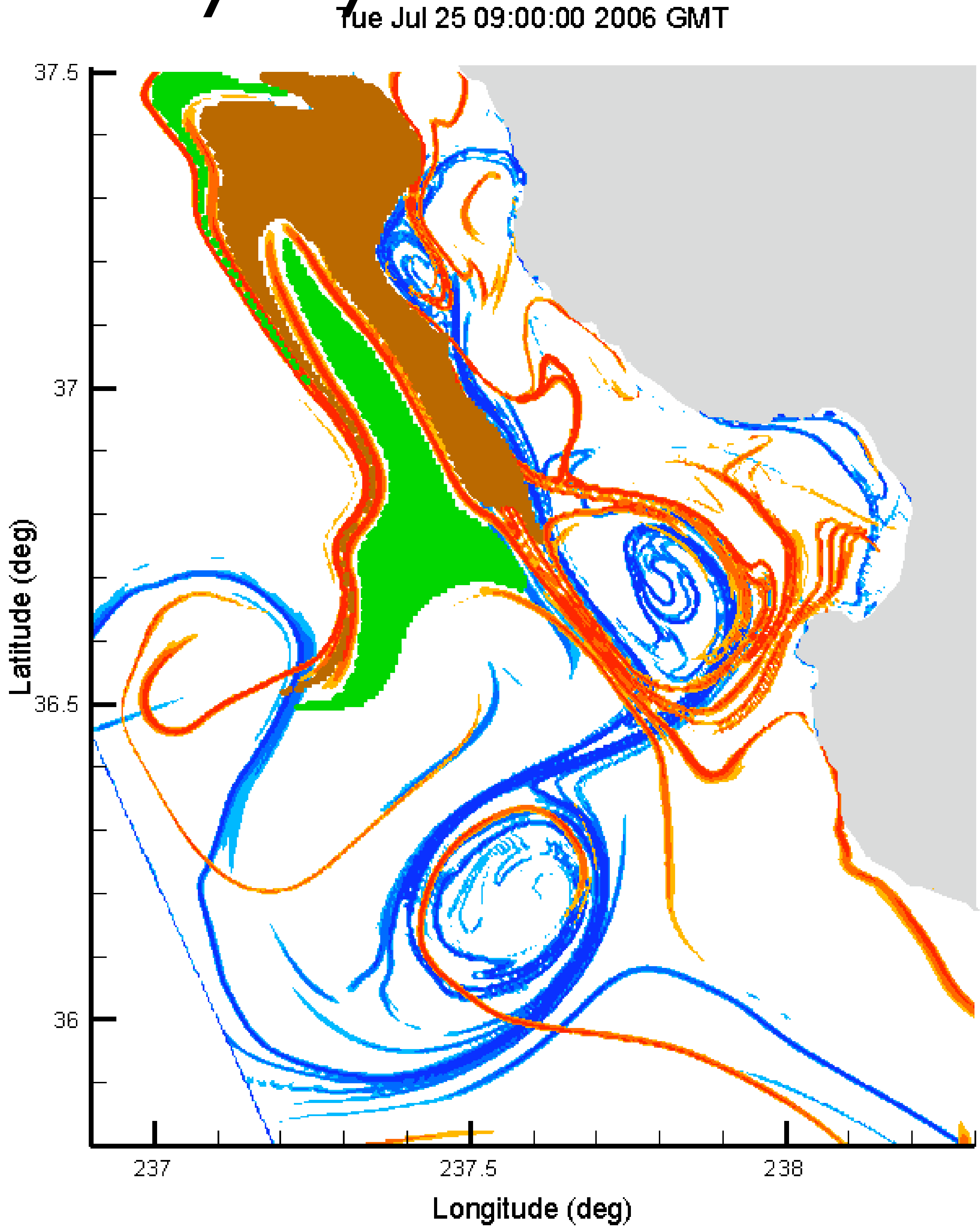
Mixing in Monterey Bay



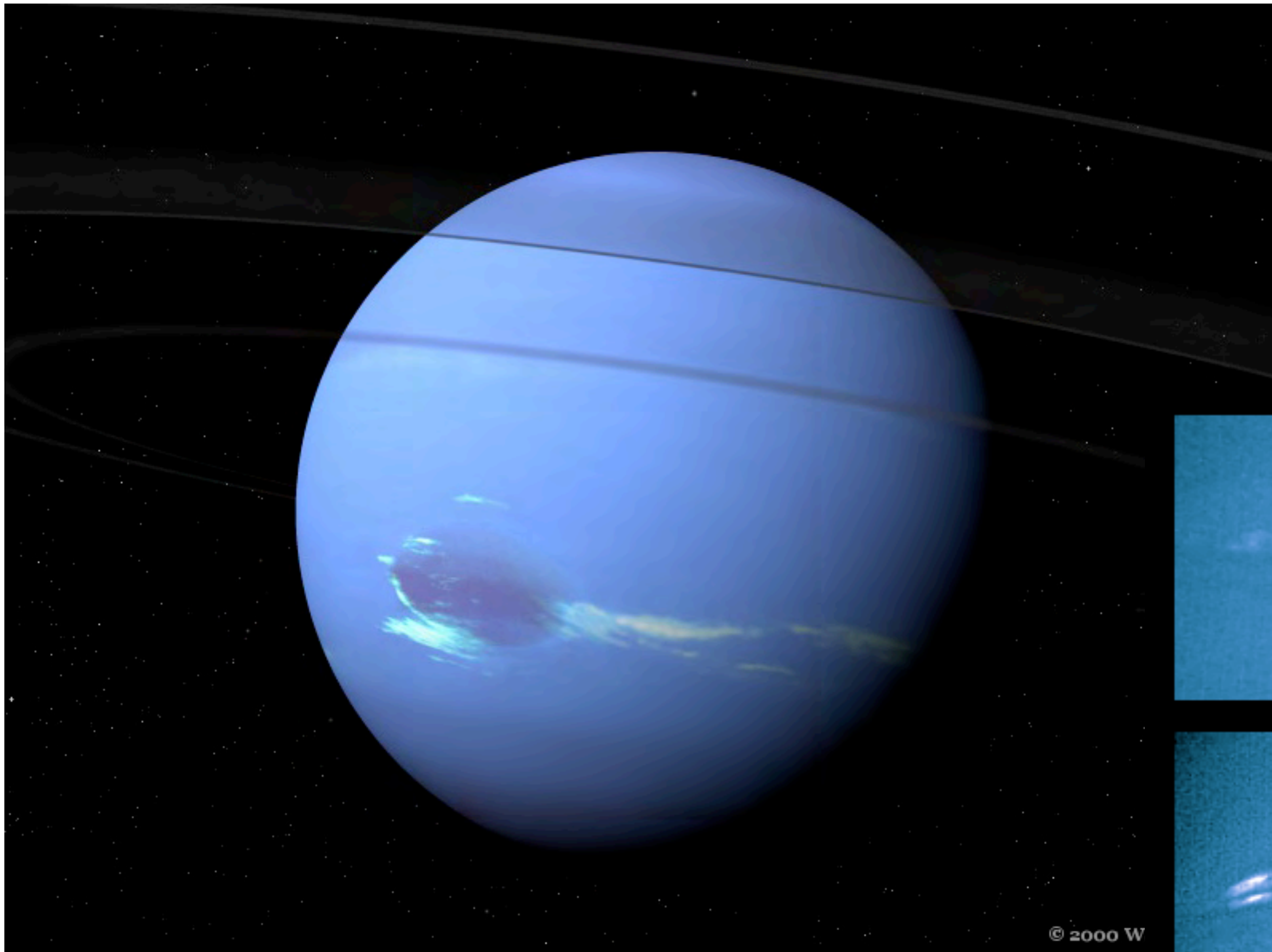
Mixing in Monterey Bay

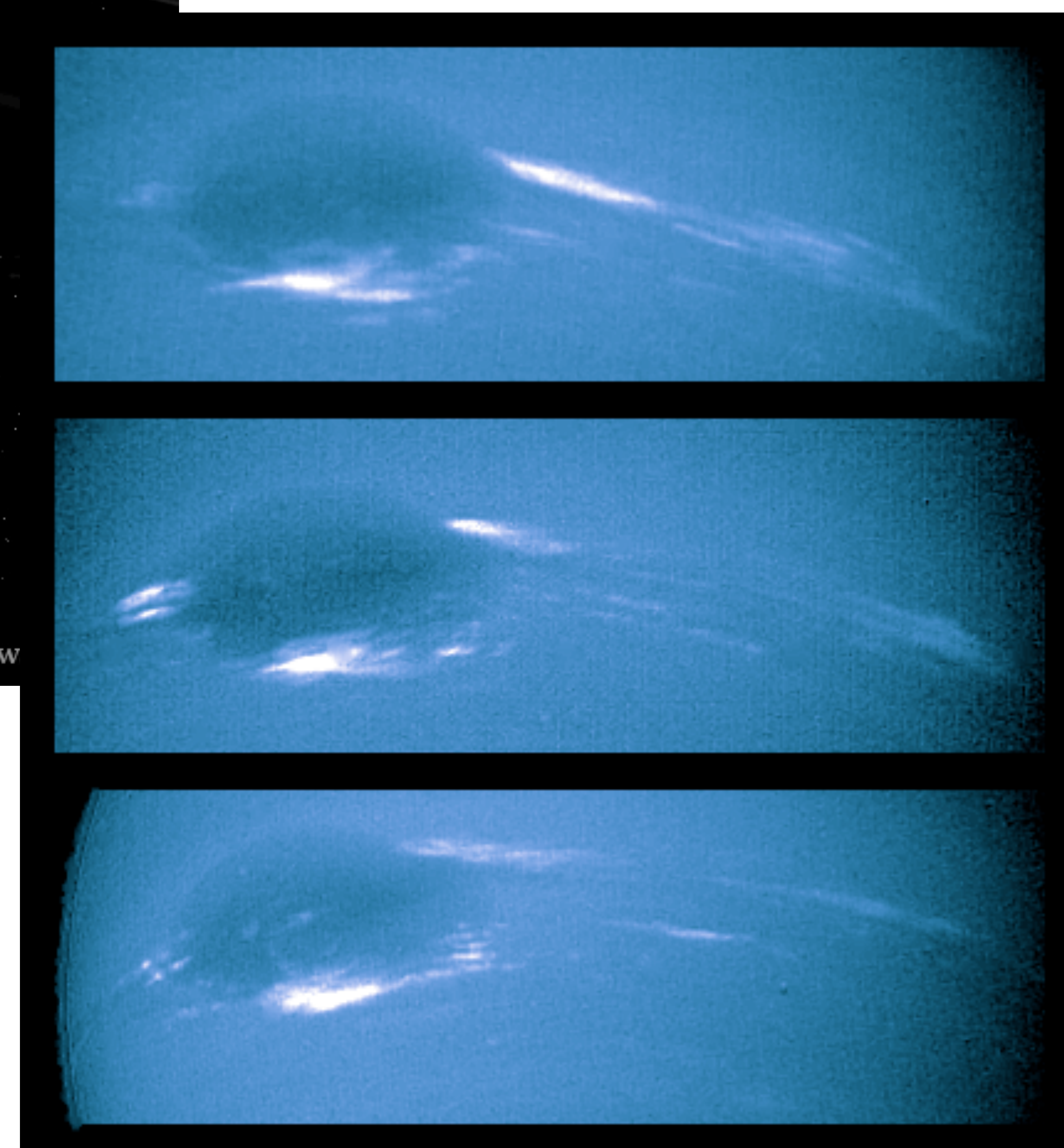


Mixing in Monterey Bay



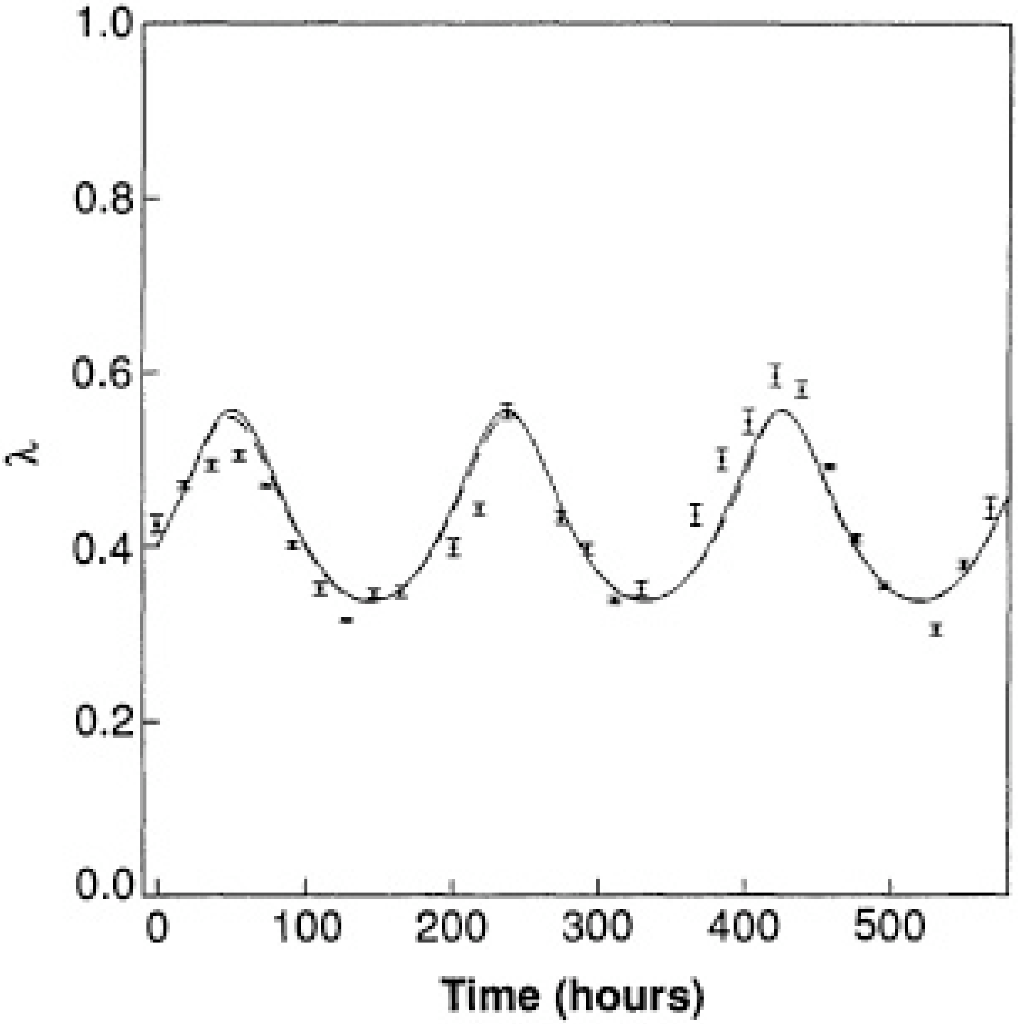
Neptune’s Great Dark Spot



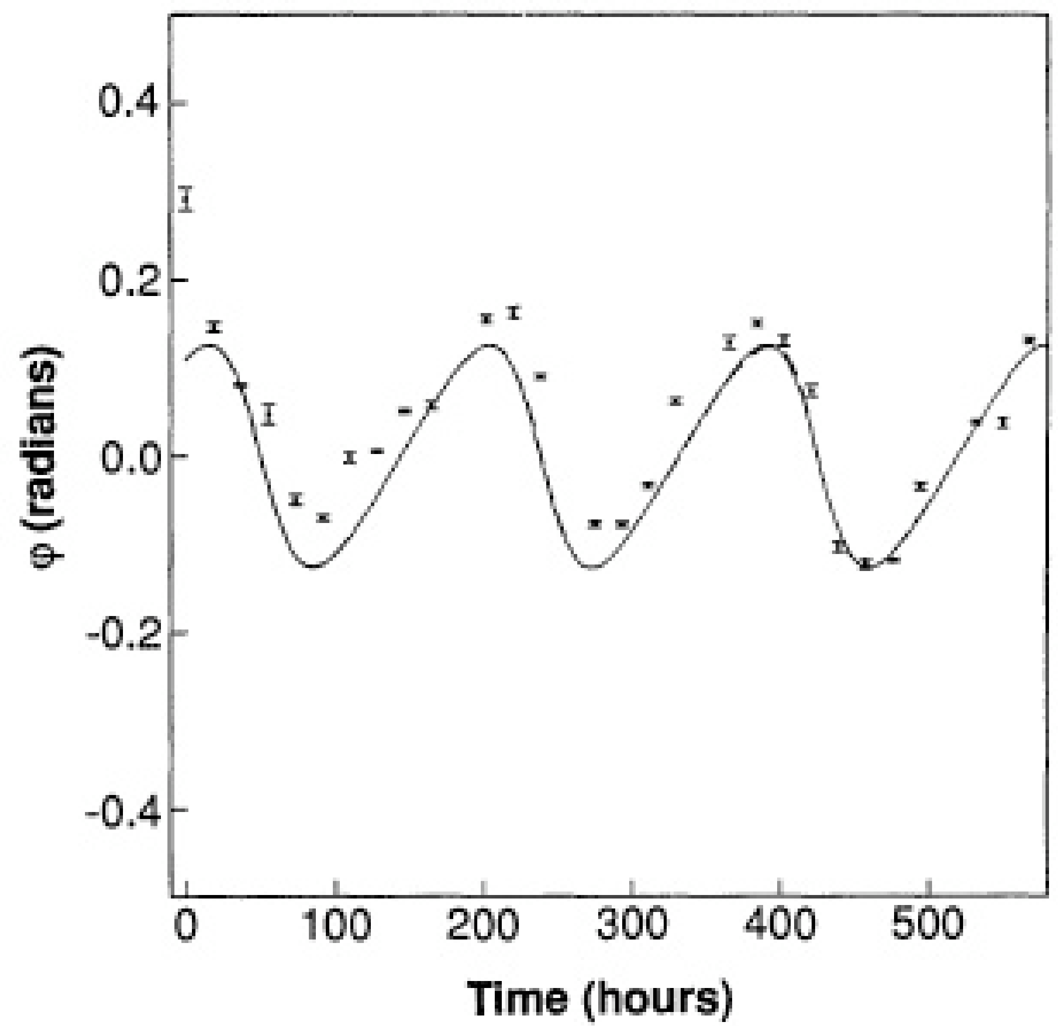


Modelling the Dark spot

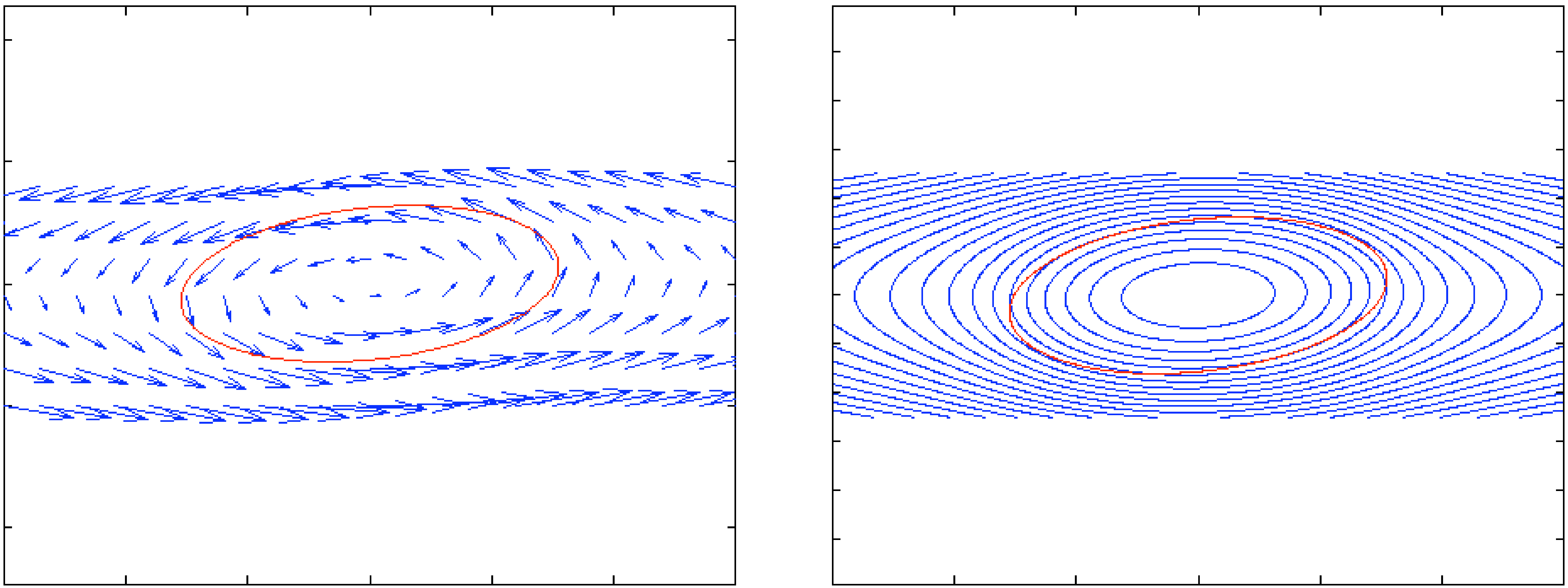
Eccentricity



Orientation



Kida vortex velocities



In the present paper I have integrated the equations of motion and plotted curves for the paths of particles in the well-known simple cases of two-dimensional motion, viz., for liquid contained in a rotating elliptic cylinder and in a rotating equilateral triangular prism, and for liquid extending to infinity and disturbed by the translation or rotation of an elliptic cylinder.

