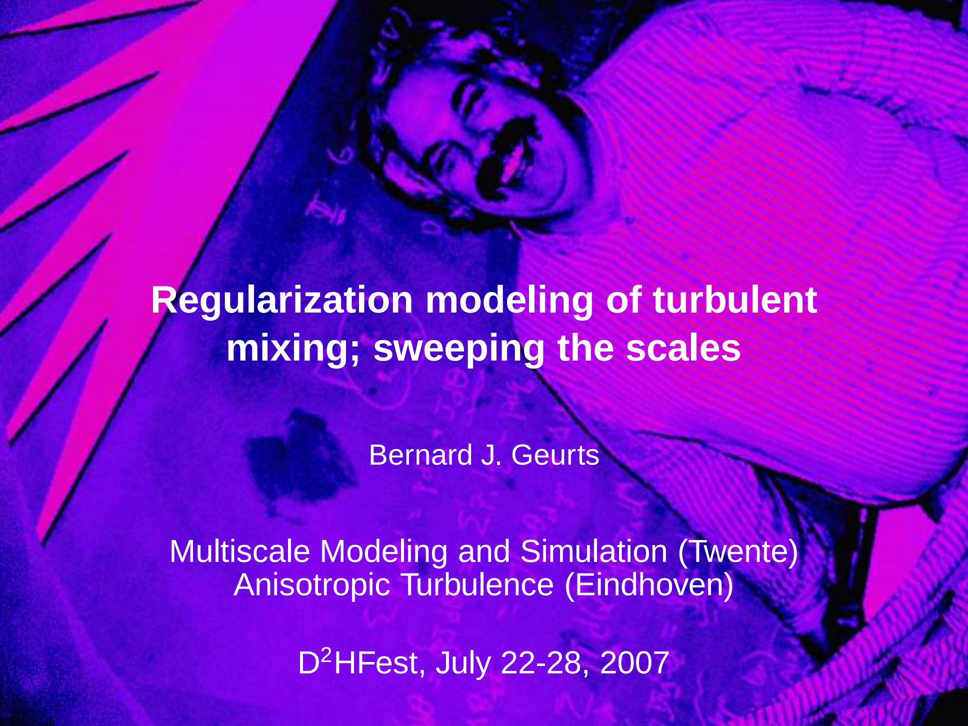


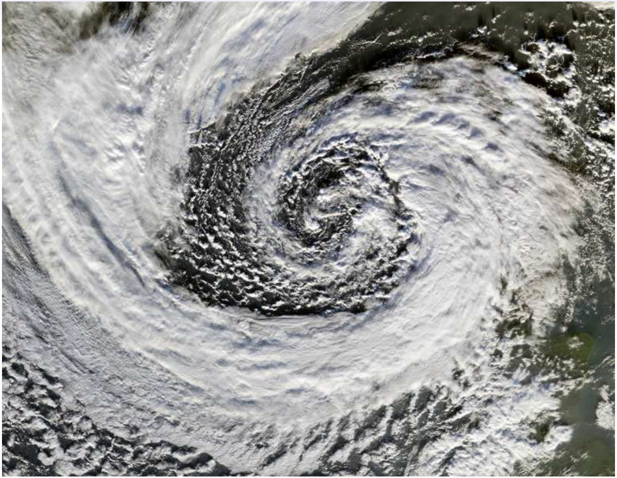
Regularization modeling of turbulent mixing; sweeping the scales

Bernard J. Geurts

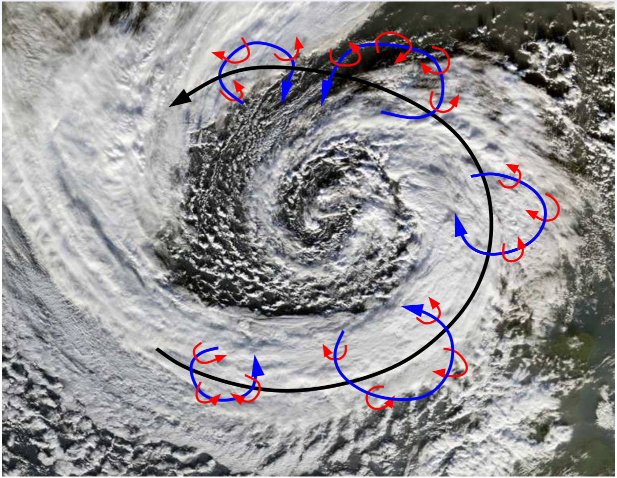
Multiscale Modeling and Simulation (Twente)
Anisotropic Turbulence (Eindhoven)

D^{2}HFest, July 22-28, 2007

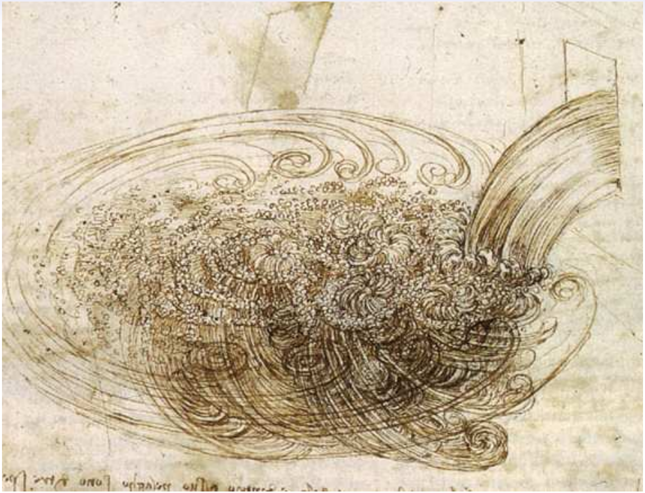
Turbulence — sweeping scales?



Cyclone near Iceland



Cyclone near Iceland



Early impression of swept scales - Da Vinci

Practical relevance - weather dynamics

Vortices in the far wake - airport congestion

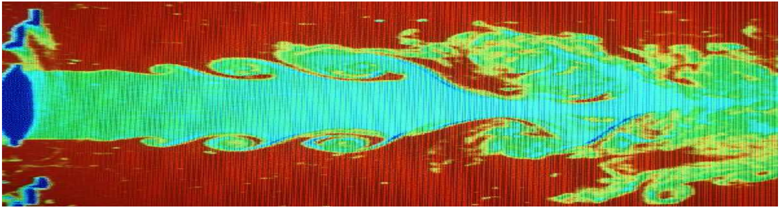
Outline

1. Turbulence and filtering
2. Regularization modeling of small scales
3. Numerics: friend or foe?
4. Mixing: combustion and stratification
5. Concluding remarks

Outline

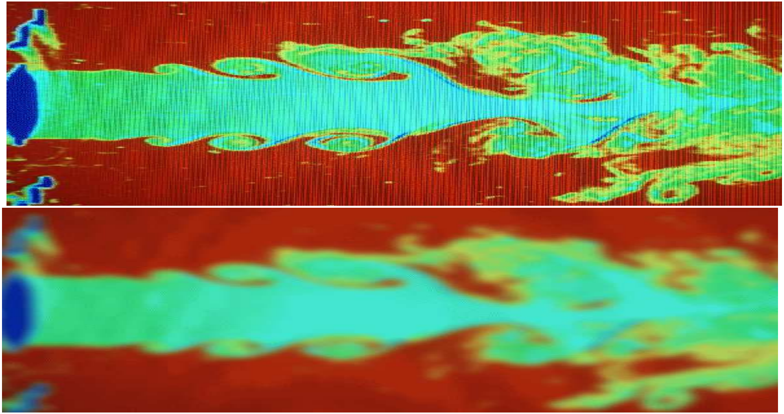
1. **Turbulence and filtering**
2. Regularization modeling of small scales
3. Numerics: friend or foe?
4. Mixing: combustion and stratification
5. Concluding remarks

DNS and LES in a picture



- capture both large and small scales: $W \sim Re^3 \sim N^4$
- Coarsening/mathematical modeling instead: LES

DNS and LES in a picture



- capture both large and small scales: $W \sim Re^3 \sim N^4$
- Coarsening/mathematical modeling instead: LES

Filtering Navier-Stokes equations

Convolution-Filtering: filter-kernel G, width Δ

$$\overline{u}_i = L(u_i) = \int G(x-\xi)u(\xi)\,d\xi$$

Application of filter:

$$\partial_t \overline{u}_i + \partial_j(\overline{u}_i\overline{u}_j) + \partial_i \overline{p} - \frac{1}{Re}\partial_{jj}\overline{u}_i = -\partial_j(\overline{u_i u_j} - \overline{u}_i\overline{u}_j)$$

Turbulent stress tensor:

$$\tau_{ij} = \overline{u_i u_j} - \overline{u}_i\overline{u}_j = L(u_i u_j) - L(u_i)L(u_j) = [L, \Pi_{ij}](\mathbf{u})$$

Properties τ:

- fully specified by filter L and NS-dynamics
- depends on u and $\overline{u}$ – closure problem

Filtering Navier-Stokes equations

Convolution-Filtering: filter-kernel G, width Δ

$$\overline{u}_i = L(u_i) = \int G(x-\xi)u(\xi)\, d\xi$$

Application of filter:

$$\partial_t \overline{u}_i + \partial_j(\overline{u}_i \overline{u}_j) + \partial_i \overline{p} - \frac{1}{Re}\partial_{jj}\overline{u}_i = -\partial_j(\overline{u_i u_j} - \overline{u}_i \overline{u}_j)$$

Turbulent stress tensor:

$$\tau_{ij} = \overline{u_i u_j} - \overline{u}_i \overline{u}_j = L(u_i u_j) - L(u_i)L(u_j) = [L, \Pi_{ij}](\mathbf{u})$$

Properties τ:

- fully specified by filter L and NS-dynamics
- depends on u and $\overline{u}$ – closure problem

Heuristic: Eddy-viscosity modeling

Obtain smoothing via increased dissipation:

$$\partial_t v_i + \partial_j(v_j v_i) + \partial_i P - \left(\frac{1}{Re} + \nu_t\right)\partial_{jj} v_i = 0$$

Damp large gradients: dimensional analysis

$$\nu_t = length \quad \times \quad velocity \sim \Delta \quad \times \quad \Delta|\partial_x v|$$

Effect: Strong damping at large filter-width and/or gradients

Heuristic: Eddy-viscosity modeling

Obtain smoothing via increased dissipation:

$$\partial_t v_i + \partial_j(v_j v_i) + \partial_i P - \left(\frac{1}{Re} + \nu_t\right)\partial_{jj} v_i = 0$$

Damp large gradients: dimensional analysis

$$\nu_t = \textit{length} \times \textit{velocity} \sim \Delta \times \Delta|\partial_x v|$$

Effect: Strong damping at large filter-width and/or gradients

Some explicit subgrid models

Popular models:

- Dissipation: Eddy-viscosity models, e.g., Smagorinsky

$$\tau_{ij} \to -\nu_t S_{ij} = -(C_S \Delta)^2 |S| S_{ij} \quad ; \quad \text{effect } \frac{1}{Re} \to \left(\frac{1}{Re} + \nu_t\right)$$

- Similarity: Inertial range, e.g., Bardina

$$\tau_{ij} \to [L, \Pi_{ij}](\overline{\mathbf{u}}) = \overline{\overline{u}_i \overline{u}_j} - \overline{\overline{u}}_i \overline{\overline{u}}_j$$

- Mixed models ?

$$m_{ij} = Bardina + C_d Smagorinsky$$

C_d via dynamic Germano-Lilly procedure

Are these models any good?

