CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 140b

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The Moore-Greitzer equations model rotating stall and surge in gas turbine engines describe the dynamics of a compression system, such as those in gas turbine engines. The three-state "MG3" equations have the form:

$$\begin{aligned}
\frac{d\psi}{dt} &= \frac{1}{4B^2 l_c}\left(\phi - \Phi_T(\psi)\right), \\
\frac{d\phi}{dt} &= \frac{1}{l_c}\left(\Phi_c(\phi) - \psi + \frac{J}{8}\frac{\partial^2 \Psi_c}{\partial \phi^2}\right), \\
\frac{dJ}{dt} &= \frac{2}{\mu + m}\left(\frac{\partial \Phi_c}{\partial \phi} + \frac{J}{8}\frac{\partial^3 \Phi_c}{\partial \phi^3}\right) J,
\end{aligned}$$

where ψ represents the pressure rise across the compressor, ϕ represents the mass flow through the compressor and J represents the amplitude squared of the first modal flow perturbation (corresponding to a rotating stall disturbance). For the Caltech compressor rig, the parameters and characteristic curves are given by:

$$\begin{aligned}
B &= 0.2, & \Phi_T(\psi) &= \gamma\sqrt{\psi}, \\
l_c &= 6, & \Psi_c(\phi) &= 1 + 1.5\phi - 0.5\phi^3, \\
\mu &= 1.25, & m &= 2.
\end{aligned}$$

The paramter γ represents the throttle setting and typically varies between 1 and 2.

1. Compute the bifurcation diagram for the system showing the equilibrium value(s) for J as a function of γ. Your answer should match what was shown in class (i.e., make sure to get capture the hysteresis loop).

2. Suppose that we can modulate the throttle, so that $\gamma = \gamma_0 + u$. Analyze the performance of the system using the Liaw-Abed control law $u = kJ$. Show that if we choose k sufficiently large to cause the bifurcation to stall to be supercritical.

3. Suppose that we impose magnitude and rate limits on u:
$$|u| \le 1, \qquad |\dot{u}| \le 1.$$
Assume that we implement the control law
$$\dot{u} = \alpha(J) = \begin{cases} \operatorname{sat}\left(\frac{1}{\epsilon}(kJ - \operatorname{sat}(u))\right) & |u| < 1, \\ 0 & |u| = 1, \end{cases}$$
where $\operatorname{sat}(\cdot)$ is a saturation function of magnitude 1 and ϵ is a small constant. This control law limits both the magnitude and rate of the input. Using the center manifold theorem, compute an approximate model of the system at the bifurcation point in terms of u and J and use a phase portrait (computed with MATLAB or a similar tool) to describe the set of initial conditions for J (assuming $u(0) = 0$) for which the system avoids hysteresis.