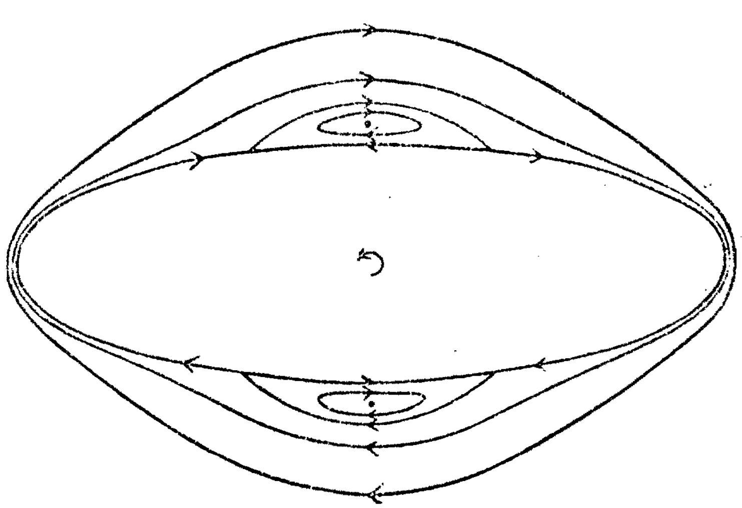


Fig. 15

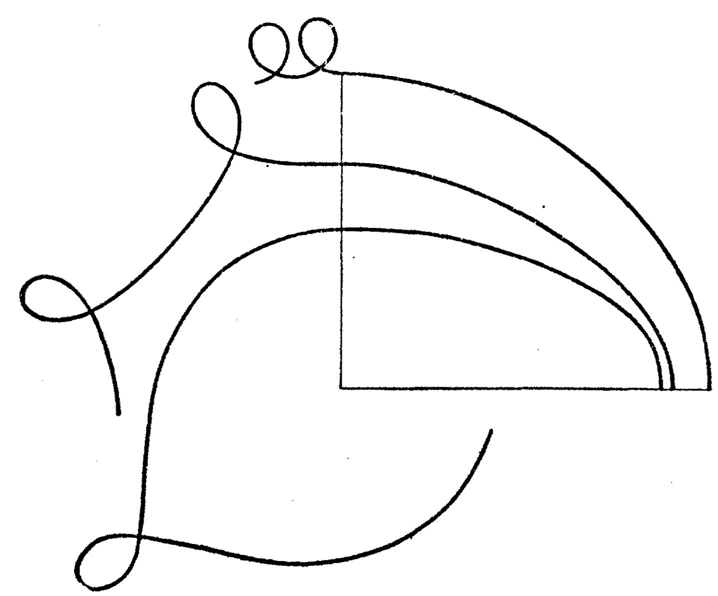


Fig. 16.

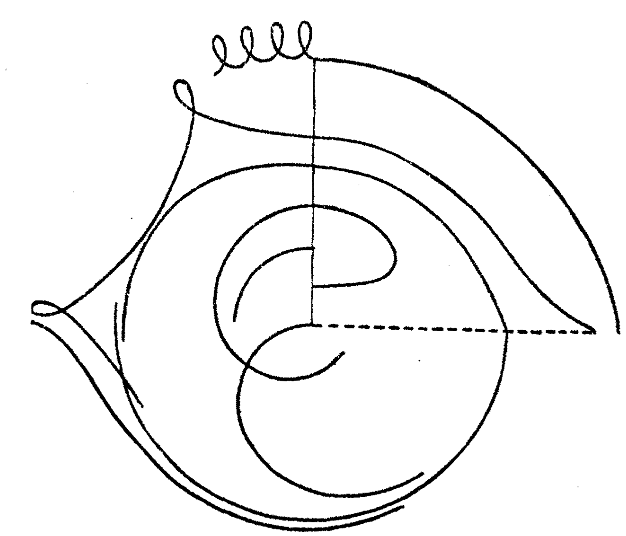
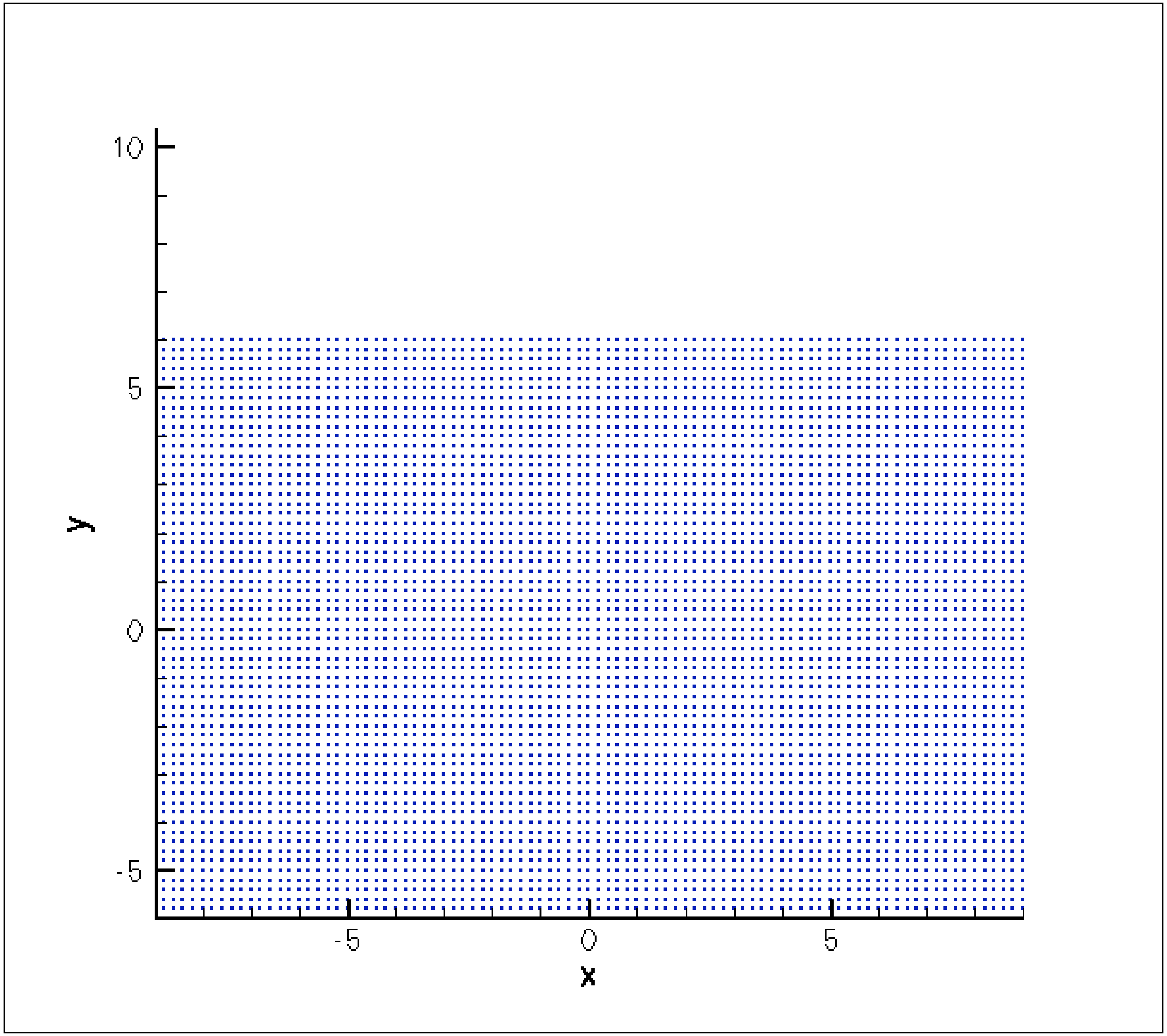
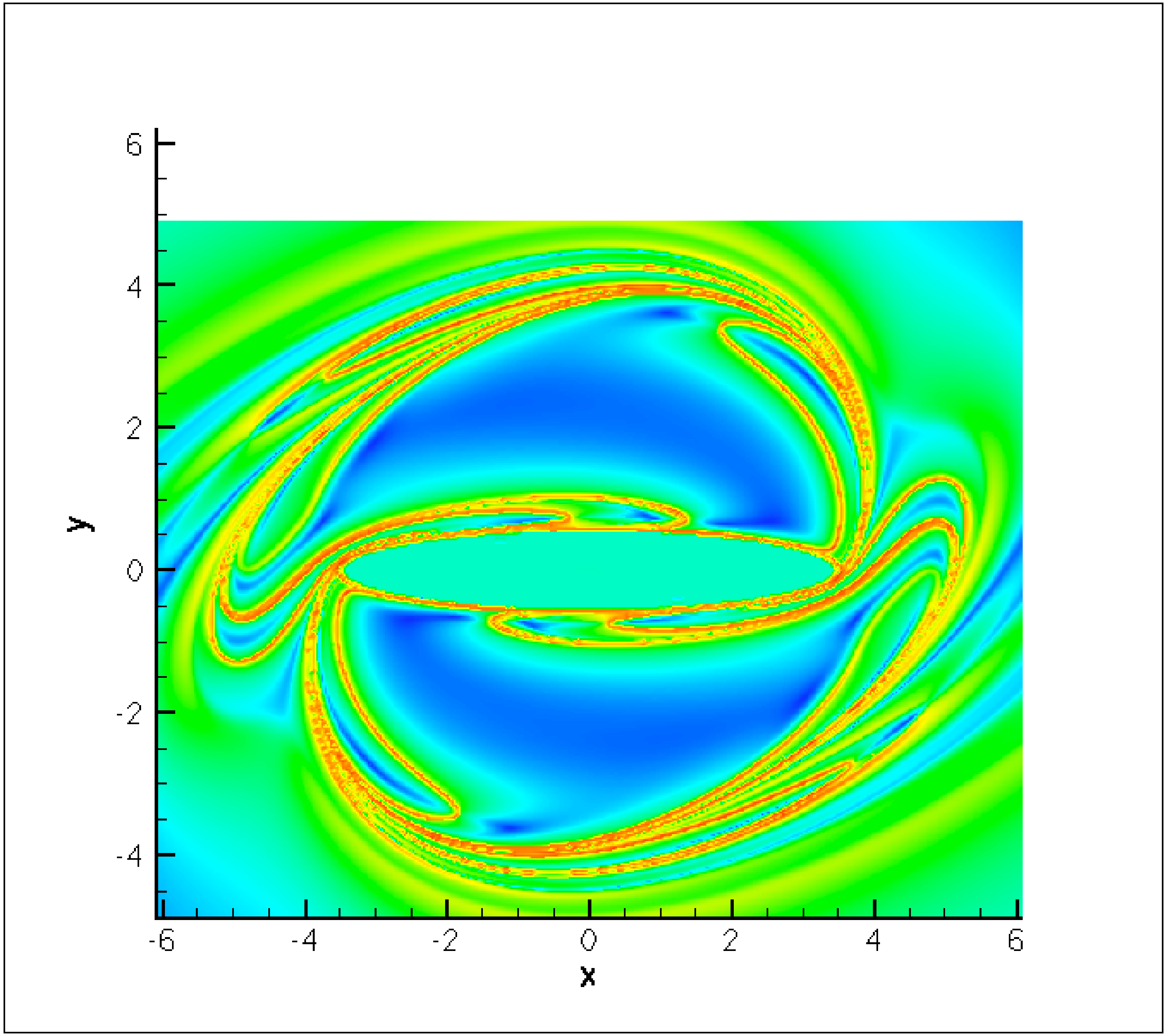


Fig. 17.