Some explicit subgrid models

Popular models:

- Dissipation: Eddy-viscosity models, e.g., Smagorinsky

$$\tau_{ij} \to -\nu_t S_{ij} = -(C_S \Delta)^2 |S| S_{ij} \quad ; \quad \text{effect } \frac{1}{Re} \to \left(\frac{1}{Re} + \nu_t\right)$$

- Similarity: Inertial range, e.g., Bardina

$$\tau_{ij} \to [L, \Pi_{ij}](\overline{\mathbf{u}}) = \overline{\overline{u}_i \overline{u}_j} - \overline{\overline{u}}_i \overline{\overline{u}}_j$$

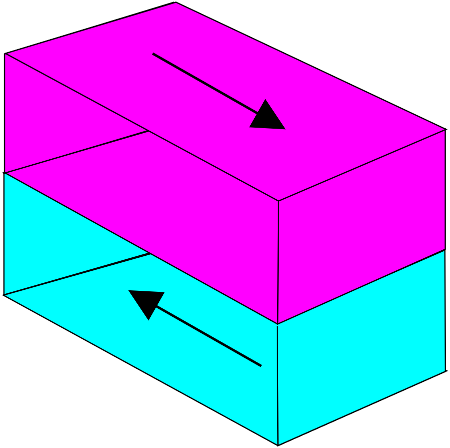
- Mixed models ?

$$m_{ij} = Bardina + C_d Smagorinsky$$

C_d via dynamic Germano-Lilly procedure

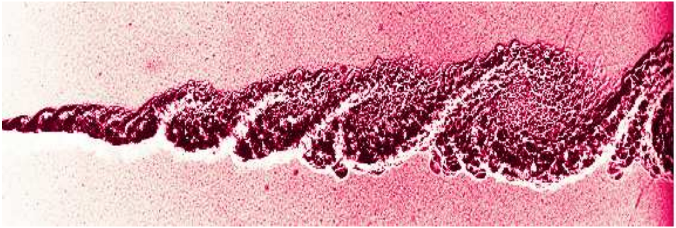
Are these models any good?

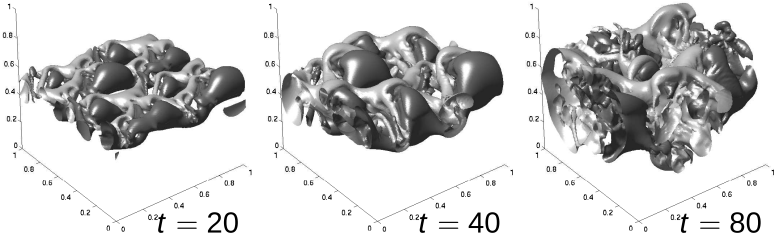
Mixing layer: testing ground for models



Temporal model for mixing layer

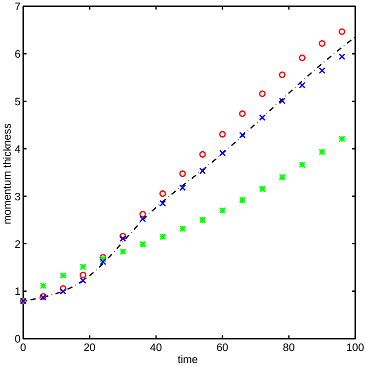
Mixing layer: testing ground for models

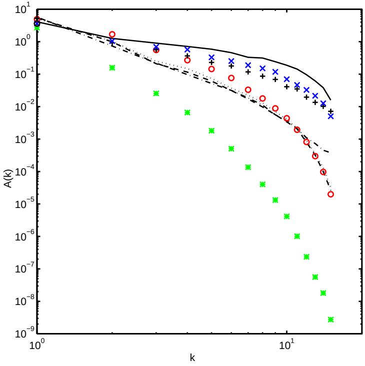




Temporal at different $t \approx$ Spatial at different x

Some basic flow properties

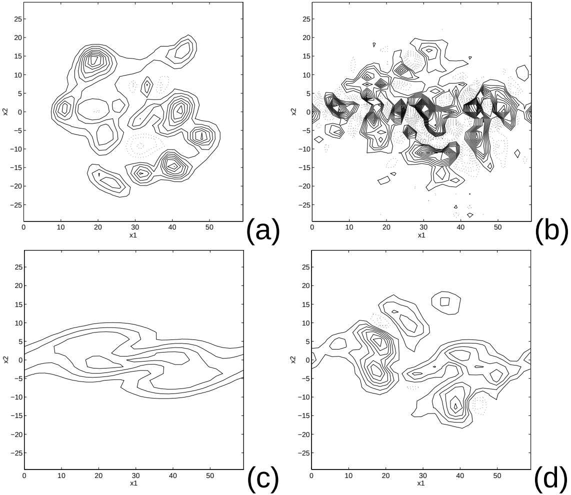




Momentum thickness and energy spectrum

- Smagorinsky too dissipative, Bardina not enough
- dynamic models quite accurate
- problems, e.g., intermediate and smallest scales

Instantaneous snapshots of spanwise vorticity



a: $\overline{DNS}$, b: Bardina, c: Smagorinsky, d: dynamic

Accuracy limited: regularization models better?

Outline

1. Turbulence and filtering
2. **Regularization modeling of small scales**
3. Numerics: friend or foe?
4. Mixing: combustion and stratification
5. Concluding remarks

Models and connection to filter

Central role of filter identified, but ...
... modeling rather heuristically, and ...
... connection with filter is only partial ..

Alternative – regularization models:

- ‘first principles’ approach – alter nonlinearity
- derive implied subgrid model
- achieve unique coupling to filter, e.g., Leray, LANS-α

Models and connection to filter

Central role of filter identified, but ...
... modeling rather heuristically, and ...
... connection with filter is only partial ..

Alternative – regularization models:

- ‘first principles’ approach – alter nonlinearity
- derive implied subgrid model
- achieve unique coupling to filter, e.g., Leray, LANS-α

Example: Leray regularization

Alter convective fluxes: sweeping by large scales

$$\partial_t u_i + \overline{u}_j \partial_j u_i + \partial_i p - \frac{1}{Re} \Delta u_i = 0$$

LES template:

$$\partial_t \overline{u}_i + \partial_j (\overline{u}_j \overline{u}_i) + \partial_i \overline{p} - \frac{1}{Re} \Delta \overline{u}_i = -\partial_j \left(m_{ij}^L \right)$$

Implied Leray model

$$m_{ij}^L = L\left(\overline{u}_j L^{-1}(\overline{u}_i)\right) - \overline{u}_j \overline{u}_i = \overline{\overline{u}_j u_i} - \overline{u}_j \overline{u}_i$$

- Rigorous analysis available ($\sim$ 1930s)
- Uniquely coupled to filter L and its inverse L^{-1}

Example: Leray regularization

Alter convective fluxes: sweeping by large scales

$$\partial_t u_i + \overline{u}_j \partial_j u_i + \partial_i p - \frac{1}{Re} \Delta u_i = 0$$

LES template:

$$\partial_t \overline{u}_i + \partial_j (\overline{u}_j \overline{u}_i) + \partial_i \overline{p} - \frac{1}{Re} \Delta \overline{u}_i = -\partial_j \left(m_{ij}^L \right)$$

Implied Leray model

$$m_{ij}^L = L\left(\overline{u}_j L^{-1}(\overline{u}_i) \right) - \overline{u}_j \overline{u}_i = \overline{\overline{u}_j u_i} - \overline{u}_j \overline{u}_i$$

- Rigorous analysis available ($\sim$ 1930s)
- Uniquely coupled to filter L and its inverse L^{-1}

TU/e

Example: Leray regularization

Alter convective fluxes: sweeping by large scales

$$\partial_t u_i + \overline{u}_j \partial_j u_i + \partial_i p - \frac{1}{Re}\Delta u_i = 0$$

LES template:

$$\partial_t \overline{u}_i + \partial_j(\overline{u}_j \overline{u}_i) + \partial_i \overline{p} - \frac{1}{Re}\Delta \overline{u}_i = -\partial_j \left(m_{ij}^L \right)$$

Implied Leray model

$$m_{ij}^L = L\left(\overline{u}_j L^{-1}(\overline{u}_i)\right) - \overline{u}_j \overline{u}_i = \overline{\overline{u}_j u_i} - \overline{u}_j \overline{u}_i$$

- Rigorous analysis available ($\sim$ 1930s)
- Uniquely coupled to filter L and its inverse L^{-1}

Approximate inversion procedures

- Geometric series: repeated filtering

$$L^{-1}(v) = (I - (I - L))^{-1}(v) \approx \sum_{n=0}^{N} (I - L)^n (v)$$

For example:

$$N = 0: \quad u = L_1^{-1}(v) = v$$

$$N = 1: \quad u = L_2^{-1}(v) = v + (I - L)(v) = 2v - \overline{v}$$

- Exact inversion of Simpson-top-hat: 3-point filters $(a, 1 - 2a, a)$

$$L^{-1}(v_m) \approx \sum_{j=-n}^{n} \left(\frac{a - 1 + \sqrt{1 - 2a}}{a} \right)^{|j|} \frac{v_{m+rj/2}}{(1 - 2a)^{1/2}}$$

Use: $a = 1/3$ and $r = \Delta/h = 2, 4, \ldots$ Convergence?

Approximate inversion procedures

- Geometric series: repeated filtering

$$L^{-1}(v) = (I - (I - L))^{-1}(v) \approx \sum_{n=0}^{N} (I - L)^n (v)$$

For example:

$$N = 0 : \qquad u = L_1^{-1}(v) = v$$

$$N = 1 : \quad u = L_2^{-1}(v) = v + (I - L)(v) = 2v - \overline{v}$$

- Exact inversion of Simpson-top-hat: 3-point filters $(a, 1 - 2a, a)$

$$L^{-1}(v_m) \approx \sum_{j=-n}^{n} \left(\frac{a - 1 + \sqrt{1 - 2a}}{a} \right)^{|j|} \frac{v_{m+rj/2}}{(1 - 2a)^{1/2}}$$

Use: $a = 1/3$ and $r = \Delta/h = 2, 4, \ldots$ Convergence?

Approximate inversion procedures

- Geometric series: repeated filtering

$$L^{-1}(v) = (I - (I - L))^{-1}(v) \approx \sum_{n=0}^{N} (I - L)^n (v)$$

For example:

$$N = 0: \qquad u = L_1^{-1}(v) = v$$

$$N = 1: \quad u = L_2^{-1}(v) = v + (I - L)(v) = 2v - \overline{v}$$

- Exact inversion of Simpson-top-hat: 3-point filters $(a, 1-2a, a)$

$$L^{-1}(v_m) \approx \sum_{j=-n}^{n} \left(\frac{a - 1 + \sqrt{1-2a}}{a} \right)^{|j|} \frac{v_{m+rj/2}}{(1-2a)^{1/2}}$$

Use: $a = 1/3$ and $r = \Delta/h = 2, 4, \ldots$ Convergence?

Approximate inversion procedures

- Geometric series: repeated filtering

$$L^{-1}(v) = (I - (I - L))^{-1}(v) \approx \sum_{n=0}^{N} (I - L)^n (v)$$

For example:

$$N = 0: \qquad u = L_1^{-1}(v) = v$$

$$N = 1: \quad u = L_2^{-1}(v) = v + (I - L)(v) = 2v - \overline{v}$$

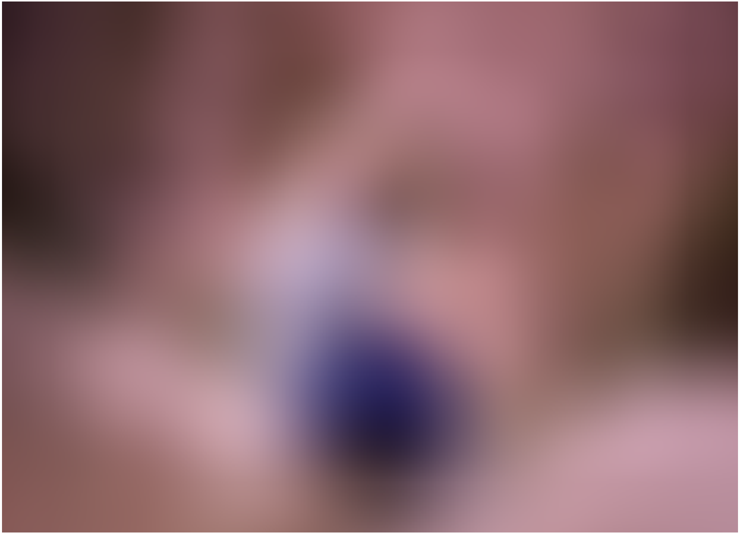
- Exact inversion of Simpson-top-hat: 3-point filters $(a, 1-2a, a)$

$$L^{-1}(v_m) \approx \sum_{j=-n}^{n} \left(\frac{a - 1 + \sqrt{1-2a}}{a} \right)^{|j|} \frac{v_{m+rj/2}}{(1-2a)^{1/2}}$$

Use: $a = 1/3$ and $r = \Delta/h = 2, 4, \ldots$ Convergence?

