Computational Dynamics for Mechanical Systems

two lectures of course CDS 140b: Introduction to Dynamics
Tuesday March 4 and Thursday March 6 2008, 10:30am-11:55am, Steele 214

Sigrid Leyendecker
sleye@caltech.edu

Homework

A pdf-document containing the required plots and text and the complete program code should be emailed to Molei (mtao@caltech.edu with Cc to myself sleye@caltech.edu) on or before Thursday March 13.
Please label plots properly (legend of plotted curves, axes and significant values).

Homework

The Lagrangian for the planar pendulum (Figure 1 of the lecture notes), with mass $m = 1$ and length $l = 1$ and gravitational constant $g = -9.81$ reads

$$L(q, \dot{q}) = \frac{1}{2}\dot{q}^2 - (g \cos q)$$

and the Hamiltonian reads

$$H(q, p) = \frac{1}{2}p^2 + (g \cos q)$$

For the initial conditions $q_0 = \frac{4}{5}\pi$ and $p_0 = 0$, implement the dynamics of the pendulum using two different schemes. You can chose one from the set {forward Euler (2), backward Euler (3)} and one from the set {energy-momentum scheme (8), variational integrator (12) with midpoint rule discrete Lagrangian (11)}. The nonlinear residual equations are to be solved up to a tolerance of 10^{-9}.

1. Use a constant time-step $\tau = 10^{-2}$ for all k and let the integration run for $N = 1000$ time-steps. For each of the two discrete trajectories:
   a) Create one plot containing the evolution of kinetic, potential and total energy and another plot showing only the evolution of the total energy (2 x 2 = 4 plots).
   b) Plot the evolution of the configuration variable (2 plots).
   c) Plot the evolution of the value of the Jacobi-determinant of the discrete flow map (2 plots).
   d) Check whether the schemes are time-reversible by first performing the forward dynamical simulation to compute the discrete trajectory $\{q_k\}_{k=0}^{N}$. Then, using the final state as an initial condition (use the last and the prelast configuration in case of the two-step scheme), compute the reverse trajectory $\{\tilde{q}_k\}_{k=N}^{0}$ using the

negative time-step. Plot both trajectories in the same plot using different colours and linestyles and return the value $\max_{t_k \in \{t_0,\ldots,t_N\}} |q_k - \tilde{q}_k|$ and the time-node where the maximal error occurs (index k and t_k) (2 plots and 6 values).

2. Verify the convergence of each method numerically by computing the discrete trajectory until $t_N = 1$ with different time-steps $\tau \in \{10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}\}$. Using the trajectory corresponding to $\tau = 10^{-5}$ as a reference solution, plot the mesh error ϵ_Δ over the time-step size in a double logarithmic plot and return the order of convergence which is the slope of the resulting curve (please return exact value) (2 plots and 2 values).

3. Comment on the qualitative and quantitative behaviour of all curves plotted.

Project

Dynamic optimisation of a falling cat turning over using the constrained version of DMOC and a 3-dimensional model of the cat, where the front and back parts of the body are connected by a spherical joint.