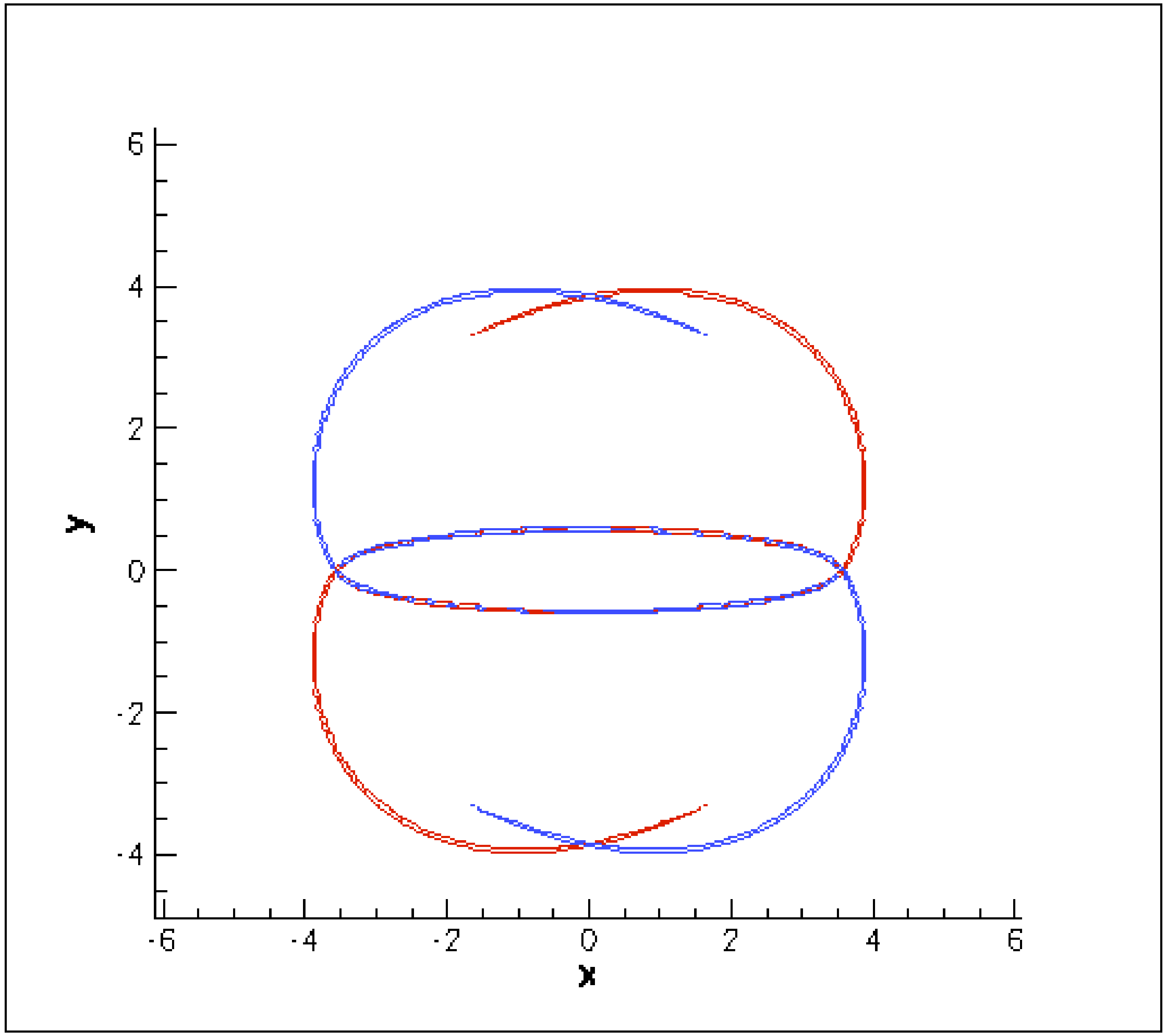
Trajectories in the Kida vortex



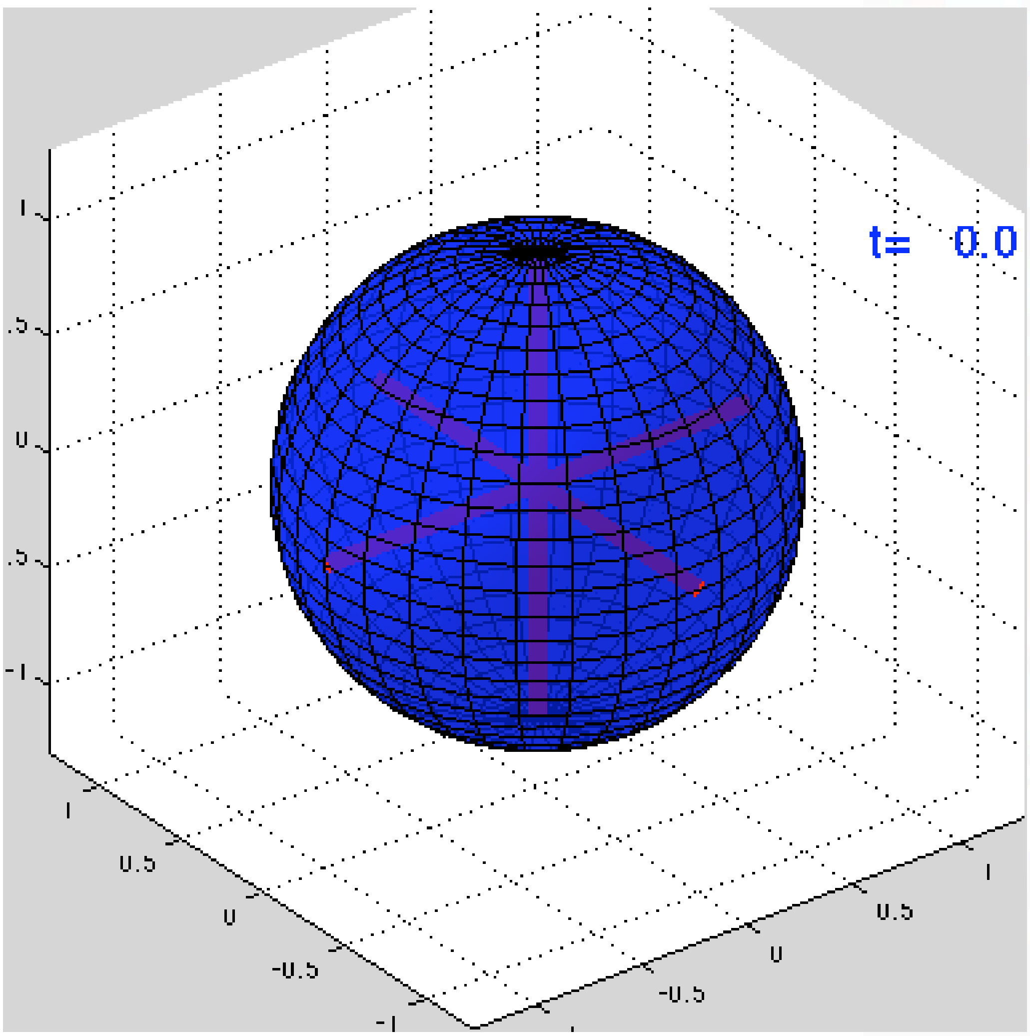
LCS in the Kida vortex

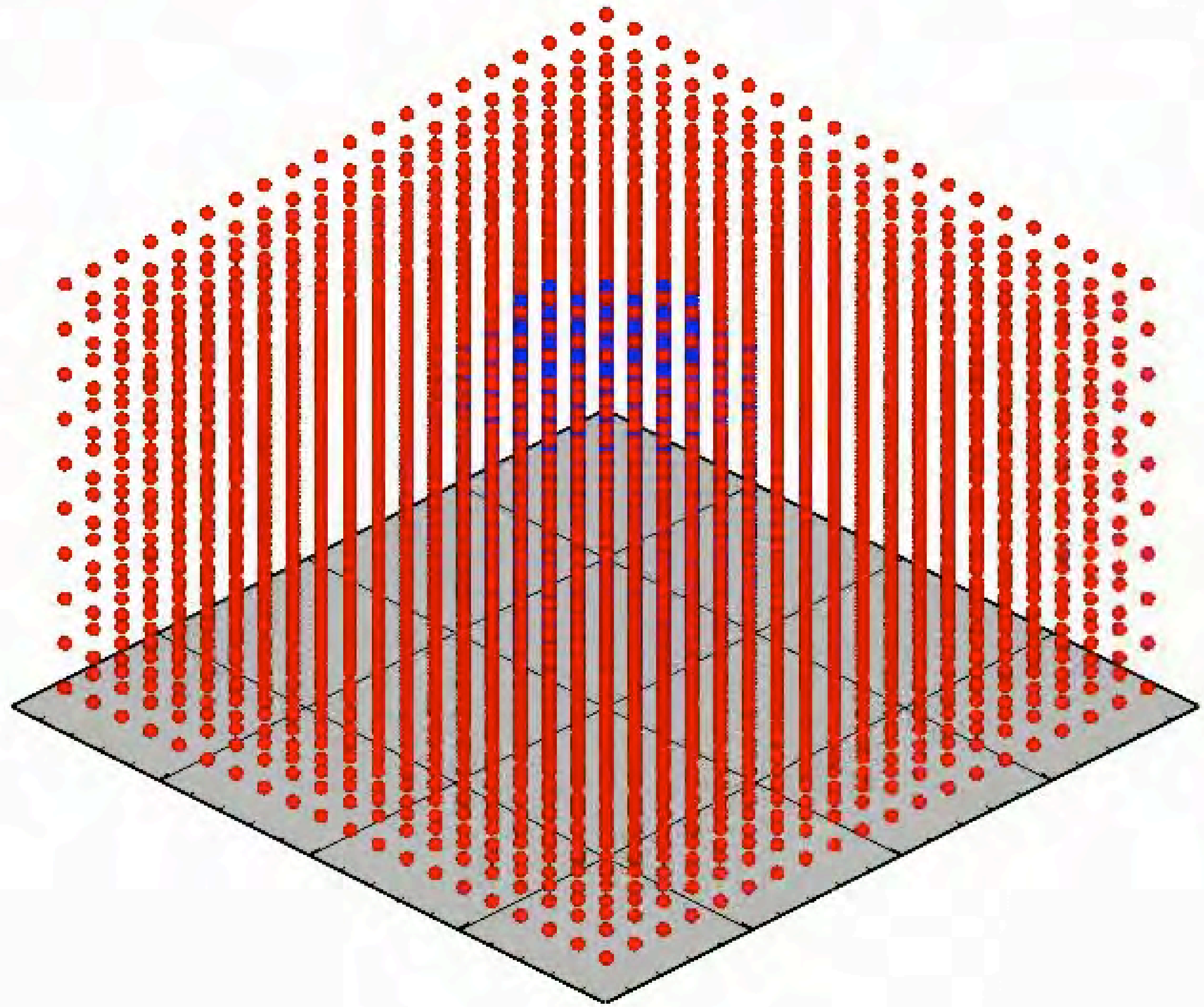


Lobes in the kida vortex

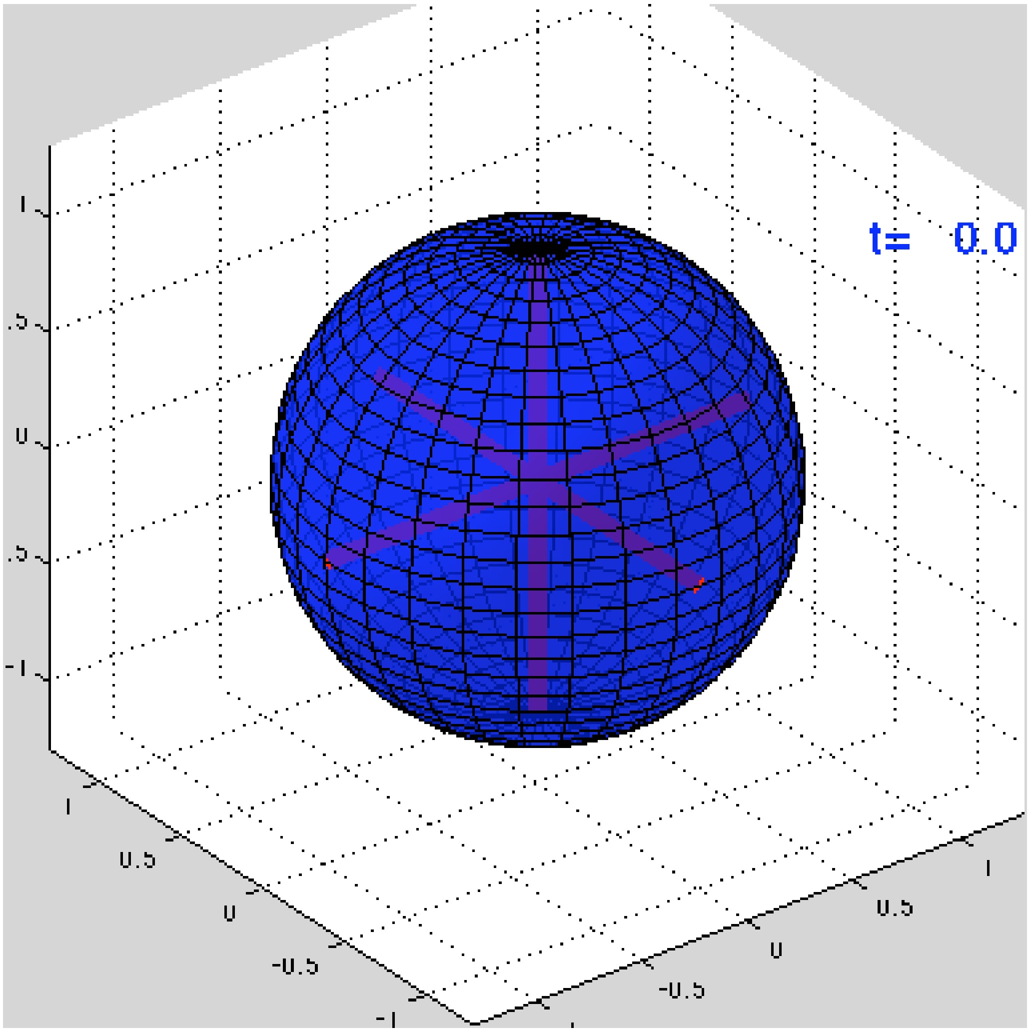


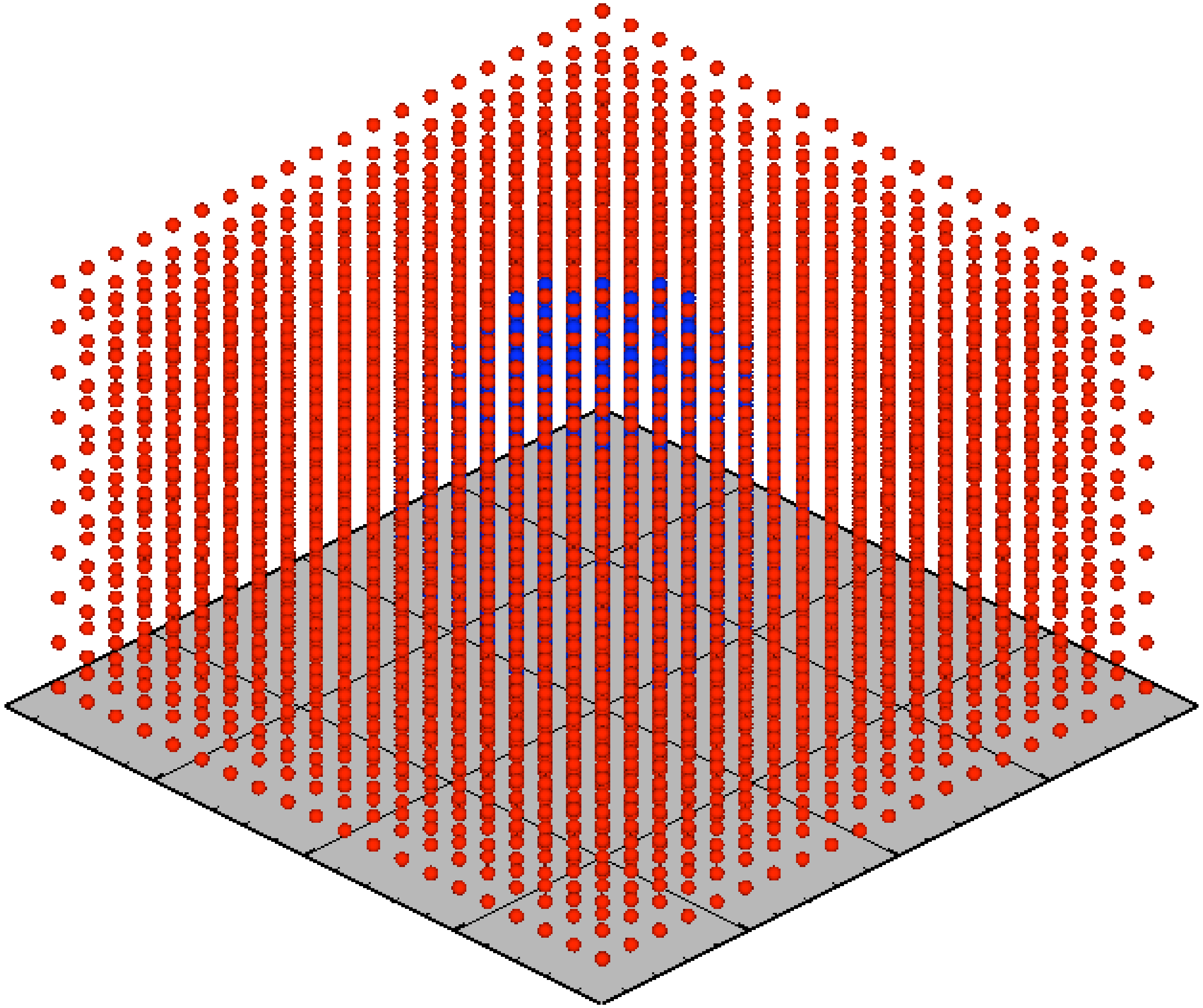
Mediterranean Salt Lenses (Meddies)



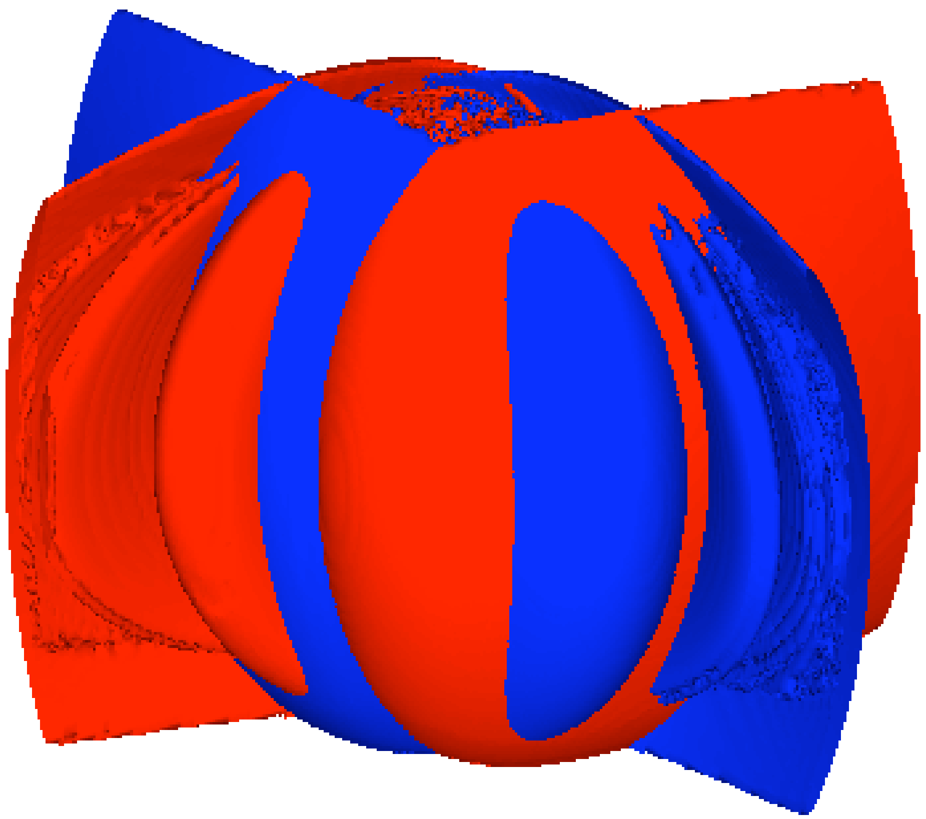


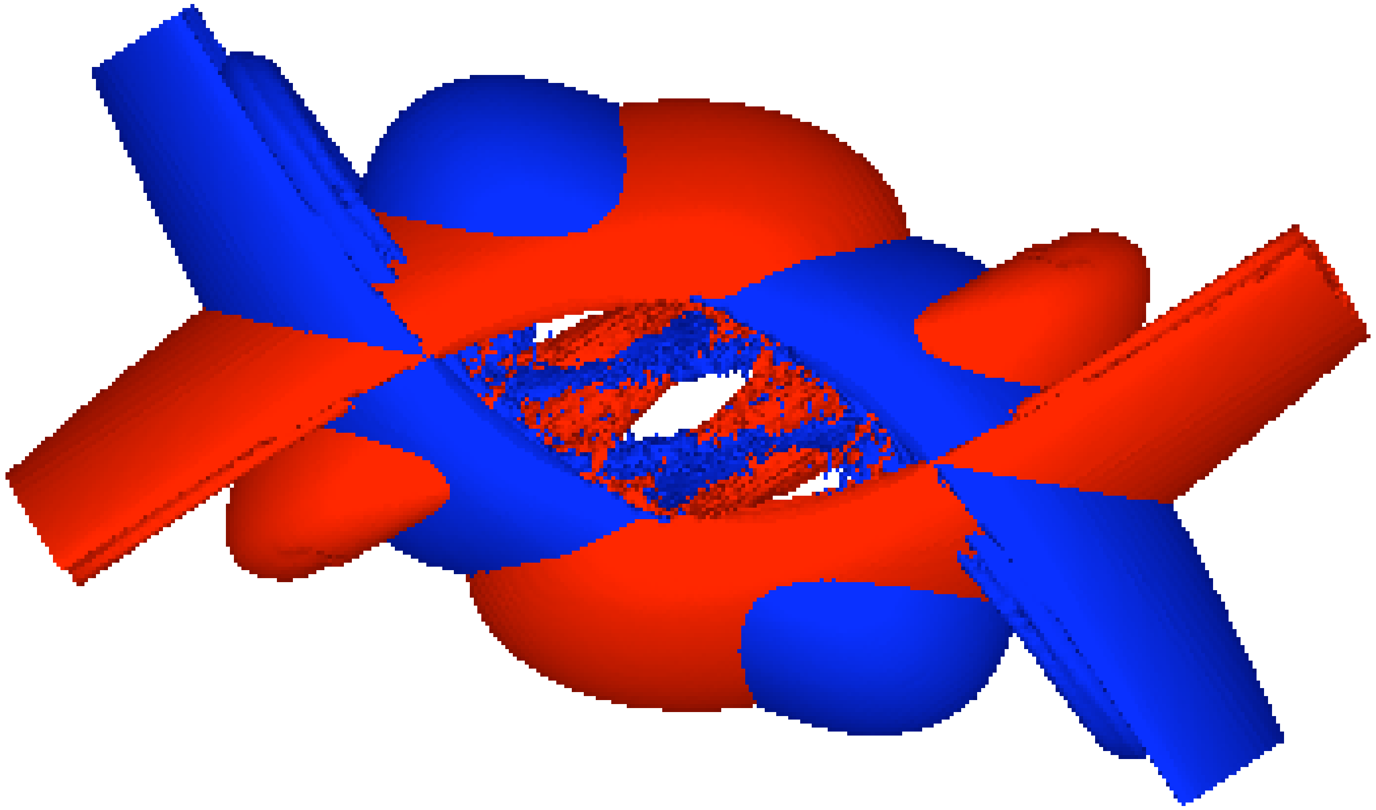
Mediterranean Salt Lenses (Meddies)



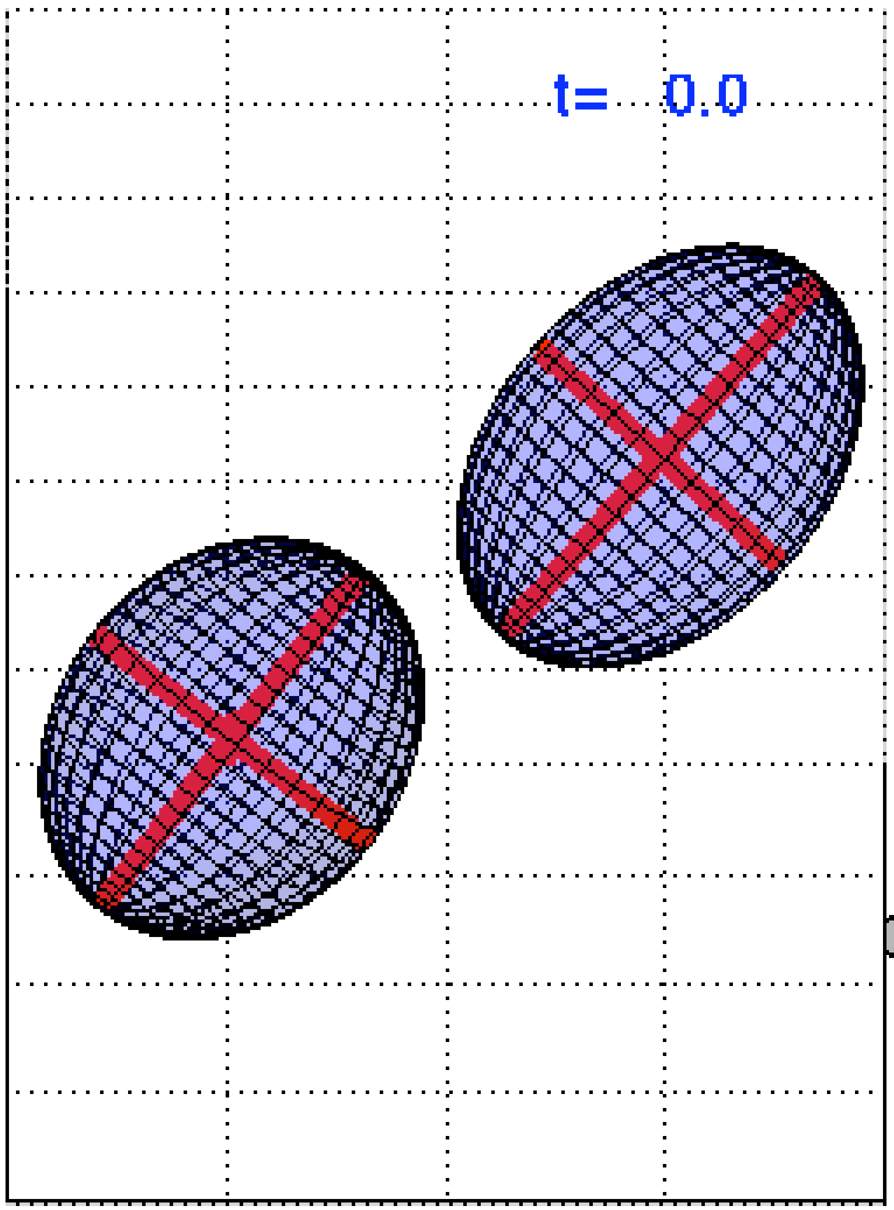


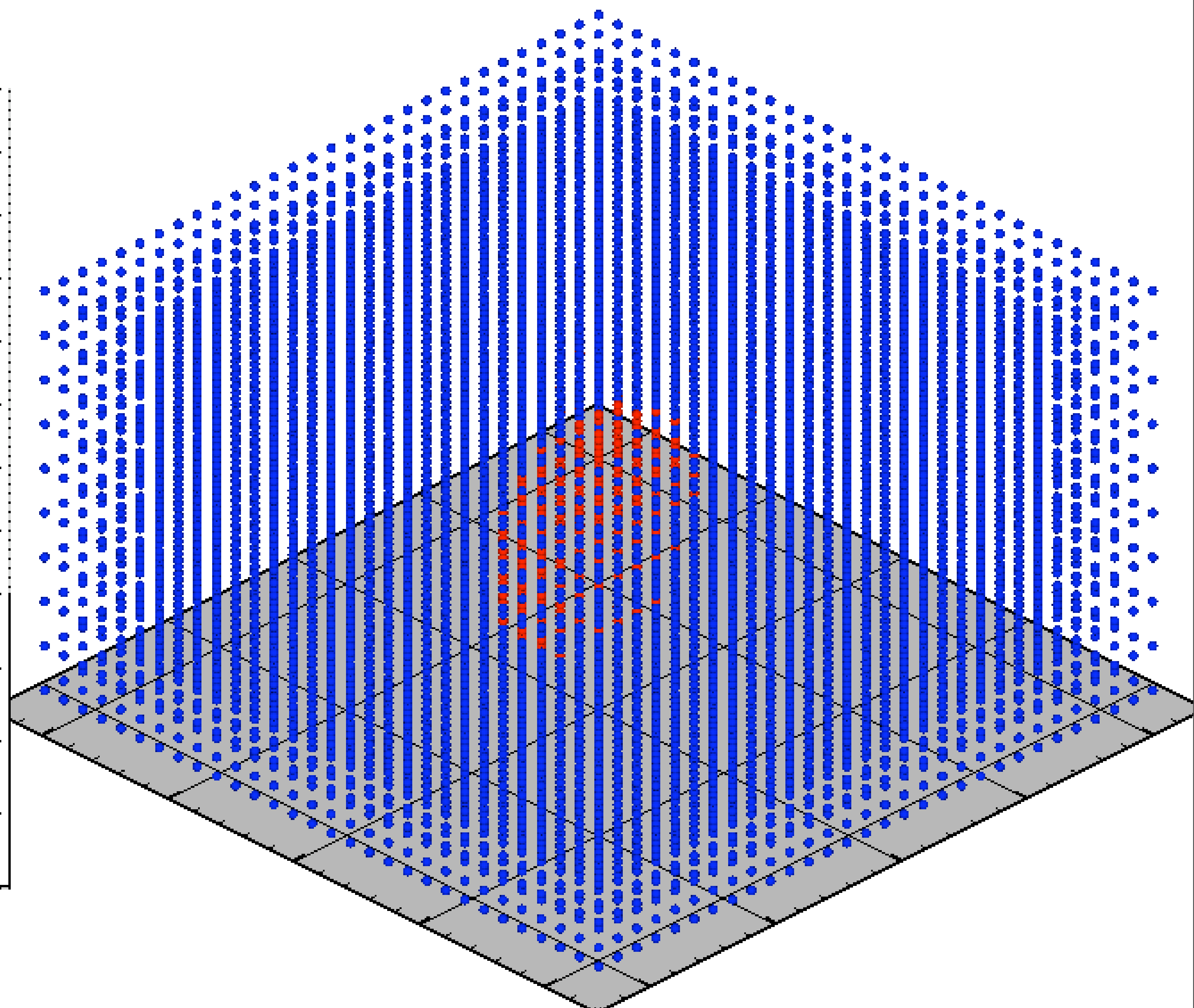
Lobes in Meddies.