Refining - refining

Refining - refining



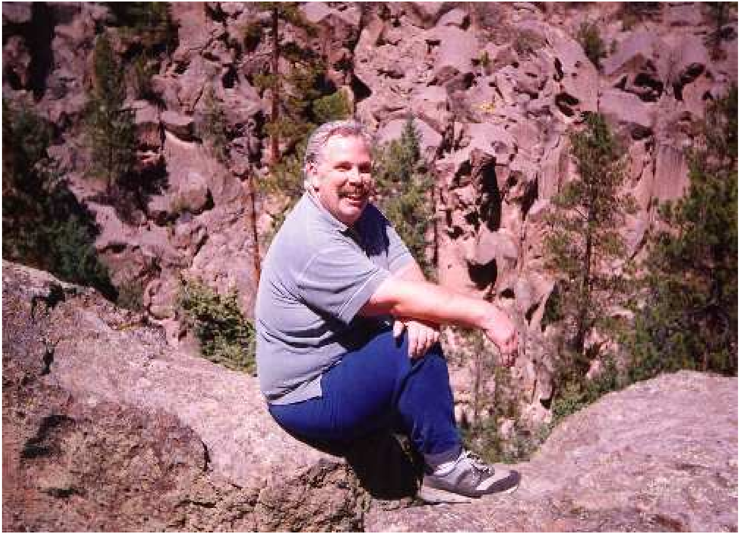
Refining - refining

Refining - refining

Refining - refining

Refining - refining

Refining - refining



NS-α regularization

Kelvin's circulation theorem

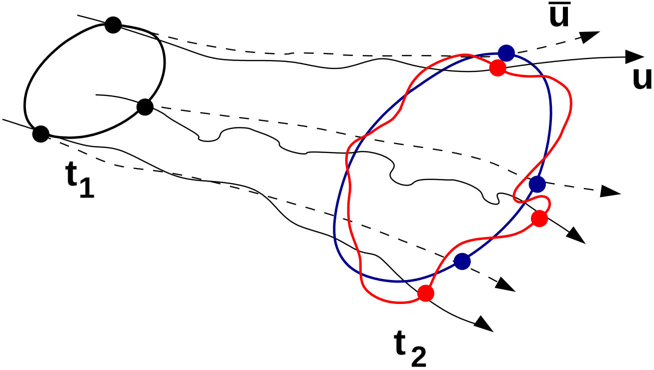
$$\frac{d}{dt}\left(\oint_{\Gamma(\mathbf{u})} u_j\,dx_j\right)-\frac{1}{Re}\oint_{\Gamma(\mathbf{u})}\partial_{kk}u_j\,dx_j=0\quad\Rightarrow\quad NS-eqs$$

NS-α regularization

Kelvin's circulation theorem

$$\frac{d}{dt}\Big(\oint_{\Gamma(\mathbf{u})} u_j\, dx_j\Big) - \frac{1}{Re}\oint_{\Gamma(\mathbf{u})} \partial_{kk} u_j\, dx_j = 0 \quad \Rightarrow \quad NS - eqs$$

Filtered Kelvin theorem ($\Gamma(\mathbf{u}) \to \Gamma(\overline{\mathbf{u}})$) extends Leray



NS-α regularization

α-principle: $\Gamma(\mathbf{u}) \to \Gamma(\overline{\mathbf{u}}) \Rightarrow$ Euler-Poincaré

$$\partial_t u_j + \overline{u}_k \partial_k u_j + u_k \partial_j \overline{u}_k + \partial_j p - \partial_j\left(\frac{1}{2}\overline{u}_k u_k\right) - \frac{1}{Re}\partial_{kk} u_j = 0$$

Rewrite into LES template: Implied subgrid model

$$\begin{aligned} \partial_t \overline{u}_i \quad &+ \quad \partial_j(\overline{u}_j \overline{u}_i) + \partial_i \overline{p} - \frac{1}{Re}\partial_{jj}\overline{u}_i \\ &= \quad -\partial_j\left(\overline{\overline{u}_j u_i} - \overline{u}_j \overline{u}_i\right) - \frac{1}{2}\left(\overline{u_j \partial_i \overline{u}_j - \overline{u}_j \partial_i u_j}\right) = -\partial_j m_{ij}^{\alpha} \end{aligned}$$

Alternative expression of large-scale sweeping

TU/e

NS-α regularization

α-principle: $\Gamma(\mathbf{u}) \to \Gamma(\overline{\mathbf{u}}) \Rightarrow$ Euler-Poincaré

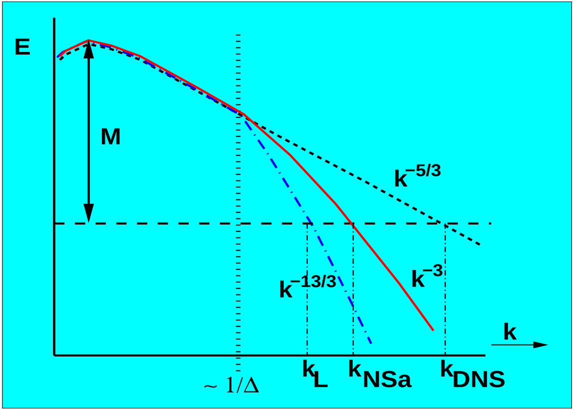
$$\partial_t u_j + \overline{u}_k \partial_k u_j + u_k \partial_j \overline{u}_k + \partial_j p - \partial_j (\frac{1}{2}\overline{u}_k u_k) - \frac{1}{Re}\partial_{kk} u_j = 0$$

Rewrite into LES template: Implied subgrid model

$$\begin{aligned}\partial_t \overline{u}_i &+ \partial_j(\overline{u}_j \overline{u}_i) + \partial_i \overline{p} - \frac{1}{Re}\partial_{jj}\overline{u}_i \\ &= -\partial_j\left(\overline{\overline{u}_j u_i} - \overline{u}_j \overline{u}_i\right) - \frac{1}{2}\left(\overline{u_j \partial_i \overline{u}_j - \overline{u}_j \partial_i u_j}\right) = -\partial_j m_{ij}^{\alpha}\end{aligned}$$

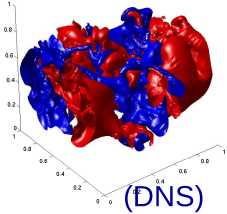
Alternative expression of large-scale sweeping

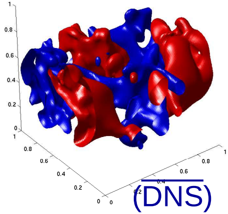
Cascade-dynamics – computability

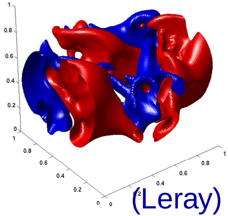


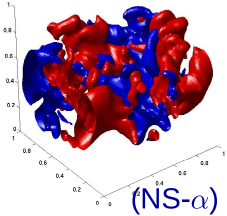
- NS-α,Leray are dispersive
- Regularization alters spectrum – controllable cross-over as $k \sim 1/\Delta$: steeper than $-5/3$

Leray and NS-α predictions: *Re* $= 50$, $\Delta = \ell/16$



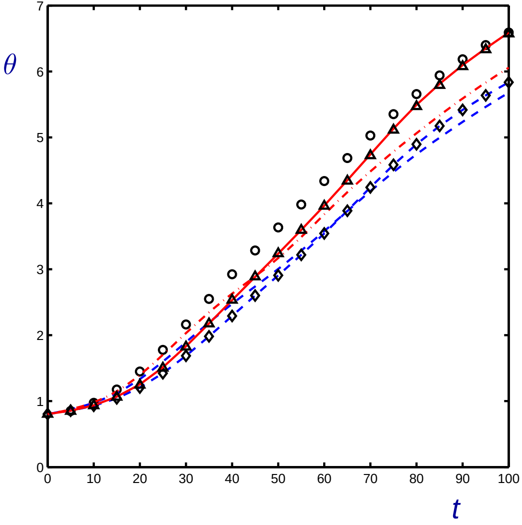






Snapshot u_2: red (blue) corresponds to up/down

Momentum thickness θ as $\Delta = \ell/16$



Filtered DNS (○)

Leray-model

32^3: dash-dotted

64^3: solid

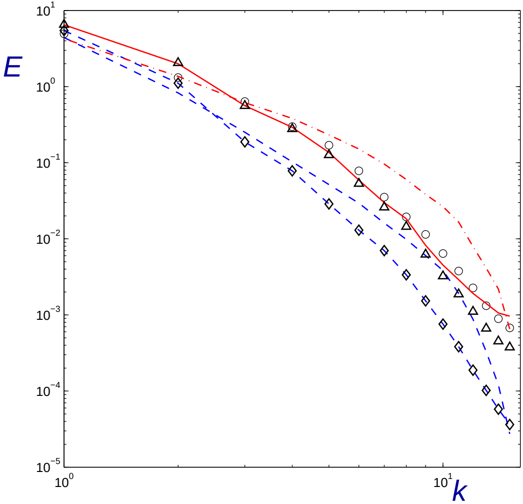
96^3: △

dynamic model

32^3: dashed

64^3: dashed with ⋄

Streamwise kinetic energy E as $\Delta = \ell/16$



Filtered DNS ($\circ$)

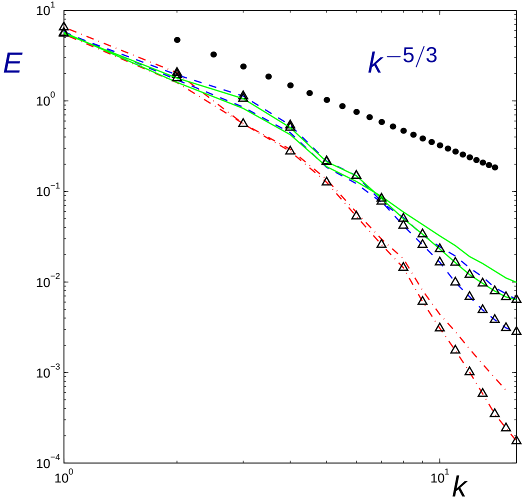
Leray-model

- 32^3: dash-dotted
- 64^3: solid
- 96^3: $\triangle$

dynamic model

- 32^3: dashed
- 64^3: dashed with $\diamond$

Robustness at arbitrary Reynolds number



$Re = 50$

64^3: dash-dotted

96^3: dash-dotted, $\triangle$

$Re = 500$

64^3: dashed

96^3: dashed, $\triangle$

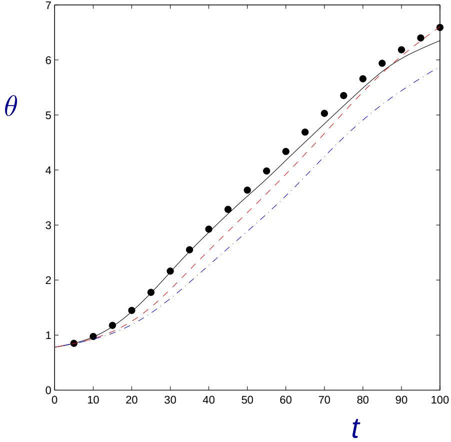
$Re = 5000$

64^3: solid

96^3: solid, $\triangle$

What about improvements arising from NS-α ?

