Problem 1 — Harmonic Oscillator (Pendulum)
In Strogatz Chapter 6 it was shown that the origin is a nonlinear center for the pendulum example.
Let $\ddot{x} + \sin x = 0$.

(a) Can you prove stability of the origin using linearization? Use an appropriate Liapunov function to prove that the origin is a stable fixed point.

(b) Let's add a “damping term” $\ddot{x} + \epsilon(1 - x^2)\dot{x} + \sin x = 0$. Study the stability of the origin for different values of $\epsilon$ ($\epsilon > 0$, $\epsilon < 0$).

Perko:
Problem 4, pg 135

Strogatz:
7.1.2, 7.1.8, 7.2.9