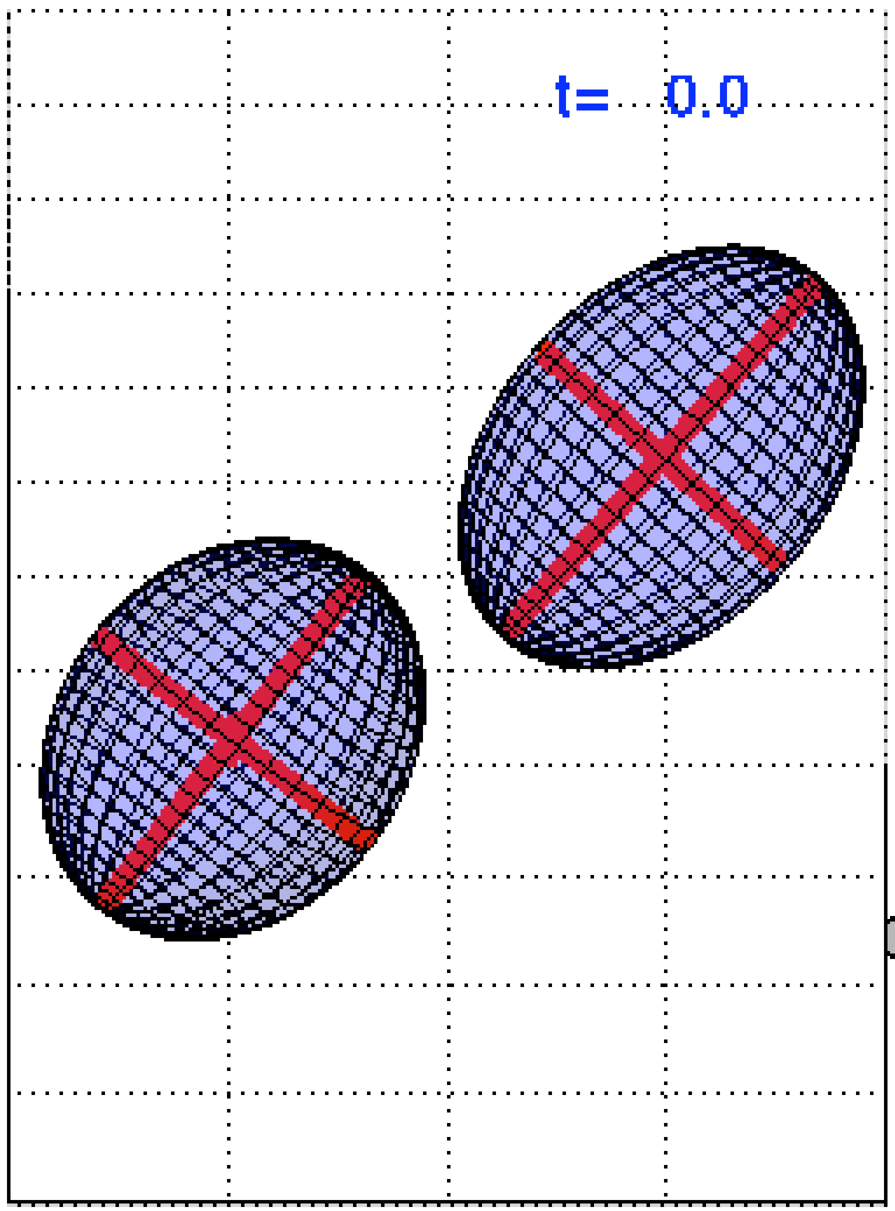


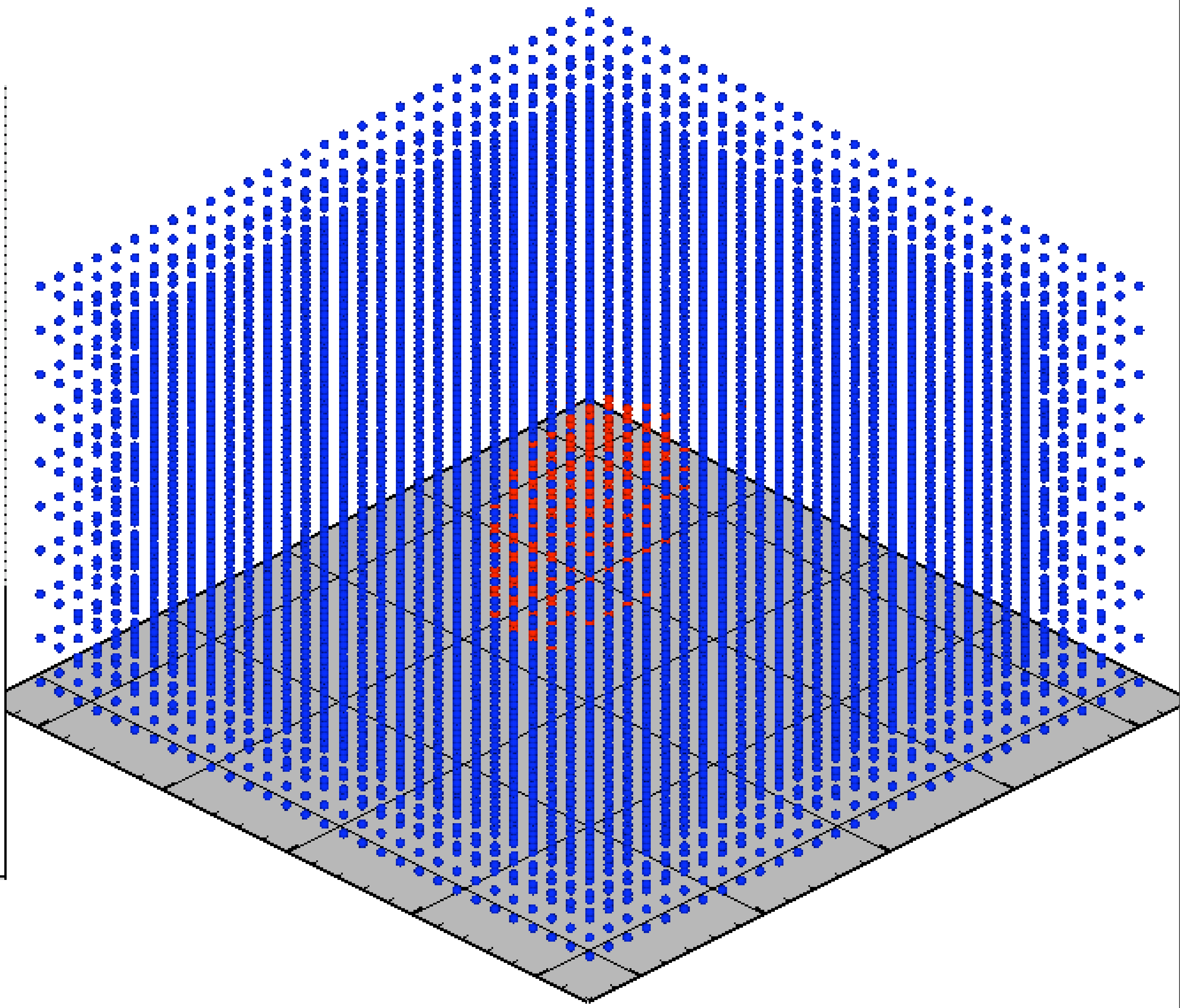
Interacting vortices



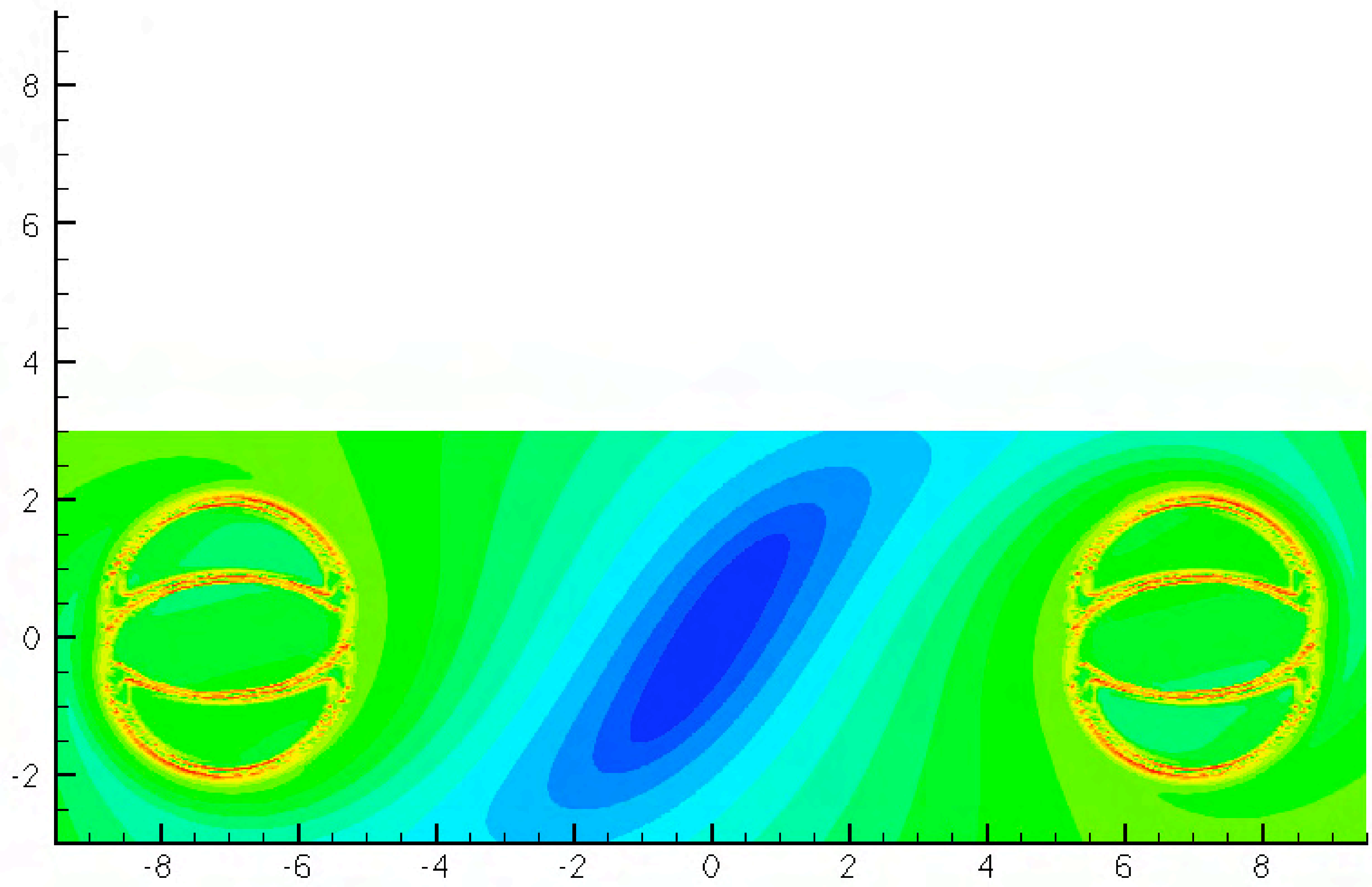


Interacting vortices





LCS in interacting vortices



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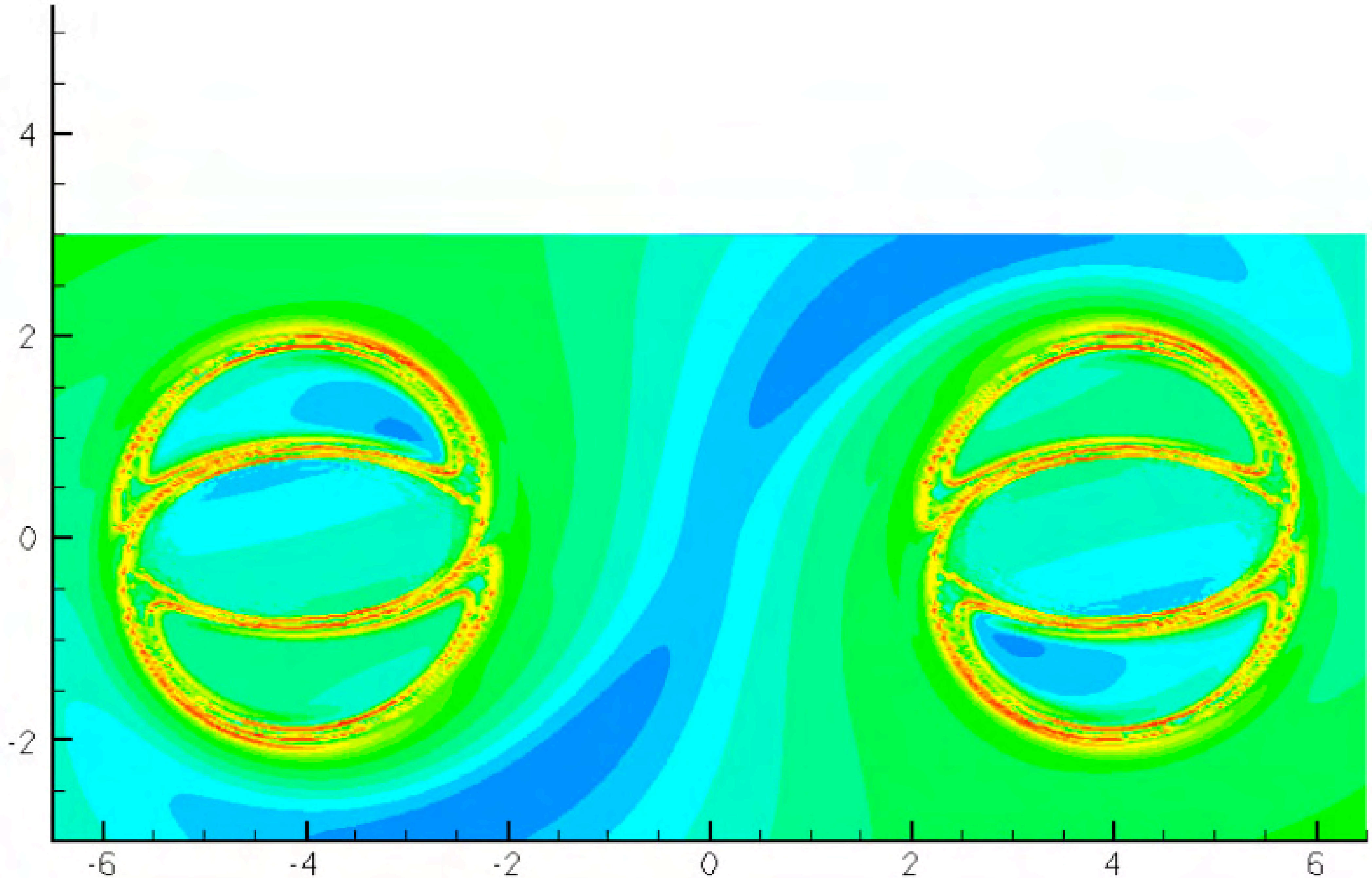

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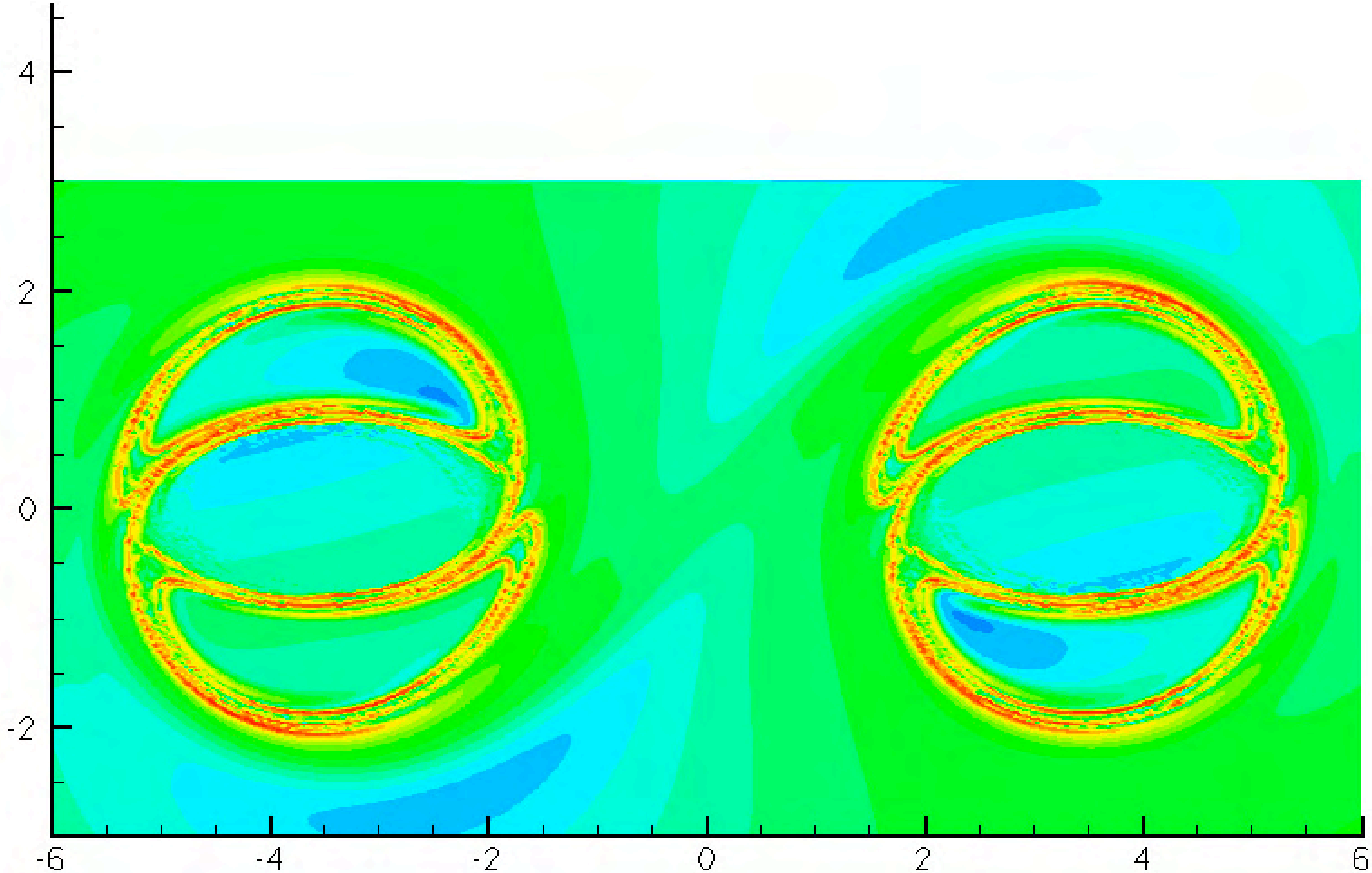
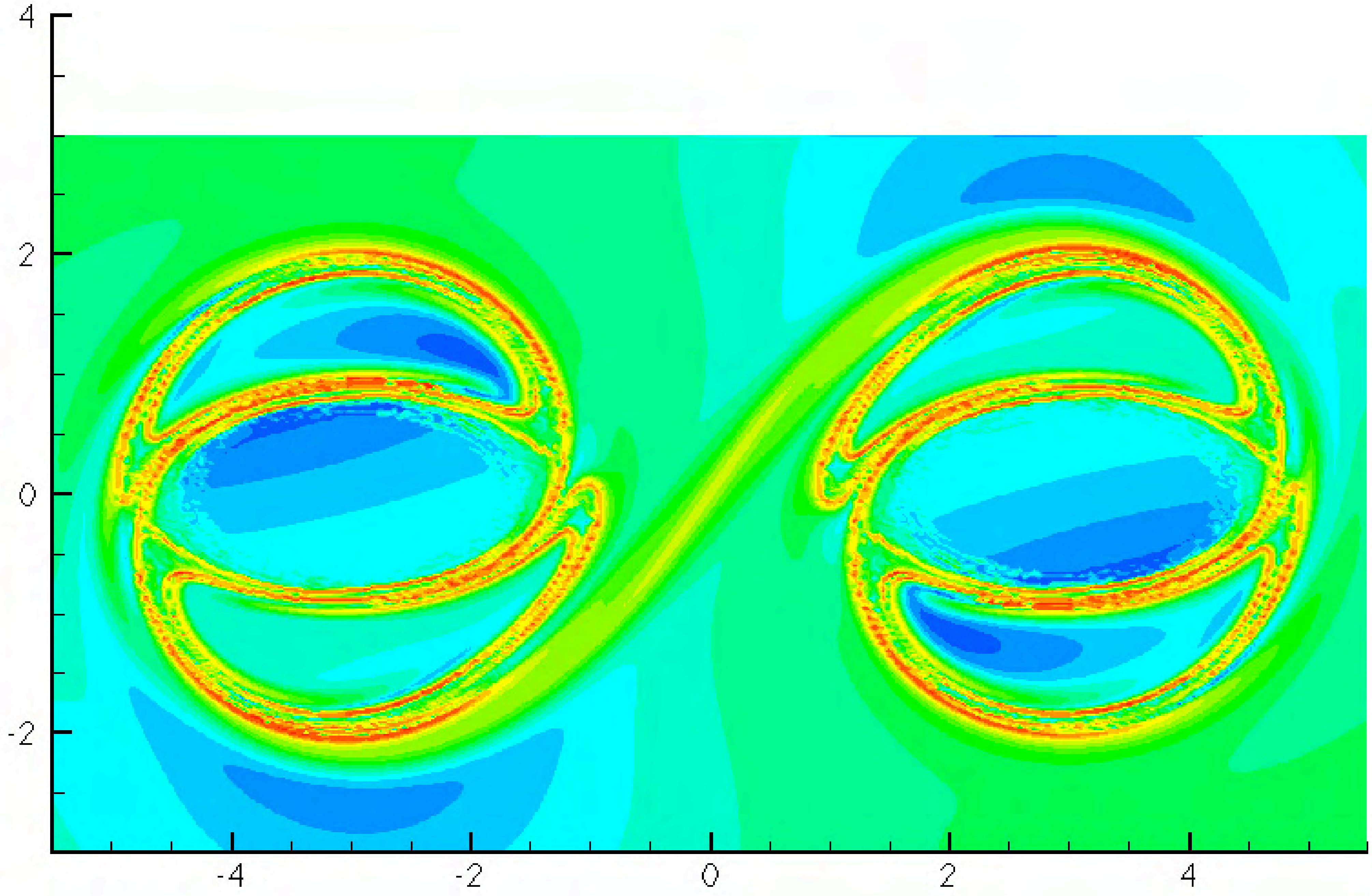
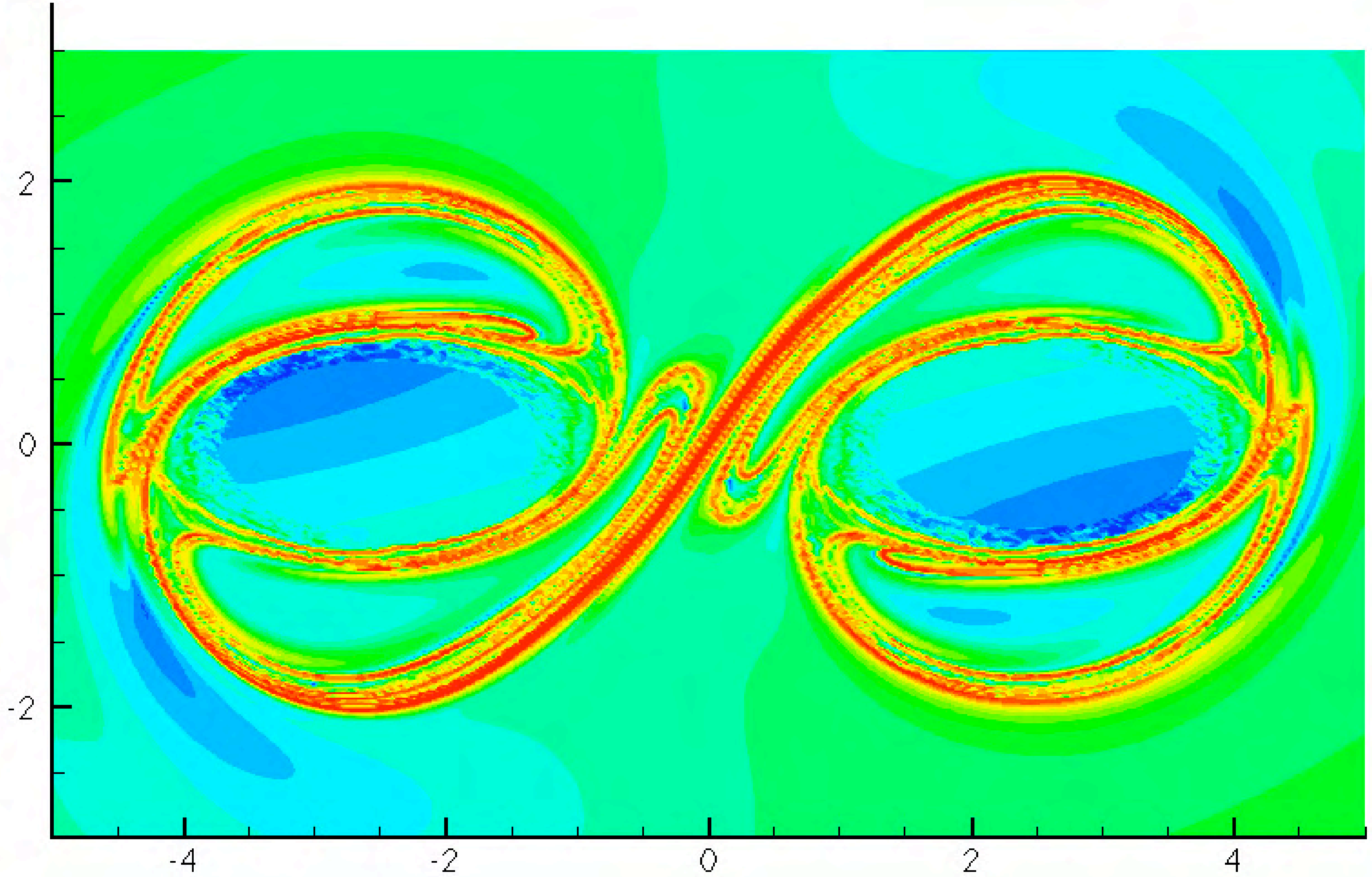