Momentum thickness θ as $\Delta = \ell/16$



- NS-α (solid), Leray (dash), dynamic (dash-dotted), DNS •
- approximately grid-independent at 96^3

NS-α accurate but converges slowest

Outline

Numerics in academic LES setting

Goal: *approximate the unique solution to system of PDE's resulting after adopting explicit closure model*

General requirements:

- Filter separates scales $> \Delta$ from scales $< \Delta$
- Computational grid provides additional length-scale h
- Require Δ/h to be sufficiently large ($\Delta/h \to \infty$)
- Good numerics: $v(x, t : \Delta, h) \to v(x, t : \Delta, 0)$ rapidly

However:

- computational costs $\sim N^4$: implies modest Δ/h
- potentially large role of numerical method in computational dynamics because of marginal resolution

Numerics in academic LES setting

Goal: *approximate the unique solution to system of PDE's resulting after adopting explicit closure model*

General requirements:

- Filter separates scales $> \Delta$ from scales $< \Delta$
- Computational grid provides additional length-scale h
- Require Δ/h to be sufficiently large ($\Delta/h \to \infty$)
- Good numerics: $v(x,t:\Delta,h) \to v(x,t:\Delta,0)$ rapidly

However:

- computational costs $\sim N^4$: implies modest Δ/h
- potentially large role of numerical method in computational dynamics because of marginal resolution

Numerics in academic LES setting

Goal: *approximate the unique solution to system of PDE's resulting after adopting explicit closure model*

General requirements:

- Filter separates scales $> \Delta$ from scales $< \Delta$
- Computational grid provides additional length-scale h
- Require Δ/h to be sufficiently large ($\Delta/h \to \infty$)
- Good numerics: $v(x,t:\Delta,h) \to v(x,t:\Delta,0)$ rapidly

However:

- computational costs $\sim N^4$: implies modest Δ/h
- potentially large role of numerical method in computational dynamics because of marginal resolution

Numerics in academic LES setting

Goal: *approximate the unique solution to system of PDE's resulting after adopting explicit closure model*

General requirements:

- Filter separates scales $> \Delta$ from scales $< \Delta$
- Computational grid provides additional length-scale h
- Require Δ/h to be sufficiently large ($\Delta/h \to \infty$)
- Good numerics: $v(x, t : \Delta, h) \to v(x, t : \Delta, 0)$ rapidly

However:

- computational costs $\sim N^4$: implies modest Δ/h
- potentially large role of numerical method in computational dynamics because of marginal resolution

Numerics in academic LES setting

Goal: *approximate the unique solution to system of PDE's resulting after adopting explicit closure model*

General requirements:

- Filter separates scales $> \Delta$ from scales $< \Delta$
- Computational grid provides additional length-scale h
- Require Δ/h to be sufficiently large ($\Delta/h \to \infty$)
- Good numerics: $v(x, t : \Delta, h) \to v(x, t : \Delta, 0)$ rapidly

However:

- computational costs $\sim N^4$: implies modest Δ/h
- potentially large role of numerical method in computational dynamics because of marginal resolution

Numerics in academic LES setting

Goal: *approximate the unique solution to system of PDE's resulting after adopting explicit closure model*

General requirements:

- Filter separates scales $> \Delta$ from scales $< \Delta$
- Computational grid provides additional length-scale h
- Require Δ/h to be sufficiently large ($\Delta/h \to \infty$)
- Good numerics: $v(x,t:\Delta,h) \to v(x,t:\Delta,0)$ rapidly

However:

- computational costs $\sim N^4$: implies modest Δ/h
- potentially large role of numerical method in computational dynamics because of marginal resolution

TU/e

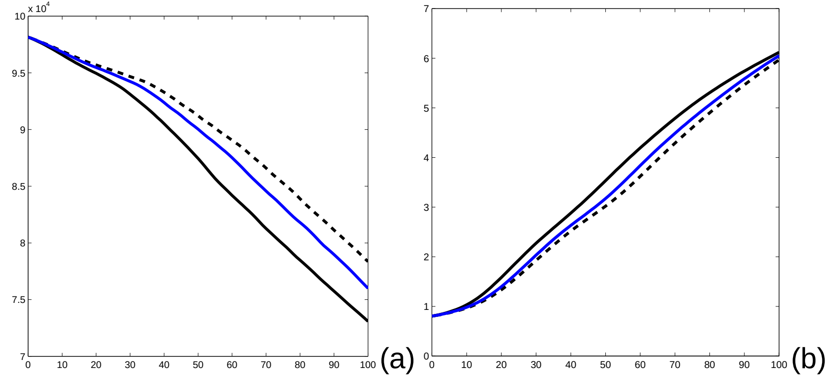
Numerical aspects

Study influence of:

- Filter inversion
- Spatial discretization method

Also study error interactions/cancellations ...

Quality of filter-inversion – Leray

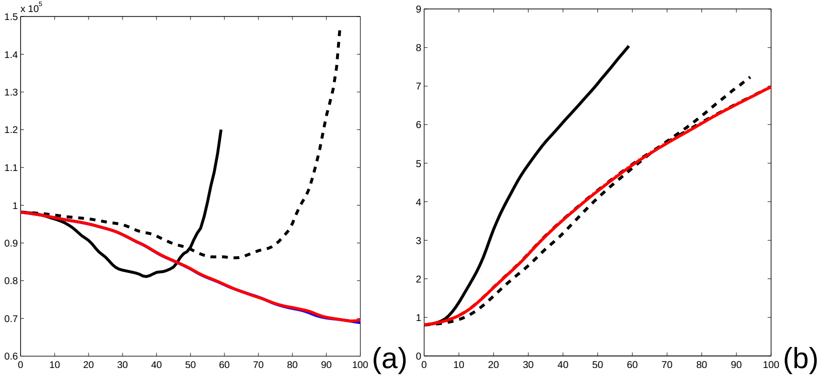


Inversion of Simpson top-hat filter:

- Kinetic energy (a) and momentum thickness (b)
- $n = 1$ (solid), $n = 2$ (dash), $n \geq 5$

Quality of filter-inversion – LANS-α

Kinetic energy E and momentum thickness D



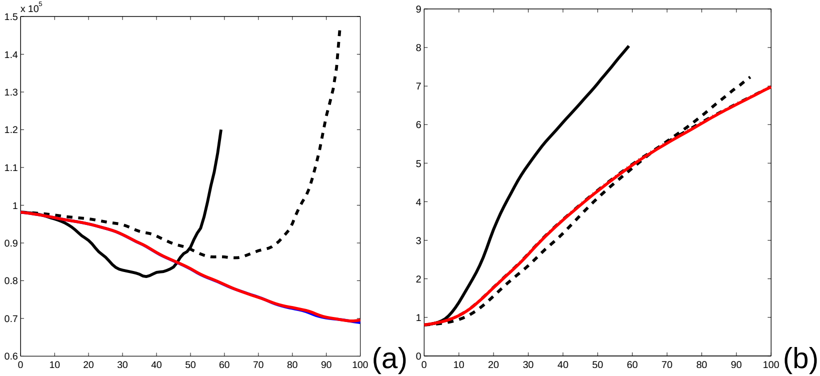
Inversion of Simpson top-hat filter:

- $n = 1$ (solid), $n = 2$ (dash), $n = 5$, $n \geq 10$

Conclusion: $n \geq 5$

Quality of filter-inversion – LANS-α

Kinetic energy E and momentum thickness D



Inversion of Simpson top-hat filter:

- $n = 1$ (solid), $n = 2$ (dash), $n = 5$, $n \geq 10$

Conclusion: $n \geq 5$

Spatial discretization

Discretization induces spatial filter:

$$\delta_x f(x) = \partial_x(\widehat{f})$$

- modified mean flux
- modified turbulent stress tensor

Compare:

- Second order FV: $\delta_x^{(2)}(f(u))$
- Fourth order FV: $\delta_x^{(4)}(f(u))$
- Combination treats small scales at second order

$$\delta_x(f(u)) = \delta_x^{(4)}(f(\overline{u})) + \delta_x^{(2)}(f(u) - f(\overline{u}))$$

Spatial discretization

Discretization induces spatial filter:

$$\delta_x f(x) = \partial_x(\widehat{f})$$

- modified mean flux
- modified turbulent stress tensor

Compare:

- Second order FV: $\delta_x^{(2)}(f(u))$
- Fourth order FV: $\delta_x^{(4)}(f(u))$
- Combination treats small scales at second order

$$\delta_x(f(u)) = \delta_x^{(4)}(f(\overline{u})) + \delta_x^{(2)}(f(u) - f(\overline{u}))$$

Spatial discretization

Discretization induces spatial filter:

$$\delta_x f(x) = \partial_x(\widehat{f})$$

- modified mean flux
- modified turbulent stress tensor

Compare:

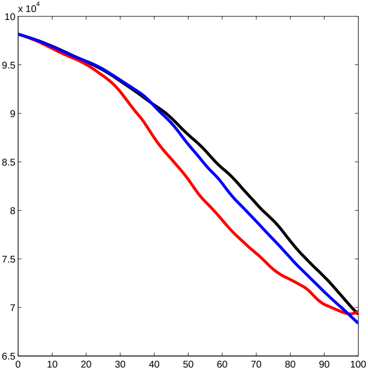
- Second order FV: $\delta_x^{(2)}(f(u))$
- Fourth order FV: $\delta_x^{(4)}(f(u))$
- Combination treats small scales at second order

$$\delta_x(f(u)) = \delta_x^{(4)}(f(\overline{u})) + \delta_x^{(2)}(f(u) - f(\overline{u}))$$

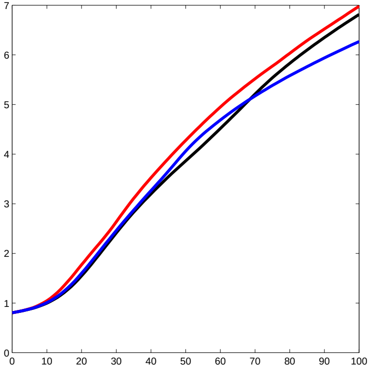
TU/e

LANS-α: sensitive to discretization

$N = 32$: Kinetic energy E and momentum thickness D



(a)

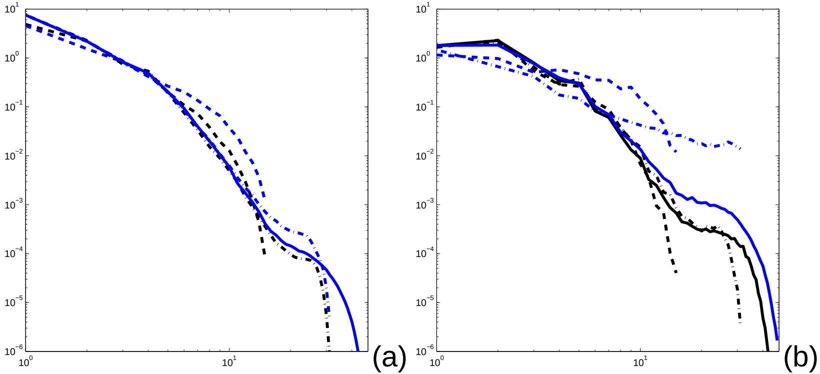


(b)

- Second order, Fourth order, (4-2) method

Spectral signature of numerics

Leray and LANS-α spectra:



- Second order, Fourth order
- $N = 32$ (dash), $N = 64$ (dash-dot), $N = 96$ (solid)

Numerics or modeling or both ?

Observe:

- At marginal resolution numerics modifies the equations
- Likewise, a subgrid model modifies these equations

Dilemma: which is to be preferred

- Not just best numerics/best model ... LES paradoxes
- Governed by error-interactions: nonlinear error-accumulation

Numerics or modeling or both ?

Observe:

- At marginal resolution numerics modifies the equations
- Likewise, a subgrid model modifies these equations

Dilemma: which is to be preferred

- Not just best numerics/best model ... LES paradoxes
- Governed by error-interactions: nonlinear error-accumulation

Numerics or modeling or both ?

Observe:

- At marginal resolution numerics modifies the equations
- Likewise, a subgrid model modifies these equations

Dilemma: which is to be preferred

- Not just best numerics/best model ... LES paradoxes
- Governed by error-interactions: nonlinear error-accumulation

Numerics or modeling or both ?

Observe:

- At marginal resolution numerics modifies the equations
- Likewise, a subgrid model modifies these equations

Dilemma: which is to be preferred

- Not just best numerics/best model ... LES paradoxes
- Governed by error-interactions: nonlinear error-accumulation

Numerics or modeling or both ?

