Problem 1:

Position of M: $(x(t),x^2(t))$
Position of m: $(x(t)+l*sin(\theta),x^2(t)-l*cos(\theta))$
Velocity of M: $(\dot{x},2x\dot{x})$
Velocity of m: $(\dot{x}+l*cos(\theta)*\dot{\theta},2*x*\dot{x}+l*sin(\theta)*\dot{\theta})$

$KE=\frac{1}{2}M\|V_M\|^2+\frac{1}{2}m\|V_m\|^2$
$=\frac{1}{2}M*(\dot{x}^2+4x^2\dot{x}^2)+\frac{1}{2}m*((\dot{x}+l*cos(\theta)*\dot{\theta})^2+(2x\dot{x}+l*sin(\theta)*\dot{\theta})^2)$
$=\frac{1}{2}(M+m)*(1+4x^2)*\dot{x}^2+\frac{1}{2}ml^2\dot{\theta}^2+ml\dot{x}\dot{\theta}(cos(\theta)+2x*sin(\theta))$
$PE=Mgx^2+mg(x^2-l*cos(\theta))$
$=(m+M)x^2-mglcos(\theta)$

Lagrangian:
$L=KE-PE$
Euler-Lagrange equations:
$\frac{d}{dt}[\frac{dL}{d\dot{q}^i}]-\frac{dL}{dq^i}=0$
$\frac{dL}{d\dot{x}}=(M+m)(1+4x^2)\dot{x}+ml\dot{\theta}(cos(\theta)+2x*sin(\theta))$
$\frac{d}{dt}[\frac{dL}{d\dot{x}}]=(M+m)(8x\dot{x}^2+(1+4x^2)\ddot{x})+ml\ddot{\theta}(cos(\theta)+2x*sin(\theta)+ml\dot{\theta}(-sin(\theta)+2\dot{x}*sin(\theta)+2x*cos(\theta))$
$\frac{dL}{dx}=\frac{1}{2}(M+m)8x\dot{x}^2+2mlx\dot{\theta}sin(\theta)-2(m+M)gx$
yielding:
$4(M+m)x\dot{x}^2+(M+m)(4x^2+1)\ddot{x}+ml\ddot{\theta}(cos(\theta)+2xsin(\theta))+ml\dot{\theta}^2(2xcos(\theta)-sin(\theta))+2(M+m)gx=0$
And for theta:
$2mlsin(\theta)\dot{x}^2+ml(2xsin(\theta)+cos(\theta))\ddot{x}+ml^2\ddot{\theta}+mglsin(\theta)=0$

Energy:
$E=\frac{dL}{d\dot{x}}\dot{x}+\frac{dL}{d\dot{\theta}}\dot{\theta}-L$
$\frac{dE}{dt}=\frac{d}{dt}(\frac{dL}{d\dot{x}})\dot{x}+\frac{dL}{d\dot{x}}\ddot{x}+\frac{d}{dt}(\frac{dL}{d\dot{\theta}})\dot{\theta}+\frac{dL}{d\dot{\theta}}\ddot{\theta}-\frac{dL}{dx}\dot{x}-\frac{dL}{d\theta}\dot{\theta}-\frac{dL}{d\dot{x}}\ddot{x}-\frac{dL}{d\dot{\theta}}\ddot{\theta}$
$=[\frac{d}{dt}(\frac{dL}{d\dot{x}})-\frac{dL}{dx}]\dot{x}+[\frac{d}{dt}(\frac{dL}{d\dot{\theta}})-\frac{dL}{d\theta}]\dot{\theta}$
$=0$
So energy is conserved.
Hamiltonian:
$H=\sum_i p\cdot\dot{q}^i-L$ where $p=\frac{dL}{d\dot{q}}$

Problem 8:

$\dot{x}=-x$
$\dot{y}=2y-5x^3$
Mus prove that x,y: $y=x^3$ is invariant: condition for invariance: vector field is always tangent to your manifold
Consider a curve with $\frac{dx}{ds}=-x$, $y=x^3$
$\frac{dy}{ds}=3x^2\frac{dx}{ds}=-3x^3=(2-5)x^3=2y-5x^3$ as required
so the curve is a trajectory of the system implying that the vector field is tangeant to this trajectory viewed as a manifold.