LCS in interacting vortices



LCS in interacting vortices



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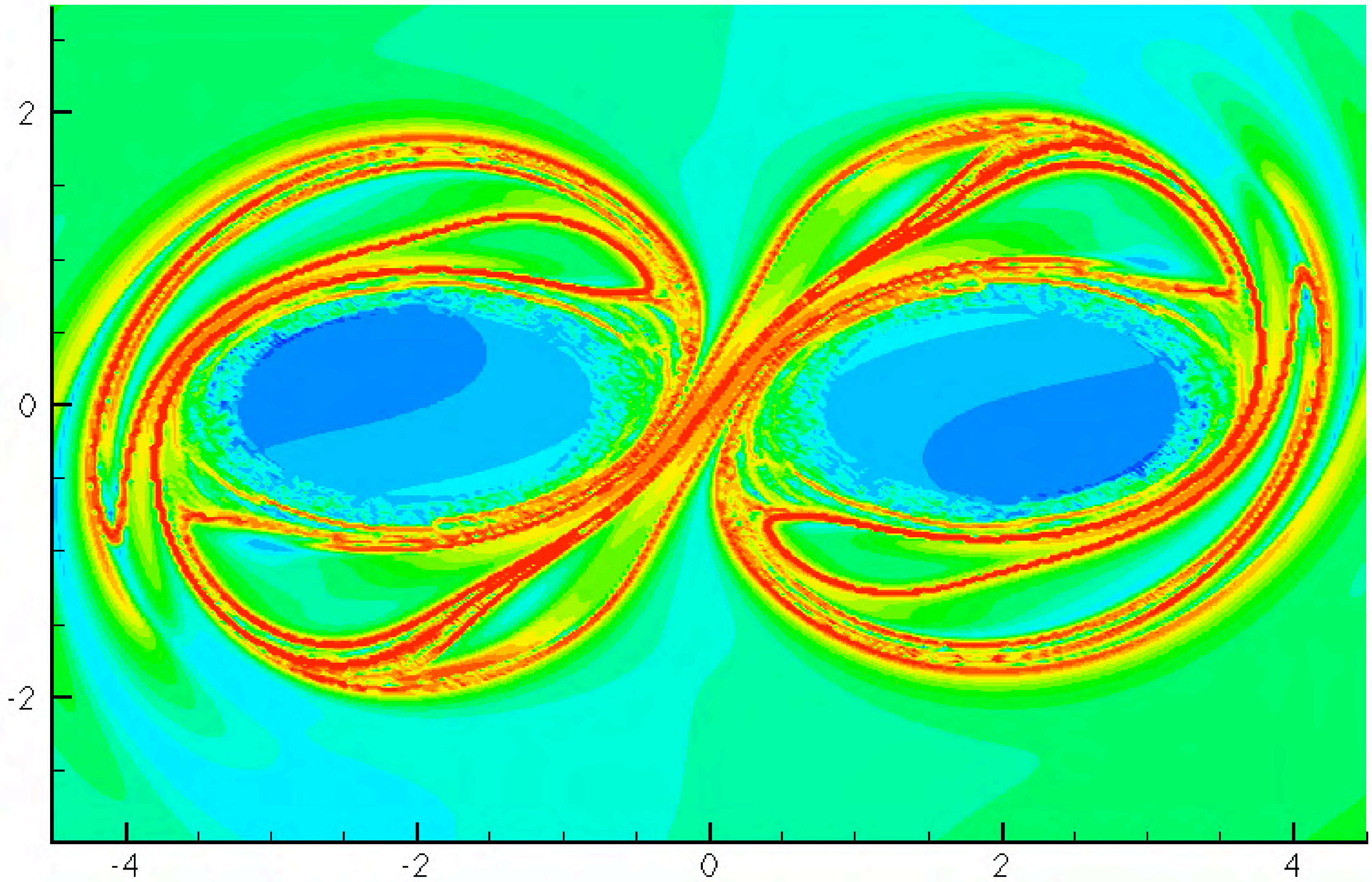
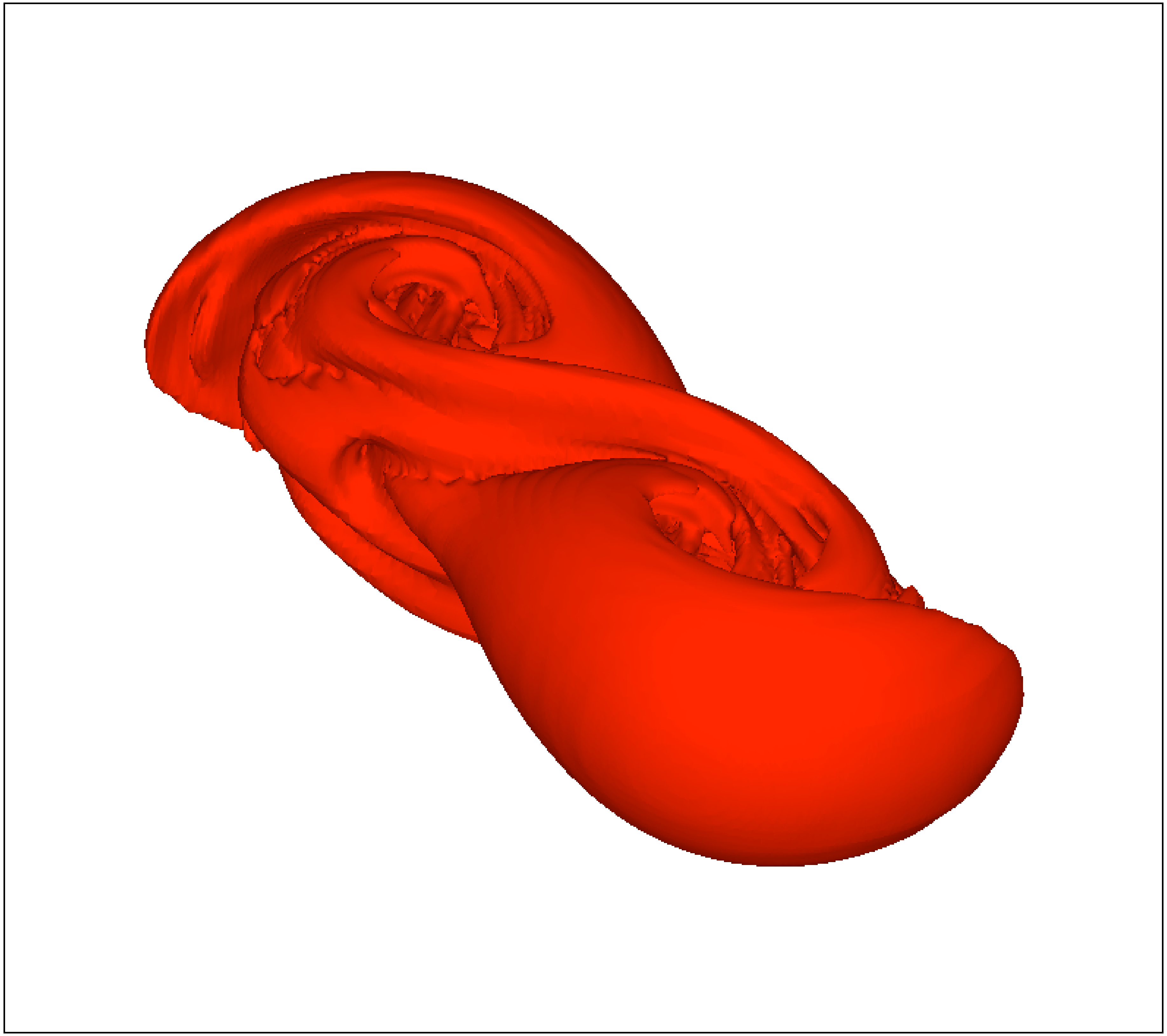
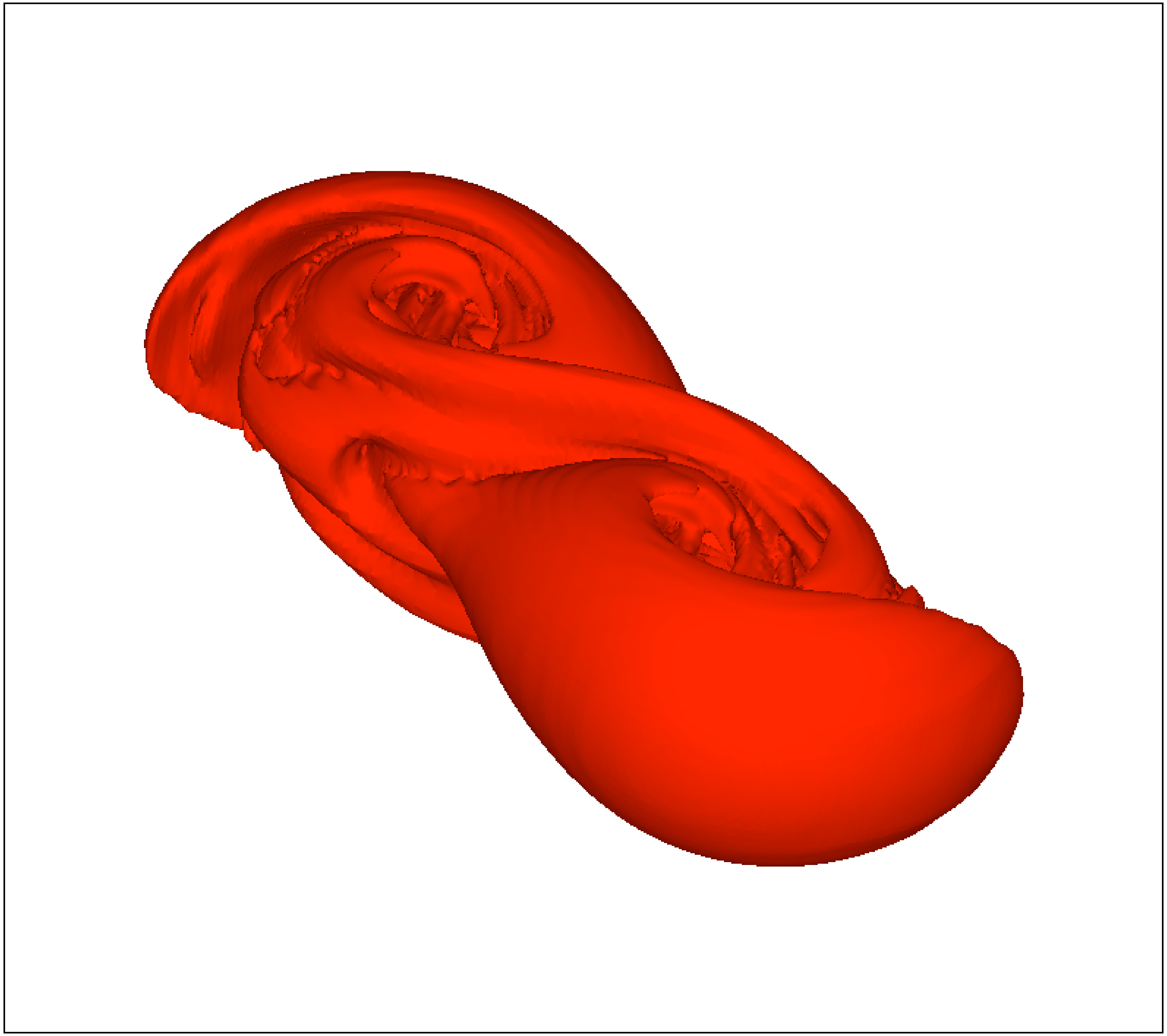