Observe:

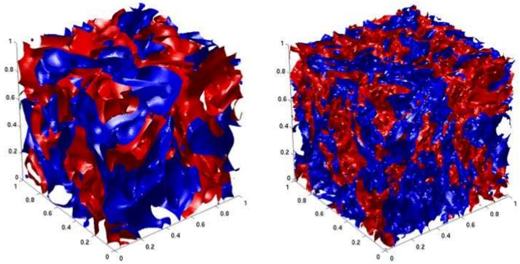
- At marginal resolution numerics modifies the equations
- Likewise, a subgrid model modifies these equations

Dilemma: which is to be preferred

- Not just best numerics/best model ... LES paradoxes
- Governed by error-interactions: nonlinear error-accumulation

Smagorinsky fluid

Homogeneous decaying turbulence at $Re_\lambda = 50,\ 100$



- Smagorinsky fluid — subgrid model:

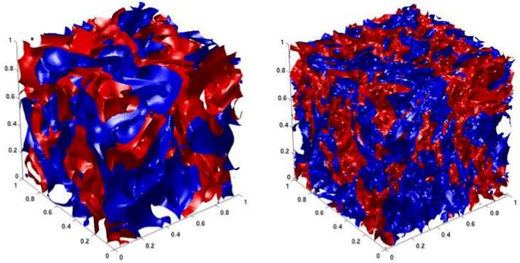
$$m_{ij}^S = -2(C_S\overline{\Delta})^2|\overline{S}|\overline{S}_{ij} = -2\ell_S^2|\overline{S}|\overline{S}_{ij}$$

introduces Smagorinsky-length ℓ_S

- Also dynamic Smagorinsky fluid with length-scale ℓ_d

Smagorinsky fluid

Homogeneous decaying turbulence at $Re_\lambda = 50,\ 100$



- Smagorinsky fluid — subgrid model:

$$m_{ij}^S = -2(C_S\overline{\Delta})^2|\overline{S}|\overline{S}_{ij} = -2\ell_S^2|\overline{S}|\overline{S}_{ij}$$

  introduces Smagorinsky-length ℓ_S

- Also dynamic Smagorinsky fluid with length-scale ℓ_d

Experimental error-assessment

Pragmatic: minimal total error at given computational costs

Discuss:

- error-landscape/optimal refinement strategy
- LES-specific error-minimization: SIPI

TU/e

Experimental error-assessment

Pragmatic: minimal total error at given computational costs

Discuss:

- error-landscape/optimal refinement strategy
- LES-specific error-minimization: SIPI

Accuracy measures

Monitor resolved kinetic energy

$$E = \frac{1}{|\Omega|} \int_{\Omega} \frac{1}{2} \overline{\mathbf{u}} \cdot \overline{\mathbf{u}} \, d\mathbf{x} = \frac{1}{2} \langle \overline{\mathbf{u}} \cdot \overline{\mathbf{u}} \rangle$$

Measure relative error: top-hat filter Δ, grid $h = \Delta / r$

$$\delta_E(\Delta, r) = \left\| \frac{E_{LES}(\Delta, r) - E_{\overline{DNS}}(\Delta, r)}{E_{\overline{DNS}}(\Delta, r)} \right\|$$

with error integrated over time

$$\|f\|^2 = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} f^2(t) dt$$

- each simulation represented by **single** number
- concise representation facilitates comparison

Accuracy measures

Monitor resolved kinetic energy

$$E = \frac{1}{|\Omega|} \int_{\Omega} \frac{1}{2} \overline{\mathbf{u}} \cdot \overline{\mathbf{u}} \, d\mathbf{x} = \frac{1}{2} \langle \overline{\mathbf{u}} \cdot \overline{\mathbf{u}} \rangle$$

Measure relative error: top-hat filter Δ, grid $h = \Delta / r$

$$\delta_E(\Delta, r) = \left\| \frac{E_{LES}(\Delta, r) - E_{\overline{DNS}}(\Delta, r)}{E_{\overline{DNS}}(\Delta, r)} \right\|$$

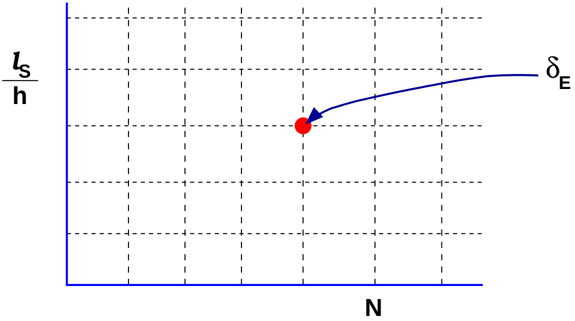
with error integrated over time

$$\|f\|^2 = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} f^2(t) dt$$

- each simulation represented by **single** number
- concise representation facilitates comparison

Error-landscape: Definition

Framework for collecting error information:



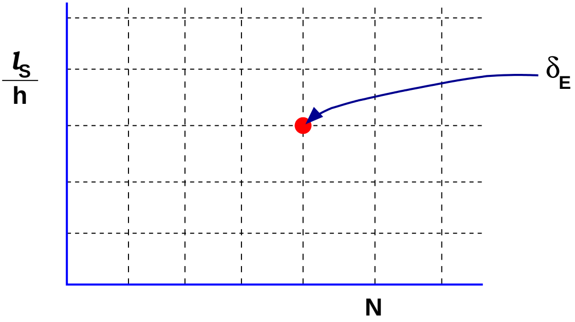
Each Smagorinsky LES corresponds to single point:

$$\left(N, \frac{\ell_S}{h}\right) \quad ; \quad error \, : \quad \delta_E$$

Contours of δ_E — fingerprint of LES

Error-landscape: Definition

Framework for collecting error information:

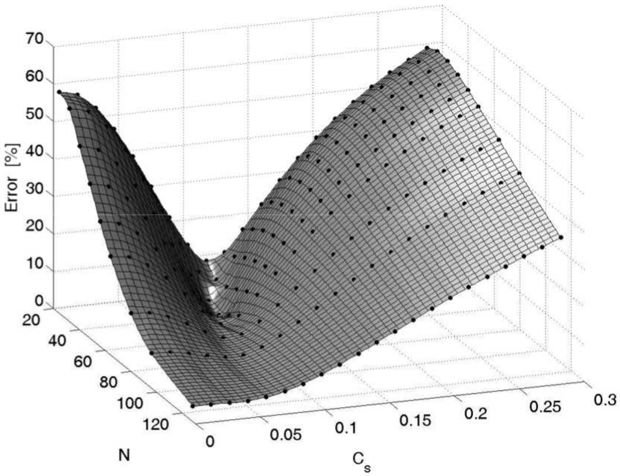


Each Smagorinsky LES corresponds to single point:

$$\left(N, \frac{\ell_S}{h}\right) \quad ; \quad error : \quad \delta_E$$

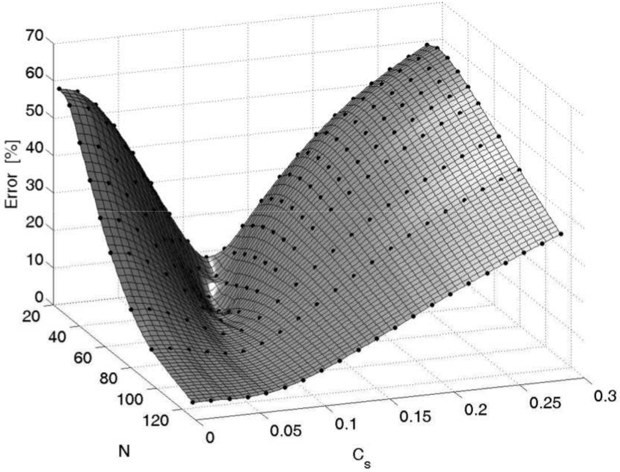
Contours of δ_E — fingerprint of LES

Total error-landscape



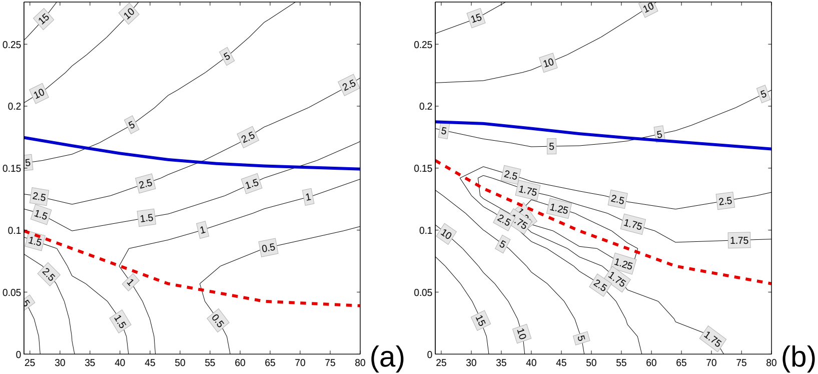
- combination of central discretization and Smagorinsky
- optimum at $C_S > 0$: SGS modeling is viable here

Total error-landscape



- combination of central discretization and Smagorinsky
- optimum at $C_S > 0$: SGS modeling is viable here

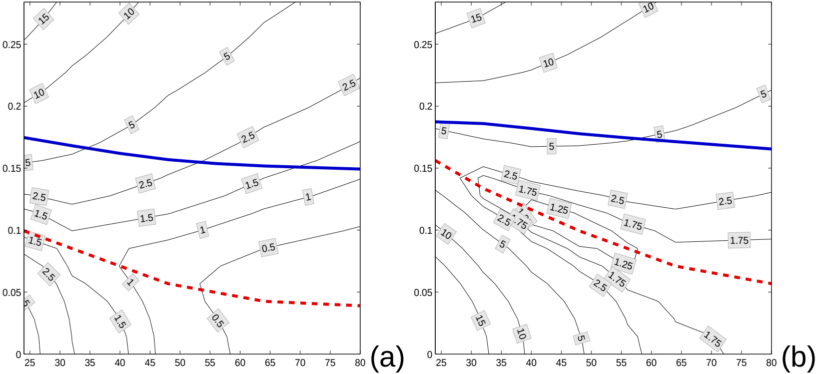
Error-landscape: optimal refinement



Optimal trajectory for $Re_\lambda = 50$ (a) and $Re_\lambda = 100$ (b)

- Under-resolution leads to strong error-increase
- Dynamic procedure over-estimates viscosity

Error-landscape: optimal refinement



Optimal trajectory for $Re_\lambda = 50$ (a) and $Re_\lambda = 100$ (b)

- Under-resolution leads to strong error-increase
- Dynamic procedure over-estimates viscosity

MILES philosophy

Observation:

- practical LES implies marginal resolution
- which implies large role of specific numerical discretization
- next to dynamics due to subgrid model
- and leads to strong interactions and complex error-accumulation

Proposal:

- obtain smoothing via appropriate numerical method alone
- accept that there is no grid-independent solution, other than DNS
- accept that predictions become discretization dependent

Is ‘no-model/just numerics’ option optimal/viable ?

Consider example: DG-FEM and homogeneous turbulence

MILES philosophy

Observation:

- practical LES implies marginal resolution
- which implies large role of specific numerical discretization
- next to dynamics due to subgrid model
- and leads to strong interactions and complex error-accumulation

Proposal:

- obtain smoothing via appropriate numerical method alone
- accept that there is no grid-independent solution, other than DNS
- accept that predictions become discretization dependent

Is ‘no-model/just numerics’ option optimal/viable ?

Consider example: DG-FEM and homogeneous turbulence

MILES philosophy

Observation:

- practical LES implies marginal resolution
- which implies large role of specific numerical discretization
- next to dynamics due to subgrid model
- and leads to strong interactions and complex error-accumulation

Proposal:

- obtain smoothing via appropriate numerical method alone
- accept that there is no grid-independent solution, other than DNS
- accept that predictions become discretization dependent

Is ‘no-model/just numerics’ option optimal/viable ?

Consider example: DG-FEM and homogeneous turbulence

MILES philosophy

Observation:

- practical LES implies marginal resolution
- which implies large role of specific numerical discretization
- next to dynamics due to subgrid model
- and leads to strong interactions and complex error-accumulation

Proposal:

- obtain smoothing via appropriate numerical method alone
- accept that there is no grid-independent solution, other than DNS
- accept that predictions become discretization dependent

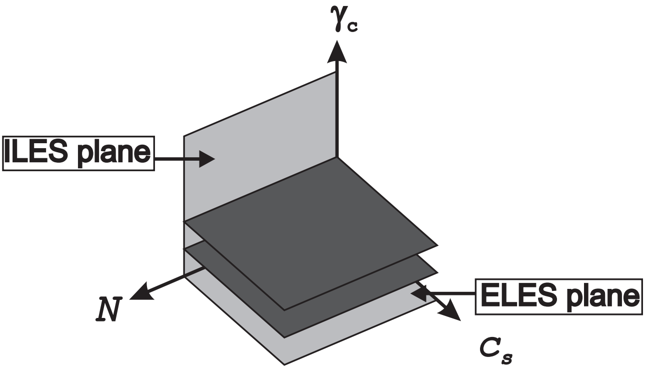
Is ‘no-model/just numerics’ option optimal/viable ?