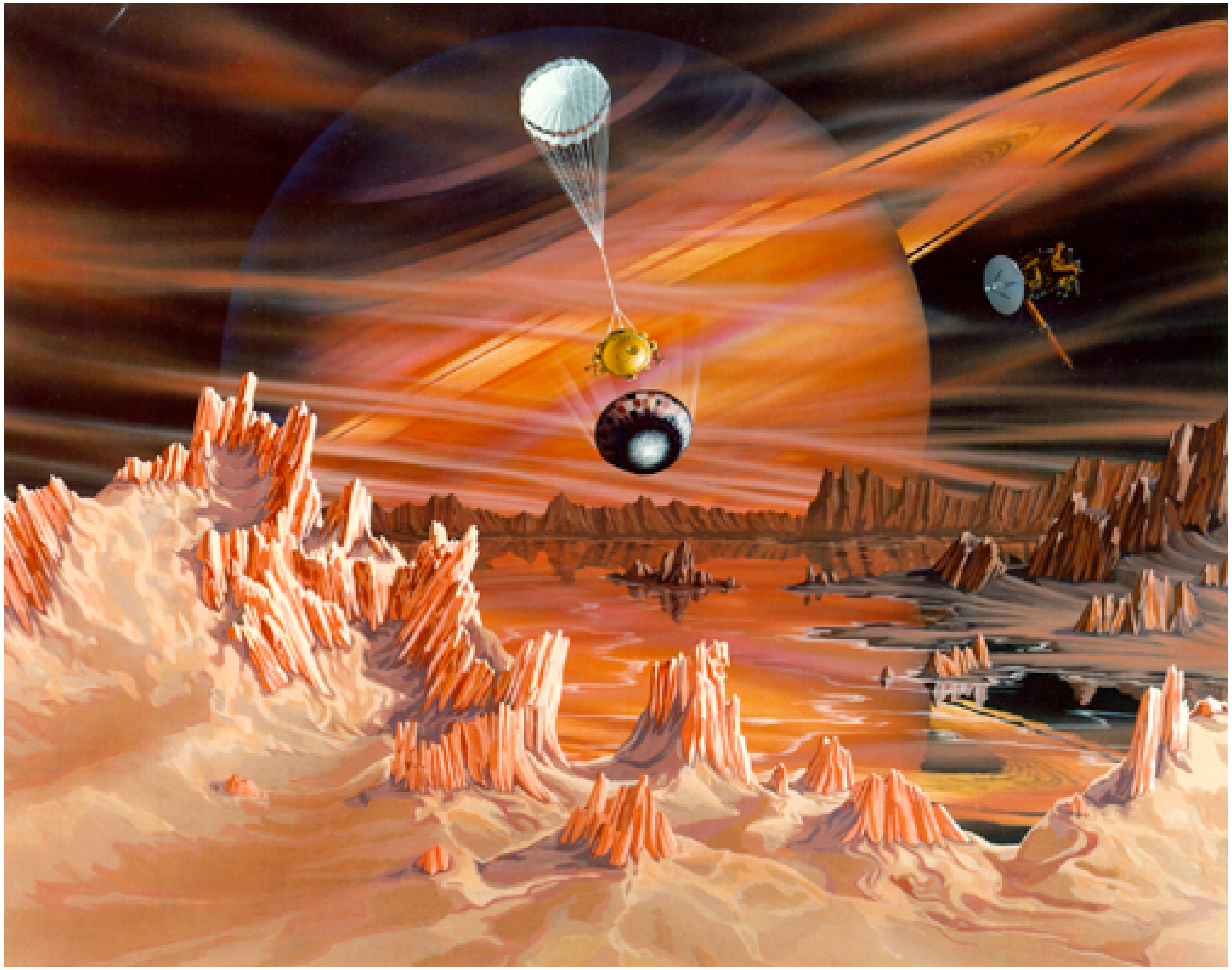
LCS in interacting vortices 3D



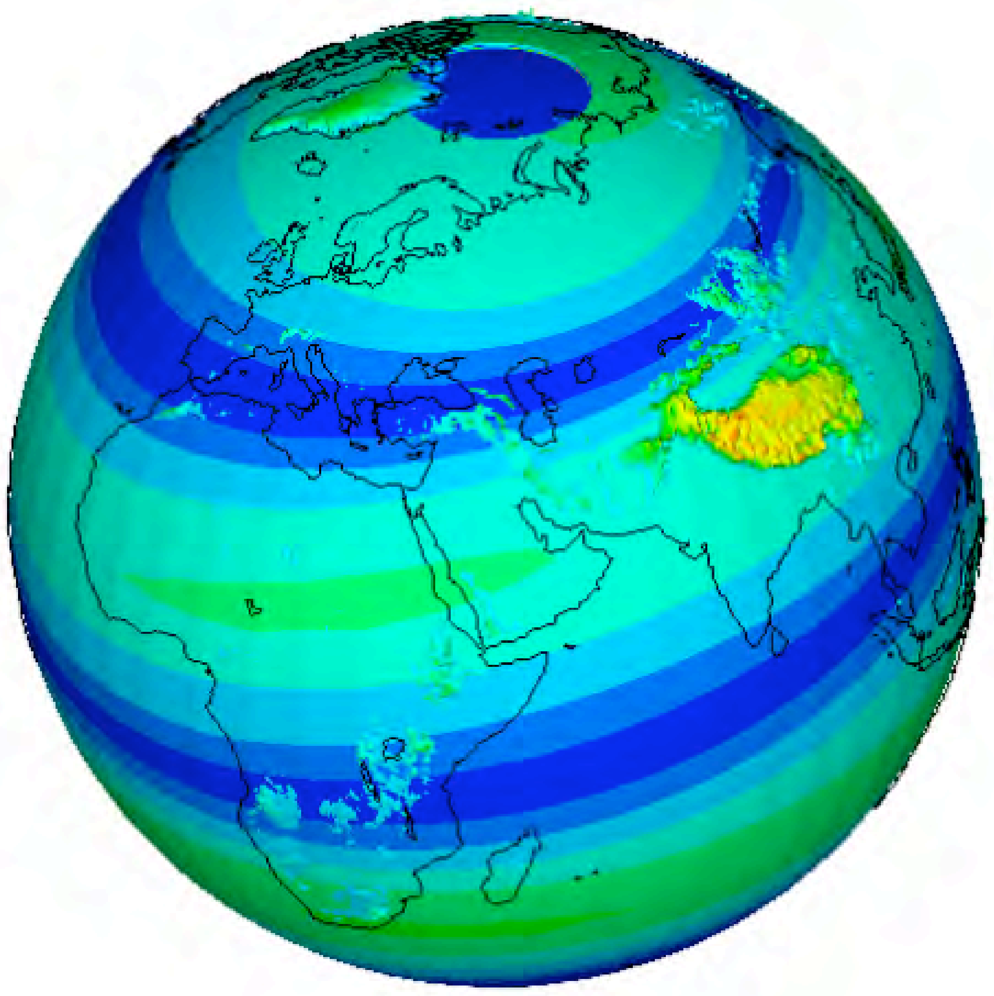
LCS in interacting vortices 3D



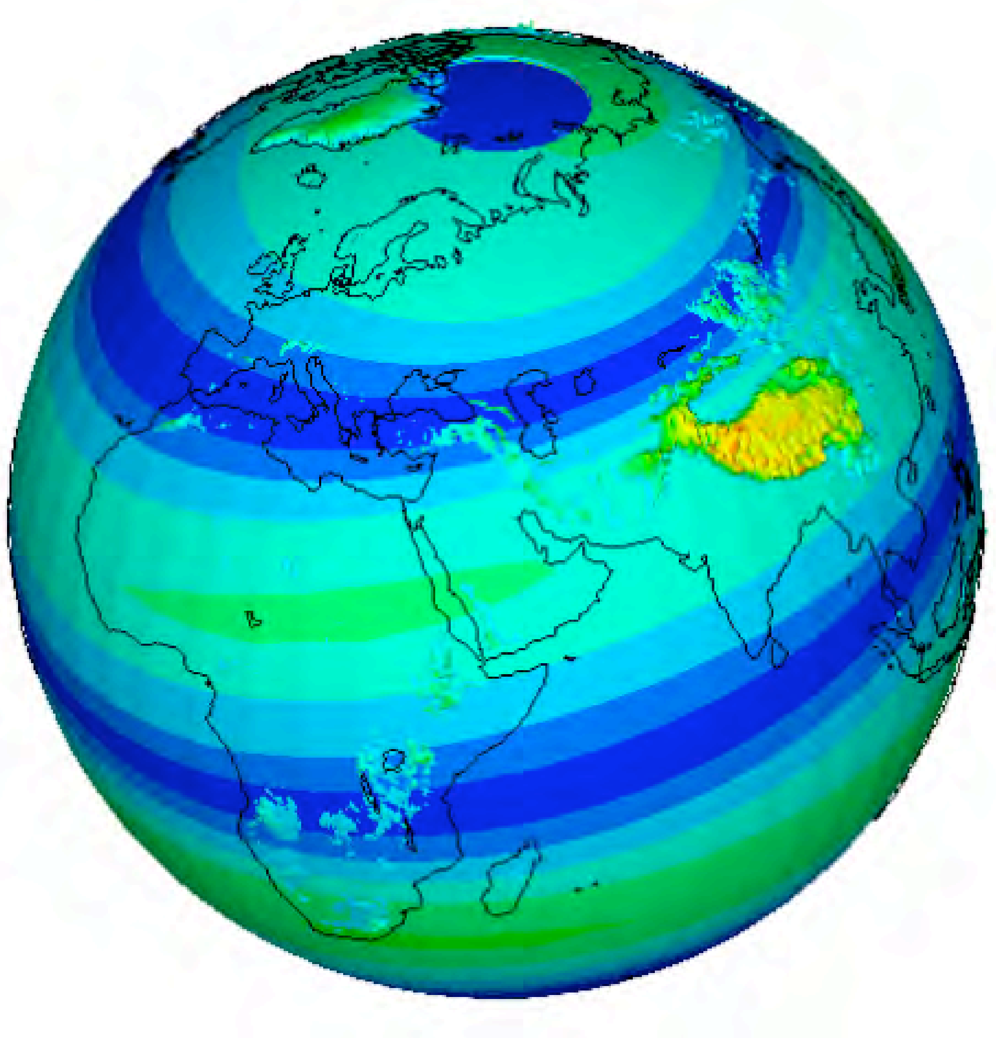
3D LCS in atmosphere of Titan

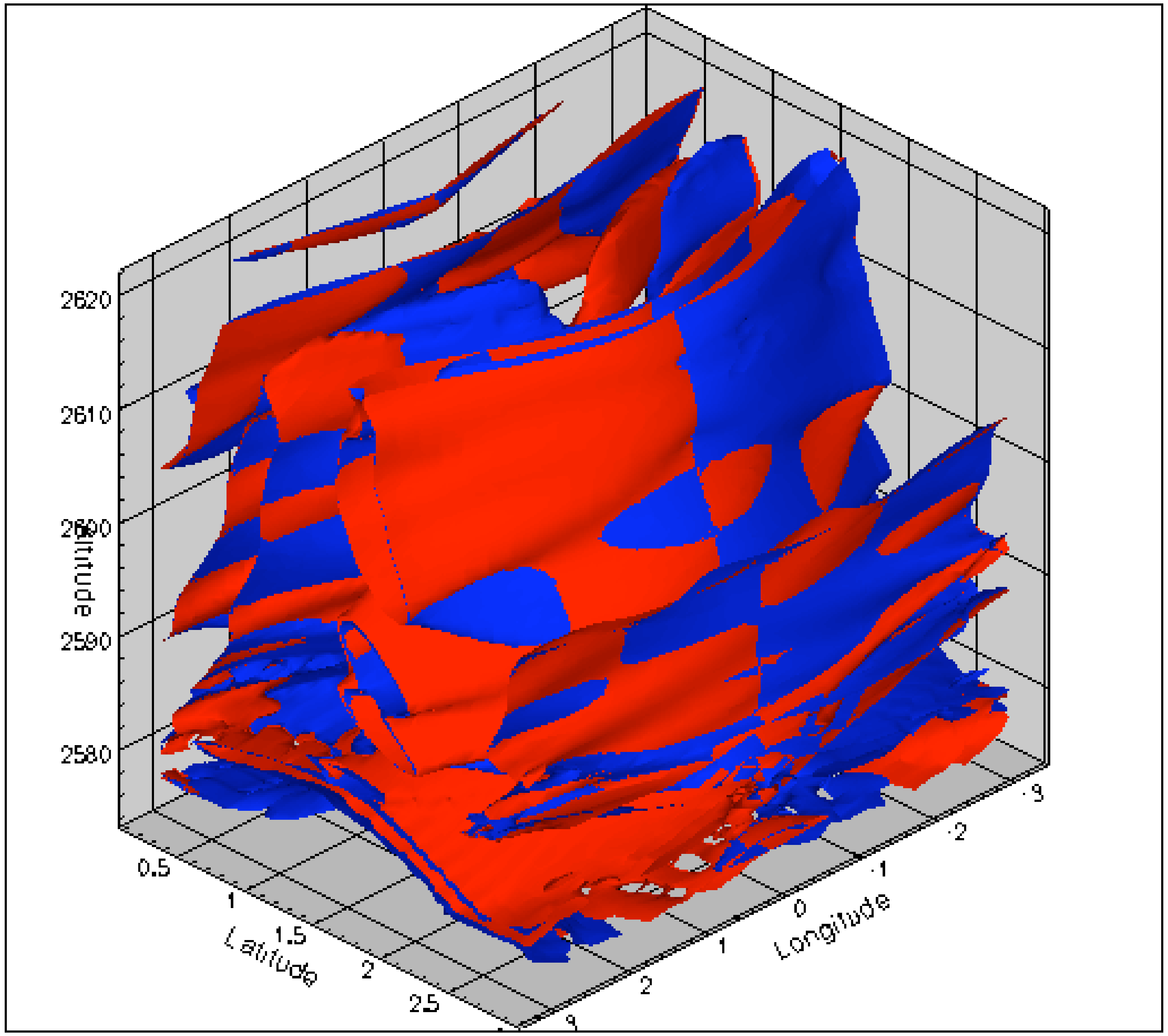


3D LCS in atmosphere of Titan



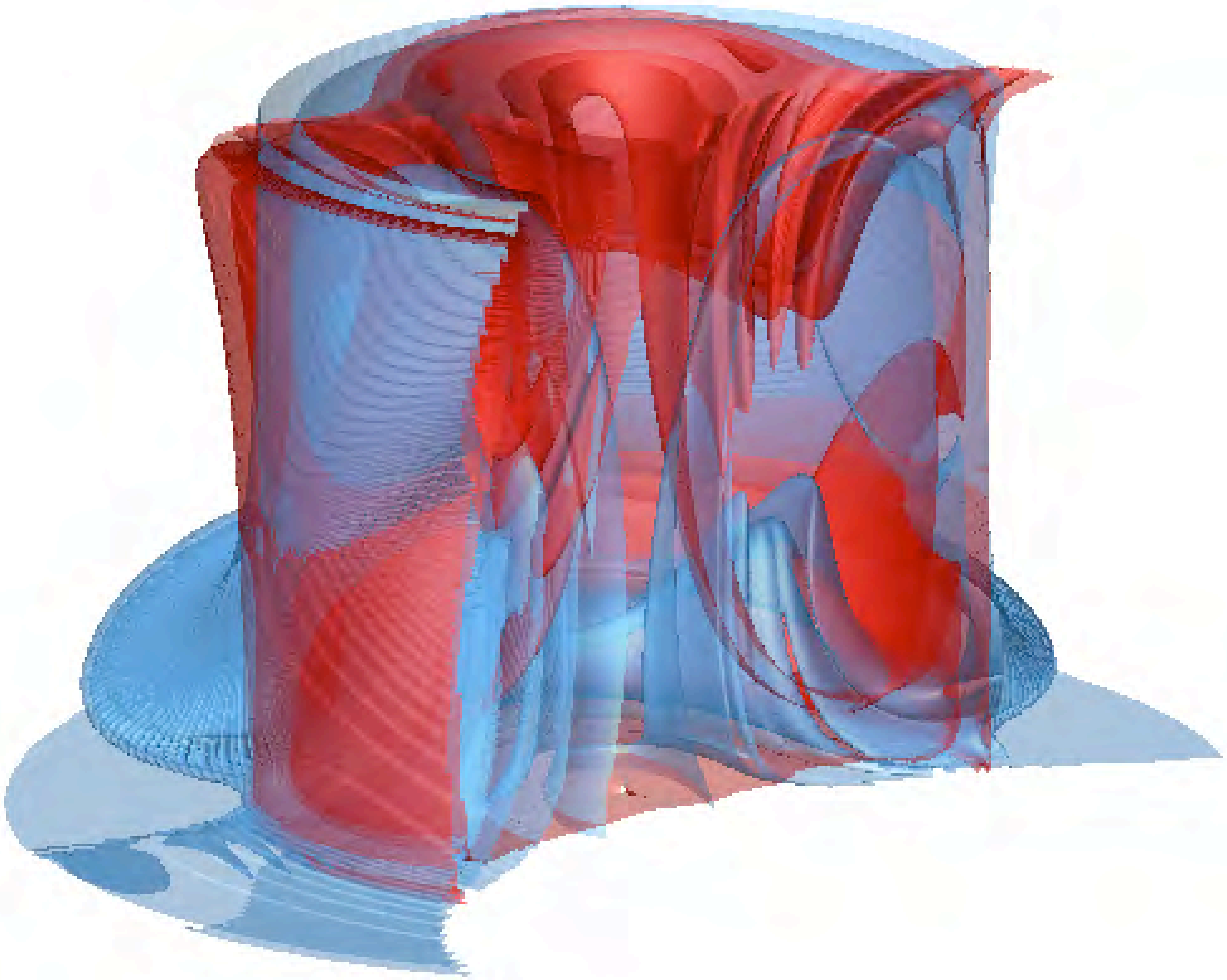
3D LCS in atmosphere of Titan





3D LCS in a convection cell

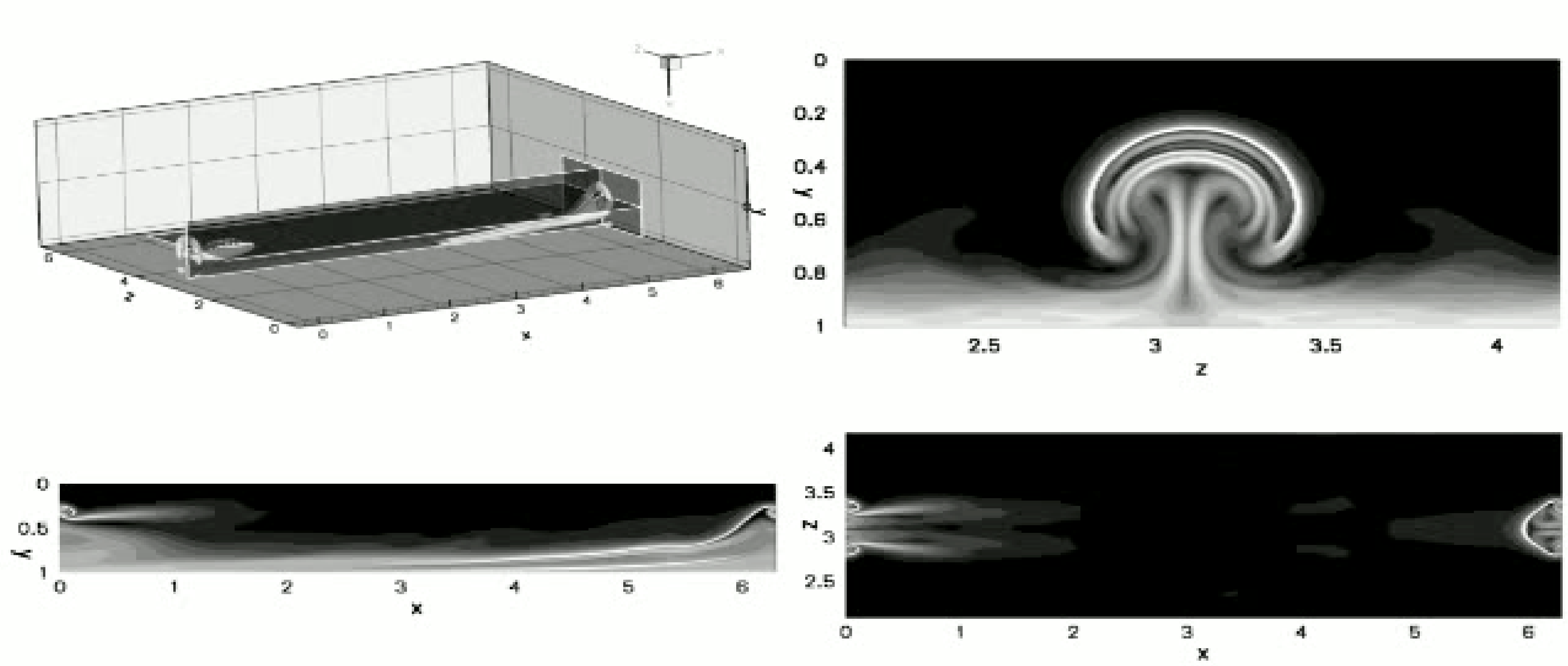
3D LCS in a convection cell



Hairpin vortices in near wall turbulent flow

Melissa Green,
Clancy Rowley,
George Haller.

Hairpin vortices in near wall turbulent flow



Melissa Green,
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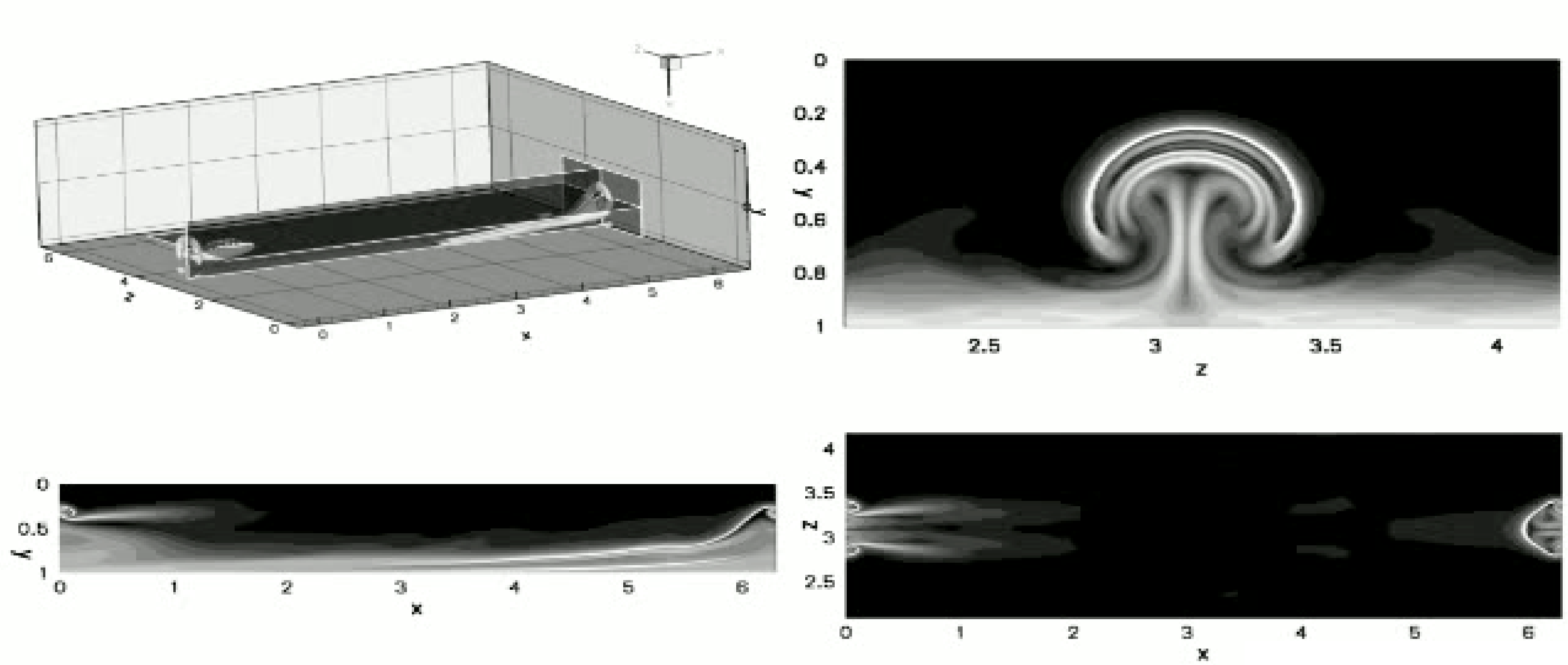
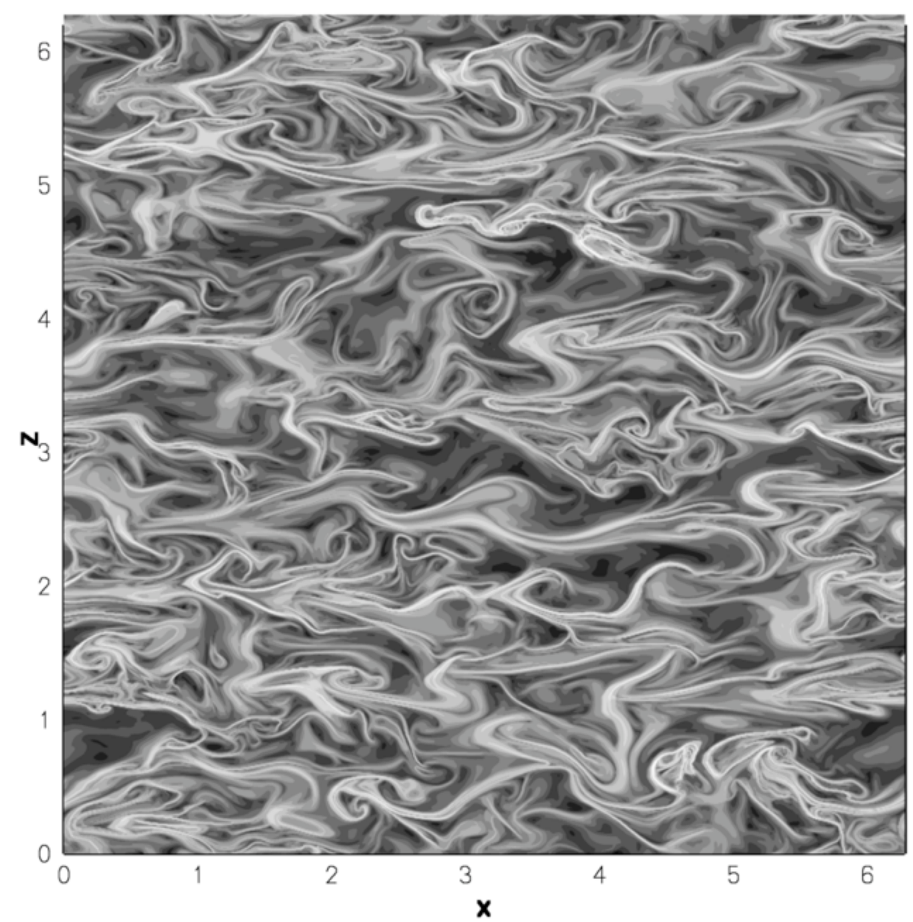
Hairpin vortices in near wall turbulent flow





Melissa Green,
Clancy Rowley,
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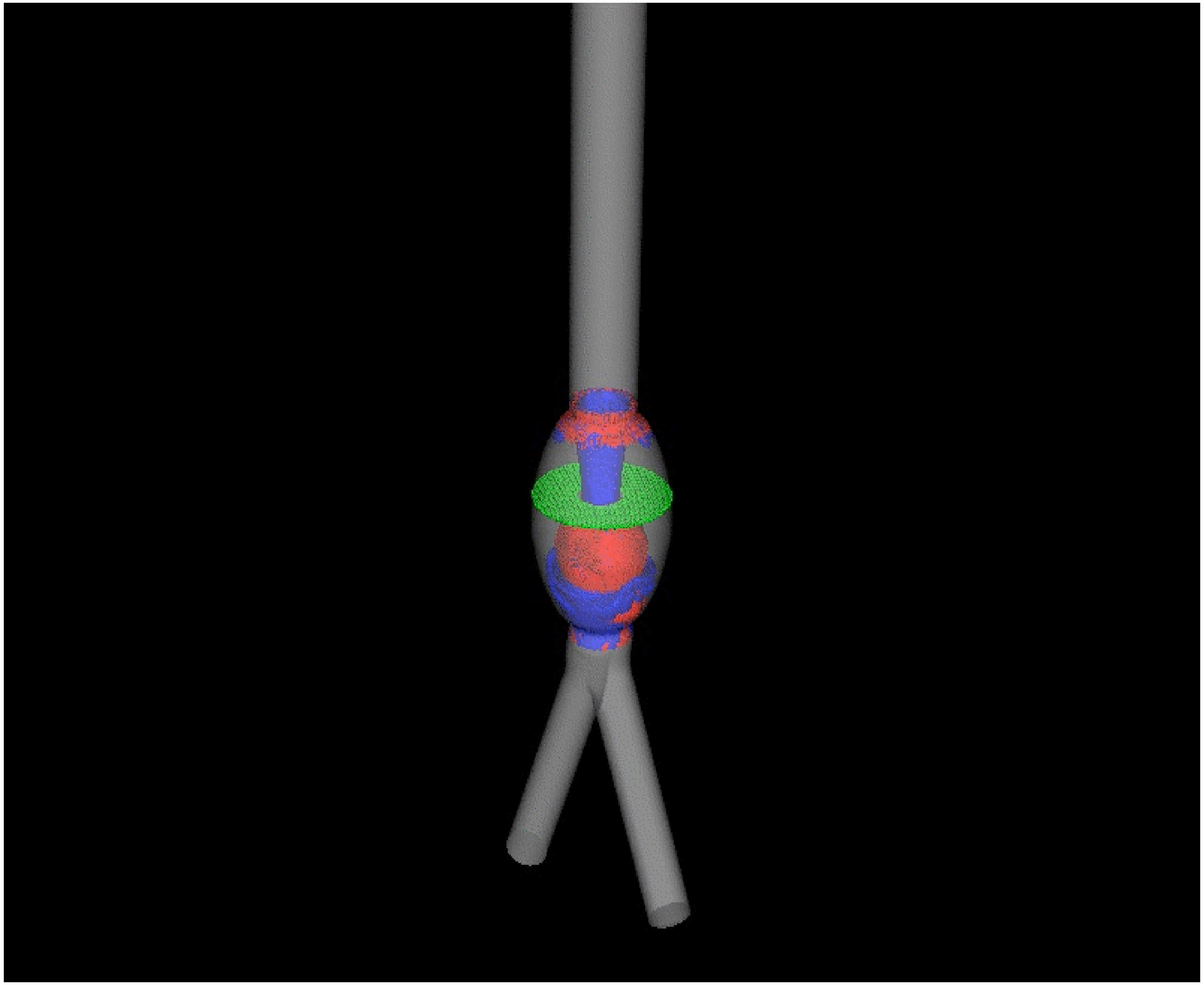
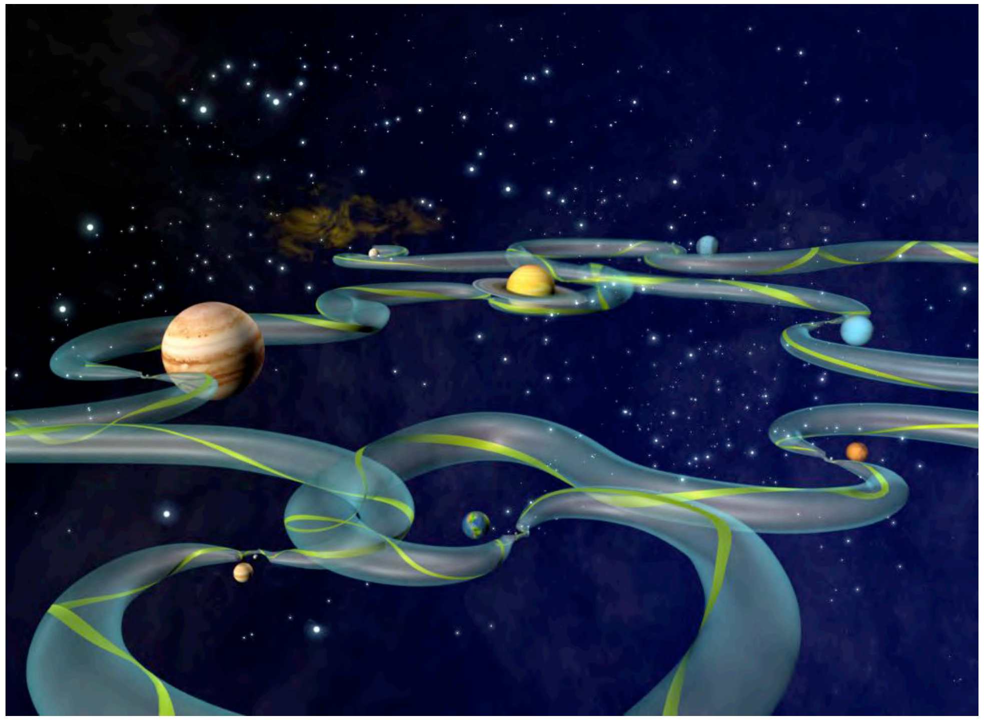
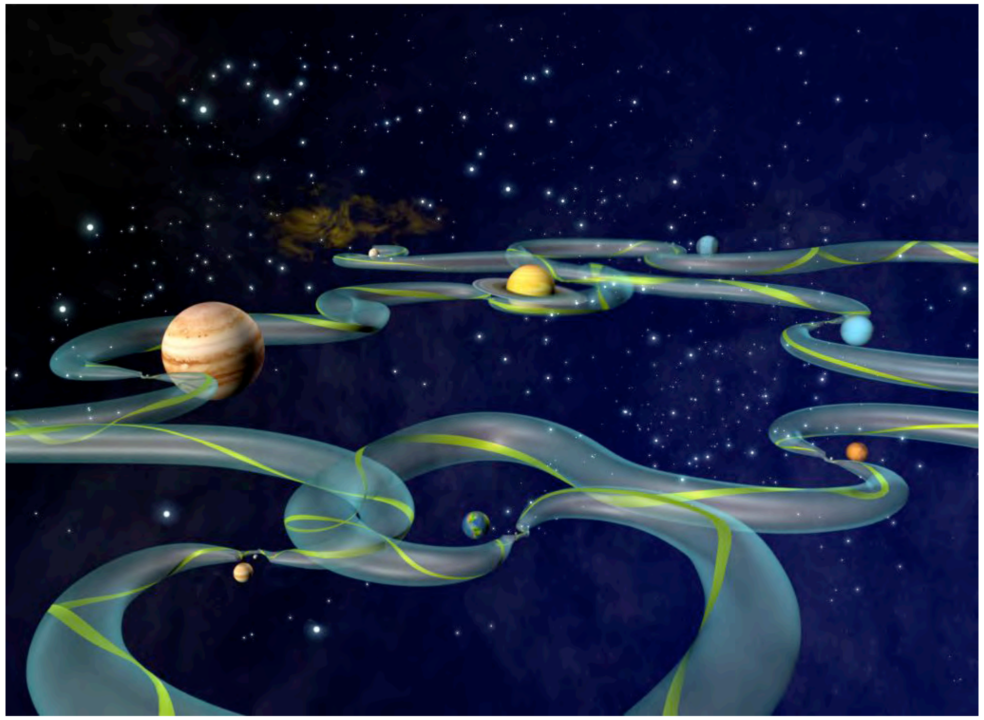
LCS in blood flow

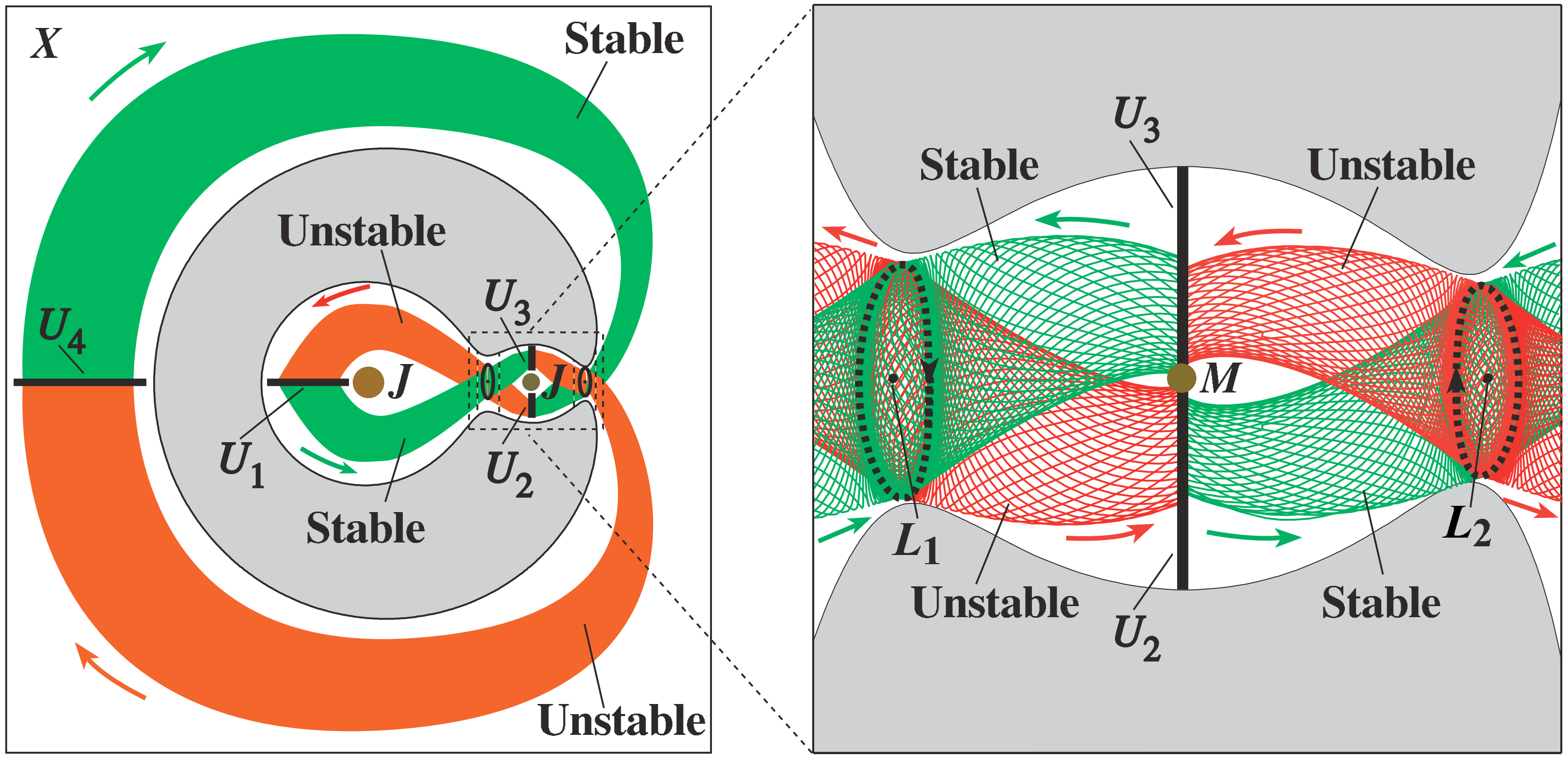


Invariant manifolds in space travel

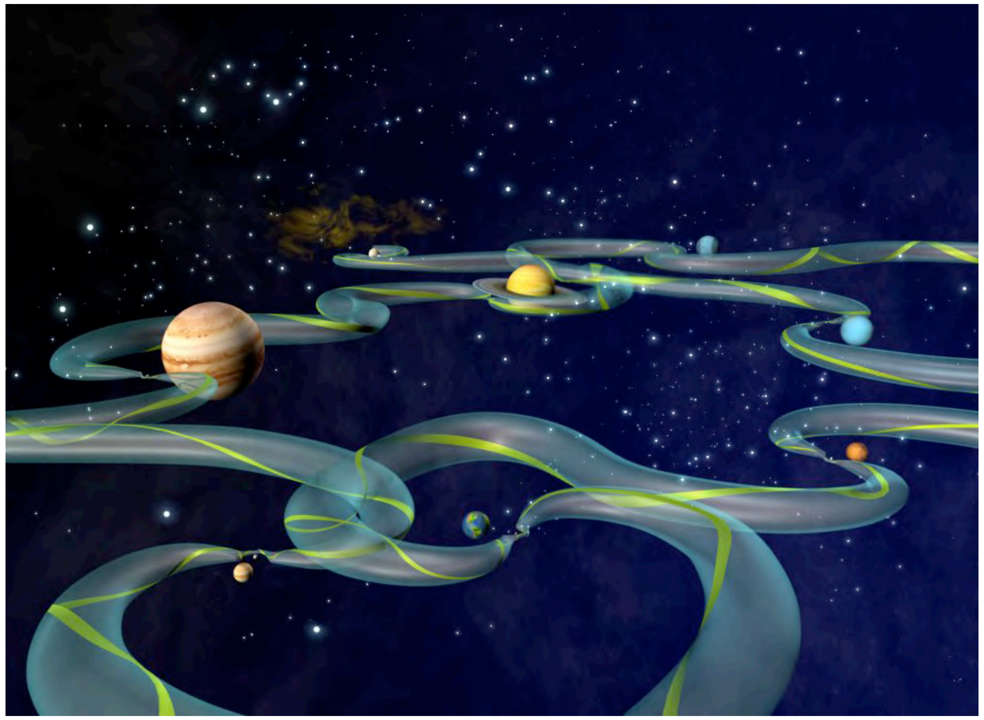


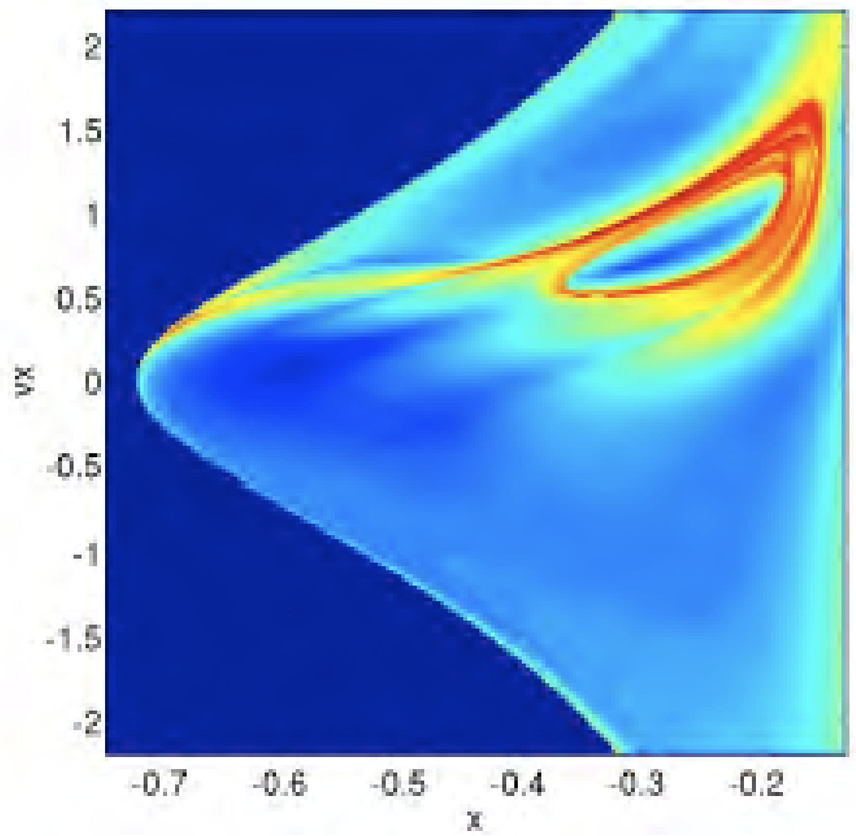
Invariant manifolds in space travel

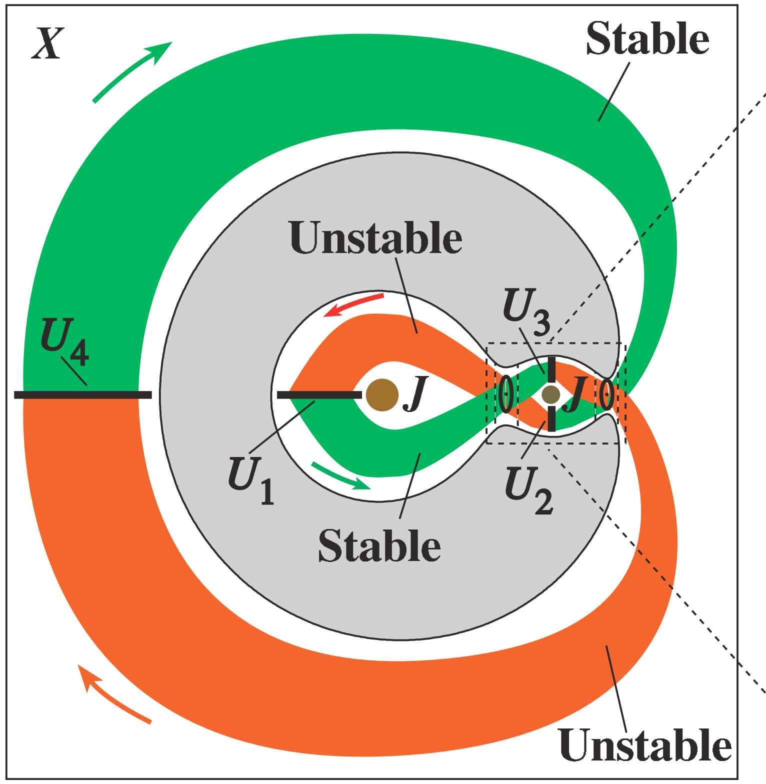


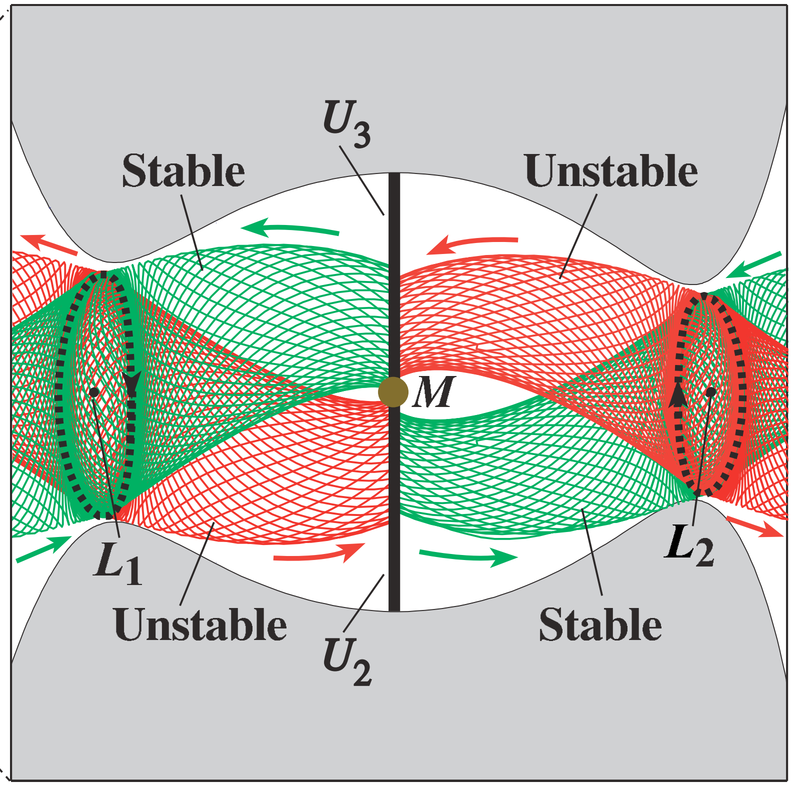


Invariant manifolds in space travel









Numerical considerations

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- Choosing integration time, T.

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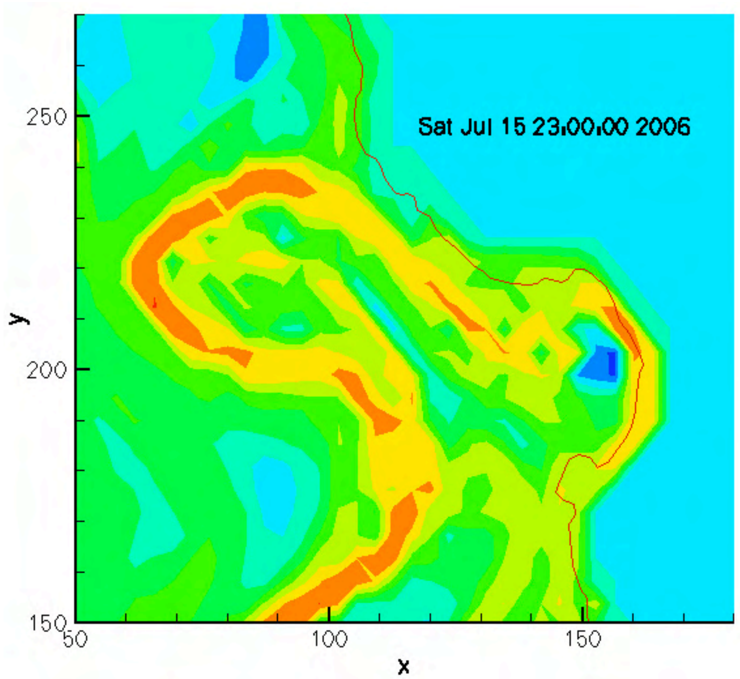

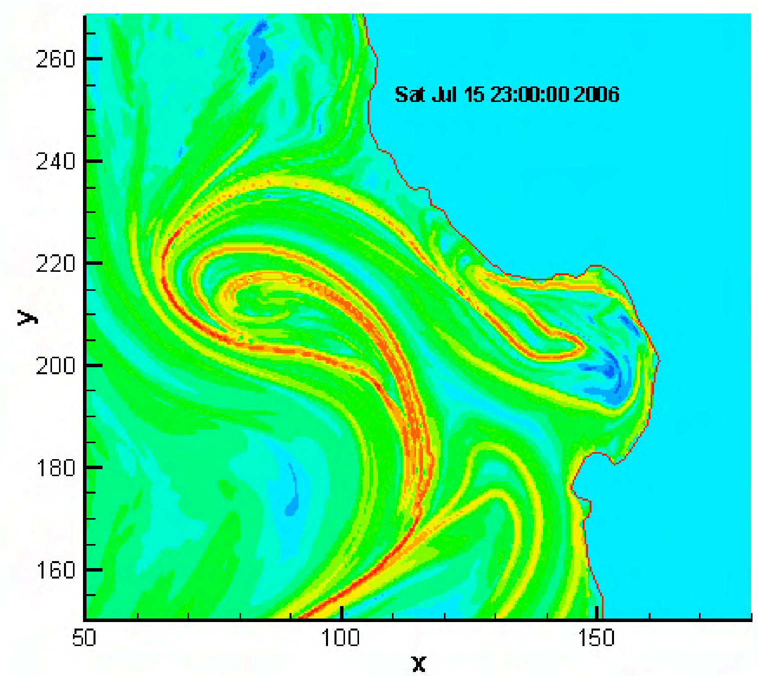

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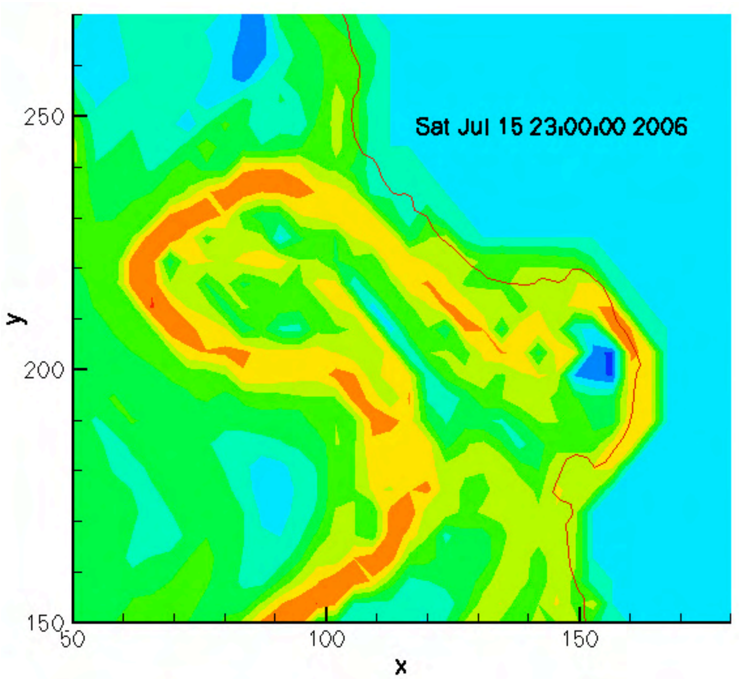

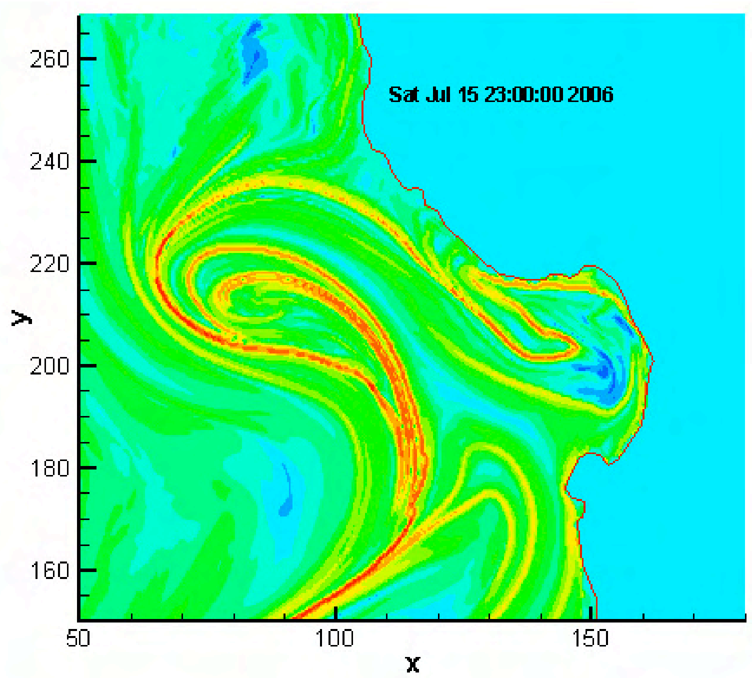

- Memory: huge data in; huge data out.