Consider example: DG-FEM and homogeneous turbulence

DG-FEM of homogeneous turbulence

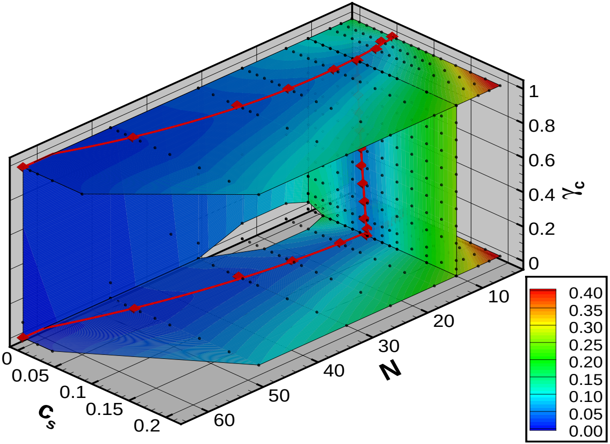
Discretization: Approximate Riemann solver

$$F = F_{central} + \gamma F_{dissipative} \quad ; \quad HLLC - flux$$



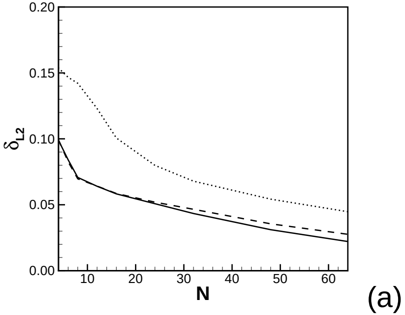
Three-dimensional accuracy charts

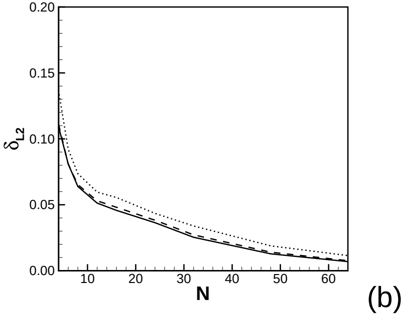
LES with DG-FEM: dissipative numerics



Third order DG-FEM at $Re_\lambda = 100$: red symbols - optimal setting

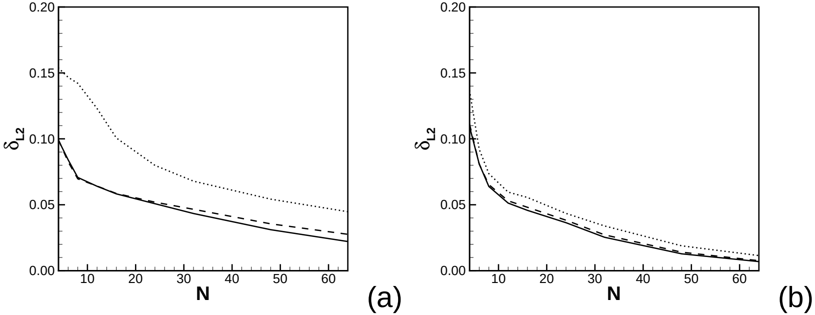
Optimality of MILES ?





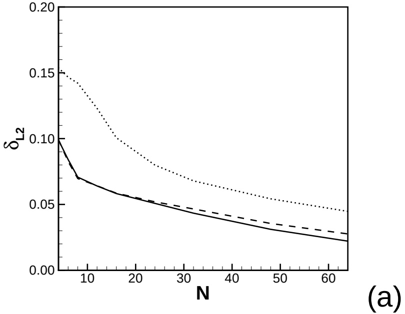
- (a): 2nd order ; (b): 3rd order
- $\gamma_c = 1.00$ (dot), $\gamma_c = 0.10$ (dash) and $\gamma_c = 0.01$ (–)
- MILES-error larger than with explicit SGS model
- Exchange of dissipation in optimum ...
- Option: directly optimize total simulation error - SIPI

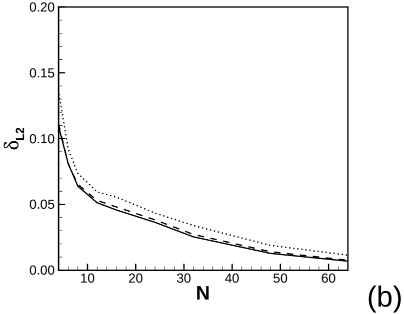
Optimality of MILES ?



- (a): 2nd order ; (b): 3rd order
- $\gamma_c = 1.00$ (dot), $\gamma_c = 0.10$ (dash) and $\gamma_c = 0.01$ (–)
- MILES-error larger than with explicit SGS model
- Exchange of dissipation in optimum ...
- Option: directly optimize total simulation error - SIPI

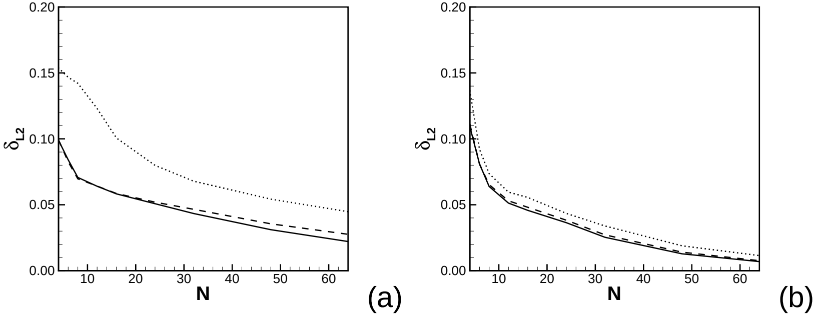
Optimality of MILES ?





- (a): 2nd order ; (b): 3rd order
- $\gamma_c = 1.00$ (dot), $\gamma_c = 0.10$ (dash) and $\gamma_c = 0.01$ (–)
- MILES-error larger than with explicit SGS model
- Exchange of dissipation in optimum ...
- Option: directly optimize total simulation error - SIPI

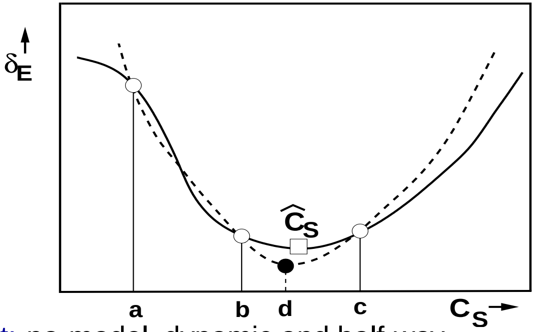
Optimality of MILES ?



- (a): 2nd order ; (b): 3rd order
- $\gamma_c = 1.00$ (dot), $\gamma_c = 0.10$ (dash) and $\gamma_c = 0.01$ (–)
- MILES-error larger than with explicit SGS model
- Exchange of dissipation in optimum ...
- Option: directly optimize total simulation error - SIPI

SIPI - basic algorithm

Goal: minimize total error at given N



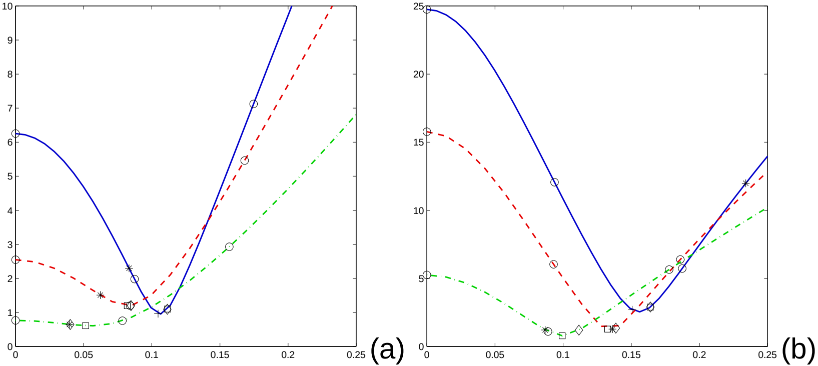
Initial triplet: no-model, dynamic and half-way

New iterand

$$d = b - \frac{1}{2} \frac{(b-a)^2[\delta_E(b) - \delta_E(c)] - (b-c)^2[\delta_E(b) - \delta_E(a)]}{(b-a)[\delta_E(b) - \delta_E(c)] - (b-c)[\delta_E(b) - \delta_E(a)]}$$

SIPI applied to homogeneous turbulence

Each iteration = separate simulation



$Re_\lambda = 50$ (a) and $Re_\lambda = 100$ (b). Resolutions $N = 24$ (solid), $N = 32$ (dashed) and $N = 48$ (dash-dotted)

Iterations: $\circ \longrightarrow * \longrightarrow \diamond \longrightarrow \Box \longrightarrow +$

Convergence example

	$Re_\lambda = 50$		$Re_\lambda = 100$	
n	$C_S^{(n)}(24)$	$C_S^{(n)}(48)$	$C_S^{(n)}(24)$	$C_S^{(n)}(48)$
2	0.17470000	0.15690000	0.18740000	0.17780000
3	0.08735000	0.07845000	0.09370000	0.08890000
4	0.08330619	0.03959082	0.23399921	0.08686309
5	0.11142326	0.04010420	0.16404443	0.11154788
6	0.11156545	0.05141628	0.16400079	0.09935076
7	0.10461386	0.05283713	0.15068033	0.09947629
8	0.10508422	0.05468589	0.15445586	0.09926872
9	0.10602797	0.05504089	0.15600828	0.09938967

- Computational overhead SIPI: CPU-time $T \sim N^4$ implies approximate optimization at N can be (almost) completed within cost of one simulation at $3N/2$

Outline

1. Turbulence and filtering
2. Regularization modeling of small scales
3. Numerics: friend or foe?
4. **Mixing: combustion and stratification**
5. Concluding remarks

Application to complex physics

Intuitive subgrid-scale modeling:

- collection of assumptions/parameters increases
- consistency becomes problem
- predictive power ?

Can first principles approach be extended?

Consider combustion and stratified mixing

Application to complex physics

Intuitive subgrid-scale modeling:

- collection of assumptions/parameters increases
- consistency becomes problem
- predictive power ?

Can first principles approach be extended?

Consider combustion and stratified mixing

Application to complex physics

Intuitive subgrid-scale modeling:

- collection of assumptions/parameters increases
- consistency becomes problem
- predictive power ?

Can first principles approach be extended?

Consider combustion and stratified mixing

Example: Turbulent combustion

Computational model:

$$\partial_t \rho + \partial_j(\rho u_j) = 0$$
$$\partial_t(\rho u_i) + \partial_j(\rho u_i u_j) + \partial_i p - \partial_j \sigma_{ij} = 0$$
$$\partial_t e + \partial_j((e+p)u_j) - \partial_j(\sigma_{ij} u_i) + \partial_j q_j - h_k \omega_k = 0$$
$$\partial_t(\rho c_k) + \partial_j(\rho c_k u_j) - \partial_j(\pi_{kj}) - \omega_k = 0 \quad ; \quad k = 1, \ldots, N_s$$

F-O-P flame: Arrhenius law $F + O \rightarrow P$

$$\omega_F = -(\rho c_F)(\rho c_O)\frac{Da}{W_O}\exp(-\frac{Ze}{T})$$
$$\omega_O = \alpha \omega_F \quad ; \quad \omega_P = -(1+\alpha)\omega_F$$

- Damköhler: Da and Zeldovich: Ze
- weight-ratio $\alpha = W_O/W_F$

Energy equation source term: $h_j \omega_j = Q\omega_F$

Example: Turbulent combustion

Computational model:

$$\partial_t \rho + \partial_j(\rho u_j) = 0$$
$$\partial_t(\rho u_i) + \partial_j(\rho u_i u_j) + \partial_i p - \partial_j \sigma_{ij} = 0$$
$$\partial_t e + \partial_j((e+p)u_j) - \partial_j(\sigma_{ij}u_i) + \partial_j q_j - h_k \omega_k = 0$$
$$\partial_t(\rho c_k) + \partial_j(\rho c_k u_j) - \partial_j(\pi_{kj}) - \omega_k = 0 \quad ; \quad k = 1, \ldots, N_s$$

F-O-P flame: Arrhenius law $F + O \to P$

$$\omega_F = -(\rho c_F)(\rho c_O)\frac{Da}{W_O}\exp(-\frac{Ze}{T})$$
$$\omega_O = \alpha\omega_F \quad ; \quad \omega_P = -(1+\alpha)\omega_F$$

- Damköhler: *Da* and Zeldovich: *Ze*
- weight-ratio $\alpha = W_O/W_F$

Energy equation source term: $h_j\omega_j = Q\omega_F$

Example: Turbulent combustion

Computational model:

$$\partial_t \rho + \partial_j(\rho u_j) = 0$$
$$\partial_t(\rho u_i) + \partial_j(\rho u_i u_j) + \partial_i p - \partial_j \sigma_{ij} = 0$$
$$\partial_t e + \partial_j((e+p)u_j) - \partial_j(\sigma_{ij} u_i) + \partial_j q_j - h_k \omega_k = 0$$
$$\partial_t(\rho c_k) + \partial_j(\rho c_k u_j) - \partial_j(\pi_{kj}) - \omega_k = 0 \quad ; \quad k = 1, \ldots, N_s$$

F-O-P flame: Arrhenius law $F + O \rightarrow P$

$$\omega_F = -(\rho c_F)(\rho c_O)\frac{Da}{W_O}\exp(-\frac{Ze}{T})$$
$$\omega_O = \alpha\omega_F \quad ; \quad \omega_P = -(1+\alpha)\omega_F$$

- Damköhler: Da and Zeldovich: Ze
- weight-ratio $\alpha = W_O/W_F$

Energy equation source term: $h_j\omega_j = Q\omega_F$

Spatial filtering and subgrid closure

Filtered model equations: ($\overline{\rho}$, $\widetilde{u}_j$, $\overline{p}$ and $\widetilde{c}_k$)

$$\partial_t \overline{\rho} + \partial_j(\overline{\rho}\widetilde{u}_j) = 0$$
$$\partial_t(\overline{\rho}\widetilde{u}_i) + \partial_j(\overline{\rho}\widetilde{u}_i\widetilde{u}_j) + \partial_i\overline{p} - \partial_j\breve{\sigma}_{ij} = -\partial(\overline{\rho}\tau_{ij}) + \partial_j(\overline{\sigma}_{ij} - \breve{\sigma}_{ij})$$
$$\partial_t\breve{e} + \partial_j((\breve{e} + \overline{p})\widetilde{u}_j) - \partial_j(\breve{\sigma}_{ij}\widetilde{u}_i) + \partial_j\breve{q}_j = \overline{h_k\omega_k} + \mathcal{A}$$
$$\partial_t(\overline{\rho}\widetilde{c}_k) + \partial_j(\overline{\rho}\widetilde{c}_k\widetilde{u}_j) - \partial_j(\breve{\pi}_{kj}) = -\partial_j(\overline{\rho}\zeta_{jk}) + \overline{\omega}_k + \partial_j(\overline{\pi}_{kj} - \breve{\pi}_{kj})$$

Terms requiring closure:

- Turbulent stress tensor: τ_{ij}
- Velocity-species stress tensor: ζ_{jk}
- Chemical source terms

Many more contributions - systematic approach needed

Investigate extended Leray approach

Spatial filtering and subgrid closure

Filtered model equations: ($\overline{\rho}$, $\widetilde{u}_j$, $\overline{p}$ and $\widetilde{c}_k$)

$$\partial_t \overline{\rho} + \partial_j(\overline{\rho}\widetilde{u}_j) = 0$$

$$\partial_t(\overline{\rho}\widetilde{u}_i) + \partial_j(\overline{\rho}\widetilde{u}_i\widetilde{u}_j) + \partial_i\overline{p} - \partial_j\breve{\sigma}_{ij} = -\partial(\overline{\rho}\tau_{ij}) + \partial_j(\overline{\sigma}_{ij} - \breve{\sigma}_{ij})$$

$$\partial_t\breve{e} + \partial_j((\breve{e} + \overline{p})\widetilde{u}_j) - \partial_j(\breve{\sigma}_{ij}\widetilde{u}_i) + \partial_j\breve{q}_j = \overline{h_k\omega_k} + \mathcal{A}$$

$$\partial_t(\overline{\rho}\widetilde{c}_k) + \partial_j(\overline{\rho}\widetilde{c}_k\widetilde{u}_j) - \partial_j(\breve{\pi}_{kj}) = -\partial_j(\overline{\rho}\zeta_{jk}) + \overline{\omega}_k + \partial_j(\overline{\pi}_{kj} - \breve{\pi}_{kj})$$

Terms requiring closure:

- Turbulent stress tensor: τ_{ij}
- Velocity-species stress tensor: ζ_{jk}
- Chemical source terms

Many more contributions - systematic approach needed

Investigate extended Leray approach

Spatial filtering and subgrid closure

Filtered model equations: ($\overline{\rho}$, $\widetilde{u}_j$, $\overline{p}$ and $\widetilde{c}_k$)

$$\partial_t \overline{\rho} + \partial_j(\overline{\rho}\widetilde{u}_j) = 0$$
$$\partial_t(\overline{\rho}\widetilde{u}_i) + \partial_j(\overline{\rho}\widetilde{u}_i\widetilde{u}_j) + \partial_i\overline{p} - \partial_j\breve{\sigma}_{ij} = -\partial(\overline{\rho}\tau_{ij}) + \partial_j(\overline{\sigma}_{ij} - \breve{\sigma}_{ij})$$
$$\partial_t\breve{e} + \partial_j((\breve{e} + \overline{p})\widetilde{u}_j) - \partial_j(\breve{\sigma}_{ij}\widetilde{u}_i) + \partial_j\breve{q}_j = \overline{h_k\omega_k} + \mathcal{A}$$
$$\partial_t(\overline{\rho}\widetilde{c}_k) + \partial_j(\overline{\rho}\widetilde{c}_k\widetilde{u}_j) - \partial_j(\breve{\pi}_{kj}) = -\partial_j(\overline{\rho}\zeta_{jk}) + \overline{\omega}_k + \partial_j(\overline{\pi}_{kj} - \breve{\pi}_{kj})$$

Terms requiring closure:

- Turbulent stress tensor: τ_{ij}
- Velocity-species stress tensor: ζ_{jk}
- Chemical source terms

Many more contributions - systematic approach needed

Investigate extended Leray approach

Spatial filtering and subgrid closure

Filtered model equations: ($\overline{\rho}$, $\widetilde{u}_j$, $\overline{p}$ and $\widetilde{c}_k$)

$$\partial_t \overline{\rho} + \partial_j(\overline{\rho}\widetilde{u}_j) = 0$$

$$\partial_t(\overline{\rho}\widetilde{u}_i) + \partial_j(\overline{\rho}\widetilde{u}_i\widetilde{u}_j) + \partial_i\overline{p} - \partial_j\breve{\sigma}_{ij} = -\partial(\overline{\rho}\tau_{ij}) + \partial_j(\overline{\sigma}_{ij} - \breve{\sigma}_{ij})$$

$$\partial_t\breve{e} + \partial_j((\breve{e} + \overline{p})\widetilde{u}_j) - \partial_j(\breve{\sigma}_{ij}\widetilde{u}_i) + \partial_j\breve{q}_j = \overline{h_k\omega_k} + \mathcal{A}$$

$$\partial_t(\overline{\rho}\widetilde{c}_k) + \partial_j(\overline{\rho}\widetilde{c}_k\widetilde{u}_j) - \partial_j(\breve{\pi}_{kj}) = -\partial_j(\overline{\rho}\zeta_{jk}) + \overline{\omega}_k + \partial_j(\overline{\pi}_{kj} - \breve{\pi}_{kj})$$

Terms requiring closure:

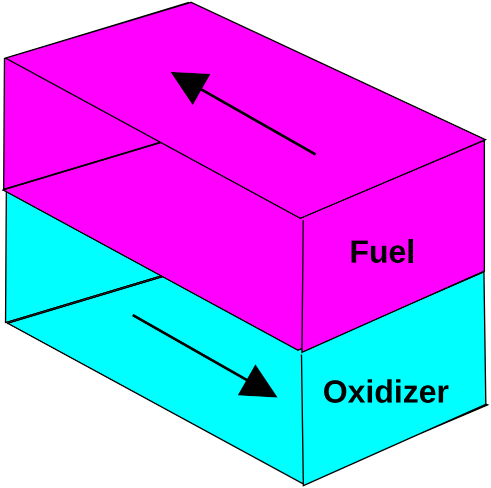
- Turbulent stress tensor: τ_{ij}
- Velocity-species stress tensor: ζ_{jk}
- Chemical source terms

Many more contributions - systematic approach needed

Investigate extended Leray approach

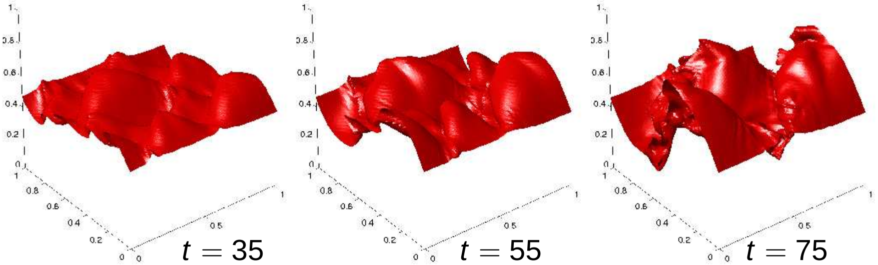
TU/e

Combustion-modulated turbulence



Stoichiometric surface

Surface $c_F = c_O$



- DNS at 256^3 resolution
- modest heat-release $Q = -1$

Turbulence combustion interaction

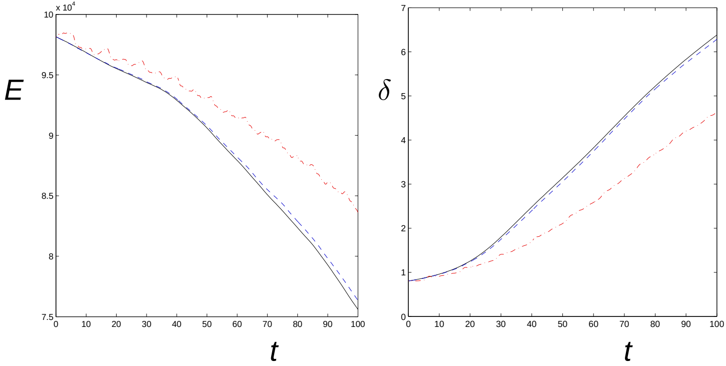
Leray predictions: 32^3 and $\Delta = \ell/16$

$$E(t) = \int_\Omega \frac{1}{2}\overline{u}_i\overline{u}_i \, d\mathbf{x} \quad ; \quad \delta = \frac{1}{4}\int_{-\ell/2}^{\ell/2} \Big(1 - \langle\overline{u}_1\rangle\Big)\Big(\langle\overline{u}_1\rangle + 1\Big) dx_2$$