Numerical considerations

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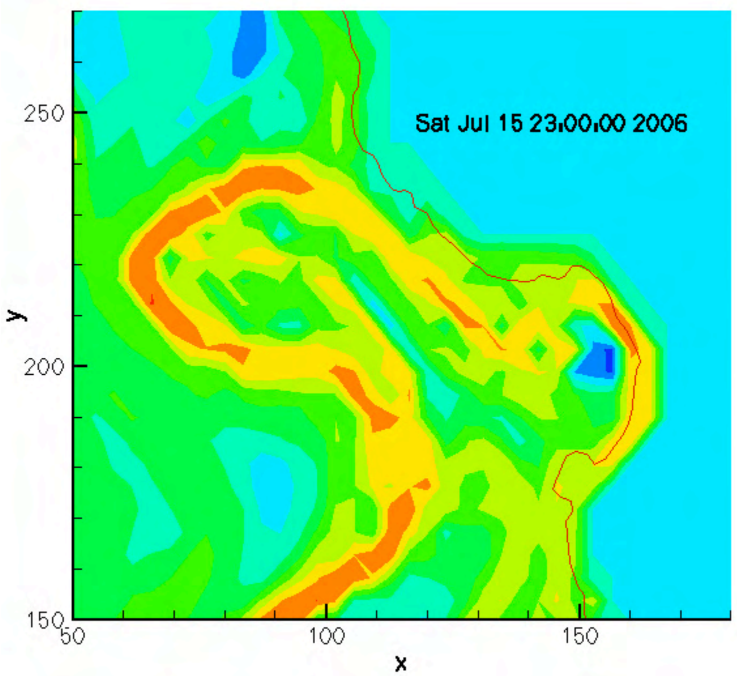

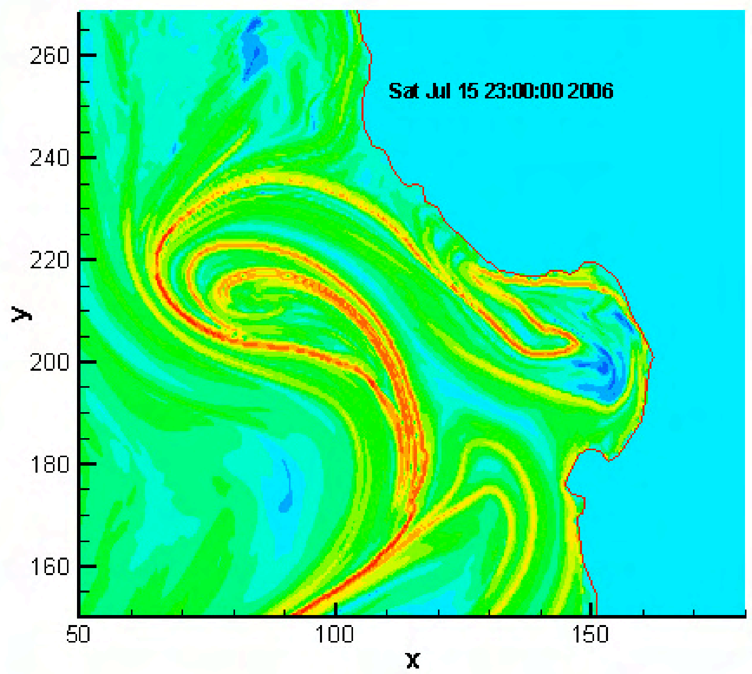

- Memory: huge data in; huge data out.
- Straightforward to parallelize.

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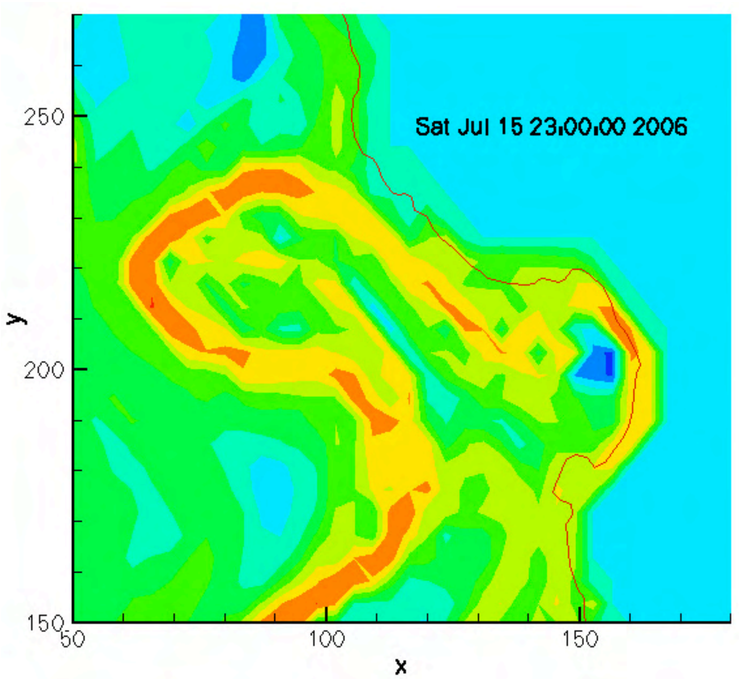

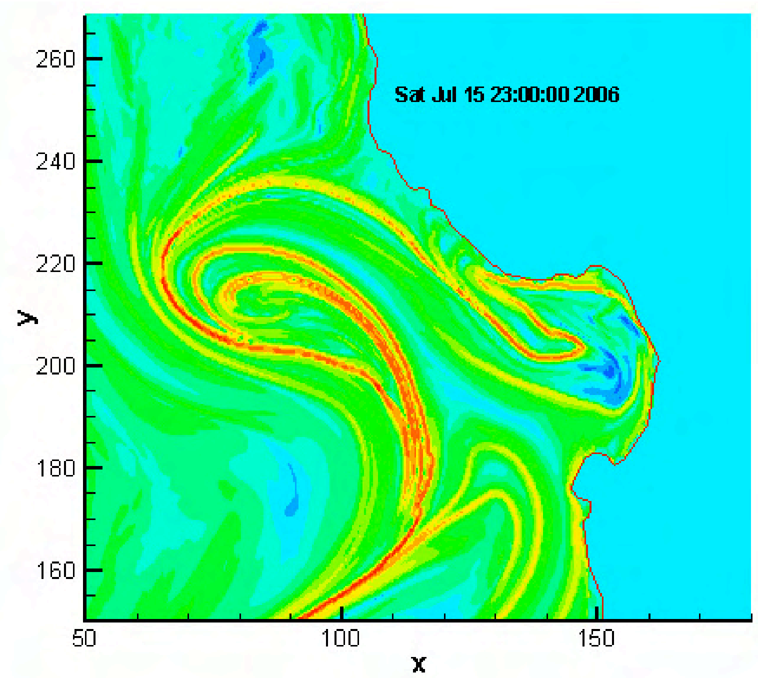

- Memory: huge data in; huge data out.
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- Computations are now "bandwidth limited."

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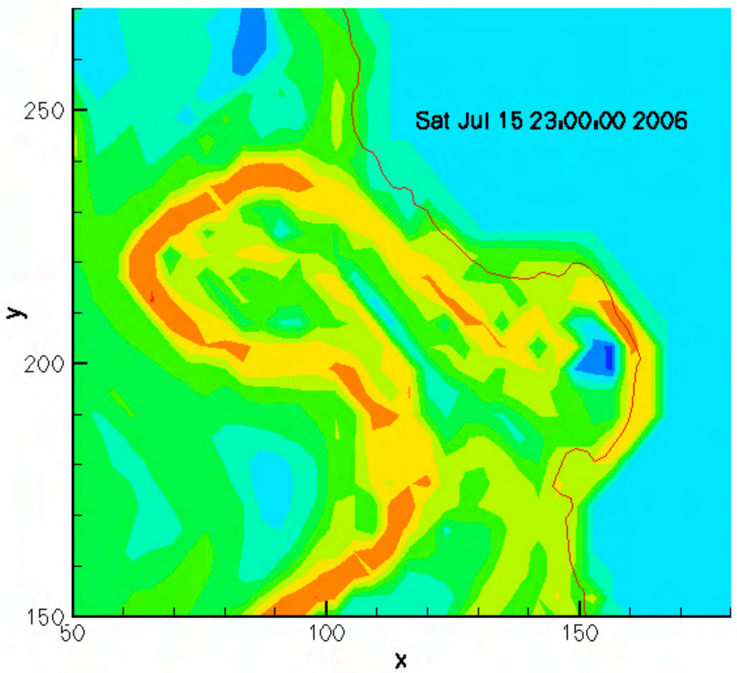
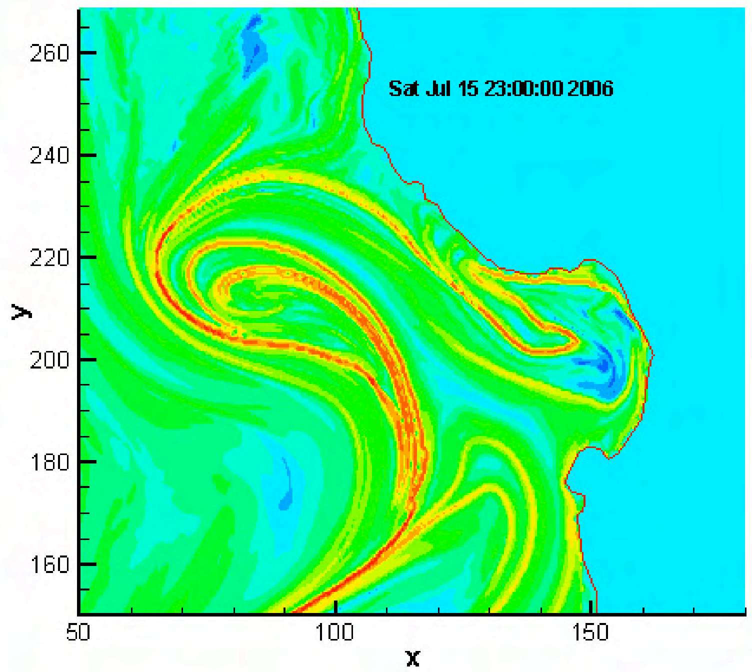
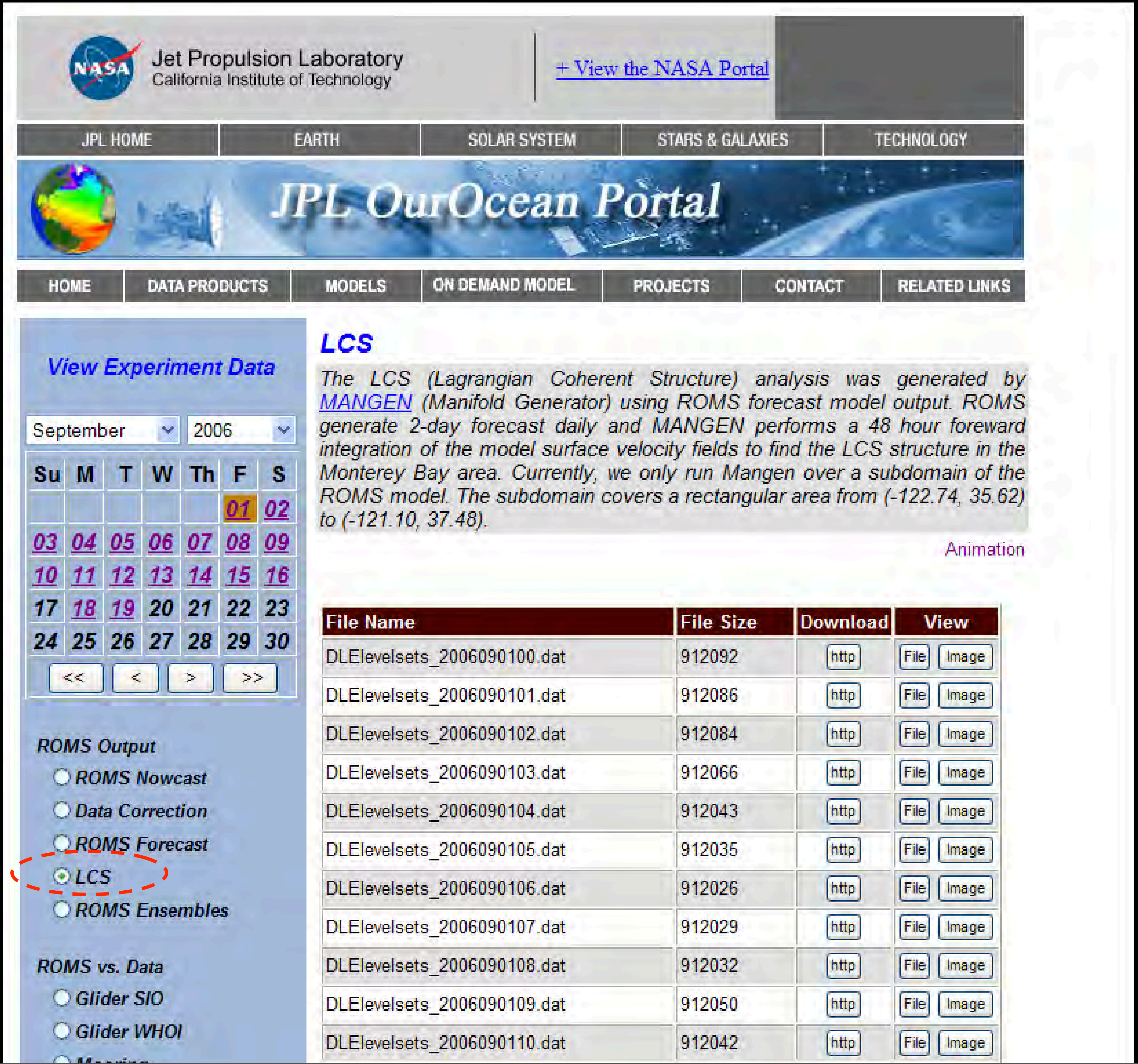




- Memory: huge data in; huge data out.
- Straightforward to parallelize.
- Computations are now "bandwidth limited."
- Computed online in near real time.

Near Real-Time Computation of LCS

http://ourocean.jpl.nasa.gov/MB06/



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View Experiment Data

September 2006

Su	M	T	W	Th	F	S
					01	02
03	04	05	06	07	08	09
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

<< < > >>

ROMS Output

- *ROMS Nowcast*
- *Data Correction*
- *ROMS Forecast*
- *LCS*
- *ROMS Ensembles*

ROMS vs. Data

- *Glider SIO*
- *Glider WHOI*

LCS

The LCS (Lagrangian Coherent Structure) analysis was generated by MANGEN (Manifold Generator) using ROMS forecast model output. ROMS generate 2-day forecast daily and MANGEN performs a 48 hour foreward integration of the model surface velocity fields to find the LCS structure in the Monterey Bay area. Currently, we only run Mangen over a subdomain of the ROMS model. The subdomain covers a rectangular area from (-122.74, 35.62) to (-121.10, 37.48).

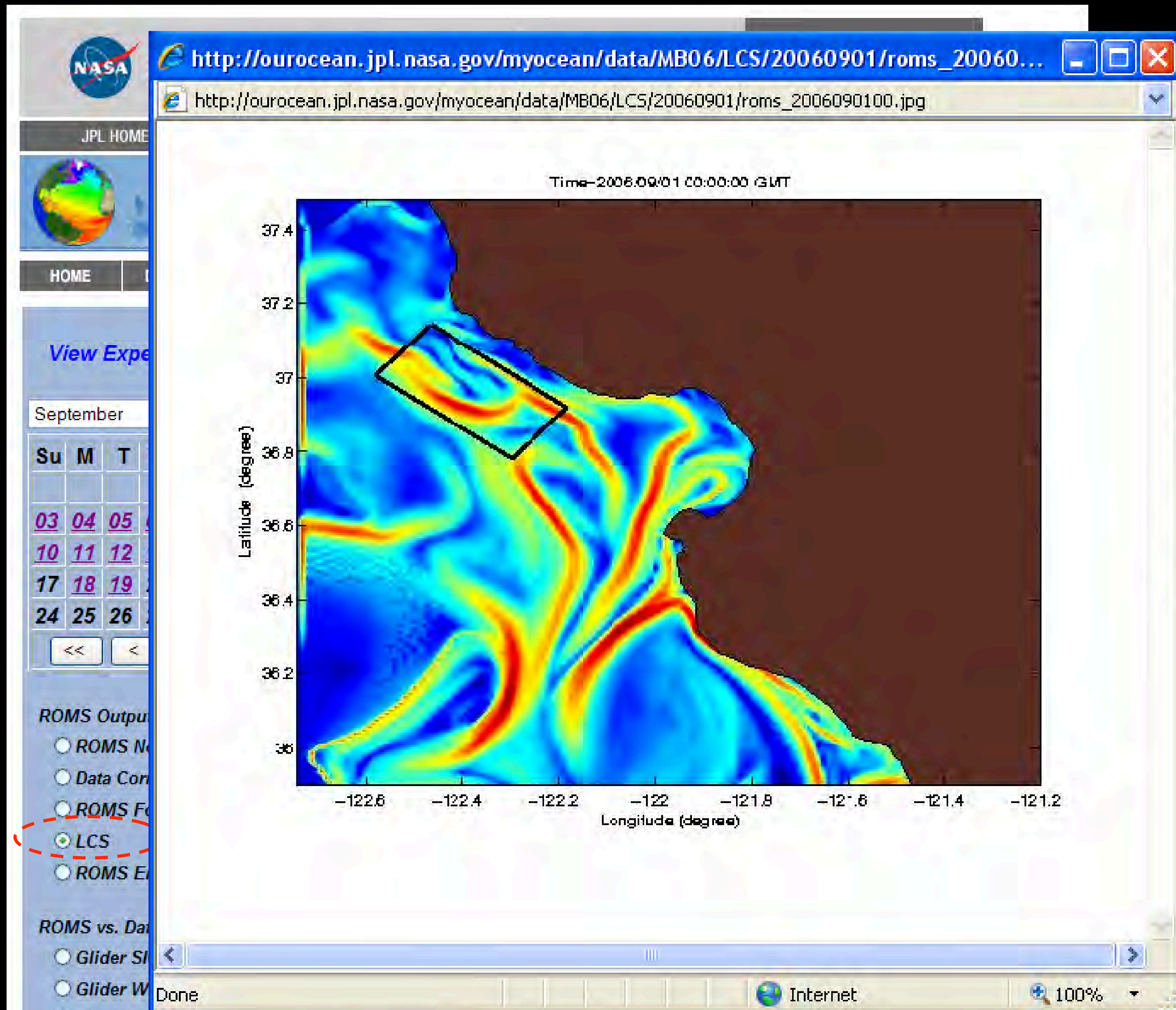
Animation

File Name	File Size	Download	View	
DLElevelsets_2006090100.dat	912092	http	File	Image
DLElevelsets_2006090101.dat	912086	http	File	Image
DLElevelsets_2006090102.dat	912084	http	File	Image
DLElevelsets_2006090103.dat	912066	http	File	Image
DLElevelsets_2006090104.dat	912043	http	File	Image
DLElevelsets_2006090105.dat	912035	http	File	Image
DLElevelsets_2006090106.dat	912026	http	File	Image
DLElevelsets_2006090107.dat	912029	http	File	Image
DLElevelsets_2006090108.dat	912032	http	File	Image
DLElevelsets_2006090109.dat	912050	http	File	Image
DLElevelsets_2006090110.dat	912042	http	File	Image

Peggy Li
Yi Chao

Near Real-Time Computation of LCS

http://ourocean.jpl.nasa.gov/MB06/



Peggy Li
Yi Chao

Upcoming LCS projects ...

Numerics:

- Visualization in 3D.
- Automatic extraction of LCS.
- Fast computation methods.

Theory:

- What are the properties of LCS?
- Relationship with transfer operator methods.

Applications:

- 3D Hurricanes.
- 3D Oceans.
- Quantified transport and mixing in geophysical flows.