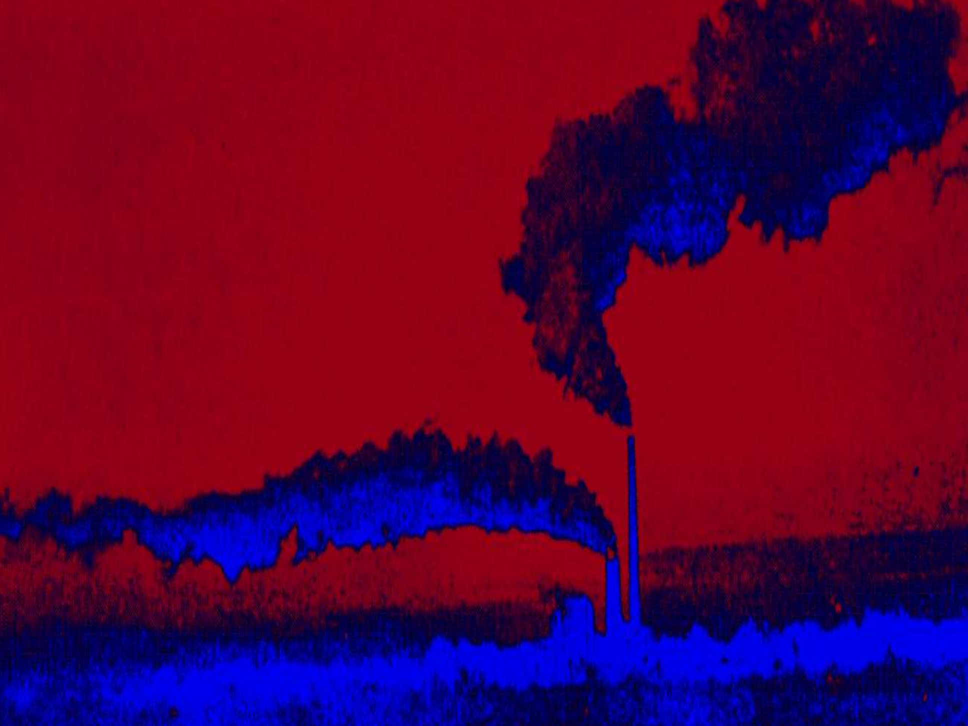
Turbulence combustion interaction

Leray predictions: 32^3 and $\Delta = \ell/16$

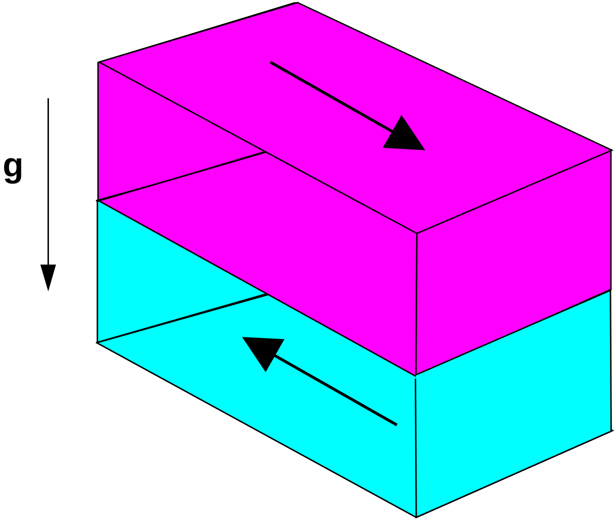
$$E(t) = \int_\Omega \frac{1}{2}\overline{u}_i\overline{u}_i \, d\mathbf{x} \quad ; \quad \delta = \frac{1}{4}\int_{-\ell/2}^{\ell/2}\Big(1 - \langle\overline{u}_1\rangle\Big)\Big(\langle\overline{u}_1\rangle + 1\Big)dx_2$$



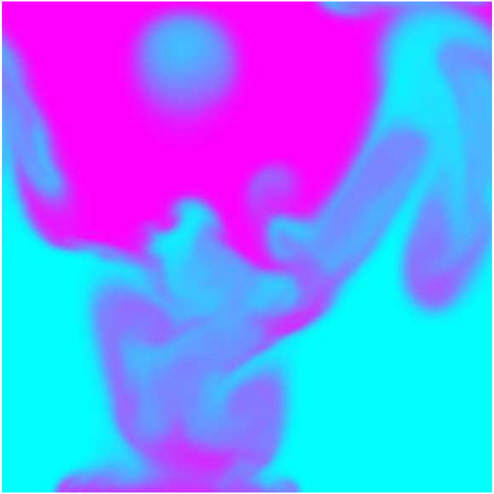
- $Q = -1$ (solid), $Q = -10$ (dash), $Q = -100$ (dash-dot)



Model: Stratified mixing layer



Turbulent plumes



- Unstable, anisotropic, inhomogeneous: go

Stratification: DNS and LES

Stratification:

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p - \frac{1}{Re} \nabla^2 \mathbf{u} = -\frac{1}{Fr^2} (c \mathbf{e_g})$$

Advection-diffusion for scalar c

$$\partial_t c + \mathbf{u} \cdot \nabla c - \frac{1}{Re\ Sc} \nabla^2 c = 0$$

Characteristic numbers: (Re, Fr, Sc)

LES: subgrid modeling in momentum and scalar equations

Stratification: DNS and LES

Stratification:

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p - \frac{1}{Re} \nabla^2 \mathbf{u} = -\frac{1}{Fr^2} (c \mathbf{e_g})$$

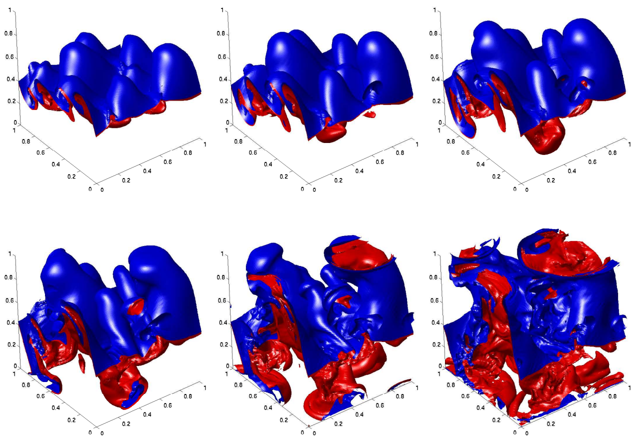
Advection-diffusion for scalar c

$$\partial_t c + \mathbf{u} \cdot \nabla c - \frac{1}{Re\ Sc} \nabla^2 c = 0$$

Characteristic numbers: (Re, Fr, Sc)

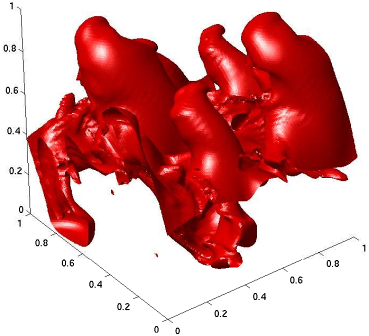
LES: subgrid modeling in momentum and scalar equations

Unstable stratification at $Fr = 2$, $Re = 50$

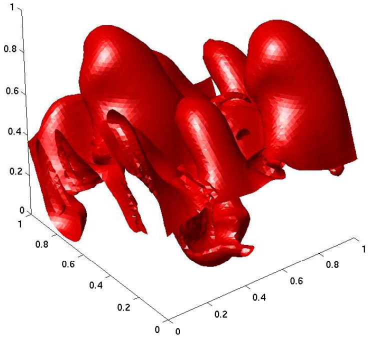


- mixing of heavy fluid on top of lighter fluid: $t = 30, ..., 55$

LES capturing of plumes



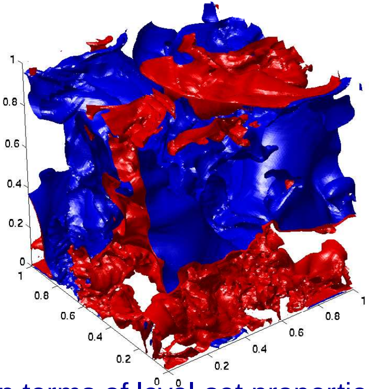
DNS ($t = 40$)



Leray LES ($t = 40$)

- Case: $Fr = 2$ and $Sc = 10$ at $\Delta = \ell/16$
- What about prediction mixing efficiency?

Level-set: surface-area and wrinkling



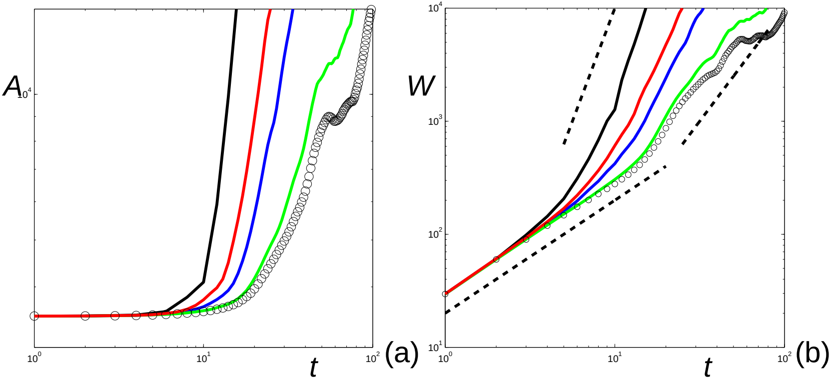
Measure mixing in terms of level-set properties

$$I = \int_{S(a,t)} dA\, f(\mathbf{x},t) = \int_V d\mathbf{x}\, \delta\Big(c(\mathbf{x},t) - a\Big) |\nabla c(\mathbf{x},t)| f(\mathbf{x},t)$$

where $S(a,t) = \{\mathbf{x} \in \mathbb{R}^3 \mid c(\mathbf{x},t) = a\} \subset V$ – Laplace transform

Effect of *Fr* on surface area and wrinkling

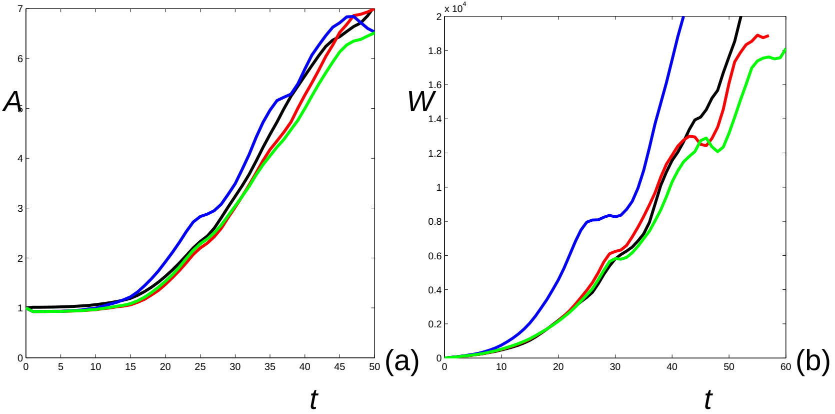
Area ($f = 1$) and wrinkling ($f = |\nabla \cdot \mathbf{n}|$)



- DNS: $Sc = 10$ and $Fr = 1, 1.5, 2, 4, 8$
- W: Initially $\sim t$ then from $\sim t^3$ to $\sim t^4$
- global features captured well with LES

Surface-area and wrinkling: LES

Localized wrinkling more difficult to capture with LES



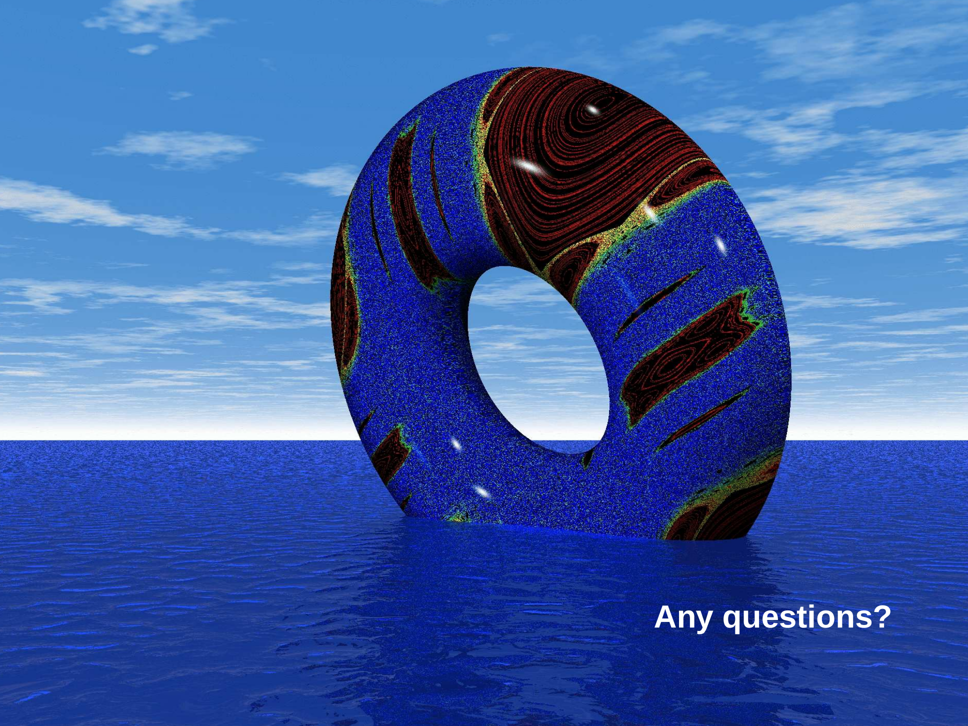
- $Fr = 2$ and $Sc = 10$: DNS, Leray, NS-α, Dynamic

Outline

Concluding Remarks

Discussed:

- turbulence and coarsening – closure problem
- heuristic modeling – eddy-viscosity
- regularization approach – large-scale sweeping
- role of spatial discretization
- Leray approach robust and generally accurate
- under-resolved LANS-α exaggerates small scales
- extended to complex flows - combustion, stratification (separation, rotation, particles, electromagnetism, ...)


Any questions?